INGENIERÍA CIVIL
UNIVERSIDAD DE CHILE

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Control 3

Ingeniería Estructural CI3211
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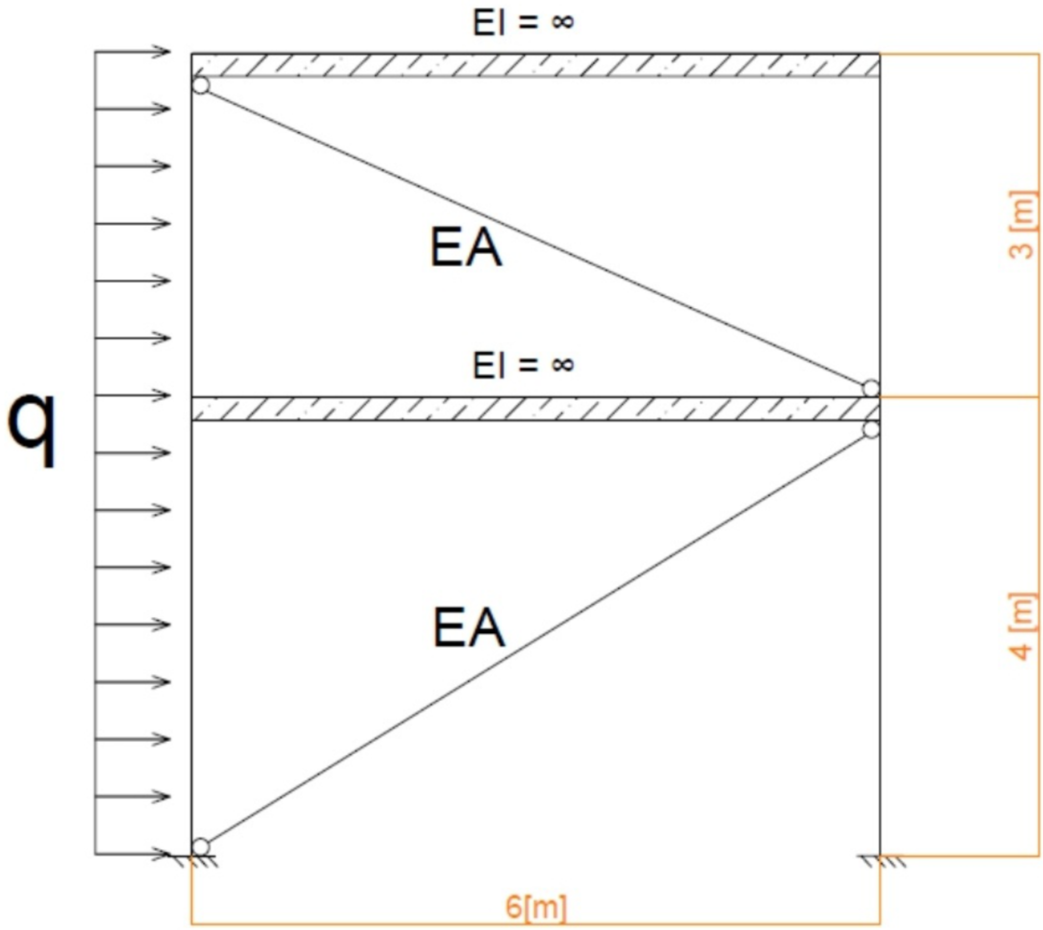
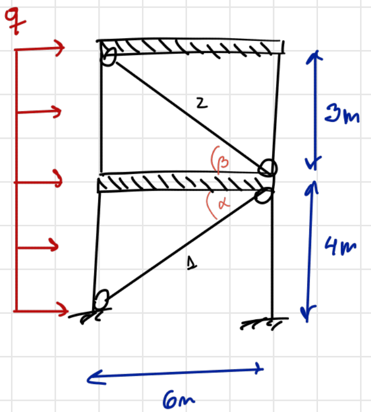
Para estudiar el efecto de un sismo en los edificios un método utilizado en Chile son los edificios de corte, en la figura 1 se representa la carga lateral que simula el actuar de un terremoto para una estructura de dos pisos. Utilizando el **método de rigidez**, determine los diagramas de esfuerzos internos de las **columnas y bielas.**

Considere:

- EA = 900 [tonf]
- EI = 1500 [tonf m2]
- q = 5 [tonf/m]
- Columnas axialmente rígidas



Control # 3 Ing Estructural



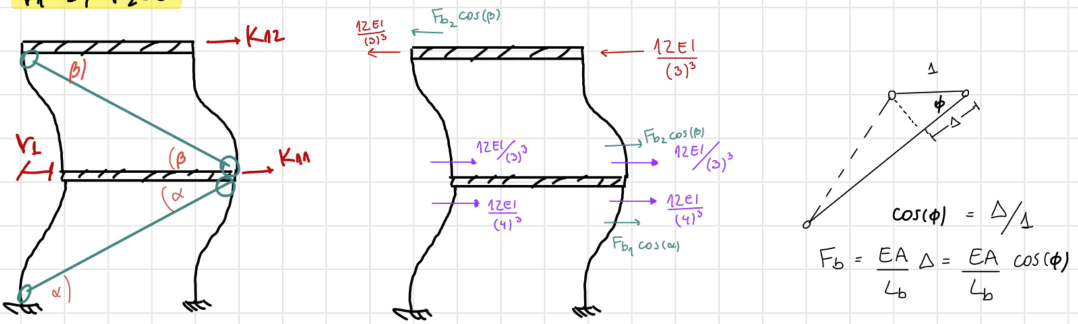
1) Calculo de GIC

- En los empotrados no se permite movimiento
- Las vigas podrían rotar, pero al ser las columnas axialmente rígidas se restringen
- Existe desplazamiento en cada piso, estos son los 2 GDL del sistema.

2) Calculo de matriz de rigidez

$\cos(\alpha) = \frac{6}{7,2}$ $\cos(\beta) = \frac{6}{6,7}$

$r_1 = 1, \quad r_2 = 0$



$\cos(\phi) = \Delta/1$

$F_b = \frac{EA}{L_b}\Delta = \frac{EA}{L_b}\cos(\phi)$

$$F_{b1} = \frac{EA}{L_{b1}}\cos(\alpha) = \frac{EA}{7,2}\cdot\frac{6}{7,2} = \frac{6\,EA}{(7,2)^2} \qquad F_{b2} = \frac{EA}{6,7}\,\frac{6}{6,7} = \frac{6\,EA}{(6,7)^2}$$

$$K_{11} = F_{b1}\cos(\alpha) + F_{b2}\cos(\beta) + \frac{12EI}{(3)^3}\times 2 + \frac{12EI}{(4)^3}\times 2$$

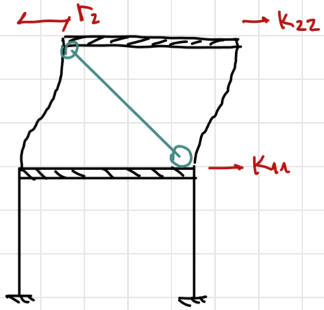
$$= \frac{6EA}{(7,2)^2}\left(\frac{6}{7,2}\right) + \frac{6EA}{(6,7)^2}\left(\frac{6}{6,7}\right) + \frac{24\,EI}{27} + \frac{24\,EI}{64}$$

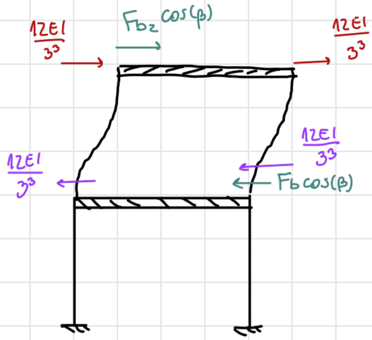
$$K_{12} = -\left(2\times\frac{12EI}{(3)^3} + F_{b2}\cos(\beta)\right) = -\frac{24}{27}EI - \frac{6EA}{(6,7)^2}\left(\frac{6}{6,7}\right)$$

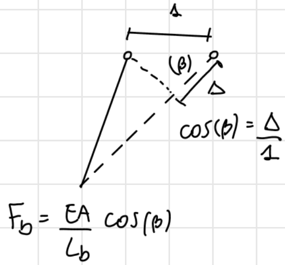
$K_{11} = 2090\,[\frac{tonf}{m}]$

$K_{12} = -1441\,[\frac{tonf}{m}]$

$r_1 = 0,\ r_2 = 1$







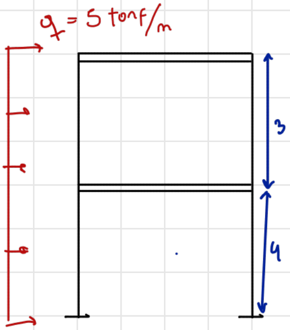
$$K_{22} = \frac{12EI}{3^3} \times 2 + F_{b2}\cos(\beta) = \frac{24EI}{27} + \frac{EA}{6{,}7}\left(\frac{6}{6{,}7}\right)^2$$

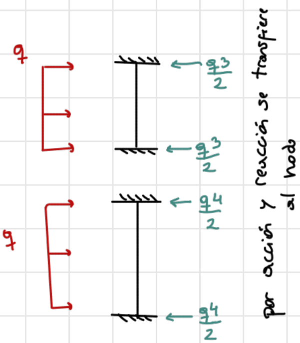
$$K_{21} = -\frac{12EI}{3^3} \times 2 - F_{b2}\cos\beta = -\frac{24EI}{27} - \frac{EA}{6{,}7}\left(\frac{6}{6{,}7}\right)^2$$

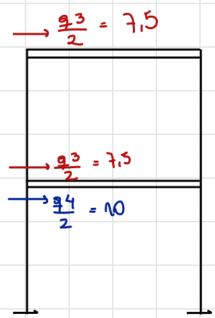
$K_{22} = 1441 \left[\frac{tonf}{m}\right]$

$K_{21} = -1441 \left[\frac{tonf}{m}\right]$

3) Vector de carga







$$\vec{F} = \begin{pmatrix} \frac{q4}{2} + \frac{q3}{2} \\ \frac{q3}{2} \end{pmatrix} = \begin{pmatrix} 17{,}5 \text{ tonf} \\ 7{,}5 \text{ tonf} \end{pmatrix}$$

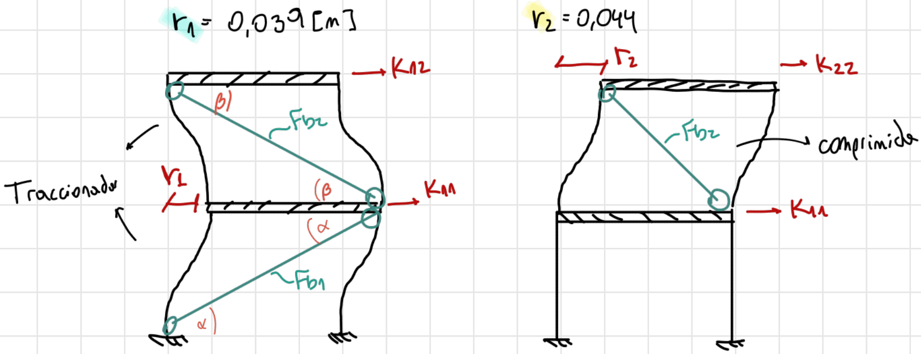
4) Aplicando metodo de rigidez

$$\vec{F} = [K]\,\vec{r}$$

$$\begin{pmatrix} 17{,}5 \text{ tonf} \\ 7{,}5 \text{ tonf} \end{pmatrix} = \begin{bmatrix} 2090 & -1441 \\ -1441 & 1441 \end{bmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \longrightarrow \begin{matrix} r_1 = 0{,}039\,[m] \\ r_2 = 0{,}044\,[m] \end{matrix}$$

5) Diagrama de esfuerzos

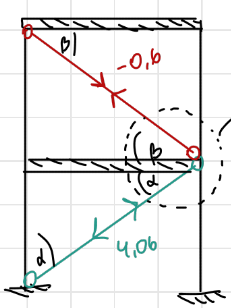
Axial Bielas



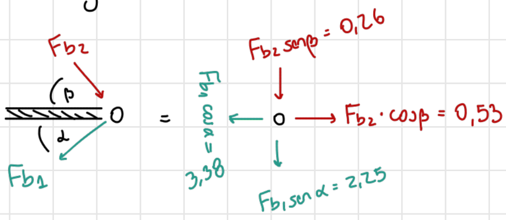
$$Fb_1 = \frac{EA}{(7{,}2)} \cdot \left(\frac{6}{7{,}2}\right) \cdot r_1 = 4{,}06\ [\text{tonf}]$$

$$Fb_2 = \frac{EA}{6{,}7}\left(\frac{6}{6{,}7}\right) \cdot r_1 = 4{,}7\ [\text{tonf}] \qquad Fb_2 = \frac{EA}{6{,}7}\left(\frac{6}{6{,}7}\right) \cdot r_2 = (-)\,5{,}3 \text{ tonf}$$

Por superposición



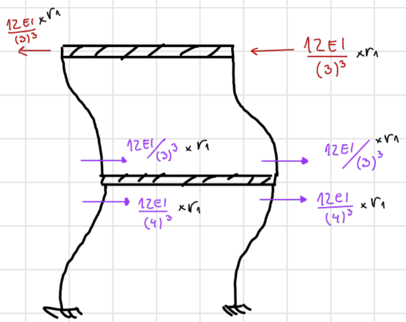
Un poco de geometria

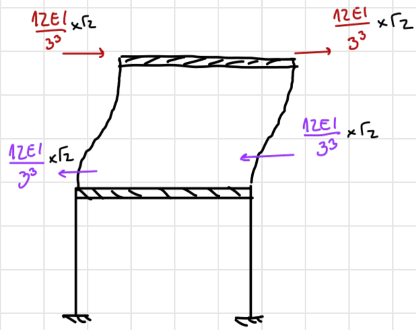


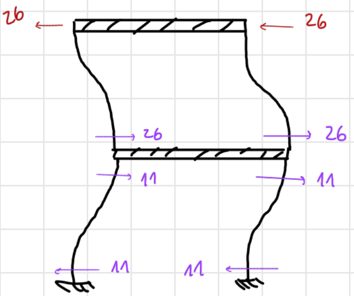
Corte [tonf]

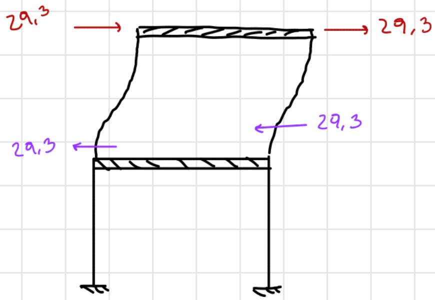
$r_1 = 0,039$ [m]

$r_2 = 0,044$ [m]

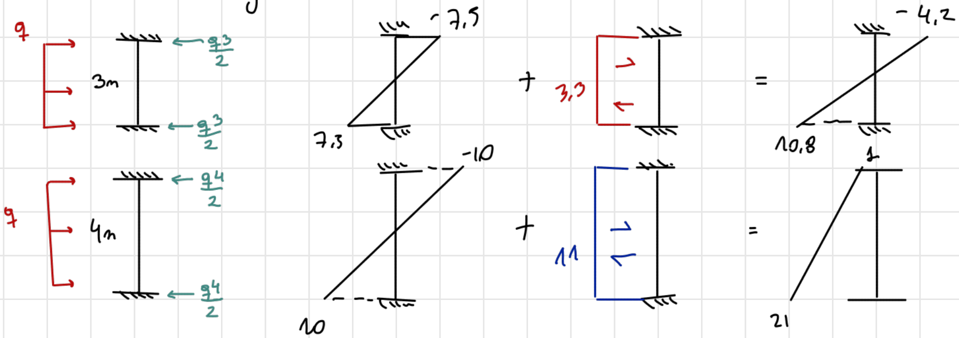




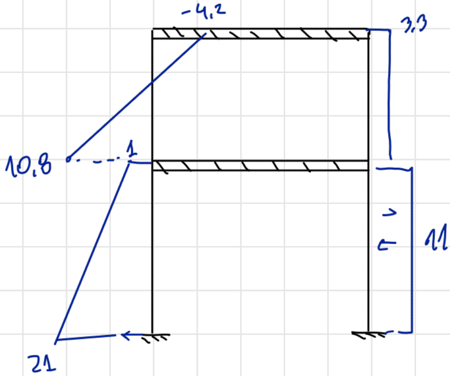




Sistema de carga



Por superposición el diagrama de Corte queda

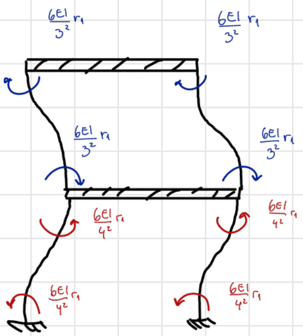


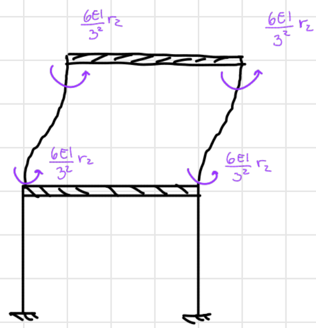
[tonf]

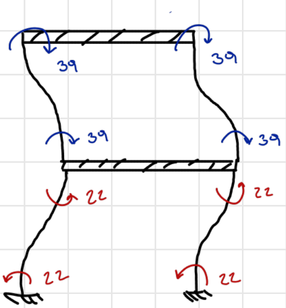
Flexión

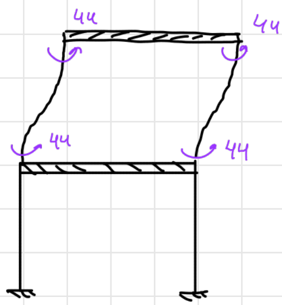
$r_1 = 0{,}039\,m$

$r_2 = 0{,}044\,m$

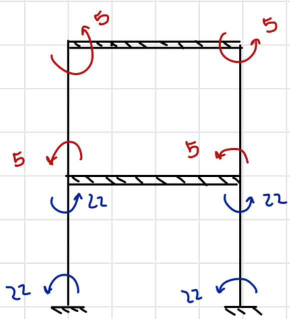






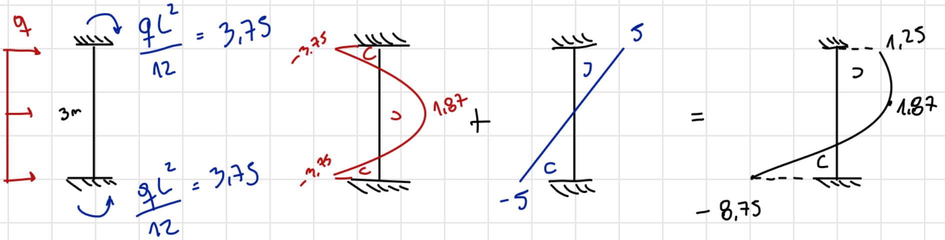


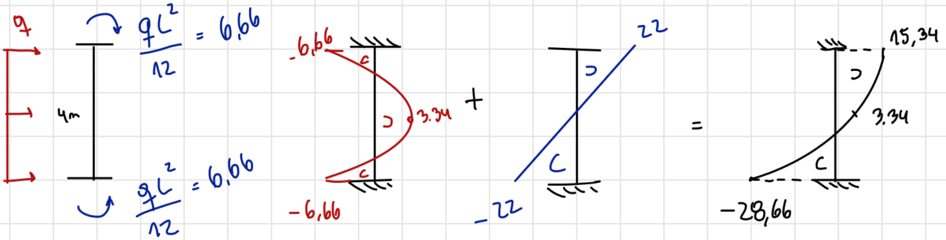
Superposición :



Nos falta considerar el sist de carga distribuida

Diagrama por carga dist





Quedando el diagrama como :

Flexión
[tonf·m]