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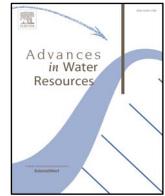
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## Parsimonious nonstationary flood frequency analysis

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### ABSTRACT

There is now widespread awareness of the impact of anthropogenic influences on extreme floods (and droughts) and thus an increasing need for methods to account for such influences when estimating a frequency distribution. We introduce a parsimonious approach to nonstationary flood frequency analysis (NFFA) based on a bivariate regression equation which describes the relationship between annual maximum floods,  $x$ , and an exogenous variable which may explain the nonstationary behavior of  $x$ . The conditional mean, variance and skewness of both  $x$  and  $y = \ln(x)$  are derived, and combined with numerous common probability distributions including the lognormal, generalized extreme value and log Pearson type III models, resulting in a very simple and general approach to NFFA. Our approach offers several advantages over existing approaches including: parsimony, ease of use, graphical display, prediction intervals, and opportunities for uncertainty analysis. We introduce nonstationary probability plots and document how such plots can be used to assess the improved goodness of fit associated with a NFFA.

### 1. Introduction

The field of flood frequency analysis (FFA) has a long and rich history which has led to several manuals of practice available for different continents and countries including the United States (IACWD, 1982; USACE, 1993), the United Kingdom (NERC, 1975; Robson and Reed, 1999; Stewart et al., 2008), Australia (Ball et al., 2016), and continental Europe (Madsen et al., 2014) as well as summaries of the field of FFA in book chapters (Stedinger et al., 1993; Kottegoda and Rosso, 1997; Stedinger, 2016), books (Rao and Hamed, 2000; Hosking and Wallis, 2005), and review articles (Greis, 1983; Bobée and Rasmussen, 1995; Castellarin et al., 2012). Critical to nearly all traditional approaches to FFA, summarized above, is the assumption of stationarity, loosely defined as the conditions under which key summary statistics of streamflows, such as their moments or L-moments, do not systematically change with time.

Ever since the seminal work by Hurst (1957), hydrologists have been keenly aware of the complex behavior of streamflow and its relationships with climatic and other watershed conditions. Over the ensuing years, numerous complex statistical representations were introduced in an effort to reproduce what became known as the “Hurst phenomenon” (see O’Connell et al. 2016). More recently, interest in the complexity of streamflow dynamics has grown exponentially due to improvements in our understanding of the impacts of numerous physical and social processes on streamflow, including: increasing urban

populations causing major stresses to landscapes as well as water infrastructures and water availability (e.g. Vörösmarty et al., 2000), global warming due to increased greenhouse gas concentrations in the atmosphere, and a better understanding of the effects of both atmospheric-oceanic processes and anthropogenic factors on the hydrological cycle (e.g. IPCC, 2013). This increased understanding, awareness and interest has led to a tremendous increase in research associated with the detection, attribution and prediction of hydrological processes such as the new scientific decadal 2013–2022 initiative of IAHS, entitled “Panta Rhei-Everything Flows”, which is dedicated to research activities on change in hydrology and society (Montanari et al., 2013). As evidenced from recent review articles and special journal issues, there are now myriad approaches under development for nonstationary flood frequency analysis (NFFA) (e.g. Petrow and Merz, 2009; Kiang et al., 2011; Salas et al., 2012; Madsen et al., 2014; Hall et al., 2014; Bayazit, 2015; Salas et al., 2017). For a general review of the literature on NFFA methods, we refer the reader to the review articles cited above. After our introductory remarks, we describe how our particular approach to NFFA differs from previous efforts.

In spite of the tremendous increase in attention given to NFFA, there is still no consensus on a methodology for performing NFFA. In fact, there is not even consensus on the need for NFFA, as described below. Nevertheless, there is some effort by government agencies to update flood protection design guidelines to account for nonstationarity (Stedinger and Griffis, 2011; Madsen et al., 2014; Prosdocimi et al.,

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2014) yet the proposed methods are by no means widely accepted nor tested. Despite tremendous interest in the development of methods of NFFA, considerable debate continues over whether stationary or nonstationary methods should be employed in practice (see [Serinaldi and Kilsby, 2015](#); [Luke et al., 2017](#) and recent discussion by [Salas et al., 2017](#)). Thus, despite the recent interest in the development of NFFA approaches ([Milly et al., 2008](#); [Milly et al., 2015](#)), considerable controversy remains as evidenced in the many papers which properly question the need for nonstationary approaches (e.g. [Cohn and Lins, 2005](#); [Villarini et al., 2009a](#); [Villarini et al., 2011](#); [Lins and Cohn, 2011](#); [Hirsch and Ryberg, 2012](#); [Matalas, 2012](#); [Montanari and Koutsoyiannis, 2014](#); [Serinaldi and Kilsby, 2015](#)). For example, [Villarini et al. \(2011\)](#) and [Villarini et al. \(2009a\)](#) summarize numerous studies including their own analyzes which did not provide statistical evidence of nonstationarity in flood series, except where major watershed changes occurred. Even if there were acceptance on an approach to NFFA, many open questions remain concerning the selection of an appropriate design event under such conditions (e.g. [Stedinger and Griffis, 2011](#); [Rootzén and Katz, 2013](#); [Obeyseker and Salas, 2014](#); [Salas and Obeyseker, 2014](#); [Read and Vogel, 2015](#); [Salas et al., 2017](#)). The lack of consensus results in part from the tremendous uncertainty associated with our ability to detect, attribute and model past trends, and the even greater uncertainty associated with our ability to predict future trends in hydrological processes.

The question of whether to perform a stationary or nonstationary flood frequency analysis remains an open question and probably should remain so, as evidenced from the findings of [O'Brien and Burn \(2014\)](#), [Serinaldi and Kilsby \(2015\)](#) and [Luke et al. \(2017\)](#). In a careful comparison of the precision (uncertainty) associated with various estimates of design flood events using both stationary and nonstationary methods, [Luke et al. \(2017\)](#) found that stationary methods were nearly always preferred over nonstationary approaches. [Vogel et al. \(2013\)](#), [Prosdocimi et al. \(2014\)](#) and [Serinaldi and Kilsby \(2015\)](#) document how one can quantify the various factors which give rise to uncertainty in our ability to detect and model trends in hydrologic series. [Obeyseker and Salas \(2014\)](#), [O'Brien and Burn \(2014\)](#) and [Serinaldi and Kilsby \(2015\)](#) describe approaches to derive confidence intervals for design events under nonstationary conditions. Yet even in situations in which there is physical knowledge of the processes causing the hydrologic nonstationarity, there is always considerable additional uncertainty associated with NFFA so that risk-based approaches to design event selection are paramount.

Risk-based decision approaches are now well-established for selection of a design event based on the expected benefits and damages avoided versus the cost of the investment required ([National Research Council, 2000](#)) and such approaches are now standard practice by U.S. Federal agencies (see [Brekke et al., 2009](#); [Stakhiv, 2011](#)). Importantly, given the additional uncertainty associated with NFFA, it is even more important to employ risk based approaches which consider both the possibility of a stationary and a nonstationary future ([Rosner et al., 2014](#)) as well as a wide range of possible forms of future nonstationarity ([Spence and Brown, 2016](#)). The need for risk-based approaches to the selection of a design event cannot be overemphasized because such approaches enable an accounting of most relevant sources of uncertainty (e.g. [Stedinger and Crainiceanu, 2001](#); [Brekke et al., 2009](#); [Sivapalan and Samuel, 2009](#); [Stakhiv, 2011](#); [Rosner et al., 2014](#); [Spence and Brown, 2016](#)).

The literature on approaches to NFFA is growing rapidly as evidenced by the numerous recent reviews on the subject ([Khaliq et al., 2006](#); [Petrow and Merz, 2009](#); [Madsen et al., 2014](#); [Ehret et al., 2014](#); [Bayazit, 2015](#); [Salas et al., 2017](#)). Such methods may be extremely important in urbanizing watersheds because, as shown by [Vogel et al. \(2011\)](#), the annual maximum flood (AMF) series associated with rivers in urbanized or urbanizing areas of the U.S. exhibited significant increases in magnitude over the past century, a finding consistent with those of others ([Konrad, 2003](#); [Moglen and Shivers, 2006](#);

[Villarini et al., 2009b](#); [Prosdocimi et al., 2015](#)), in spite of the fact that variations in stormwater infrastructure and rainfall climatology can lead to a wide spectrum of changes in urban watersheds ([Smith et al., 2013](#)). [Olsen et al. \(1999\)](#) found significant increases in flood magnitudes in tributaries and the main stem of the northern portions of the upper Mississippi river basin as well as for portions of the upper Mississippi river near St. Louis. [Mallakpour and Villarini \(2015\)](#) found significant increases in the frequency, but not the magnitude, of historic flood events in the central region of the U.S. Despite considerable uncertainty over the impact of changes in climate on the behavior of floods (e.g. [Koutsoyiannis et al., 2008](#); [Hirsch and Ryberg, 2012](#)). There is good evidence that changes in land use, land cover, agricultural practice and water infrastructure have led to considerable changes in the behavior of flood series ([Villarini et al., 2011](#)).

If an observed flood series is known to exhibit an increasing trend during some particular historical period, then the magnitude of all upper quantiles associated with that flood series will also increase over that period. Under such conditions it would not be appropriate to report a single value of the 99th percentile (i.e. flood with average return period of 100 years) or to issue a floodplain delineation map without denoting the date associated with its creation. In such situations, in which historical trends in flood series are obvious and/or overt due to knowledge of changes in historical land use, climate and/or water infrastructure, it is imperative to provide updated estimates of design floods that reflect current conditions. The goal of this study is to develop a suite of approaches to NFFA which are particularly well suited to updating design events to current conditions.

## 2. On the value of parsimony: Robustness, resistance and efficiency

A full appreciation of the methodology introduced here can only be understood with a knowledge of the concepts of parsimony, robustness, resistance and efficiency, all of which have been given little attention in the area of NFFA. Our long history of stationary FFA including a myriad of probability density functions (pdf) and parameter estimation methods introduced over 50 years ago, led [Kuczera \(1986\)](#) and others to introduce the concepts of robustness, resistance and efficiency to help hydrologists choose a suitable approach. [Kuczera \(1986\)](#) defined a robust model as “one which the analyst is confident to use as a predictive tool in the roles assigned to it”. He also introduced the properties of resistance and efficiency, two properties expected of a robust model. [Kuczera \(1986\)](#) suggests that a resistant model “must be capable of estimating extreme events, irrespective of which contending flood distribution best represents the real world, without disastrous loss of performance. Performance is indicated by some criterion such as mean squared error (MSE)”. Others had already sought resistant flood estimators such as in the seminal study by [Slack et al. \(1975\)](#) which found that in the absence of knowledge about the distribution of floods and associated economic losses, the normal distribution was preferred over several more complex and commonly used skewed alternatives.

Identification of a resistant estimator is not sufficient, because a robust estimator must also perform efficiently in the sense of exhibiting low MSE when compared to alternative estimators. It has long been known that efficient (low MSE) estimators tend to also be parsimonious ([Box and Jenkins, 1976](#)). A parsimonious estimator is one that accomplishes a desired level of prediction efficiency with as few model parameters as possible. More recently, [Laio et al. \(2009\)](#) and [Di Baldassarre et al. \(2009\)](#) argue that all model selection criteria implicitly consider the principle of parsimony which they describe in terms of the inverse trade-off between the bias and variance of an estimator (i.e. more complex estimators tend to exhibit lower bias at the cost of increased variance).

Still today, even after the concept of parsimony has pervaded the early literature on FFA, a common assumption of NFFA literature is that more accurate estimates of flood quantiles result when more complex

and improved estimators are employed. Thus, recent reviews of NFFA by [Khaliq et al. \(2006\)](#) and others describe myriad complex parametric and nonparametric NFFA methods, most of which have not been evaluated within the context of robustness, and many of which are certainly not parsimonious, as discussed below. The few studies we could find which did evaluate the robustness of alternative NFFA estimators in terms of both their resistance and efficiency, found that the more parsimonious models tended to perform best ([Serinaldi and Kilsby, 2015](#); [Luke et al., 2017](#)). As expected from our earlier experience on the value of parsimony in stationary FFA, both [Serinaldi and Kilsby \(2015\)](#) and [Luke et al. \(2017\)](#) found that in the presence of the uncertainty concerning whether or not a flood series exhibits nonstationarity, the more parsimonious stationary estimators were generally preferred. However, [O'Brien and Burn \(2014\)](#) found that regional approaches to NFFA may be preferred over regional approaches to FFA when significant nonstationarity is evident within a region.

There are many examples of the fact that parsimonious estimators often exhibit lower MSE than more complex competing estimators even when the parsimonious estimator assumes an incorrect pdf. For example, [Lu and Stedinger \(1992\)](#) consider quantile estimators for the generalized extreme value (GEV) distribution and show that for pdfs with realistic shape parameters, a two parameter GEV with fixed shape parameter (such as the Gumbel estimator) generally had lower MSE than the three-parameter GEV estimator, even if the assumed shape parameter is misrepresented. Similarly, [Kuczera \(1986\)](#) found the parsimonious two-parameter lognormal (LN2) maximum likelihood estimator to exhibit lower MSE when compared to numerous realistic three-parameter alternatives.

In summary, there exists extensive evidence of the value of parsimony in stationary FFA and after the two recent studies by [Serinaldi and Kilsby \(2015\)](#) and [Luke et al. \(2017\)](#), we expect the principle of parsimony to play an equally important role in NFFA. Similarly, [Castellarin et al. \(2012\)](#) found that relatively simple approaches to FFA with minimum data requirements were the preferred choice in many countries and by most of the institutions and agencies in charge of developing flood risk mitigation plans in Europe. The primary goal of this study is to introduce a parsimonious approach to NFFA which only requires estimation of one additional parameter over and beyond a stationary FFA yet can be applied to any pdf, regardless of the number of parameters.

### 3. Distinguishing features of methodology

Two general approaches have been applied to FFA and NFFA, consisting of the peaks over threshold approach (POT) and the use of the annual maximum flood (AMF) series approach. We only consider the AMF approach, though there may be considerable value to testing the POT approach for NFFA in future studies (i.e. see [Khaliq et al., 2006](#)). See [Stedinger et al. \(1993, Section 18.6.1\)](#) and [Stedinger \(2016, Section 76.2.3\)](#) and associated references for guidance on when an AMF analysis is preferred over a POT analysis.

Existing approaches to NFFA summarized in review articles by [Khaliq et al. \(2006\)](#), [Petrow and Merz \(2009\)](#), [Madsen et al. \(2014\)](#), [Ehret et al., \(2014\)](#), [Bayazit \(2015\)](#) and [Salas et al. \(2017\)](#) usually involve fitting pdfs whose parameters and/or moments are related to exogenous variables which are related to drivers of nonstationary behavior. A variety of estimation methods have been advanced for estimating the combined nonstationary model consisting of a pdf and one or more models relating either moments or pdf model parameters to exogenous variables. To fully understand many of the NFFA methods advanced in the above cited reviews would require advanced training in statistics such as those based on generalized maximum likelihood, generalized linear models, kernel and wavelet density estimation, quantile regression, and Bayesian approaches, just to name a few. A secondary goal of this study is to introduce a generalized approach to NFFA which would not require advanced training in statistics to fully

understand and implement. One could argue that algorithms now exist in R software ([R Core Team 2015](#)) to facilitate implementation of most of these advanced NFFA approaches (e.g. see GAMLSS software by [Stasinopoulos and Rigby, 2007](#); extRemes software by [Gilleland and Katz, 2011](#); and NEVA software by [Cheng et al., 2014](#)). However, unlike the methodology introduced here, a complete understanding of those methods requires advanced training in statistics and some of these tools are not useful for distributions other than the generalized extreme value (GEV) and generalized Pareto pdfs.

Our approach differs from all previous work on NFFA because it employs a single regression equation, based on an exogenous variable, from which the conditional mean, conditional variance and conditional skewness of both series of annual maximum floods (AMF):  $x$  and its logarithms  $y = \ln(x)$  may be derived to ensure consistency among the resulting moments and distributional model parameters as well as to ensure a parsimonious approach. Since only a single regression model is used to derive the conditional moments, a parsimonious NFFA can be developed with only one additional model parameter required to convert a stationary FFA into a NFFA, another contribution which sets this work apart from all previous work on methods for NFFA.

When combining a pdf with models which relate pdf model parameters or moments to covariates, it is important to ensure that the resulting NFFA behaves in accord with flood observations, another critical and distinguishing feature of our approach. Consider an example using a nonstationary GEV pdf where the location, scale and shape parameters are given by  $\xi$ ,  $\alpha$ , and  $\kappa$ , respectively (using notation from [Stedinger et al., 1993](#); and [Stedinger 2016](#)). Consider the parsimonious model proposed by [Cheng et al. \(2014\)](#) and others, in which only the location parameter is linearly related to time  $t$ , so that  $\xi = a + bt$  with the scale and shape parameters fixed. One can pose such a model and even show that the linear relationship is in accord with flood observations, however, such an approach could not make physical sense, because holding  $\alpha$  constant, implies that the variance of the flood series does not change over time. This results in a coefficient of variation of the flood series,  $C$ , which changes over time - an unlikely result for flood series and a result inconsistent with many recent studies which have documented that the assumption of a constant value of  $C$  is reasonable at thousands of rivers (i.e. [Vogel et al., 2011](#); [Prosdocimi et al., 2014](#), and [Hecht 2017](#)). Our parsimonious approach based also on a single model of the flood series versus a covariate, employs conditional moments and conditional pdf model parameters derived from that model, which ensure that the resulting NFFA will always be in accord with the original flood observations.

Another unique aspect of our proposed approach to NFFA involves the myriad benefits which arise from the combination of a single (possibly multivariate) regression equation along with its corresponding exogeneous variable(s) and associated conditional moments derived for use with any pdf, although herein we only apply it to four commonly used in FFA: generalized extreme value (GEV), lognormal (LN2), lognormal (LN3) and log Pearson type III (LP3). Those benefits, summarized in the next section, range from the numerous advantages of regression methods including: rigorous graphical displays, parsimony, prediction intervals associated with trend extrapolations, accommodation of complex multivariate nonlinear relationships, and an ability to account for missing observations, abrupt changes, the impact of serial correlation and changes in the coefficient of variation of flood series on resulting statistical inference associated with the NFFA. We then describe our parsimonious approach to NFFA which begins by introducing the theoretical regression framework for NFFA, including a derivation of the conditional moments of  $x$  and  $y = \ln(x)$ . We then combine those conditional moments with the four pdfs considered: GEV, LN2, LN3, and LP3 resulting in four different NFFA models. We then apply the resulting NFFA models to AMF series for two urbanizing basins near Boston, Massachusetts. Finally, we introduce nonstationary probability plots and associated goodness-of-fit metrics, documenting how such procedures can be used to evaluate and possibly improve the practice of

NFFA.

#### 4. Advantages of regression for nonstationary flood frequency analysis

Regression offers a generalized multivariate statistical approach for relating the behavior of flood series to covariate explanatory variables which can explain a portion of the variability of those floods. Conditional moments associated with a single bivariate regression model are derived and combined with a pdf to create a parsimonious NFFA. Below, we outline a dozen advantages of using regression within NFFA which, taken together, offer significant advantages over alternative approaches.

**Ease of application:** Ordinary least squares (OLS) regression is a topic in most introductory statistics courses and is easy to implement in existing statistical software packages.

**Effective graphical communication:** Regression is useful for communicating results because the goodness-of-fit of trend models can be conveyed both quantitatively and qualitatively in a single graphical image.

**Parsimonious estimation of conditional moments:** Most previous studies on NFFA develop separate, independent, regression equations for each moment and/or model parameter, leading to NFFA with several additional parameters over stationary FFA. Instead, as we suggest, one may derive all conditional moments needed for NFFA from a single regression model resulting in a parsimonious approach.

**Accommodation of changes in coefficient of variation of flood series:** A promising extension to our analysis would be through the use of weighted least squares (WLS) regression for accommodating heteroscedastic regression residuals resulting from situations in which the coefficient of variation of flood series is changing. (see [Strupczewski and Kaczmarek, 2001](#); and [Hecht, 2017](#)).

**Accommodation of nonlinear relationships:** A wide range of monotonic non-linear functions can be linearized with ladder-of-powers transformations ([Mosteller and Tukey, 1977](#); [Helsel and Hirsch, 2002](#)), enabling the application of OLS and WLS linear regression methods for fitting highly nonlinear relationships.

**Analytical prediction Intervals:** Both confidence intervals corresponding to the true regression relationship, as well as prediction intervals associated with future predicted floods are easily computed from analytical relations (e.g. [Helsel and Hirsch, 2002](#)).

**Multivariate statistical modeling:** Multivariate regression can be used to model the multiple and interacting impacts of various covariates which quantify the impact of climate, land-use and other factors on flood series ([Kwon et al., 2008](#); [Delgado et al., 2010](#); [López and Francés, 2013](#); [Prosdocimi et al., 2014](#); [Prosdocimi et al., 2015](#); [Condon et al., 2015](#); [Šraj et al., 2016](#)). Hypothesis tests are also available for evaluating various alternative hypotheses.

**Model selection and evaluation:** Numerous procedures such as cross validation, random forests, partial least squares, ridge regression, and best subsets selection methods are available for selection of covariates in multivariate regression models. In addition, influence statistics are available for evaluating the influence of outliers on regression coefficients ([Helsel and Hirsch, 2002](#)). Such model selection procedures are of critical importance to ensure development of parsimonious multivariate models.

**Step changes or abrupt shifts:** Changes in flood series may be either gradual or abrupt due to either anthropogenic perturbations, such as reservoir construction, or climatic shifts ([McCabe and Wolock, 2002](#); [Villarini et al., 2009a](#)). To test for hypothesized abrupt step changes in flood series, binary indicator variables that account for whether a covariate lies below or above a threshold can be added to a regression model ([Bates et al., 2012](#)).

**Handling of missing and historical information:** The conditional regression approach described here, can easily account for intermittent or missing observations and in addition, offers a parsimonious

alternative to the expected moments algorithm ([Cohn et al., 1997](#); [Lamontagne et al., 2016](#)) for integrating non-instrumental historical flood records into NFFA if they are believed to represent an unbiased sample of floods during the historical period. These tasks are accomplished by simply accounting for the time of occurrence associated with each flood and associated covariate observation.

**Accounting for persistence in hydrologic series:** Given the short records typically available in hydrology, it is often difficult to distinguish trends from short- and long-term persistence ([Vogel et al., 1998](#)), both of which have been documented in AMF series ([Villarini et al., 2009a](#); [Tan and Gan, 2015](#)). Fortunately, [Matalas and Sankarasubramanian \(2003\)](#) developed analytical formulae to adjust standard errors of regression model coefficients for numerous common classes of persistence.

#### 5. Regression model of annual maximum flood series

Our central goal is to combine pdfs, which are widely accepted for modeling stationary AMF series, with conditional moments derived from a regression model which can describe the nonstationary behavior of AMF series. A distinguishing feature of this work is that very careful attention is given to the structure and theoretical properties of the regression models which all take the form

$$y = f(x) = \beta_0 + \beta_1 w_1 + \beta_2 w_2 + \dots + \varepsilon \quad (1)$$

where  $x$  denotes the AMF series, and  $y = f(x)$  is some transformation of the AMF series, with  $w_1, w_2, \dots$  denoting covariate climate, land use and possibly other variables,  $\beta_1, \beta_2, \dots$  denoting model coefficients and  $\varepsilon$  denoting model error which is assumed to be independent, homoscedastic and normally distributed.

A model of the form given in (1), with a logarithm transformation  $y = \ln(x)$  was carefully evaluated by [Vogel et al. \(2011\)](#) for AMF series at thousands of rivers across the U.S. and by [Prosdocimi et al. \(2014\)](#) for rivers in the United Kingdom. In this initial study, we employ a linear regression model using a logarithmic transformation of the AMF series  $x$  in (1), leading to an exponential model for  $x$  which was shown by [Vogel et al. \(2011\)](#) to be an excellent representation of AMF series in the U.S., regardless of whether the series was determined to exhibit a significant trend.

The basic premise of this work is that a regression of the form given in (1) can be developed for relating the AMF series to one or more explanatory variables, each of which would describe some portion of the (possibly) nonstationary behavior of the series. Another basic premise of this work is that the single regression model can be used to derive conditional moments needed to convert the stationary LN2, LN3, GEV, LP3 and other models into their nonstationary counterparts. In this section, we describe the initial regression model employed here along with its theoretical properties and associated conditional moments. The goal of this study is to introduce a generalized approach to NFFA based on the integration of a single regression model with various stationary pdfs, thus, we consider only a bivariate regression model here; yet we encourage others to extend our work using multivariate (yet parsimonious) regression models which include covariates known to be drivers of nonstationary behavior.

To simplify the introduction of our methodology, we consider only a single explanatory variable, time, selected as a surrogate for all the possible time-dependent nonstationary influences on the AMF series considered in two case studies. We recommend that in any future application of our methodology, investigators incorporate explanatory variables which can describe the physical influence of whatever nonstationary behavior is hypothesized. For example, in the case of the two urbanizing watersheds considered later, relevant explanatory variables might be the area of directly connected impervious surfaces and other metrics which represent the influence of urbanization on the behavior of flood magnitudes. We recommend development of multivariate models which include numerous additional covariates because this

approach has been shown to improve upon our ability to describe future nonstationary behavior of flood series (Kwon et al., 2008; Delgado et al., 2010; López and Francés, 2013; Prosdociami et al., 2014; Prosdociami et al., 2015; Condon et al., 2015).

5.1. The regression and its implied conditional and unconditional moments

To introduce our generalized approach to NFFA we employ the bivariate regression:  $y = \beta_0 + \beta_1 w + \varepsilon$  where  $y = \ln(x)$ , with  $x$  representing the AMF series and  $w$  represents an explanatory variable, with other variables as defined in (1). Both Vogel et al. (2011) and Prosdociami et al. (2014) found this model, with  $w$  equal to time, to be useful for examining flood trends for rivers in the U.S. and the U.K., regardless of whether or not trends exist. Hirsch and Ryberg (2012) also used this model to evaluate AMF series at 200 long term rivers in the US with the covariate  $w$  equal to carbon dioxide concentrations. We emphasize that the form of the regression model in (1) and its simpler form given below make no assumption regarding the probability distribution of  $y$ . We also note that the results presented in this section for a bivariate regression would need to be extended for the multivariate case, as is advocated elsewhere in this paper and by others.

The bivariate regression model can be rewritten in the form:

$$y = \mu_y + \beta(w - \mu_w) + \varepsilon \tag{2}$$

where  $y$  is conditioned upon the explanatory variable  $w$ ,  $\mu_y$  and  $\mu_w$  are the mean of the variables  $y$  and  $w$ , respectively,  $\beta$  is the regression coefficient, and  $\varepsilon$  represents the model error which is assumed to be independent in time, normally distributed and to have zero mean and constant variance with

$$\sigma_\varepsilon^2 = (1 - \rho^2)\sigma_y^2 \tag{3}$$

where  $\rho$  denotes the cross-correlation coefficient between  $y$  and  $w$ . Estimates of the single regression coefficient in (2) may be obtained from:

$$\hat{\beta} = \hat{\rho} \frac{\hat{\sigma}_y}{\hat{\sigma}_w} \tag{4}$$

with  $\hat{\rho} = \frac{\sum_{i=1}^n (w_i - \bar{w})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (w_i - \bar{w})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$ ,  $\hat{\sigma}_y = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2}$ ,  $\hat{\sigma}_w = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (w_i - \bar{w})^2}$ ,  $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ , and  $\bar{w} = \frac{1}{n} \sum_{i=1}^n w_i$

Note for this very special case in which we are employing the explanatory variable  $w = \text{time}$ , which is not a random variable, Prosdociami et al. (2014, Appendix A3) derive expressions for its sample mean and variance. Assuming no missing observations between the first and last years, sample estimates of the mean and variance of  $w$  are equal to  $\bar{w} = (n + 1)/2$  and  $\hat{\sigma}_w^2 = n(n + 1)/12$ .

5.2. Unconditional moments

When performing a stationary analysis the moments of  $x$  and  $y = \ln(x)$  will not depend on the explanatory variable  $w$  so that the unconditional moments of  $x$  and  $y$  are defined by their means  $\mu_x$  and  $\mu_y$ ; their standard deviations  $\sigma_x$  and  $\sigma_y$ , and their skewnesses  $\gamma_x$  and  $\gamma_y$ . Suitable sample at-site and/or regional estimators of those moments will depend on the assumed pdf of  $x$  and  $y$ , which are given in the Appendix A.

5.3. Conditional moments of  $y$

Here we use the regression in (2) to derive conditional moments of both  $x$  and  $y$  for use in NFFA, without making any assumptions regarding the probability distribution of either  $x$  or  $y$ . Taking the expectation of (2) leads to an expression for the expected value of  $y$ , conditioned upon  $w$ :

$$\mu_{y|w} = \mu_y + \beta(w - \mu_w) \tag{5}$$

Similarly, the conditional variance of  $y$  is obtained by taking the variance of (2) which leads to:

$$\sigma_{y|w}^2 = \sigma_\varepsilon^2 = \sigma_y^2(1 - \rho^2) \tag{6}$$

Note that the conditional variance of  $y$  decreases as the explanatory power of the trend model increases with the limit approaching zero as  $\rho$  approaches unity. Note also that the conditional variance of  $y$  reduces to its original, unconditional variance when the regression in (2) has no explanatory power, i.e.  $\rho = 0$ . Note that numerous previous studies which have employed this same regression model in NFFA did not account for the fact that the conditional variance of  $y$  given in (6) is generally smaller than its unconditional value (see for example, Stedinger and Griffis, 2011; Vogel et al., 2011; Prosdociami et al., 2014 and Luke et al., 2017; just to name a few).

Analogous to the conditional mean and variance of  $y$  in (5) and (6), one can show that the conditional skewness of  $y$  is given by:

$$\gamma_{y|w} = \gamma_y - \beta^3 \gamma_w \tag{7}$$

The conditional skewness is equal to the unconditional skewness when the skewness of the explanatory variable is equal to zero, which is the case when time is used as the explanatory variable  $w$ , as employed in this study.

5.4. Conditional moments of  $x$

If one takes the exponential of both sides of the regression for  $y$  in (2) we obtain an expression for  $x$ :

$$x_w = \exp(y_w) = \exp(\mu_y + \beta(w - \mu_w) + \varepsilon) \tag{8}$$

where here the notation  $x_w$  and  $y_w$  emphasizes that the values of  $x_w$  and  $y_w$  are conditioned upon the value of the explanatory variable  $w$ . Note that we have not yet made any assumptions regarding the pdf of  $x$ ,  $y$ ,  $x_w$  or  $y_w$  given the form of the regression in (2), the conditional values  $y_w$  and  $x_w$  are likely to be well approximated by a normal and lognormal distribution, respectively, regardless of the original pdf associated with the variable of interest here,  $x$ . This is true, because the only random variable on the right hand side of (8) are the model errors  $\varepsilon$  and  $\exp(\varepsilon)$ , which were found to be well approximated by normal and lognormal distributions, respectively, for thousands of rivers in the U.S. regardless of whether, or not, the estimated trend model coefficient  $\beta$  was found to be statistically significant (see results of normality hypothesis tests in Appendix A2 of Vogel et al. 2011).

Thus, only for the purposes of deriving an approximation to the conditional moments of  $x$ , we use the fact that,  $x_w$  is likely to be well approximated by a lognormal distribution with conditional moments in log space given by  $\mu_{y|w}$  and  $\sigma_{y|w}$  in (5) and (6), respectively, regardless of the true underlying distribution of  $x$ . Thus, the conditional moments of  $x$  can be approximated using the standard expressions which relate the mean, variance and skewness of  $x$  and  $y = \ln(x)$  for a lognormal variate, which yields:

$$\mu_{x|w} = \exp\left(\mu_{y|w} + \frac{\sigma_{y|w}^2}{2}\right) \tag{9a}$$

$$\sigma_{x|w}^2 = \exp(2\mu_{y|w} + \sigma_{y|w}^2)(\exp(\sigma_{y|w}^2) - 1) \tag{9b}$$

$$C_{x|w} = \frac{\sigma_{x|w}}{\mu_{x|w}} = \sqrt{\exp(\sigma_{y|w}^2) - 1} \tag{9c}$$

$$\gamma_{x|w} = 3C_{x|w} + C_{x|w}^3 \tag{9d}$$

where  $C_{x|w}$  denotes the coefficient of variation of  $x$  conditioned on  $w$ . Eq. (9) can be simplified by combining it with (5) and (6) so that

$$\mu_{x|w} = \exp\left(\mu_y + \beta(w - \mu_w) + \frac{\sigma_y^2(1 - \rho^2)}{2}\right) \quad (10a)$$

$$\sigma_{x|w}^2 = \exp(2\mu_y + 2\beta(w - \mu_w) + \sigma_y^2(1 - \rho^2))(\exp(\sigma_y^2(1 - \rho^2)) - 1) \quad (10b)$$

$$C_{x|w} = \sqrt{\exp(\sigma_y^2(1 - \rho^2)) - 1} \quad (10c)$$

$$\gamma_{x|w} = 3C_{x|w} + C_{x|w}^3 \quad (10d)$$

Here we note that both the conditional mean and the conditional variance of  $x$ , depend on the value of the explanatory variable  $w$ , whereas both the conditional coefficient of variation and the conditional coefficient of skewness, are constant (see discussion below). It is very important to understand the impact of nonstationarity in the moments of the flood series  $x$  on overall flow variability, which can be explained by contrasting the unconditional coefficient of variation,  $C_x = \sigma_x/\mu_x$  of the flood series given in (6) with its conditional value  $C_{x|w}$  given in (10c). In general,  $C_{x|w} < C_x$  as long as  $|\rho| > 0$ , which will always be the case for a nonstationary series. This can also be seen for the case when  $x$  follows an LN2 model, where Read and Vogel (2015) show that the conditional coefficient of variation of  $x$  is given by

$$C_{x|w} = \sqrt{(C_x^2 + 1)^{(1+\rho^2)} - 1} \quad (11)$$

Under nonstationary conditions, there is always the question of whether or not the variability (defined by the coefficient of variation of  $x$ ) is changing over time. The LN2 case provides an excellent background for understanding this issue so often confused in the literature. For the LN2 case, the unconditional coefficient of variation of the flood series is given by  $C_x = \sqrt{\exp(\sigma_y^2) - 1}$ , which can be rewritten using (6) as  $C_x = \sqrt{\exp(\sigma_y^2/(1 - \rho^2)) - 1}$  which is only constant if the regression in (2) exhibits homoscedastic residuals or residuals with constant variance. Thus, one could employ a standard test of homoscedasticity of the residuals such as the one introduced by Breusch and Pagan (1979) or White (1980) to evaluate whether or not the coefficient of variation of the flood series is changing.

Note that if the variance of the residuals  $\sigma_\varepsilon^2$  changes over time, then  $C_x$  is no longer constant, and a heteroscedastic regression model is needed to characterize the conditional coefficient of variation of  $x$ , as recommended by Hecht (2017). One may also employ WLS regression in the case of heteroscedastic model errors as described by Strupczewski and Kaczmarek (2001). Interestingly, after looking at hundreds of rivers across the U.S. using a 5% level modified Breusch-Pagan (1979) test, Hecht (2017) found few examples of rivers, even in urbanizing regions, which exhibited values of  $C_x$  which appear to change over time.

## 6. Probability distributions of annual maximum flood series

Vogel and Wilson (1996) provide a review of the results of local, regional, continental and global studies which sought to determine a suitable pdf for modeling series of annual maximum floods. They summarize the results of studies available as of that date, along with the results of a survey of pdfs used by various agencies and countries worldwide. On the basis of those surveys and literature review, combined with their own national evaluation based on L-moment diagrams for 1490 rivers in the U.S., Vogel and Wilson (1996) recommend the use of three distributions for modeling AMF series including: GEV, LP3 and the LN3 distributions. In the most recent summary of FFA methods, Stedinger (2016) suggests that the GEV and LP3 models are the most widely accepted. Among all the pdfs considered by the Interagency Advisory Committee on Water Data (1982, see Appendix 14) the LN2 and LP3 distributions performed best in their comparisons and were the only two distributions which did not exhibit significant bias in observed flood frequencies. We note that the LN2 model is a

special case of both the LP3 and LN3 models.

On the bases of these extensive national and global comparisons, we develop nonstationary versions of the following four pdfs: LN2, LN3, LP3 and GEV. These choices are also consistent with other recent studies including: the large-scale pan European evaluation by Salinas et al. (2014) which recommends the GEV model, England Jr et al. (2017) which recommends the LP3 model for the U.S. and Ball et al. (2016) which recommends both the LP3 and GEV models for the Australian continent. Although use of regional information is generally preferred over use of at-site methods in both stationary (Hosking and Wallis, 2005; Ouarda, 2016), and nonstationary (O'Brien and Burn, 2014) FFA, in this initial study we employ at-site parameter estimation methods, which have been proven to be efficient in previous studies, for each of the pdfs considered. The parsimonious approach introduced here could be combined with the index flood assumption to enable extension to regional NFFA.

### 6.1. Stationary and nonstationary quantile functions

In this section, we describe the stationary and nonstationary quantile functions corresponding to the LN2, LN3, GEV and LP3 distributions. Since we employ these quantile functions with actual data for two urbanizing rivers in the following section, we introduce both the stationary and the nonstationary quantile estimators to clarify how to implement the procedures introduced here. The challenge here is to develop nonstationary quantile estimators which account for BOTH the mean and variance of the AMF series  $x$ , being functions of the explanatory variable  $w$ . Thus, our estimation methods must be flexible enough to enable the distributional parameters, such as the lower bound of the LN3 and GEV distributions, to be functions of  $w$  yet be readily and easily implemented. Our experience indicates that whenever a trend in flood series is apparent, there appears to be a noticeable trend in the corresponding lower bound of its pdf.

#### 6.1.1. Lognormal –LN2

The lognormal distribution is one of the most widely used distributions in hydrology. Stedinger (1980) provides guidance on the most efficient estimation methods for the LN2 and LN3 pdfs. If the AMF series  $x$  follows an LN2 distribution, then  $y = \ln(x)$  follows a normal distribution. Stedinger (1980) shows that an attractive estimator of the quantile of a stationary LN2 variate is the maximum likelihood estimator given by:

$$\hat{x}_p = \exp(\bar{y} + z_p v_y) \quad (12)$$

where  $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$  and  $v_y = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2}$  and  $z_p$  is the inverse of a standard normal variable with nonexceedance probability  $p$ . Substitution of estimates of the conditional moments of  $y$  in (5) and (6) into the stationary quantile function of an LN2 variate leads to the nonstationary LN2 quantile estimator:

$$\hat{x}_{p|w} = \exp(\hat{\mu}_{y|w} + z_p \hat{\sigma}_{y|w}) = \exp\left(\bar{y} + \hat{\beta} \cdot (w_i - \bar{w}) + z_p \sqrt{\hat{\sigma}_y^2 - \hat{\beta}^2 \hat{\sigma}_w^2}\right) \quad (13)$$

where  $z_p$  is the quantile function for a standard normal variable of nonexceedance probability  $p$  and the estimators  $\bar{w}$ ,  $\hat{\sigma}_w^2$ ,  $\hat{\beta}$ ,  $\bar{y}$  and  $\hat{\sigma}_y$  are given in (4). Note that for the stationary case  $\hat{\beta} = 0$ , in which case (13) reduces to the well-known stationary LN2 quantile estimator in (12). Note also that numerous previous studies which have combined the same regression model used here, with the LN2 pdf did not account for the reduction in the conditional standard deviation of  $y$  as is done here (see for example, Stedinger and Griffis, 2011; Vogel et al., 2011; Prosdocimi et al., 2014; and Luke et al., 2017; just to name a few).

#### 6.1.2. Log Pearson type III–LP3

If the AMF series  $x$  follows an LP3 distribution, then  $y = \ln(x)$

follows a Pearson type III (P3) distribution (also termed a three-parameter Gamma pdf) which can be characterized by its mean  $\mu_y$ , standard deviation  $\sigma_y$  and skewness  $\gamma_y$ . The LP3 is an extremely flexible distribution that can assume a wide range of distributional forms ranging from an exponential pdf when  $\gamma_y = 2$  to an LN2 model when  $\gamma_y = 0$ . Detailed procedures for estimation of the LP3 model under stationary conditions, based on either at-site gaged data (as is assumed here) or based on regional skew information, are provided by both England Jr et al. (2017) and Ball et al. (2016). The nonstationary quantile function for an LP3 variate is obtained in a very similar fashion to that of the LN2 case. Again, substitution of the conditional moments of  $y$  into the stationary quantile function of an LP variate leads to the nonstationary LP3 quantile estimator:

$$\hat{x}_{p|w} = \exp(\hat{\mu}_{y|w} + k_p \hat{\sigma}_{y|w}) = \exp\left(\bar{y} + \hat{\beta} \cdot (w_i - \bar{w}) + k_p \sqrt{\hat{\sigma}_y^2 - \hat{\beta}^2 \cdot \hat{\sigma}_w^2}\right) \tag{14}$$

where

$$k_p = A \left\{ \max\left[ H, 1 - \left(\frac{F}{6}\right)^2 + z_p \cdot \left(\frac{F}{6}\right) \right]^3 \right\} - B$$

$$A = \max\left(\frac{2}{\hat{\gamma}_{y|w}}, 0.4\right)$$

$$B = 1 + 0.0144 \cdot \max(0, \hat{\gamma}_{y|w} - 2.25)^2$$

$$F = \hat{\gamma}_{y|w} - 0.063 \cdot \max(0, \hat{\gamma}_{y|w} - 1)^{1.85}$$

$$H = \left(B - \frac{2}{\hat{\gamma}_{y|w} \cdot A}\right)^{1/3}$$

which reduces to the stationary LP3 quantile function  $\hat{x}_{p|w} = \exp(\bar{y} + k_p \sigma_y)$  when  $\hat{\beta} = 0$ . Here  $\hat{\beta}$  and  $k_p$  are the inverse of a standard normal and a three-parameter Gamma variate, respectively, with nonexceedance probability  $p$ . While estimates of  $z_p$  are widely available in most any mathematical software package,  $k_p$  is more difficult to estimate but may be approximated using the computational method recommended by Kirby (1972) and given in (14), which is accurate for values of  $\gamma_y \leq |9|$  limits well beyond the skew values considered in this and most other studies. For the case when the explanatory variable  $w$  in the regression does not exhibit skewness, as is the case here using the variable time, an at-site estimator of conditional skewness  $\gamma_{y|w}$  in (7) may be estimated using the nearly unbiased estimator introduced by Tasker and Stedinger (1986) for the LP3 distribution:

$$\hat{\gamma}_{y|w} = \left(1 + \frac{6}{n}\right) \cdot \frac{n \sum_{i=1}^n (y_i - \bar{y})^3}{(n-1) \cdot (n-2) \cdot \hat{\sigma}_y^3} \tag{15}$$

Although this initial study only employs at-site estimators, we emphasize that at-site estimates of sample skew are highly variable which is why England Jr et al. (2017) and others recommend computing a regional skew estimator which weighs estimates of at-site and regional skews.

### 6.1.3. Generalized extreme value-GEV

The GEV distribution is a generalization of the Type-I, II, and III extreme value distributions, also known as the Gumbel, Frechet and Weibull pdfs, respectively. It may be the most widely used pdf for modeling annual series of natural hazards and other extremes, perhaps because it is the asymptotic distribution of the maximum value in an infinite sample with an extensive extreme value theory to support its application (Ghil et al., 2011). The AMF series represents the series of maxima, each drawn from a very large number of discharges measured in each year. Following the notation from Stedinger et al. (1993) and Stedinger (2016), the stationary quantile function of a GEV variate is usually written in terms of the three-parameters of the distribution  $\alpha$ ,  $\xi$  and  $\kappa$  which characterize its scale, location and shape, respectively. An estimate of the stationary GEV quantile estimator is given by

$$\hat{x}_p = \hat{\xi} + \frac{\hat{\alpha}}{\hat{\kappa}} (1 - [-\ln(p)]^{\hat{\kappa}}) \tag{16}$$

Although the generalized MLEs introduced by Martins and Stedinger (2000) are likely to be the most attractive parameter estimators for a GEV variate, especially if one has prior information, we employ method of moments estimators here, which have been shown to be more efficient than either L-moment or MLE's (see Stedinger, 2016 for discussion and references). Stedinger (2016, Table 76.4) reports expressions for the mean, standard deviation and skewness of  $x$ , denoted  $\mu_x$ ,  $\sigma_x$  and  $\gamma_x$ , respectively, as function of the pdf model parameters  $\xi$ ,  $\alpha$  and  $\kappa$ . Bhunya et al. (2007) report approximations for estimating the parameters  $\xi$ ,  $\alpha$  and  $\kappa$  from GEV samples based on sample estimates of the moments  $\mu_x$ ,  $\sigma_x$  and  $\gamma_x$  which we denote as  $\bar{x}$ ,  $\hat{\sigma}_x$  and  $\hat{\gamma}_x$ , respectively; which are reproduced in Appendix A. Substitution of the method of moment estimators of the stationary GEV parameters given in (A-1), (A-2) and (A-3) and denoted as  $\hat{\xi}$ ,  $\hat{\alpha}$  and  $\hat{\kappa}$ , into (16) leads to the stationary GEV quantile estimator. Similarly, a nonstationary GEV quantile function is obtained by replacing the sample moments  $\mu_x$ ,  $\sigma_x$  and  $\gamma_x$  with their conditional moments  $\mu_{x|w}$ ,  $\sigma_{x|w}$  and  $\gamma_{x|w}$  given in (10). After substitution of estimates of those conditional moments into the method of moment estimators given in Appendix A, one obtains estimates of the conditional parameters denoted  $\hat{\xi}_{x|w}$ ,  $\hat{\alpha}_{x|w}$  and  $\hat{\kappa}_{x|w}$ , leading to the nonstationary quantile function:

$$\hat{x}_{p|w} = \hat{\xi}_{x|w} + \frac{\hat{\alpha}_{x|w}}{\hat{\kappa}_{x|w}} (1 - [-\ln(p)]^{\hat{\kappa}_{x|w}}) \tag{17}$$

Again, we emphasize that much more attractive estimators of the GEV distribution would employ regional information for estimation of the shape parameter  $\kappa$  for both stationary and nonstationary cases, as is described in Martins and Stedinger (2000), El Adlouni et al. (2007) and Stedinger (2016).

### 6.1.4. Three-parameter lognormal -LN3

If the AMF series  $x$  follows an LN3 distribution, then the variable  $u = \ln(x - t_x)$  follows a normal distribution with mean and standard deviation equal to  $\mu_u$  and  $\sigma_u$ . The stationary quantile function for an LN3 variate has a very similar form to the stationary quantile function for an LN2 variate in (12). Stedinger (1980) documents the most efficient estimators of the LN3 distribution under various conditions of interest to a hydrologist. We take an analogous approach to the one we took with the GEV distribution where we employed the method of moments for both the nonstationary and stationary cases. Here our goal is to fit both and stationary and nonstationary lognormal model using the method of moments since both unconditional and conditional estimates of those moments have been derived here for the nonstationary case. A method of moment estimator of the stationary LN3 quantile estimator was given by Singh et al. (1990) as

$$\hat{x}_p = \hat{t}_x + \exp(\hat{\mu}_u + z_p \hat{\sigma}_u^2) \tag{18}$$

where

$$\hat{\mu}_u = \frac{1}{2} \ln \left[ \frac{\hat{\sigma}_x^2}{\lambda(\lambda-1)} \right] \hat{\sigma}_u^2 = \ln(\lambda) \hat{t}_x = \bar{x} - \frac{\hat{\sigma}_x}{\theta}$$

$$B = \frac{1}{2} [-\hat{\gamma}_x + \sqrt{\hat{\gamma}_x^2 + 4}] \theta = \frac{1 - B^{2/3}}{B^{1/3}} \lambda = \theta^2 + 1$$

with  $z_p$  defined previously and the sample moments  $\bar{x}$ ,  $\hat{\sigma}_x$  and  $\hat{\gamma}_x$  given in Appendix A.

Similarly, the nonstationary LN3 quantile estimator is obtained from:

$$\hat{x}_{p|w} = \hat{t}_{x|w} + \exp(\hat{\mu}_{u|w} + z_p \hat{\sigma}_{u|w}^2) \tag{19}$$

where

$$\hat{\mu}_{u|w} = \frac{1}{2} \ln \left[ \frac{\hat{\sigma}_{x|w}^2}{\lambda(\lambda - 1)} \right] \quad \hat{\sigma}_{u|w}^2 = \ln(\lambda) \quad \hat{\epsilon}_{x|w} = \hat{\mu}_{x|w} - \frac{\hat{\sigma}_{x|w}}{\theta}$$

$$B = \frac{1}{2} \left[ -\hat{\gamma}_{x|w} + \sqrt{\hat{\gamma}_{x|w}^2 + 4} \right] \quad \theta = \frac{1 - B^{2/3}}{B^{1/3}} \quad \lambda = \theta^2 + 1$$

and  $z_p$  is the quantile function for a standard normal variable of non-exceedance probability  $p$  and the estimators  $\hat{\mu}_{x|w}$ ,  $\hat{\sigma}_{x|w}$  and  $\hat{\gamma}_{x|w}$  are given in (A-4a), (A-4b) and (A-4d), respectively. Note that for the stationary case  $\hat{\beta} = 0$  in (2) in which case the nonstationary LN3 quantile function in (19) reduces to the stationary LN3 quantile function in (18).

### 7. Case study–Regression Results

In this initial study, we consider the Aberjona and Neponset rivers, which drain two urbanizing suburban watersheds near Boston, Massachusetts. Both have long term U.S. Geological Survey gaging records. The Aberjona River (USGS Gage 01102500) drains a 24 mi.<sup>2</sup>. watershed in which impervious cover increased from 6% to 36% during the gaging period from 1940 to 2015 (75 years). Fig. 2 in Allaire et al. (2015) summarizes changes in watershed population and water withdrawals over this period. The watershed area has no major flow regulations, impoundments, withdrawals or storm water detention systems. See Allaire et al. (2015) for further information on the hydrologic properties of the Aberjona River. The Neponset River (USGS Gage 01105000) at Norwood, MA has measured discharge from 1938 to 2015 (77 years). Approximately 16% of the 34.7 mi.<sup>2</sup>. watershed is currently impervious. Flow in the Neponset river is regulated by small mills and reservoirs upstream and is subject to municipal and industrial diversions. Both the Aberjona and Neponset river watersheds are obvious laboratories to test the application of NFFA methodologies because of the gradual anthropogenic land cover changes that have occurred over the past century.

Figs. 1 and 2 use open circles to illustrate the relationship between the natural logarithm of the AMF series  $y = \ln(x)$  and the explanatory variable  $w = \text{time}$ , for the Aberjona River and the Neponset River watersheds, respectively. Using a 5% significance level we were unable to reject the null hypothesis of no trend, for either river, based on a  $t$ -test of the regression slope coefficient. Also, shown in each figure is the fitted regression model along with various other metrics associated with the model, including the sample size  $n$ , the goodness-of-fit metric  $R^2$  (which in this case of bivariate regression is simply equal to  $\hat{\rho}^2$  defined in (4)), as well as the attained significance level  $p$ , associated with the estimated regression coefficient  $\hat{\beta}$ . The attained significance level  $p$  is

computed from the data, and indicates the probability that one would obtain a value of  $\hat{\beta}$  equal to or greater than the value obtained, if the null hypothesis of no trend ( $\beta = 0$ ) were correct. In Table 2, for both rivers, we compare values of  $\hat{\beta}$ , based on OLS regression with non-parametric estimates based on the nonparametric Kendall–Theil slope estimator described by Helsel and Hirsch (2001, Chapter 10). The attained significance levels in Table 2 for the parametric and nonparametric cases are computed using a  $t$ -test and the Mann–Kendall trend test, respectively. We note the near equivalence of the parametric and nonparametric estimates  $\hat{\beta}$  and very similar attained significance levels using both estimators. It is important to consider both nonparametric and parametric estimates of the regression slope, because this parameter plays such a critical role in our subsequent NFFA, and because as we show below, the regression model residuals are poorly behaved, which raises questions about our ability to perform statistical inference on the resulting models.

The extremely low attained significance levels for both rivers ( $p < 0.001$ ), summarized in Table 2, indicate that proof for a trend is so strong that there is very little possibility for evidence this strong, or stronger, to arise by chance if there were truly no trend. It is equally important to consider the likelihood that we would reject a trend, if it really existed, because as Vogel et al. (2013) emphasize, rejecting a trend when it really exists could have much greater societal consequences than accepting a trend when it does not exist. See Vogel et al. (2013), Rosner et al. (2014) and Prosdocimi et al. (2014) for a discussion of the likelihood of type I and II errors within the context of trend detection and the factors which could lead us to reject a trend when it really exists, leading to substantial societal consequences.

We emphasize that one should never take the result of an individual hypothesis test, based on a sample of limited length, too seriously. Neyman and Pearson (1933) suggest that “no test based upon a theory of probability can by itself provide any valuable evidence of the truth or falsehood of a hypothesis. But we may look at the purpose of tests from another viewpoint. Without hoping to know whether each separate hypothesis is true or false, we may search for rules to govern our behavior with regard to them, in following which we insure that, in the long run of experience, we shall not often be wrong”. It is in this light that we interpret the above results.

The fitted regression models in Figs. 1 and 2 represent the model of the conditional mean  $\hat{\mu}_{y/w} = \bar{y} + \hat{\beta}(w - \bar{w})$ . To highlight the uncertainty associated with future predictions of  $y = \ln(x)$ , we also illustrate 95% prediction intervals, in Figs. 1 and 2, which are intervals within which 95% of all flood observations  $y$ , are expected to lie (see Section 9.4.5 in Helsel and Hirsch, 2002, for an analytical formula for

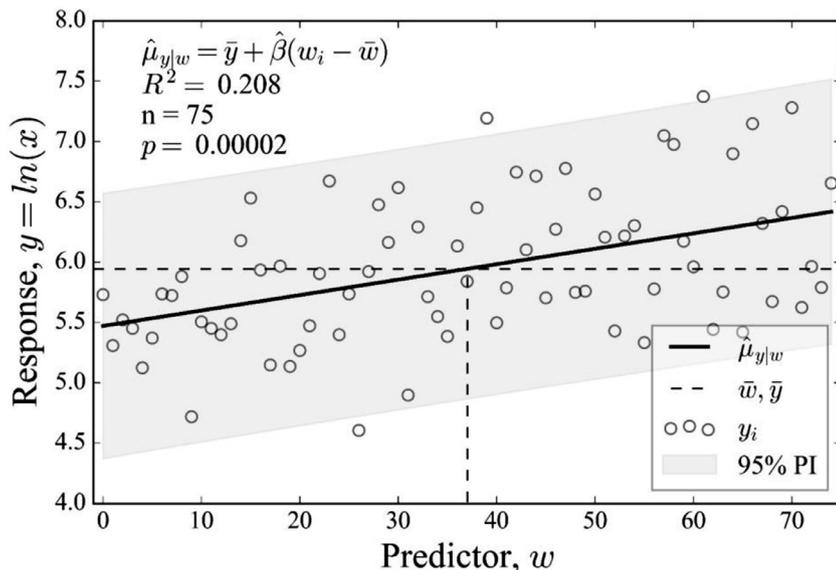


Fig. 1. Natural logarithm  $y$ , of annual maximum streamflow  $x$  of the Aberjona River at Winchester, MA, 1940–2014 versus  $w = \text{time}$  in years. Solid line is a bivariate linear regression which represents the conditional mean of  $y = \ln(x)$  as a function of  $w$ , denoted  $\hat{\mu}_{y/w}$ . Shaded region is the estimated 95% prediction interval for future values of  $y = \ln(x)$ .

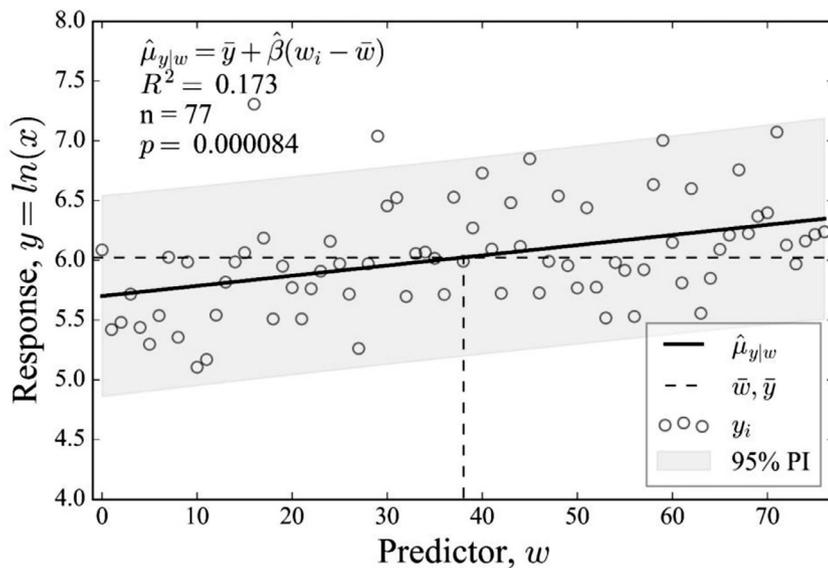


Fig. 2. Natural logarithm  $y$  of annual maximum streamflow  $x$ , on the Neponset River at Norwood, MA, 1938–2014 versus  $w = \text{time}$  in years. Solid line is a bivariate linear regression which represents the conditional mean of  $y = \ln(x)$  as a function of  $w$  denoted  $\hat{\mu}_{y|w}$ . Shaded region is the estimated 95% prediction interval for future values of  $y = \ln(x)$ .

computing prediction intervals). Note that such prediction intervals widen considerably under extrapolation, highlighting another important advantage of our approach, because it emphasizes the increasing uncertainty inherent in extrapolation of trends. Future studies may wish to derive the variance and/or confidence intervals associated with quantile estimates based on the various nonstationary pdf models introduced in this study analogous to the work of Obeysekera and Salas (2014). We refer the reader to Section 9.4.4 of Helsel and Hirsch (2002) which provides estimates of the variance and/or confidence intervals associated with the conditional mean regression estimator  $\hat{\mu}_{y|w}$ .

### 7.1. Regression model evaluations

A necessary condition for any statistical inference to be performed on a fitted trend model based on ordinary least squares (OLS) regression requires that model residuals be approximately homoscedastic, normally distributed and independent in time. Figs. 3 and 4 illustrate diagnostic plots which summarize the behavior of the regression model residuals for the regressions corresponding to the Aberjona and Neponset Rivers, respectively. The following subplots are provided for each regression: Subplot *a*) illustrates a histogram of the model residuals which should appear Gaussian with a mean of zero. Subplot *b*) displays the residuals  $y_i - \hat{y}_i$  versus  $w_i$  and should appear to exhibit constant variance. A Breusch-Pagan (1979) test was performed which enables us to reject the null hypothesis of homoscedastic residuals (constant variance) at the 5% significance level for the Aberjona River and fail to reject the null hypothesis of homoscedasticity for the Neponset River. The attained significance levels associated with the Breusch-Pagan test were:  $p = 0.031$  and  $p = 0.628$  for the Aberjona and Neponset River, respectively. A normal probability plot is illustrated in subplot *c*) as well as the probability plot correlation coefficient (PPCC) and its associated attained significance level  $p$  (see Vogel, 1986). Here the attained significance level,  $p$ , is interpreted a bit differently than for other tests. Again,  $p$  is computed from the observations, and indicates the probability of obtaining a value of PPCC smaller than the computed value, if the null hypothesis of normality were correct. The attained significance levels were  $p = 0.725$  and  $p = 0.0062$  for the Aberjona and Neponset River residuals, respectively, which implies that the Aberjona probability plot is typical of what probability plots would look like if the samples were normal, whereas, very few normal probability plots, for normal samples, would look like the result obtained for the Neponset river. Thus, we cannot reject the null hypothesis of normality of the trend model residuals at the 5% level for the Aberjona River, but we do reject it for the Neponset River. Finally, subplot *d*) is a correlogram displaying the lag- $k$  serial correlation

coefficient of the residuals for lags of 1–50, along with 95% confidence intervals for the case of independence (no serial correlation).

For the Aberjona River, the results in Fig. 3, combined with the Breusch-Pagan and PPCC normality hypothesis tests reveal that model residuals appear to exhibit some heteroscedasticity, yet they appear to be approximately independent and normally distributed. For the Neponset River, the results in Fig. 4, combined with associated Breusch-Pagan and PPCC normality hypothesis tests, reveal that model residuals are approximately homoscedastic and independent, but are poorly approximated by a normal distribution. Thus, our diagnostic analysis of model residuals raises questions concerning our ability to perform statistical inference concerning nonstationary behavior in the AMF at both sites. For this reason, we included nonparametric estimates of the regression model slope coefficient and associated attained significance levels, for both rivers, in Table 2.

### 7.2. Stationary and nonstationary probability plots

Probability plots (Helsel and Hirsch, 2002) are a graphical technique to illustrate the goodness-of-fit of different pdfs to a set of data. The degree of linearity of the observations in a probability plot corresponding to a hypothesized distribution is a measure of the goodness-of-fit of that particular pdf to the observations. Thus, the linearity of the probability plot, measured using a PPCC provides both a quantitative measure of the goodness-of-fit of the hypothesized pdf to the observations and can also be used to perform a hypothesis test for two-parameter distributional alternatives (Vogel, 1986; Stedinger et al., 1993; Heo et al., 2008), though hypothesis test results for three-parameter distributional alternatives are questionable (Vogel and McMartin, 1991). Until this study, all procedures for developing probability plots and for computing PPCC test statistics have assumed stationarity. We introduce a procedure for constructing nonstationary probability plots and for computing the PPCC to assess the possible improvement in goodness-of-fit which results when there is good evidence of nonstationary behavior in the AMF series.

When performing diagnostic evaluations, such as probability plots, Coles (2001) suggests working with a standardized version of the data, conditional on the fitted parameter values. Standardization can remove the trend from the data, effectively resulting in a transformation of the time-series from nonstationary to stationary. Thus, by standardizing either the  $x$  or  $y$  series using their conditional moments introduced here, one can apply a stationary probability plot to the standardized series, resulting in a nonstationary probability plot.

The steps used to construct stationary and nonstationary probability plots are enumerated below:

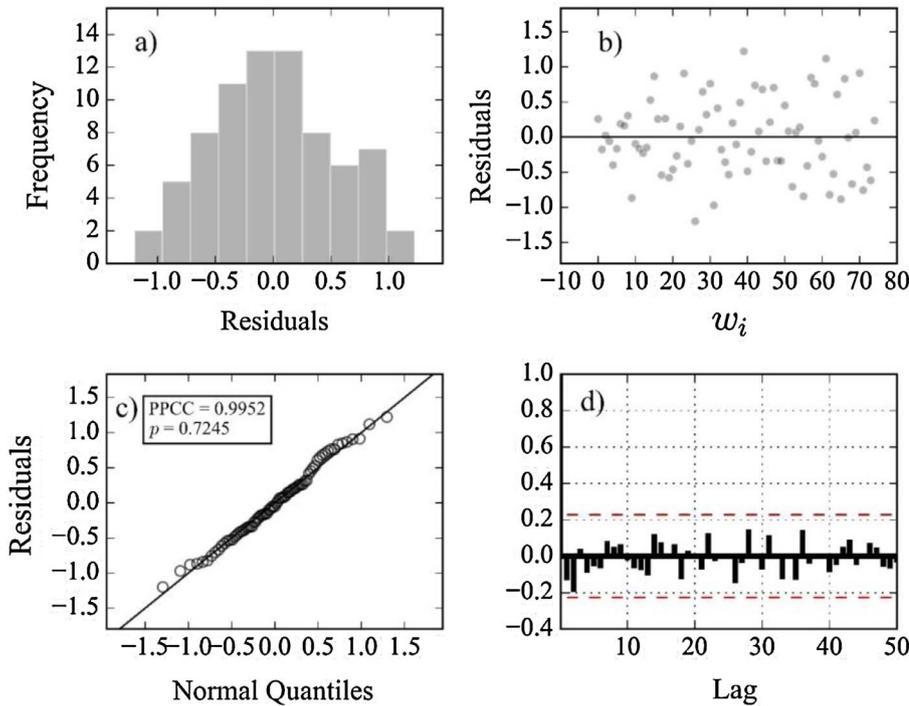


Fig. 3. Diagnostic plots for the Aberjona River regression residuals which enable evaluation of the assumptions of independence, homoscedasticity and normality. Subplots display (a) Histogram of residuals (b) Scatter plot of residuals over time (c) Normal probability plot with corresponding PPCC and attained significance level  $p$ , associated with a PPCC normality test and (d) Correlogram exhibiting lag- $k$  correlation for first 50 lags along with 95% confidence intervals under the null hypothesis of no serial correlation.

1) Standardize the observations of either  $x$  for GEV and LN3 cases or  $y = \ln(x)$  for the LN2 and LP3 cases using the conditional moments for  $x$  and  $y$  given previously:

$$\hat{Z}_i = \begin{cases} \frac{y_i - \hat{\mu}_{y|w}}{\hat{\sigma}_{y|w}} & \text{for LN2 and LP3} \\ \frac{x_i - \hat{\mu}_{x|w}}{\hat{\sigma}_{x|w}} & \text{for GEV} \\ \frac{\ln(x_i - \hat{\tau}_{x|w}) - \hat{\mu}_{u|w}}{\hat{\sigma}_{u|w}} & \text{for LN3} \end{cases}$$

- 2) Rank the standardized observations  $\hat{Z}_i$  in ascending order such that  $\hat{Z}_{(i)}$  denotes the ordered values for  $i = 1, \dots, n$ . For the stationary series,  $\hat{\beta} = 0$  and for the nonstationary series,  $\hat{\beta} \neq 0$ .
- 3) Select a suitable plotting position  $p_i$  from Table 1, corresponding to the hypothesized distribution considered.
- 4) Plot the ordered set  $\{\hat{Z}_{p_i}, \hat{Z}_{(i)}; i = 1, \dots, n\}$  for the stationary and nonstationary case using the appropriate plotting position  $p_i$  from Table 1 to obtain estimates of the standardized variate  $Z$  corresponding to each of the pdfs considered using:

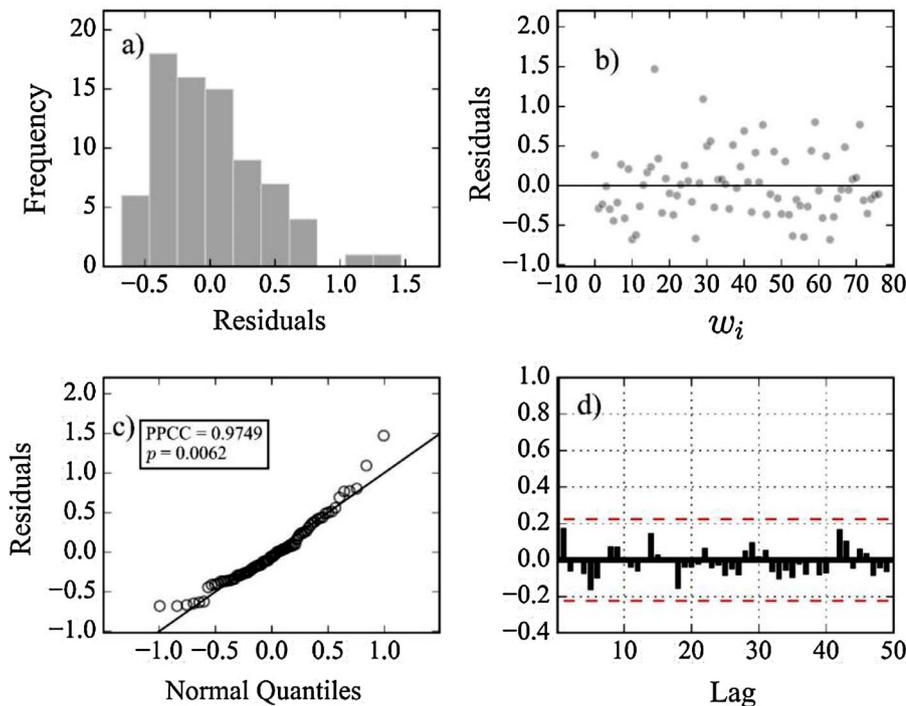


Fig. 4. Diagnostic plots for the Neponset River regression residuals which enable evaluation of the assumptions of independence, homoscedasticity and normality. Subplots display (a) Histogram of residuals (b) Scatter plot of residuals over time (c) Normal probability plot with corresponding PPCC and attained significance level  $p$  associated with a PPCC normality test. and (d) Correlogram exhibiting lag- $k$  correlation for first 50 lags along with 95% confidence intervals associated under the null hypothesis of no serial correlation.

**Table 1**  
Plotting position formulas used to construct probability plots and to estimate PPCC test statistics. Selections based on suggestions by Stedinger et al. (1993) and others.

Position name	Formula, $p_i$	pdf
Blom	$\frac{i - 0.375}{n + 0.25}$	LN2, LN3, LP3
Gringorten	$\frac{i - 0.44}{n + 0.12}$	GEV

$$\hat{Z}_{p_i} = \begin{cases} z_{p_i} & \text{for LN2 and LN3} \\ \frac{\hat{x}_{p_i|w} - \hat{\mu}_{x|w}}{\hat{\sigma}_{x|w}} & \text{for GEV} \\ k_{p_i} & \text{for LP3} \end{cases}$$

where  $\hat{x}_{p_i|w}$  is an estimate of a nonstationary GEV variate given in (17),  $z_{p_i}$  is the standardized normal quantile used in the LN2 and LN3 quantile functions in (13) and (18), and  $k_{p_i}$  is the standardized Pearson type III given in (14).

5) Calculate the PPCC test statistic which is the product moment correlation:

$$PPCC = \frac{\sum_{i=1}^n (\hat{Z}_{p_i} - \bar{\hat{Z}}) \cdot (\hat{Z}_{(i)} - \bar{Z})}{\sqrt{\sum_{i=1}^n (\hat{Z}_{p_i} - \bar{\hat{Z}})^2 \cdot \sum_{i=1}^n (\hat{Z}_{(i)} - \bar{Z})^2}}$$

where  $\bar{\hat{Z}} = \frac{1}{n} \sum_{i=1}^n \hat{Z}_{p_i}$  and  $\bar{Z} = \frac{1}{n} \sum_{i=1}^n \hat{Z}_{(i)}$ , with  $\hat{Z}_{(i)}$  and  $\hat{Z}_{p_i}$  defined in steps 1 and 4, respectively.

7.2.1. Probability plot results

Probability plots corresponding to the standardized variable  $\hat{Z}_{(i)}$  for the four distributions under both stationary and nonstationary assumptions, and the corresponding PPCCs, are displayed in Figs. 5 and 6 for the Aberjona and Neponset watersheds, respectively. For the Aberjona River, the probability plots for the four nonstationary models are slightly more linear than the corresponding probability plots for the four stationary cases, as evidenced by their slightly higher PPCC values. In contrast, for the Neponset River, the probability plots for the four nonstationary models are slightly less linear, than the corresponding probability plots for the stationary cases, as evidenced by their slightly lower PPCC values.

7.3. Discussion of hypothesis test results

There are several important lessons to be learned from the above analysis. First and foremost, there is considerable physical evidence of drivers of nonstationarity for these two urbanized watersheds which led us to hypothesize that significant population growth and associated increases in impervious cover over the period of streamflow record led to increases in the AMF series. Importantly, this conjecture is supported by our fitted trend model and associated diagnostics. The extremely low values of attained significance levels ( $p < 0.001$ ) associated with the trend model slope coefficient at both the Aberjona (Fig. 1) and Neponset (Fig. 2) rivers corresponding to both the parametric  $t$ -test and the nonparametric Mann–Kendall test (Table 2) led to a high level of confidence that both AMF series exhibited an increasing trend over the historical period.

Diagnosis of the regression model residuals led us to question our ability to perform statistical inference concerning trends in both AMF series because the Aberjona River model residuals exhibited some heteroscedasticity and the Neponset River model residuals exhibited some nonnormality. These results lead us to question conclusions derived from statistical inference on both models including hypothesis

tests and prediction intervals. Nevertheless, since the nonparametric test results support our parametric test results, we remain confident that both rivers exhibit nonstationary behavior which should be considered in any subsequent flood frequency analysis.

Our analysis only considers a trend in the mean of  $y$  which implies that the coefficient of variation of  $x$  is fixed (thus the mean and standard deviation of  $x$  both exhibit the identical trends). The apparent heteroscedasticity of the model residuals for the Aberjona River indicates that a more complex model which allows for a time varying coefficient of variation of  $x$ , should be considered. When model residuals exhibit heteroscedasticity, OLS estimates of regression model parameters are unbiased, but the standard  $t$ -test of model parameter significance can lead to incorrect inferences (Long and Ervin, 2000). To ensure correct inferences concerning regression model parameters in the presence of heteroscedasticity of an unknown form, Long and Ervin (2000) and others have suggested the use of a heteroscedasticity consistent covariance matrix and associated standard errors. While such approaches enable improved statistical inference in the presence of heteroscedasticity, they do not provide an approach to model changes in the coefficient of variation.

Analogous to the regression approach we have introduced to describe the time variation in  $y$  corresponding to a trend in both the mean and variance of  $x$  (holding coefficient of variation constant), it is possible to introduce a time varying function for the coefficient of variation of  $x$ , and to integrate that function into the quantile function of  $x$  to enable improved estimation of design events. Hecht (2017) summarizes several attractive approaches for considering time variations in the coefficient of variation, given an assumed model form, including the use of weighted least squares regression (Strupczewski and Kaczmarek, 2001), heteroscedasticity consistent standard errors (Long and Ervin, 2000) as well as by simply fitting independent regression models for the mean and standard deviation of  $x$ . Our parsimonious approach to modeling a trend in the mean and variance of  $x$  (assuming constant coefficient of variation), provides a reasonable initial approach for estimating the nonstationary 100 year flood, however further research is needed to develop a parsimonious approach for determining the impact of changes in the mean, variance and coefficient of variation of  $x$  on a design event.

7.4. Comparisons of stationary and nonstationary design events

Of critical interest in practice are estimates of selected quantiles of the distribution of the AMF series under both stationary and nonstationary conditions to enable one to update design events to reflect conditions in a particular year. One simple, yet effective way to understand the impact of the trend model on resulting design events is to compute the decadal magnification factor  $M = \exp(\Delta t \cdot \hat{\beta}) = \exp(10\hat{\beta})$  (Vogel et al., 2011) corresponding to the nonstationary LN2 model. The decadal magnification factors for the Neponset and Aberjona rivers are 1.089 and 1.136, respectively. This indicates that all flood quantiles have increased at the rate of about 9% and 14% per decade, over the historical period for the Neponset and Aberjona Rivers, respectively.

Table 3 summarizes estimates of the 100-year flood denoted  $x_{100}$  corresponding to each of the four hypothesized pdfs, for both rivers, along with corresponding estimates of the PPCC statistics, for both the stationary and nonstationary analyses. Estimates of the 100 year flood based on nonstationary conditions are obtained for the most current year on record  $w_n$ , so that they reflect the 100 year flood under current conditions as of 2015. In contrast, estimates of the 100 year flood under stationary conditions are not updated to current conditions but reflect average conditions over the observed period of flows.

Numerous conclusions can be drawn from Table 3, combined with our previous trend model diagnostics. If one ignores any possibility of nonstationarity, using a traditional (stationary) PPCC lognormal hypothesis test we were unable to reject the LN2 hypothesis for the AMF

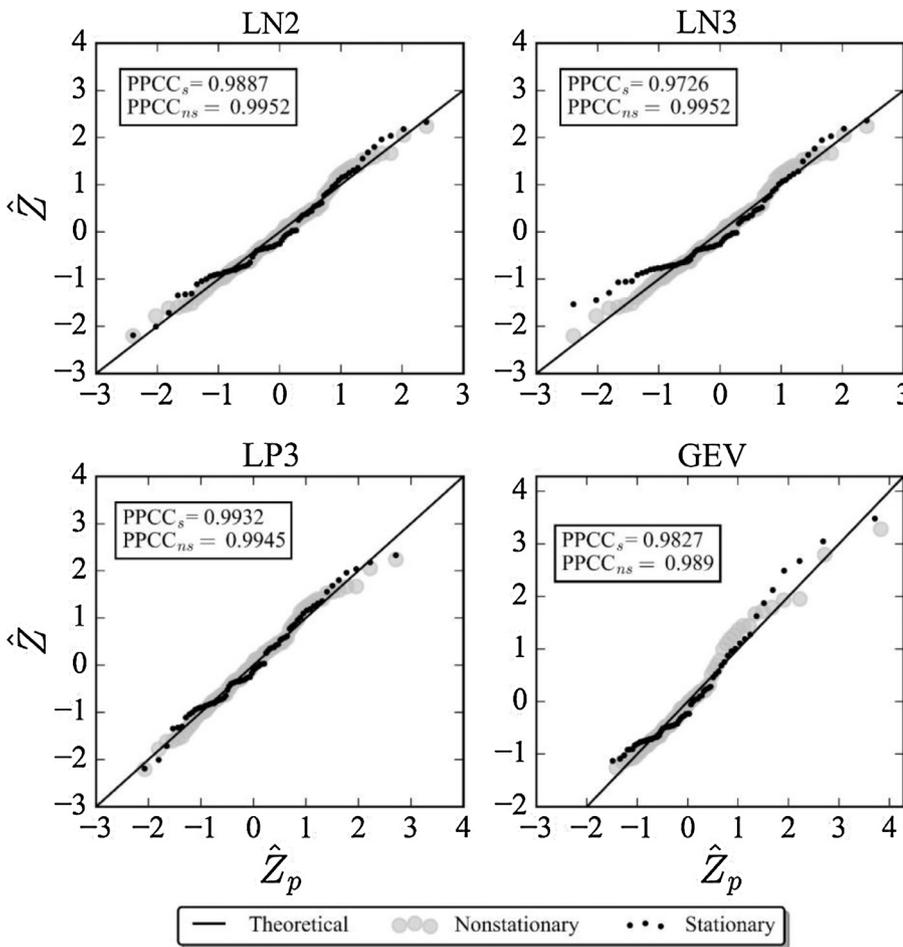


Fig. 5. Standardized probability plots for the Aberjona River. The line represents a theoretical one-to-one relationship. The points represent corresponding fits for the four hypothesized pdfs. Ordered standardized observations are plotted on the vertical axis against standardized quantile estimates. PPCC values are quantitative measures of the goodness of fit of the hypothesized distribution corresponding to the stationary (PPCC<sub>s</sub>) and nonstationary (PPCC<sub>ns</sub>) cases.

series at both rivers at the 5% level. A nonstationary analysis led to uniform improvements in the goodness-of-fit (as evidenced by increases in PPCC values) for all four models considered for the Aberjona River. In contrast, for the Neponset River, the nonstationary analysis did not result in an improvement in goodness-of-fit over a stationary analysis, though that may be due, in part, to the occurrence of an unusually large flood of record in the middle of the period of record (see Fig. 2). This is because when there is a positive trend in the flow series, the nonstationary quantile function will produce the largest design floods at the end of the historical period. Table 3 documents the considerable increases in estimates of the 100-year flood which result from using NFFA to update design events to reflect current 2015 conditions. For both rivers, engineers would be advised to consider the nonstationary estimates of  $x_{100}$  as an improvement over the stationary estimates, because there is both good statistical evidence and plausible physical drivers for the increases in AMF series. Physical evidence is provided by the considerable increases in urbanization that have occurred in both basins. Statistical evidence is provided by the extremely low attained significance levels associated with the nonparametric Mann–Kendall trend test results in Table 2. Interestingly, the design flood estimates in 2015 corresponding to the nonstationary LN2, LN3 and GEV models are nearly equal, at both rivers. It is only the design flood corresponding to the LP3 nonstationary model which differs from the other nonstationary models. To enable rigorous comparisons among the precision of these alternative design flood estimates, studies of the type performed by Obeysekera and Salas (2014), Serinaldi and Kilsby (2015) and Luke et al. (2017) are needed to compare the uncertainty associated with such stationary and nonstationary design flood estimates. Given the relatively short series of observations available corresponding to potential nonstationary hydrologic conditions, there will always be

considerable uncertainty as to whether or not a stationary or nonstationary analysis is warranted. That fact further highlights the need to include both the stationary and nonstationary results in any decision oriented analysis as recommended and documented by Rosner et al. (2014).

### 8. Conclusions

Despite widespread awareness of the impact of anthropogenic influences on extreme floods (and droughts) there remains considerable controversy and debate concerning the development of a generalized, sensible, and practical methodology for NFFA to enable updating of design events to reflect current or future hydrologic conditions. The lack of consensus results in part from the tremendous uncertainty associated with our ability to detect, attribute and model past trends, and the even greater uncertainty associated with our ability to predict future trends in hydrological processes. Nevertheless, risk-based decision making (RBDM) approaches have been used for decades, and are now a well-established methodology for the determination of appropriate levels of investment based on the expected damages avoided versus the cost of the infrastructure required (National Research Council, 2000; USACE, 2000; Tung, 2005). Incorporation of nonstationarity into such RBDM analysis only increases the level of uncertainty associated with such analysis, but does not change our overall approach to selection of a suitable design event (see Rosner et al., 2014, for an example).

Our brief review and discussion of the state-of-the-art of NFFA revealed a wide range of modeling approaches including extremely sophisticated statistical methods, with little attention given to the concept of parsimony. We reviewed historical literature which documents the tremendous value of parsimonious models for stationary FFA and we

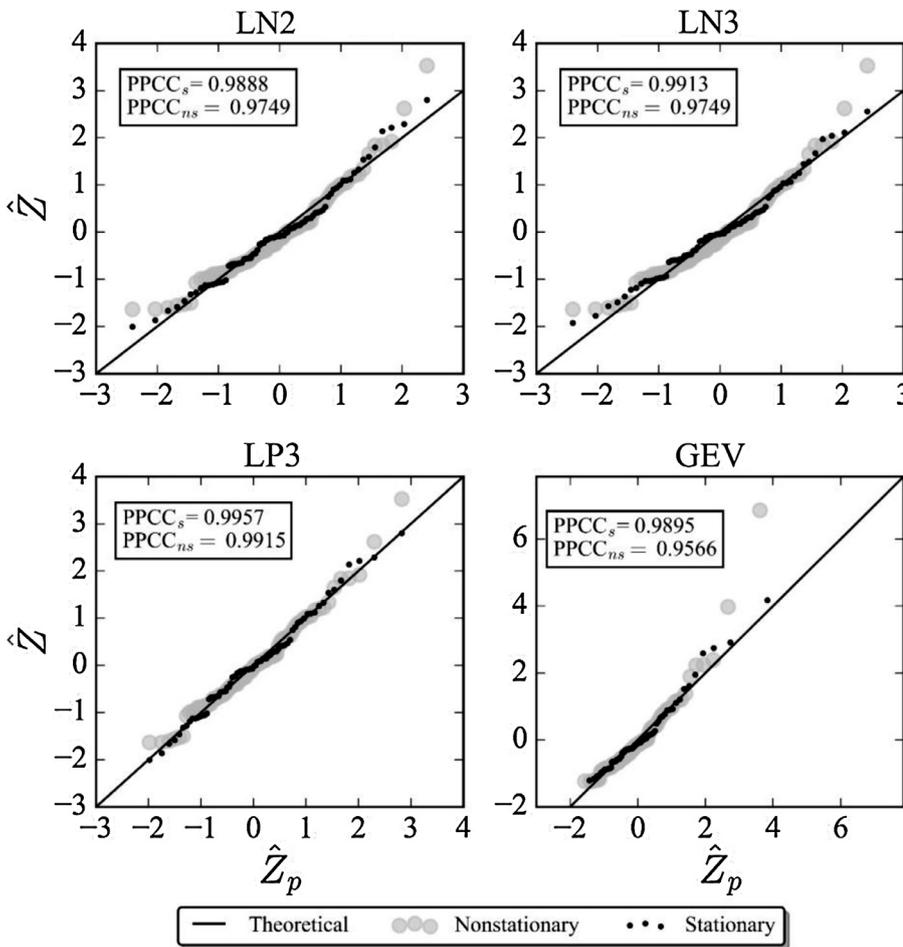


Fig. 6. Standardized probability plots for the Neponset River. The line represents a theoretical one-to-one relationship. The points represent corresponding fits for the four hypothesized pdfs. Ordered standardized observations are plotted on the vertical axis against standardized quantile estimates. PPCC values are quantitative measures of the goodness of fit of the hypothesized distribution corresponding to the stationary (PPCC<sub>s</sub>) and nonstationary (PPCC<sub>ns</sub>) cases.

Table 2

Values of the estimated regression model slope coefficient using parametric (OLS) and non-parametric (Kendall-Theil) estimation methods for both rivers. Attained significance levels for the parametric and nonparametric cases are obtained using a *t*-test and the Mann–Kendall trend test, respectively.

	Aberjona		Neponset	
	$\hat{\beta}$	<i>p</i>	$\hat{\beta}$	<i>p</i>
OLS	0.0128	0.00002	0.0085	0.000084
Kendall–Theil	0.0116	0.00014	0.0088	0.000036

have good reason to suspect similar value to parsimony in NFFA. Our primary goal was to introduce a generalized, sensible, parsimonious and practical approach to NFFA to enable the determination of a flood quantile when hydrologic change is an obvious concern. Our

Table 3

Estimates of the 100 year streamflow quantile  $x_{100}$ , in cubic-feet per second for the Aberjona River at Winchester and Neponset River at Norwood estimated using the stationary and nonstationary approaches. The estimates of  $x_{100}$  based on the nonstationary analysis are updated to current conditions at the end of the flow record in 2015. Critical PPCC values for the LN2 at the 5% level are 0.9841 and 0.9845 for the Aberjona and Neponset, respectively.

	Aberjona				Neponset			
	$x_{100}$		PPCC		$x_{100}$		PPCC	
	Stationary	Nonstationary	Stationary	Nonstationary	Stationary	Nonstationary	Stationary	Nonstationary
LN2	1580	2180	0.9887	0.9952	1200	1500	0.9888	0.9749
LN3	1560	2180	0.9726	0.9952	1320	1500	0.9913	0.9749
LP3	1900	2550	0.9932	0.9945	1430	1770	0.9957	0.9915
GEV	1570	2170	0.9827	0.9890	1320	1520	0.9895	0.9566

nonstationary analysis differs from most previous approaches because it only requires one additional parameter (trend model slope) over and beyond a stationary analysis. Our approach is based on derived conditional moments of a single trend regression which accounts for possible changes to the first three conditional moments of streamflow with only the addition of a single additional model parameter.

We have outlined how integration of a single bivariate regression into NFFA offers several advantages over existing approaches including: parsimony, ease of use, graphical display, prediction intervals, accommodating missing and/or historical information, accounting for abrupt shifts, accounting for persistence, extensions to multivariate trend models and opportunities for uncertainty analysis. We have also introduced nonstationary probability plots and associated goodness-of-fit statistics to quantitatively assess the improved goodness of fit associated with a NFFA over a stationary analysis.

Many packages written in R software (R Core Team, 2015) facilitate

the application of, advanced NFFA approaches (e.g. see GAMLSS software by Stasinopoulos and Rigby 2007; extRemes software by Gilleland and Katz, 2011; and NEVA software by Cheng et al., 2014), however, unlike the methodology introduced here, a complete understanding of those methods requires advanced training in statistics. Furthermore, unlike our methodology which could be extended to any pdf, some of the above referenced software tools are not useful for distributions other than the generalized extreme value (GEV) and generalized Pareto pdfs.

Our parsimonious approach to nonstationary flood frequency analysis (NFFA) is based on a single bivariate regression equation which describes the relationship between annual maximum floods,  $x$ , and an exogenous variable which can explain the nonstationary behavior of  $x$ . In this initial study, we only consider a single explanatory variable, time, selected as a surrogate for all the possible time-dependent nonstationary influences on the AMF series considered in two case studies. We recommend development of multivariate models which include numerous additional covariates because this approach has been shown to improve upon our ability to describe future nonstationary behavior of flood series (Kwon et al., 2008; Delgado et al., 2010; López and Francés, 2013; Prosdocimi et al., 2014, 2015; Condon et al., 2015; Šraj et al., 2016).

Using a single bivariate regression model, the conditional mean, variance and skewness of both  $x$  and  $y = \ln(x)$  are derived, and combined with numerous common probability distributions including the lognormal, generalized extreme value and log Pearson type III models, resulting in a very simple and parsimonious yet very general approach to NFFA. Numerous natural extensions to our initial bivariate regression

analysis are possible including: consideration of other probabilistic models, derivation of conditional moments for multivariate regression, derivation of confidence intervals for nonstationary quantile estimators, and application to drought and low flow frequency analysis. Perhaps the most promising extension to our initial work would be to develop regional estimates of the conditional skew and possibly the conditional coefficient of variation to enable improvements in the precision of resulting nonstationary quantile estimators.

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### Appendix A

#### Stationary and nonstationary method of moment estimators of parameters of a GEV variate

##### Stationary GEV case

Bhunya et al. (2007) report the following approximations for computing method of moment estimates of the location, scale and shape parameters  $\xi$ ,  $\alpha$  and  $\kappa$ , respectively, of a stationary GEV variate  $x$  from sample estimates of the mean, standard deviation and skewness, denoted  $\bar{x}$ ,  $\hat{\sigma}_x$  and  $\hat{\gamma}_x$ , respectively:

$$\hat{\kappa} = \begin{cases} 0.0087 \cdot \hat{\gamma}_x^3 + 0.0582 \cdot \hat{\gamma}_x^2 - 0.32 \cdot \hat{\gamma}_x + 0.2778 & -0.7 \leq \hat{\gamma}_x \leq 1.15 \\ -0.31158 \cdot [1 - \exp[-0.4556(\hat{\gamma}_x - 0.97134)]] & 1.15 \leq \hat{\gamma}_x \end{cases} \quad (A-1)$$

$$\hat{\alpha} = \hat{\sigma}_x(-0.1429\hat{\kappa}^3 - 0.7631\hat{\kappa}^2 + 1.0145\hat{\kappa} + 0.7795) \quad -0.5 \leq \hat{\kappa} \leq 0.5 \quad (A-2)$$

$$\frac{\hat{\xi} - \bar{x}}{\hat{\sigma}_x} = \begin{cases} 0.514075\hat{\kappa}^{1.33199} - 0.44901 & 0.01 \leq \hat{\kappa} \leq 0.5 \\ 19.357\hat{\kappa}^4 + 13.749\hat{\kappa}^3 + 4.484\hat{\kappa}^2 + 0.5212\hat{\kappa} - 0.04427 & -0.5 \leq \hat{\kappa} \leq 0.01 \end{cases} \quad (A-3)$$

Substitution of sample estimates of the unconditional moments  $\bar{x}$ ,  $\hat{\sigma}_x$  and  $\hat{\gamma}_x$ ,  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ ,  $\hat{\sigma}_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$  and  $\hat{\gamma}_x = \frac{n \sum_{i=1}^n (x_i - \bar{x})^3}{(n-1)(n-2)\hat{\sigma}_x^3}$  into (A-1), (A-2) and (A-3), yields method of moments estimates of the stationary GEV model parameters.

**Nonstationary GEV case:** When sample estimates of the conditional moments  $\hat{\mu}_{x|w}$ ,  $\hat{\sigma}_{x|w}$  and  $\hat{\gamma}_{x|w}$ , defined below, are substituted into (A-1), (A-2) and (A-3), in place of the unconditional moment estimators  $\bar{x}$ ,  $\hat{\sigma}_x$  and  $\hat{\gamma}_x$ , estimates of the conditional GEV model parameters result which we term  $\hat{\xi}_{x|w}$ ,  $\hat{\alpha}_{x|w}$  and  $\hat{\kappa}_{x|w}$ . Thus, sample estimates of the conditional moments of  $x$  may be obtained by replacing true values with sample estimators in (10) leading to:

$$\hat{\mu}_{x|w} = \exp\left(\bar{y} + \hat{\beta}(w - \bar{w}) + \frac{\hat{\sigma}_y^2(1 - \hat{\rho}^2)}{2}\right) \quad (A-4a)$$

$$\hat{\sigma}_{x|w}^2 = \exp(2\bar{y} + 2\hat{\beta}(w - \bar{w}) + \hat{\sigma}_y^2)(\exp(\hat{\sigma}_y^2(1 - \hat{\rho}^2)) - 1) \quad (A-4b)$$

$$\hat{C}_{x|w} = \sqrt{\exp(\hat{\sigma}_y^2(1 - \hat{\rho}^2)) - 1} \quad (A-4c)$$

$$\hat{\gamma}_{x|w} = 3\hat{C}_{x|w} + \hat{C}_{x|w}^3 \quad (A-4d)$$

where the estimators  $\bar{y}$ ,  $\bar{w}$ ,  $\hat{\sigma}_y$ , and  $\hat{\rho}$ , are given in (4). Note that since both conditional moment estimators  $\hat{\mu}_{x|w}$  and  $\hat{\sigma}_{x|w}^2$  are functions of the explanatory variable  $w$ , the conditional GEV model parameters  $\hat{\alpha}_{x|w}$  and  $\hat{\xi}_{x|w}$  will also be functions of the explanatory variable  $w$ . Since the conditional skew estimator  $\hat{\gamma}_{x|w}$  does not depend on  $w$ , for the case considered here, the resulting conditional GEV shape parameter  $\hat{\kappa}_{x|w}$  will not depend on  $w$ . Substitution of the resulting expressions for the conditional GEV model parameters  $\hat{\alpha}_{x|w}$ ,  $\hat{\xi}_{x|w}$ , and  $\hat{\gamma}_{x|w}$  into the quantile function in (17) leads to the nonstationary GEV quantile function.

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