Pauta P1 - C2 MA3403

Esta pregunta tiene una ponderación del 40%

$$X \sim Par(c,\alpha) \iff f(x) = \alpha c^{\alpha} x^{-\alpha-1} \qquad x > c \quad (c,\alpha>0)$$

a) 1pto $\quad X \sim Par(c,\alpha) \quad$ calcule $\mathbb{E}(X)$

$$\mathbb{E}(X) = \underbrace{\int_c^{\infty} x\cdot\alpha c^{\alpha} x^{-\alpha-1}\,dx}_{+0{,}4} = \alpha c^{\alpha}\int_c^{\infty} x^{-\alpha}\,dx = \underbrace{\frac{\alpha c^{\alpha}}{(1-\alpha)x^{\alpha-1}}\Bigg|_c^{\infty}}_{+0{,}2}$$

$$= \frac{\alpha c}{\alpha-1} \quad \text{si } \alpha>1 \quad \Rightarrow \mathbb{E}(X) = \begin{cases} \infty & \text{si } \alpha<1 \\ \frac{\alpha c}{\alpha-1} & \text{si } \alpha>1 \end{cases} \quad +0{,}4$$

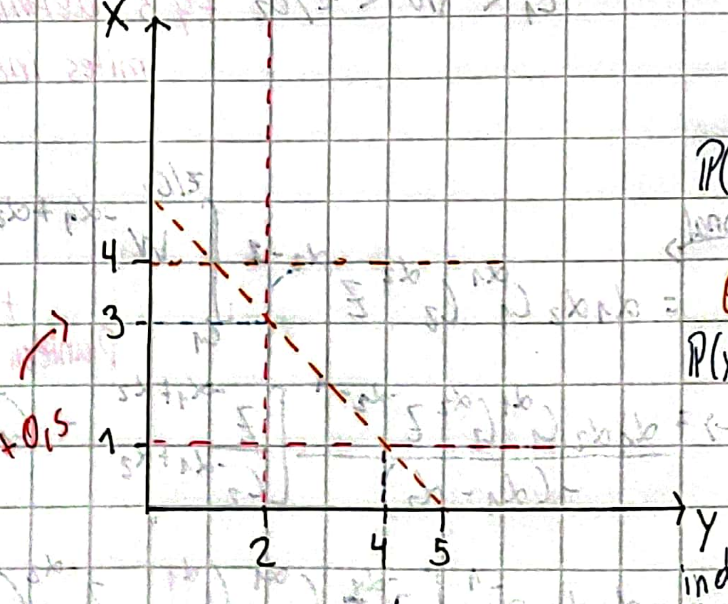
importante de identificar

b) 2 pts $\quad X\sim Par(1,\alpha_1),\ Y\sim Par(2,\alpha_2)$ ind.

- Plantee las integrales
- Grafique el dominio de integración

$$\mathbb{P}(X+Y>5 \mid X<4)$$

- $$\mathbb{P}(X+Y>5 \mid X<4) = \underbrace{\frac{\mathbb{P}(X+Y>5;\ X<4)}{\mathbb{P}(X<4)}}_{+0{,}5} = \frac{\mathbb{P}(5-Y<X<4)}{\mathbb{P}(X<4)}$$



Opción 1:

$$\mathbb{P}(X+Y>5;\ X<4) = \int_1^3\int_{5-x}^{\infty} f_{xy}\,dy\,dx + \int_3^4\int_2^{\infty} f_{xy}\,dy\,dx$$

Opción 2:

$$\mathbb{P}(X+Y>5;\ X<4) = \int_2^4\int_{5-y}^{4} f_{xy}\,dx\,dy + \int_4^{\infty}\int_1^4 f_{xy}\,dx\,dy \quad +0{,}4$$

$$\underbrace{\mathbb{P}(X<4) = \int_1^4 f_x\,dx}_{+0{,}3} \ \wedge\ \underbrace{f_{xy} \overset{\text{indep}}{=} f_x f_y}_{+0{,}3} = \alpha_1 x^{-\alpha_1-1}\,\alpha_2 2^{\alpha_2} y^{-\alpha_2-1}$$

c) 3pts $X \sim Par(c_1, \alpha_1)$, $Y \sim Par(c_2, \alpha_2)$
Usar TCV para determinar $f_Z(z)$ con $Z = X \cdot Y$.

$\bullet$ $Z = X \cdot Y$ $\quad W = X$ $\Rightarrow$ $X = W$, $\quad Y = Z/W$ +0,4

$$\Rightarrow |J| = \begin{vmatrix} 1 & 0 \\ \frac{-z}{w^2} & \frac{1}{w} \end{vmatrix} = 1/w \quad \Rightarrow f_{WZ} = f_{XY}(w, z/w) \cdot 1/w$$

+0,4 $\quad$ ~~+0,4~~ $\quad$ +0,2

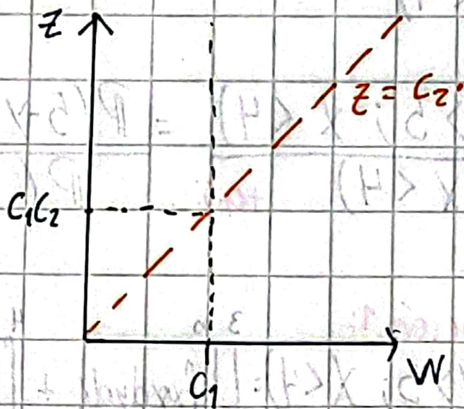
Independendencia

$$\Rightarrow f_X(w)\, f_Y(z/w) \cdot \frac{1}{w} = \alpha_1 c_1^{\alpha_1} w^{-\alpha_1-1} \cdot \alpha_2 c_2^{\alpha_2} \frac{z^{-\alpha_2-1}}{w^{-\alpha_2}}$$

+0,5 Indepencia

$$= \alpha_1 \alpha_2 c_1^{\alpha_1} c_2^{\alpha_2} \frac{z^{-\alpha_2-1}}{w^{-\alpha_2+\alpha_1+1}}$$ +0,2 expresión

Luego $\quad c_1 < x < \infty$, $\quad c_2 < y < \infty$ $\quad \Rightarrow \quad c_1 < w < \infty$, $\quad c_2 w < z < \infty$ +0,5 planter limites del cambio



$\therefore$ $c_1 c_2 < z < \infty$
$c_1 < w < z/c_2$ +0,3 definir limites integral

$$\Rightarrow f_Z = \int_{c_1}^{z/c_2} f_{WZ}\, dw \xrightarrow{\text{opcional}} = \alpha_1 \alpha_2 c_1^{\alpha_1} c_2^{\alpha_2} z^{-\alpha_2-1} \int_{c_1}^{z/c_2} w^{-\alpha_1+\alpha_2-1}$$

si $c_1 c_2 < z$ $\quad$ +0,5 Plantear integral

$$\hookrightarrow = \frac{\alpha_1 \alpha_2 c_1^{\alpha_1} c_2^{\alpha_2} z^{-\alpha_2-1}}{-(\alpha_1-\alpha_2)} \left[\frac{z^{-\alpha_1+\alpha_2}}{c_2^{-\alpha_1+\alpha_2}} - c_1^{-\alpha_1+\alpha_2} \right]$$

$$= \frac{-\alpha_1 \alpha_2 z^{-1}}{\alpha_1 - \alpha_2} \left[z^{-\alpha_1} c_2^{\alpha_1} c_1^{\alpha_1} - z^{-\alpha_2} c_2^{\alpha_2} c_1^{\alpha_2} \right]$$

//