Auxiliar Masivo

P1

(a). $f(x) = x^4 + 3x^2 - 6$

$f'(x) = 4x^3 + 6x$ ✓

$$S \to ()' \quad -C \quad C \quad -S$$

(b). $f(x) = (a+x)\cdot\sqrt{a-x}$

$f'(x) = (a+x)'\cdot\sqrt{a-x} + (a+x)\cdot(\sqrt{a-x})'$

$= 1\cdot\sqrt{a-x} + (a+x)\cdot\dfrac{1}{2\sqrt{a-x}}\cdot -1$

$= \sqrt{a-x} - \dfrac{(a+x)}{2\sqrt{a-x}}$ ✓

(c). $f(x) = \tan(x)$

$f'(x) = \dfrac{sen(x)}{\cos(x)}'$

$= \dfrac{[sen(x)]'\cdot\cos(x) - sen(x)\cdot[\cos(x)]'}{\cos^2(x)}$

$= \dfrac{\cos(x)\cdot\cos(x) - sen(x)\cdot -sen(x)}{\cos^2(x)}$

$= \dfrac{\cos^2(x) + sen^2(x)}{\cos^2(x)}$

$= \dfrac{1}{\cos^2(x)} = \sec^2(x)$ ✓

(d) $f(x) = sen(x) + \dfrac{1}{2x} - \dfrac{1}{3x^2}$

$f'(x) = \cos(x) + [(2x)^{-1}]' - [(3x^2)^{-1}]'$

$= \cos(x) + -2x^{-2} - \frac{2}{3}\cdot\frac{1}{x^3}$

$= \cos(x) - \dfrac{1}{2x^2} + \dfrac{2}{3x^3}$ ✓

(e). $f(x) = \dfrac{x^2}{x - sen(x)}$

$f'(x) = \dfrac{(x^2)'\cdot(x - sen(x)) - x^2\cdot(x - sen(x))'}{(x - sen(x))^2}$

$= \dfrac{2x\cdot(x - sen(x)) - x^2\cdot(1 - \cos(x))}{(x - sen(x))^2}$ ✓

(f). $f(x) = \dfrac{2x^5 + 4x}{\cos(x)}$

$f'(x) = \dfrac{(2x^5+4x)'\cdot\cos(x) - (2x^5+4x)\cdot[\cos(x)]'}{\cos^2(x)}$

$= \dfrac{(10x^4+4)\cdot\cos(x) - (2x^5+4x)\cdot -sen(x)}{\cos^2(x)}$ ✓

(g) $f(x)=\dfrac{1}{\sqrt{x}}+\sqrt{x}+\tan(x)+\dfrac{1}{\tan(x)}$

$$f'(x)=\frac{\cancel{1'\cdot\sqrt{x}}+1\cdot\frac{1}{2\sqrt{x}}}{x}+\frac{1}{2\sqrt{x}}+\sec^2(x)+\frac{\cancel{1'\cdot\tan(x)}-1\cdot(\tan(x))'}{\tan^2(x)}$$

$$=\frac{1}{2x\sqrt{x}}+\frac{1}{2\sqrt{x}}+\sec^2(x)+\frac{-\sec^2(x)}{\tan^2(x)}\ ✓$$

(h). $f(x)=\dfrac{x^2\ln(x)}{\cos(x)}$

$$f'(x)=\frac{(x^2\ln(x))'\cdot\cos(x)-(x^2\ln(x))\cdot[\cos(x)]'}{\cos^2(x)}$$

$$=\frac{2\cancel{x}\cdot\frac{1}{\cancel{x}}\cdot\cos(x)-(x^2\ln(x))\cdot -\operatorname{sen}(x)}{\cos^2(x)}$$

$$=\frac{2\cos(x)+x^2\ln(x)\operatorname{sen}(x)}{\cos^2(x)}$$

(i). $f(x)=\dfrac{e^x+\operatorname{sen}(x)}{xe^x}$

$$f'(x)=\frac{(e^x+\operatorname{sen}(x))'\cdot xe^x-(e^x+\operatorname{sen}(x))\cdot(xe^x)'}{(xe^x)^2}$$

$$=\frac{(e^x+\cos(x))\cdot xe^x-(e^x+\operatorname{sen}(x))\cdot(x\cdot e^x)'}{(xe^x)^2}$$

$$=\frac{(e^x+\cos(x))\cdot xe^x-(e^x+\operatorname{sen}(x))\cdot e^x\cdot x}{(xe^x)^2}$$

(j). $f(x)=\dfrac{e^x\cdot\cos(x)}{1-\operatorname{sen}(x)}$

$$f'(x)=\frac{(e^x\cdot\cos(x))'\cdot(1-\operatorname{sen}(x))-(e^x-\cos(x))\cdot(1-\operatorname{sen}(x))'}{(1-\operatorname{sen}(x))^2}$$

$$=\frac{(e^x\cos(x)-e^x\operatorname{sen}(x))\cdot(1-\operatorname{sen}(x))-(e^x-\cos(x))\cdot(-\cos(x))}{(1-\operatorname{sen}(x))^2}\ ✓$$

P2

(a). $f(x)=e^{3x^2}$

$$f'(x)=e^{3x^2}\cdot(3x^2)'$$
$$=e^{3x^2}\cdot 3\cdot 2x$$

(b). $f(x)=(x^2+4x+6)^4$

$$=4(x^2+4x+6)^3\cdot(x^2+4x+6)'$$
$$=4(x^2+4x+6)^3\cdot(2x+4)$$

(c). $f(x)=\operatorname{sen}^2(x)$

$$f'(x)=(\operatorname{sen}(x))^2$$
$$=2\operatorname{sen}(x)\cdot\cos(x)$$
$$=\operatorname{sen}(2x),$$

(d). $f(x) = \sqrt[3]{3x^3+4x} = (3x^3+4x)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3} \cdot (3x^3+4x)^{-\frac{2}{3}} \cdot (3x^3+4x)'$

$= \frac{1}{3} \cdot (3x^3+4x)^{-\frac{2}{3}} \cdot (9x^2+4)$

(e). $f(x) = \sqrt{1+\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{1+\sqrt{x}}} \cdot (1+\sqrt{x})'$

$= \frac{1}{2\sqrt{1+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}}$

(f). $f(x) = sen(sen(x^2+1))$

$f'(x) = cos(sen(x^2+1) \cdot [sen(x^2+1)]'$

$= cos(sen(x^2+1)) \cdot cos(x^2+1) \cdot (x^2+1)'$

$= cos(sen(x^2+1)) \cdot cos(x^2+1) \cdot 2x$

(g). $f(x) = \tan(\frac{1}{x})$

$f'(x) = sec^2(\frac{1}{x}) \cdot (\frac{1}{x})'$

$= sec^2(\frac{1}{x}) \cdot -\frac{1}{x^2}$

(h). $f(x) = e^{\tan(x^2)+x^2} = exp(\tan(x^2)+x^2)$

$f'(x) = exp(\tan(x^2)+x^2) \cdot [(\tan(x^2)+x^2)]$

$= exp(\tan(x^2)+x^2) \cdot [(-sec(x^2) \cdot 2x) + 2x]$ //

(i). $f(x) = \ln(x+\sqrt{1+x^2})$

$f'(x) = \frac{1}{(x+\sqrt{1+x^2})} \cdot (x+\sqrt{1+x^2})'$

$= \frac{1}{(x+\sqrt{1+x^2})} \cdot \left[1 + \frac{1}{2\sqrt{1+x^2}} \cdot 2x\right]$

(j). $f(x) = \sqrt[3]{\frac{x^2+3x+1}{\ln(3x)}} = \left(\frac{x^2+3x+1}{\ln(3x)}\right)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}\left(\frac{x^2+3x+1}{\ln(3x)}\right)^{-\frac{2}{3}} \cdot \left(\frac{x^2+3x+1}{\ln(3x)}\right)'$

$= \frac{1}{3}\left(\frac{x^2+3x+1}{\ln(3x)}\right)^{-\frac{2}{3}} \cdot \left(\frac{(x^2+3x+1)' \cdot \ln(3x) - (x^2+3x+1) \cdot (\ln(3x))'}{(\ln(3x))^2}\right)$

$= \frac{1}{3}\left(\frac{x^2+3x+1}{\ln(3x)}\right)^{-\frac{2}{3}} \cdot \left(\frac{(2x+3) \cdot \ln(3x) - (x^2+3x+1) \cdot \frac{1}{\cancel{3}x} \cdot \cancel{3}}{(\ln(3x))^2}\right)$ //

P3

$(e^x)' = e^x$
$(e^{-x})' = -1 \cdot e^{-x}$

(a) Derive cosh(x)

$$\cosh'(x) = \left(\frac{e^x + e^{-x}}{2}\right)' = \frac{1}{2} \cdot (e^x - e^{x}) = \operatorname{senh}(x)$$

(b) Derive sinh(x)

$$\sinh'(x) = \left(\frac{e^x - e^{-x}}{2}\right)' = \frac{1}{2} \cdot (e^x + e^{-x}) = \cos(x)$$

(c) Usando lo anterior, deduzca tanh(x)

$$\tanh'(x) = \left(\frac{\operatorname{senh}(x)}{\cosh(x)}\right)'$$
$$= \frac{(\operatorname{senh}(x))' \cdot \cosh(x) - \operatorname{senh}(x) \cdot (\cosh(x))'}{(\cosh(x))^2}$$
$$= \frac{\cosh(x) \cdot \cosh(x) - \operatorname{senh}(x) \cdot \operatorname{senh}(x)}{(\cosh(x))^2}$$
$$= \frac{\cosh^2(x) - \operatorname{senh}(x)}{(\cosh(x))^2}$$
$$= \frac{1}{(\cosh(x))^2} = (\operatorname{sech}(x))^2$$

P4

(a) $f(x)=\sqrt{x}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$=\lim_{h\to 0}\frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}_{//}$$

(b). $h(z)=z^2+2$

$$h'(z)=\lim_{h\to 0}\frac{f(z+h)-f(z)}{h}$$

$$=\lim_{h\to 0}\frac{(z+h)^2+2-z^2-2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{z^2}+2zh+h^2+\cancel{2}-\cancel{z^2}-\cancel{2}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2z+h)}{\cancel{h}}=2z_{//}$$

(c). $g(x)=e^x$

$$g'(x)=\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$=\lim_{h\to 0}\frac{e^{x+h}\cdot e^x}{h}$$

$$=\lim_{h\to 0}\frac{e^x\cdot e^h-e^x}{h}$$

$$=\lim_{h\to 0}\frac{e^x(e^h-1)}{h}$$

$$=e^x\cdot\lim_{h\to 0}\underbrace{\frac{(e^h-1)}{h}}_{\to 1}=e^x$$

(d). $u(x)=\ln(x)$

$$u'(x)=\lim_{h\to 0}\frac{u(x+h)-u(x)}{h}$$

$$=\lim_{h\to 0}\frac{\ln(x+h)-\ln(x)}{h}$$

$$=\lim_{h\to 0}\underbrace{\frac{\ln\left(\frac{x+h}{x}\right)}{\frac{h}{x}}}_{\to 1}\cdot\frac{1}{x}=\frac{1}{x}_{//}$$

$$\frac{x+h}{x}-\frac{x}{x}=\frac{h}{x}$$

P6

(a). $f(x) = \dfrac{x^2 - \tan^2(x)}{2x - \sec(x)}$

$$f'(x) = \frac{(x^2 - \tan^2(x))' \cdot (2x - \sec(x)) - (x^2 - \tan^2(x)) \cdot (2x - \sec(x))'}{(2x - \sec(x))^2}$$

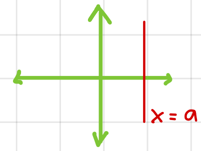
$$= \frac{[2x - (2\tan(x) \cdot \sec^2(x)] \cdot (2x - \sec(x)) - (x^2 - \tan^2(x)) \cdot (2 + \frac{1}{\text{sen}(x)})}{(2x - \sec(x))^2}$$

(b). $\ell(x) = \sin(x^{\cos(x)}) + \cos(x^{\sin(x)})$

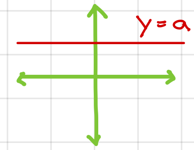
$= \sin(\exp(\cos(x) \cdot \ln(x)) + \cos(\exp(\sin(x) \cdot \ln(x))$

Recuerdo Asíntotas

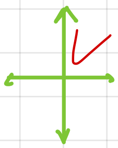
→ **A.V** $x = a$ es a.v $\iff \lim_{x \to a^+} f(x) = \pm\infty \lor \lim_{x \to a^-} f(x) = \pm\infty$



→ **A.H** $y = a$ es a.h $\iff \lim_{x \to +\infty} f(x) = a \lor \lim_{x \to -\infty} f(x) = a$

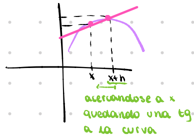


→ **A.O.** $y = mx + n$ es a.o $\iff \lim_{x \to +\infty} f(x) - (mx + n) = 0 \lor \lim_{x \to -\infty} f(x) - (mx + n) = 0$



AUX Preparacion examen

$$f'(x) = \lim_{h \to 0} \underbrace{\frac{f(x+h) - f(x)}{x + h - x}}$$



representa la pendiente
de la recta tg a la curva f en x

a) $f(x) = x^4 + 3x^2 - 6$

$$(x^4)' + (3x^2)' - (6)'$$
$$= 4x^3 + 6x - 0$$
$$= 4x^3 + 6x$$ //

b) $$\left((a+x)\sqrt{a-x}\right)' = \underbrace{(a+x)'}_{1}\sqrt{a-x} + (a+x)\underbrace{\left(\sqrt{a-x}\right)'}_{\left((a-x)^{1/2}\right)'}$$

$$\frac{1}{2}(a-x)^{-1/2} \cdot \underbrace{(a-x)'}_{-1}$$

c) $$(\tan(x))' = \left(\frac{\text{sen}(x)}{\cos(x)}\right)'$$

$$= \frac{(\text{sen}(x))' \cdot \cos(x) - \text{sen}(x) \cdot (\cos(x))'}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \text{sen}^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

id. pitagorica

d) $$\left(\text{sen}(x) + \frac{1}{2x} - \frac{1}{3x^2}\right)'$$

$$= (\text{sen}(x))' + \left(\frac{1}{2x}\right)' - \left(\frac{1}{3x^2}\right)'$$

$$= \cos(x) + \frac{1}{2}\cdot(x^{-1})' - \frac{1}{3}\left(\cancel{\frac{1}{x^2}}\right)'(x^{-2})'$$

$$= \cos(x) - \frac{1}{2x^2} + \frac{2}{3x^3}$$ //

e) $$\left(\frac{x^2}{x - \text{sen}(x)}\right)' = \frac{(x^2)'(x - \text{sen}(x)) - x^2 \cdot (x - \text{sen}(x))'}{(x - \text{sen}(x))^2}$$

$$= \frac{2x \cdot (x - \text{sen}(x)) - x^2 \cdot (1 - \cos(x))}{(x - \text{sen}(x))^2}$$

h) $\left(\dfrac{x^2 \ln(x)}{\cos(x)}\right)'$

$-\text{sen}(x)$

$$= \frac{(x^2\ln(x))' \cdot \cos(x) - x^2\ln(x)\cdot(\cancel{\cos}(x))'}{(\cos(x))^2}$$

$$= (x^2\ln(x))' = (x^2)'\cdot \ln(x) + x^2(\ln(x))'$$
$$= 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x}$$

g) $\left(\dfrac{1}{\sqrt{x}} + \sqrt{x} + \tan(x) + \dfrac{1}{\tan(x)}\right)'$

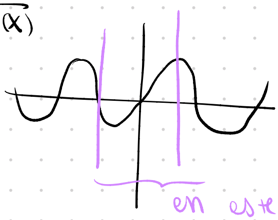
$$= \left(\frac{1}{\sqrt{x}}\right)' + \left(\sqrt{x}\right)' + \left(\tan(x)\right)' + \left(\frac{1}{\tan(x)}\right)'$$

$$= (x^{-1/2})' + (x^{1/2})' + \sec^2(x) - \operatorname{cosec}^2(x)$$

$$= -\frac{1}{2}x^{-3/2} + \frac{1}{2}x^{-1/2} + \sec^2(x) - \operatorname{cosec}^2(x)$$

$\text{sen}^{-1}(x) \neq \text{arcsen}(x)$

$= \dfrac{1}{\text{sen}(x)}$



Por sen fuera del tramo no es biy.

en este tramo es iny. y sobrey. ¡tiene inversa!

$x = f(f^{-1}(x))$

como quiero $(\text{arcsen}(x))'$

* $x = \text{sen}(\text{arcsen}(x)) \quad / ()'$

$1 = \cos(\text{arcsen}(x)) \cdot (\text{arcsen}(x))'$

$$\frac{1}{\cos(\text{arcsen}(x))} = (\text{arcsen}(x))'$$

sabemos $\text{sen}^2(x) + \cos^2(x) = 1$

$\cos(\alpha) = \sqrt{1-\text{sen}^2(x)}$

$\alpha = \text{arcsen}(x)$

$$= \frac{1}{\sqrt{1-\text{sen}^2(\text{arcsen}(x))}} = \frac{1}{\sqrt{1-x^2}}$$

$(\arctan(x))' = \dfrac{1}{1+x^2}$

$x = f(f^{-1}(x))$
$x = \tan(\operatorname{arctg}(x))$

$x = tg(\operatorname{arctg}(x)) \quad / ()'$

$1 = \sec^2(\operatorname{arctg}(x)) \cdot (\operatorname{arctg}(x))'$

$\dfrac{1}{\sec^2(\operatorname{arctg}(x))} = (\operatorname{arctg}(x))'$

sabemos $1 + tg^2(\alpha) = \sec^2(\alpha)$
$\alpha = \operatorname{arctg}(x)$

$\dfrac{1}{1+tg^2(\operatorname{arctg}(x))} = (\operatorname{arctg}(x))'$

$\dfrac{1}{1+x^2} = (\operatorname{arctg}(x))'$

- $\operatorname{sen} h(x) = \dfrac{e^x - e^{-x}}{2}$

$*\ \cos h^2(x) - \operatorname{sen} h^2(x) = 1$

Veamos $(\operatorname{sen} h^{-1}(x))'$ → inversa derivada

$(-1) \to \dfrac{1}{\operatorname{sen} h(x)}$

↳ inverso

$(\operatorname{sen} h^{-1}(x))'$

$x = f(f^{-1}(x))$

$x = \operatorname{sen} h(\operatorname{sen}h^{-1}(x)) \quad / ()'$

$1 = \cos h(\operatorname{sen}h^{-1}(x)) \cdot (\operatorname{sen}h^{-1}(x))'$

$\dfrac{1}{\cos h(\operatorname{sen}h^{-1}(x))} = (\operatorname{sen}h^{-1}(x))'$

$*$ senh (x) biyectiva ⇒ tiene inversa.
$\alpha = \operatorname{sen} h^{-1}(x)$

$\dfrac{1}{\sqrt{1+\operatorname{senh}^2(\operatorname{senh}^{-1}(x))}} = \dfrac{1}{\sqrt{1+x^2}}$

P4

a) $f(x) = \sqrt{x}$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \rightarrow \lim_{h \to 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \lim_{h \to 0} \frac{1}{2\sqrt{x}}$$

continuidad de la función
f es continua en $x = a$ si

$$\lim_{x \to a} f(x) = f(a)$$

c) $g(x) = e^x$

$$g'(x) = \lim_{h \to 0} \frac{e^x e^h - e^x}{h} = \lim_{h \to 0} \frac{e^x (e^h - 1)}{h}$$

$$= e^x \lim_{h \to 0} \cancel{\frac{e^h - 1}{h}}^{\,1} = e^x \cdot 1 = e^x$$

d) $u(x) = \ln(x)$

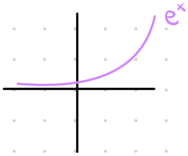
$$u'(x) = \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h}$$

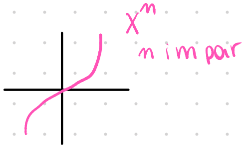
$$\lim_{h \to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h}$$

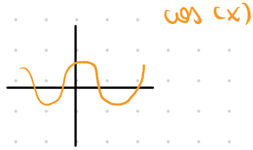
$$\lim_{h \to 0} \frac{\ln\left(1 + \frac{h}{x}\right)}{h} \begin{array}{l} / \cdot \frac{1}{x} \\ / \cdot \frac{1}{x} \end{array}$$

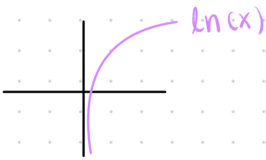
$$\lim_{h \to 0} = \frac{\frac{\ln\left(1 + \frac{h}{x}\right)}{x}}{\frac{h}{x}} = \frac{1}{x} \cdot 1$$

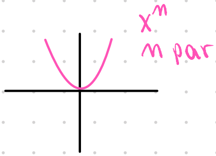
GRAFICOS

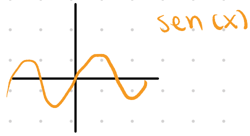












P6)

b) $f(x) = \text{sen}\,(x^{\cos(x)}) + \cos\,(x^{\text{sen}(x)})$

$= (\text{sen}\,(x^{\cos(x)}))' + (\cos\,(x^{\text{sen}(x)}))'$

$= \cos\,(x^{\cos(x)}) \cdot (x^{\cos(x)})' - \text{sen}\,(x^{\text{sen}(x)}) \cdot (x^{\text{sen}(x)})'$

$\exp(\ln(x^{\cos(x)}))'$
$\exp(\cos(x)\cdot \ln(x))'$

$\exp(\ln(x^{\text{sen}(x)}))$
$\exp(\text{sen}(x)\cdot \ln(x))'$

$(f\cdot g)' = f'g + fg'$

$= \cos\left(x^{\cos(x)}\right) \cdot e\,(\cos(x)\cdot \ln(x)) \cdot \left(-\text{sen}\,x \cdot \ln x + \cos x \cdot \frac{1}{x}\right)$

$- \text{sen}\left(x^{\text{sen}(x)}\right) \cdot e\,(\text{sen}(x)\cdot \ln(x)) \cdot \left(\cos x \cdot \ln(x) + \text{sen}(x) \cdot \frac{1}{x}\right)$

P9) $f(x) = \dfrac{x^2}{x^2-1}$

a) dom, sec., ceros, signos, paridad.

$x^2-1 \neq 0$ $\quad$ $Dom(f) = \mathbb{R} - \{\pm 1\}$

$x \neq \pm 1$

ceros. $f(x)=0$ $\quad \dfrac{x^2}{x^2-1} = 0$

$x^2 = 0$

$x = 0$

signo $\quad f(x) = \dfrac{x^2}{(x-1)(x+1)}$

	$-\infty$ a -1	-1 a 0	0 a 1	1 a $+\infty$
x^2	+	+	+	+
$x-1$	−	−	−	+
$x+1$	−	+	+	+
	+	−	−	+

$f(x) \geq 0, \quad x \in (-\infty,-1) \cup (1,+\infty)$

$f(x) < 0, \quad x \in (-1,1)$

paridad:

par $\quad f(x) = f(-x)$

$f(-x) = \dfrac{(-x)^2}{(-x)^2-1} = \dfrac{x^2}{x^2-1} = f(x) \quad \therefore$ Par.

b) asintotas (Puedo analizar rec. al ser par lo que pasa a la izq es = a la der)

Solo $x \in \mathbb{R}^{\circ}_{+}$, pues $f(x)$ es PAR

A. horizontal

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x^2}{x^2-1} = 1$$

luego por paridad el $\lim_{x\to-\infty} f(x) = 1$

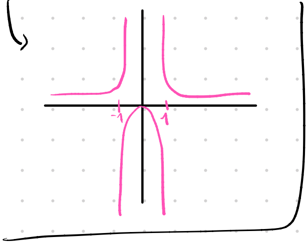
A. horizontal $y = 1$.

A. vertical (indeterminación)

$$\lim_{x\to 1} \frac{x^2}{x^2-1} = \frac{1}{0}$$

acercando se x la der e izq

$\lim_{x\to 1^+} \dfrac{x^{2\,(+)}}{(x-1)_{(+)}(x+1)_{(+)}} = +\infty$ $\quad\Big|\quad$ $\lim_{x\to 1^-} \dfrac{x^{2\,(+)}}{(x-1)_{(-)}(x+1)_{(+)}} = -\infty$ $\quad\Big|\quad$ $x = 1$ por paridad



crecimiento:

Caso 1: $x \in [0, 1)$

Caso 2: $x \in (1, +\infty)$

SPG $x_1 < x_2$

$$f(x_2) - f(x_1) = \frac{x_2^2}{x_2^2-1} - \frac{x_1^2}{x_1^2-1} = \frac{x_2^2(x_1^2-1) - x_1^2(x_2^2-1)}{(x_2^2-1)(x_1^2-1)}$$

$$f(x_2) - f(x_1) = \frac{-x_2^2 + x_1^2}{(x_2^2-1)(x_1^2-1)} = \frac{(x_1 - x_2)(x_1 + x_2)}{(x_2^2-1)(x_1^2-1)}$$

Caso 1, $x_1 x_2 \in [0,1)$

$$\frac{\overset{(-)}{(x_1 - x_2)}\overset{(+)}{(x_1 + x_2)}}{\underset{(-)}{(x_2^2-1)}\underset{(-)}{(x_1^2-1)}} = (-)$$

$f(x_2) - f(x_1) = (-)$

decreciente

$x \in [0,1)$ $f(x)$

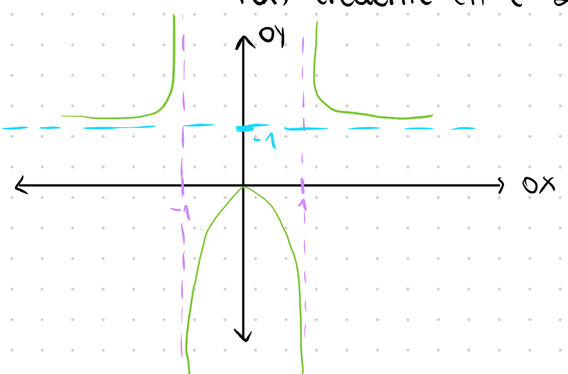
Caso 2, $x_1 x_2 \in (1, +\infty)$

$$\frac{\overset{(-)}{(x_1 - x_2)}\overset{(+)}{(x_1 + x_2)}}{\underset{(+)}{(x_2^2-1)}\underset{(+)}{(x_1^2-1)}} = (-)$$

$x \in (1, +\infty)$ $f(x)$ decreciente.

Por paridad $f(x)$ creciente en $(-1, 0)$

$f(x)$ creciente en $(-\infty, -1)$



asintota v.

asintota h.

$\text{Rec} f(x) = (-\infty, 0) \cup (1, \infty)$

$$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2} = y$$

$$\cosh^2(x) - \operatorname{senh}^2(x) = 1$$

$$e^x - e^{-x} = 2y \quad / \cdot e^x$$

$$e^x e^x - e^{-x} \cdot e^x = 2y e^x$$

$$e^{2x} - 1 - 2ye^x = 0$$

$$z = e^x$$

$$z^2 - 2yz - 1 = 0$$

$a = 1 \quad b = 2y \quad c = -1$

$$z_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$z_1 = \frac{-2y + \sqrt{4y^2 + 4}}{2}$$

$$z_1 = y + \sqrt{y^2 + 1}$$

$$z = e^x = y + \sqrt{y^2 + 1}$$

$$x = \ln(y + \sqrt{y^2 + 1})$$

$$f^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$$

$$\operatorname{senh}^{-1}(x) = \ln(x + \sqrt{x^2 + 1})$$

$$(\operatorname{senh}^{-1}(x))' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot (x + \sqrt{x^2 + 1})'$$

$$= \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left(1 + \frac{1}{\cancel{2}\sqrt{x^2 + 1}} \cdot \cancel{2}x\right)$$

$$= \frac{1}{\cancel{x + \sqrt{x^2 + 1}}} \cdot \frac{\cancel{\sqrt{x^2 + 1} + x}}{\sqrt{x^2 + 1}}$$

$$= \frac{1}{\sqrt{x^2 + 1}}$$

¡mejor la otra opcion!

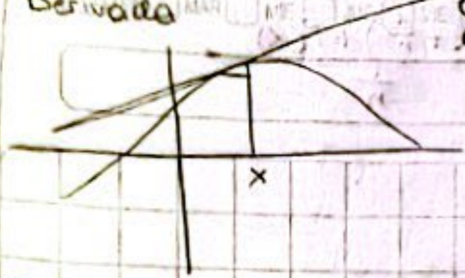
$$y = \frac{2x + 1}{3}$$

$$f^{-1}(x) = \frac{3x - 1}{2}$$

$e^x = x + 1$ → no se intenta despejar.

$x = 0$

Aux extra
Derivada



$g = mx + n$, $m \downarrow$ derivada

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$\lim_{x\to \bar{x}} \frac{f(x)-f(\bar{x})}{x-\bar{x}} = f'(\bar{x})$$

$S \to C \to -S \to -C \to S$ $\quad ()'$

Regla de la cadena $(\lambda f(x) + g(x))' = \lambda f'(x) + g'(x)$

$(f \cdot g)' = f'g + fg$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \; g \neq 0$

P1 Calcule las siguientes derivadas:

a) $f(x) = x^4 + 3x^2 - 6$

$4x^3 + 3 \cdot 2 \cdot x$ ✓

b) $f(x) = (a+x)\sqrt{a-x}$

$(a+x)' \cdot \sqrt{a-x} + (a+x)(\sqrt{a-x})'$

$1 \cdot \sqrt{a-x} + (a+x)\frac{1}{2\sqrt{a-x}} \cdot (a-x)'$

$1\sqrt{a-x} + (a+x)\frac{1}{2\sqrt{a-x}} \cdot -1$

$\sqrt{a-x} - \frac{a+x}{2\sqrt{a-x}}$ //

c) $f(x) = \tan(x)$

$$\left(\frac{sen(x)}{cos(x)}\right)' = \frac{sen(x)' cos(x) - sen(x) cos(x)'}{cos^2(x)}$$

$$= \frac{cos(x)cos(x) + sen(x)sen(x)}{cos^2(x)}$$

$$= \frac{cos^2(x) + sen^2(x)}{cos^2(x)}$$

$$= \frac{1}{cos^2(x)}$$

$$= sec^2(x)$$ //

d) $f(x) = sen(x) + \frac{1}{2x} - \frac{1}{3x^2}$

$$= cos(x) - \frac{1}{2x^2} + \frac{2}{3x^3}$$

$\frac{1}{2x} \quad \left(\frac{1}{2}x^{-1}\right)'$

$= \frac{1}{2} \cdot -1 \cdot x^{-2} \cdot$

$= -\frac{1}{2x^2}$

$\left(\frac{1}{2x}\right)' = \frac{\overset{0}{1'}\, 2x - 1 \cdot (2x)'}{4x^2}$

$= \frac{-2}{4x^2}$

$= -\frac{1}{2x^2}$

e) $f(x) = \frac{x^2}{x - sen(x)}$ $\quad \frac{2x(x-sen(x)) - x^2(x-sen(x))'}{(x-sen(x))^2} = \frac{2x(x-sen(x)) - x^2(1-cos(x))}{(x-sen(x))^2}$

f) $f(x)=\frac{2x^5+4x}{\cos(x)}=\frac{(2x^5+4x)'\cos(x)-(2x^5+4x)(\cos(x))'}{\cos^2(x)}$

$=\frac{(10x^4+4)\cos(x)+(2x^5+4x)\operatorname{sen}(x)}{\cos^2(x)}$ //

g) $f'(x)=\frac{1}{\sqrt{x}}+\sqrt{x}+\tan(x)+\frac{1}{\tan(x)}=x^{-\frac{1}{2}}+x^{\frac{1}{2}}+\tan(x)+(\tan(x))^{-1}$

$=-\frac{1}{2}\cdot x^{-\frac{3}{2}}+\frac{1}{2}x^{-\frac{1}{2}}+\sec^2(x)-1\cdot\tan^{-2}(x)\cdot\sec^2(x)$

j) $f'(x)=\frac{e^x\cos(x)}{1-\operatorname{sen}(x)}=\frac{(e^x\cos(x))'(1-\operatorname{sen}(x))-(e^x\cos(x))(1-\operatorname{sen}(x))'}{(1-\operatorname{sen}(x))^2}$

$=\frac{(e^x\cdot\cos(x)-e^x\operatorname{sen}(x))-(e^x\cos(x))(-\cos(x))}{(1-\operatorname{sen}(x))^2}$ //

Derivada de la función inversa

$(f^{-1}(x))'=\frac{1}{f'(f^{-1}(x))}$

P4 Calcule las derivadas por definición

a) $F(x)=\sqrt{x}$ $\quad f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$=\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$

$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$

f es continua en $x=a$ si $\lim_{x\to a}f(x)=f(a)$

$\lim_{h\to 0}\frac{1}{\sqrt{x}+\sqrt{x}}=\boxed{\frac{1}{2\sqrt{x}}}$ //

c) $g(x)=e^x$ $\quad g'(x)=\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$

$=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$

$=\lim_{h\to 0}\frac{e^x\cdot e^h-e^x}{h}$

$=\lim_{h\to 0}\frac{e^x(e^h-1)}{h}$ → límite conocido

$\lim_{h\to 0}e^x\cdot 1=\boxed{e^x}$

a) $u(x)=\ln(x)$ $\lim_{h\to0}\frac{u(x+h)-u(x)}{h}$

$\lim_{h\to0}\frac{\ln(x+h)-\ln(x)}{h}$

$=\lim_{h\to0}\frac{\ln\left(\frac{x+h}{x}\right)}{h}$

$=\lim_{h\to0}\frac{\ln\left(1+\frac{h}{x}\right)}{h}$ → cambio de variable $u=1+\frac{h}{x}\xrightarrow[h\to0]{}u\to1$

$=\lim_{u\to1}\frac{\ln(u)}{x(u-1)}=\lim_{u\to1}\frac{1}{x}\cdot\frac{\ln(u)}{u-1}=\frac{1}{x}\cdot1=\boxed{\frac{1}{x}}$ //

Calcularemos $(\operatorname{senh}^{-1}(x))$ usaremos $x=\exp(\ln(x))$

$x=f(f^{-1}(x))\Rightarrow \operatorname{senh}(\operatorname{senh}^{-1}(x))=x\ /()'$

$(\operatorname{senh}(x))'\cdot\cosh(\operatorname{senh}^{-1}(x))=1$

$(\operatorname{senh}(x))'=\frac{1}{\cosh(\operatorname{senh}^{-1}(x))}$

$\cosh^2(x)+\operatorname{senh}^2(x)=1$

$\cosh(x)=\sqrt{1-\operatorname{senh}^2(x)}$

$(\operatorname{senh}(x))'=\frac{1}{\sqrt{1-\operatorname{senh}^2(\operatorname{senh}^{-1}(x))}}$

$\boxed{(\operatorname{senh}(x))'=\frac{1}{\sqrt{1-x^2}}}$ //

propuesto: $(\cosh^{-1}(x))'$

Calcular: $\left(\cos(x)^{\frac{1}{\operatorname{sen}^2(x)}}\right)'$ $x=\exp(\ln(x))$

$\exp\left(\ln\left(\cos(x)^{\frac{1}{\operatorname{sen}^2(x)}}\right)\right)=\left(\exp\left(\frac{1}{\operatorname{sen}^2(x)}\ln(\cos(x))\right)\right)'$

$=\exp\left(\frac{1}{\operatorname{sen}^2(x)}\ln(\cos(x))\right)\left(-2\operatorname{sen}^{-3}(x)\cos(x)\ln(\cos(x))+\frac{1}{\operatorname{sen}^2(x)}\cdot-\frac{\operatorname{sen}(x)}{\cos(x)}\right)$ //

$x=a$ es asíntota vertical de f si $\lim_{x\to a^+}f(x)=\infty \vee \lim_{x\to a^-}f(x)=\infty$ → ver puntos de indefinición (hay que comprobarlo)

$y=a$ es asíntota horizontal de f si $\lim_{x\to\pm\infty}f(x)=a \vee \lim_{x\to-\infty}f(x)=a$

$y=mx+n$ es asíntota oblicua de f si $\lim_{x\to\pm\infty}f(x)-(mx+n)=0$ donde

$\boxed{n=\lim_{x\to\pm\infty}f(x)-mx}$ $\boxed{m=\lim_{x\to\pm\infty}\frac{f(x)}{x}}$

P9 sea $f(x)=\frac{x^2}{x^2-1}$

a) Determine dominio

$x^2-1\neq0\to x^2\neq1\Rightarrow x\neq1,\ x\neq-1\to \operatorname{Dom}(f)=\mathbb{R}/\{-1,1\}$

Ceros $x=0$ $\frac{0^2}{0^2-1}=\frac{0}{0-1}=0$ //

Signos: en $]1,+\infty[\to f(x)>0$

en $]-1,1[\to f(x)\leq0$

en $]-\infty,-1[\to f(x)>0$

Paridad: $f(-x)=\frac{(-x)^2}{(-x)^2-1}=\frac{x^2}{x^2-1}=f(x)$ La función es par, es simetrica con respecto al eje y

Asíntotas verticales: Ver puntos de indefinición $x=-1 \wedge x=1$

$\lim_{x\to -1^+} f(x)=\lim_{x\to -1^+}\frac{x^2}{x^2-1}=\left(\frac{1}{0^-}\right)=-\infty$ / $\lim_{x\to -1^-} f(x)=\lim_{x\to -1^-}\frac{x^2}{x^2-1}=\left(\frac{1}{0^+}\right)=+\infty$ $\boxed{x=-1 \text{ es asíntota vertical}}$

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\frac{x^2}{x^2-1}=\left(\frac{1}{0^+}\right)=+\infty$ / $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\frac{x^2}{x^2-1}=\left(\frac{1}{0^-}\right)=-\infty$ $\boxed{x=1 \text{ es asíntota vertical}}$

Asíntotas horizontales: $\lim_{x\to\infty} F(x)=\lim_{x\to+\infty}\frac{x^2}{x^2-1}=\underbrace{\lim_{n\to\infty}\frac{n^2}{n^2-1}}_{\text{Límite conocido}}=\frac{1}{1}=\boxed{1}$

$\boxed{\text{Asíntota horizontal } y=1}$

hacia $+\infty$ y $-\infty$ infinito porque $f(x)$ es par

No pueden haber asíntotas oblicuas.

Crecimiento: Analizar $x\geq 0 \Rightarrow [0,1[\cup]1,\infty[$

Para $[0,1[$ sean $0\leq x_1\leq x_2<1$

$$f(x_2)-f(x_1)=\frac{x_2^2}{x_2^2-1}-\frac{x_1^2}{x_1^2-1}=\frac{x_2^2(x_1^2-1)-x_1^2(x_2^2-1)}{(x_2^2-1)(x_1^2-1)}=\frac{-x_2^2+x_1^2}{(x_2^2-1)(x_1^2-1)}$$

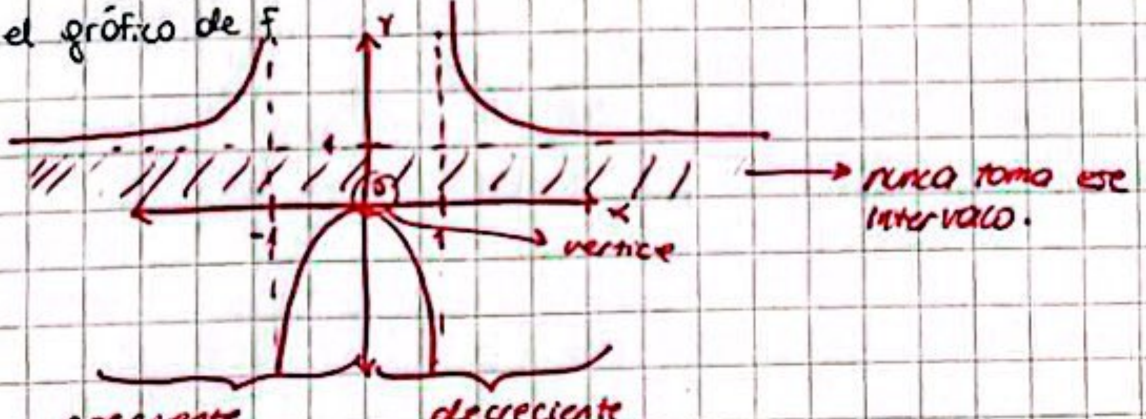
$$=\frac{\overset{<0}{(x_1-x_2)}\overset{>0}{(x_1+x_2)}}{\underset{>0}{(x_2^2-1)}\underset{>0}{(x_1^2-1)}}<0 \rightarrow f(x) \text{ es decreciente en } [0,1[$$

Para $]1,\infty[$ sean $1<x_1<x_2$

$$f(x_2)-f(x_1)=\frac{\overset{<0}{(x_1-x_2)}\overset{>0}{(x_1+x_2)}}{\underset{>0}{(x_1^2-1)}\underset{>0}{(x_2^2-1)}}<0 \rightarrow F(x) \text{ es decreciente en }]1,\infty[$$

Para $x\leq 0$ $f(x)$ es creciente en $[0,1[\cup]1,\infty[$ debido a su paridad

Esboce el gráfico de f



Recorrido: $]-\infty,0] \cup]1,+\infty[$

Scanned with CamScanner

$$\cosh^2(x) - \operatorname{senh}^2(x) = 1$$

$$\operatorname{senh}'(x) = \left(\frac{e^x - e^{-x}}{2}\right)' = \frac{1}{2}(e^x + e^{-x}) = \boxed{\cosh(x)}$$