Auxiliar extra 1 Pablo Martiniello R.

P2) a) $\frac{dy}{dx} + 2xy^2 = 0$

$\Leftrightarrow \frac{dy}{dx} = -2xy^2 \quad / \text{ si } y \neq 0$ obs: $y \equiv 0$ también es solución de la EDO

$\frac{1}{y^2}\frac{dy}{dx} = -2x \quad / \int dx$

$\int \frac{1}{y^2}\frac{dy}{dx}\,dx = \int -2x\,dx \quad /$ cv: $z = y$, $dz = \frac{dy}{dx}dx$

$\int \frac{1}{z^2}\,dz = -x^2 + c_1$

$-\frac{1}{z} = -x^2 + c_1 \quad / \ z = y, \ c_2 = -c_1$

$\frac{1}{y} = x^2 + c_2$

$y = \frac{1}{x^2 + c_2}$

y está bien definido en $x \in \mathbb{R}$ si $c_2 > 0$ y en $x \in (-\infty, -c_2) \cup (-c_2, \infty)$ en el caso contrario.

b) $dx + e^{3x}dy = 0$

$dy = -e^{-3x}dx \quad / \int$

$y = \frac{e^{-3x}}{3} + C$

y está bien definido $\forall x \in \mathbb{R}$

c) $y' = y(1-y) \quad / \text{ si } y \neq 0, \ y \neq 1$ obs $y \equiv 0$, $y \equiv 1$ son solución de la EDO

$\frac{dy}{y(1-y)} = dx \quad / \int$

$\int \frac{1}{y(1-y)}\,dy = \int dx$

$$\int \frac{1}{y} + \frac{1}{1-y}\, dy = x + c$$

$$\ln(|y|) - \ln(|1-y|) = x + c$$

$$\ln\left(\left|\frac{y}{1-y}\right|\right) = x + c \quad / \, e^{()}$$

$$\left|\frac{y}{1-y}\right| = k_1 e^x \quad / \text{ con } k_1 = e^c$$

$$\frac{y}{1-y} = k_2 e^x \quad / \text{ con } k_2 \in \mathbb{R}$$

$$y = \frac{k_2 e^x}{k_2 e^x + 1}$$

Si $k_2 \geq 0$ y está bien definido $\forall x \in \mathbb{R}$

$\boxed{P3}$ a) $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$ / notamos que el lado derecho es $\dfrac{N(x,y)}{M(x,y)}$ con N, M funciones homogéneas de grado 1.

CV $y = ux \Rightarrow y' = u'x + u$

$$u'x + u = \frac{ux - x}{ux + x}$$

$$u'x + u = \frac{u-1}{u+1}$$

$$u'x = \frac{u - 1 - (u^2 + u)}{u+1}$$

$$u'x = \frac{-(u^2+1)}{u+1}$$

$$\frac{u+1}{u^2+1}\frac{du}{dx} = -\frac{1}{x}$$

$$\frac{u+1}{u^2+1}\, du = -\frac{1}{x}\, dx \quad / \int$$

$$\int \frac{u}{u^2+1}\, du + \int \frac{1}{u^2+1}\, du = -\int \frac{1}{x}\, dx$$

$$\frac{1}{2}\ln(u^2+1) + \operatorname{arctg}(u) = -\ln(x) + C \iff \frac{1}{2}\ln\left(\left(\frac{y}{x}\right)^2 + 1\right) + \operatorname{arctg}\left(\frac{y}{x}\right) = -\ln(x) + C$$

b) $t^2 y' + y^2 = t y \quad / \cdot \frac{1}{t^2} \qquad \text{si } t \neq 0$

$y' + \frac{y^2}{t^2} = \frac{y}{t} \quad /$ ec de Bernoulli orden 2
C.V. $u = y^{-1} \qquad y' = \frac{u'}{-u^2}$

$-\frac{u'}{u^2} + \frac{1}{u^2 t^2} = \frac{1}{ut} \quad / \cdot u^2$

$-u' + \frac{1}{t^2} = \frac{u}{t}$

$u' + \frac{u}{t} = \frac{1}{t^2} \quad /$ f.i. $e^{\int \frac{1}{t} dt} = t$

$tu' + u = \frac{1}{t}$

$(tu)' = \frac{1}{t} \quad / \int dt$

$t u = \ln(t) + c$

$u = \frac{\ln(t) + c}{t}$

$y = \frac{t}{\ln(t) + c}$

c) $y' - y = e^x y^2 \quad /$ Ec de Bernoulli orden 2
C.V. $u = y^{-1} \quad y' = \frac{u'}{-u^2}$

$\frac{u'}{u^2} + \frac{1}{u} = -\frac{e^x}{u^2} \quad / \cdot u^2$

$u' + u = -e^x \quad /$ f.i. e^x

$(ue^x)' = -e^x \cdot e^x \quad / \int$

$u e^x = -\int e^{2x} dx$

$u e^x = -\frac{e^{2x}}{2} + c$

$u = -\frac{e^x}{2} + c e^{-x}$

$y = \frac{1}{-e^x/2 + c e^{-x}}$

d) $y\,dx = 2(x+y)\,dy$ / es homogénea de orden 1
c.v. $y = ux$, $y' = u'x + u$

$$\frac{y}{2(x+y)} = \frac{dy}{dx} \iff \frac{ux}{2x(1+u)} = u'x + u$$

$$\frac{u - 2u(1+u)}{2(1+u)} = u'x$$

$$-\frac{(u+2u^2)}{2(1+u)} = x\frac{du}{dx}$$

$$-\frac{1}{x}\,dx = \frac{2(1+u)}{u(1+2u)}\,du \Big/ \int$$

$$\int \frac{1}{x}\,dy = 2\left(\int \frac{1+u}{u(1+2u)}\,du\right)$$

$$-\int \frac{1}{x}\,dx = 2\left(\int \frac{1}{u} - \frac{1}{1+2u}\,du\right)$$

$$-\ln(x) + C = 2\left(\ln(u) - \frac{1}{2}\ln(1+2u)\right)$$

$$-\ln(x) + C = 2\left(\ln\left(\frac{y}{x}\right) - \frac{1}{2}\ln\left(1 + \frac{2y}{x}\right)\right)$$ / se podría simplificar más

e) Como la ecuación es una ec. de Ricatti y tenemos la solución $y_1 = \frac{2}{x}$
usamos el c.v. $y = y_1 + u$ $\quad y' = u' - \frac{2}{x^2}$

$$-\frac{2}{x^2} + u' = -\frac{4}{x^2} - \frac{1}{x}\left(\frac{2}{x} + u\right) + \left(\frac{2}{x} + u\right)^2$$

$$\cancel{-\frac{2}{x^2}} + u' = \cancel{-\frac{4}{x^2}} \cancel{-\frac{2}{x^2}} - \frac{u}{x} + \cancel{\frac{4}{x^2}} + \frac{4u}{x} + u^2$$

$u' = \frac{3u}{x} + u^2$ / ec de Bernoulli de orden 2
c.v. $w = u^{-1}$ $\quad u' = \frac{w'}{-w^2}$

$$-\frac{w'}{w^2} = \frac{3}{xw} + \frac{1}{w^2} \Big/ \cdot -w^2$$

$$w' = -\frac{3w}{x} - 1$$

$w' + \frac{3w}{x} = -1$ / f.i. $e^{3\int \frac{1}{x}dx} = x^3$

$$w'x^3 + 3wx^2 = -x^3$$

$$(wx^3)' = -x^3 \quad / \int$$

$$wx^3 = -\frac{x^4}{4} + c$$

$$w = -\frac{x}{4} + \frac{c}{x^3}$$

$$\frac{1}{u} = \frac{c - x^4}{4x^3}$$

$$u = \frac{4x^3}{c - x^4}$$

$$y + \frac{2}{x} = \frac{4x^3}{c - x^4}$$

$$y = \frac{4x^3}{c - x^4} - \frac{2}{x}$$

f) La ecuación es también una ec. de Bernoulli.

$$u = p^{-1} \qquad p' = -\frac{u'}{u^2}$$

$$-\frac{u'}{u^2} = \frac{a}{u} - \frac{b}{u^2} \quad / \cdot -u^2$$

$$u' = -au + b$$

$$u' + au = b \quad / \text{ f.i } e^{\int a\,dt} = e^{at}$$

$$u'e^{at} + ae^{at}u = be^{at}$$

$$(ue^{at})' = be^{at} \quad / \int dt$$

$$ue^{at} = b\int e^{at}\,dt$$

$$ue^{at} = \frac{b}{a}e^{at} + c \quad / \cdot \frac{1}{e^{at}}$$

$$\frac{1}{p} = u = \frac{b}{a} + ce^{-at}$$

$$p = \frac{a}{b + cae^{-at}}$$