

Formulario de Mecánica

Autor: Edgardo Rosas C.

1 Coordenadas Cartesianas: (x, y, z)

1.1 Cinemática

Posición:	$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
Velocidad:	$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$
Aceleración:	$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$

1.2 Diferenciales

Línea:	$d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$
Superficie:	$d\vec{S} = dydz\hat{i} + dx dz\hat{j} + dx dy\hat{k}$
Volumen	$dV = dx dy dz$

1.3 Operadores

Gradiente:	$\nabla\psi = \frac{\partial\psi}{\partial x}\hat{i} + \frac{\partial\psi}{\partial y}\hat{j} + \frac{\partial\psi}{\partial z}\hat{k}$
Divergencia:	$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$
Rotor:	$\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$
Laplaciano:	$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$

1.4 Transformación de coordenadas

Cilíndricas:	$x = \rho \cos \phi$	$y = \rho \sin \phi$	$z = z$
Esféricas:	$x = r \sin \theta \cos \phi$	$y = r \sin \theta \sin \phi$	$z = r \cos \phi$

2 Coordenadas Cilíndricas: (ρ, ϕ, z)

2.1 Transformación de vectores unitarios

$$\hat{\rho} = \cos \phi \hat{i} + \sin \phi \hat{j} \quad \hat{\phi} = -\sin \phi \hat{i} + \cos \phi \hat{j} \quad \hat{k} = \hat{k}$$

2.2 Cinemática

$$\begin{aligned} \text{Posición:} \quad \vec{r} &= \rho \hat{\rho} + z \hat{k} \\ \text{Velocidad:} \quad \vec{v} &= \dot{\rho} \hat{\rho} + \rho \dot{\phi} \hat{\phi} + \dot{z} \hat{k} \\ \text{Aceleración:} \quad \vec{a} &= (\ddot{\rho} - \rho \dot{\phi}^2) \hat{\rho} + (2\dot{\rho} \dot{\phi} + \rho \ddot{\phi}) \hat{\phi} + \ddot{z} \hat{k} \\ \text{Aceleración:} \quad \vec{a} &= (\ddot{\rho} - \rho \dot{\phi}^2) \hat{\rho} + \frac{1}{\rho} \frac{d}{dt} (\rho^2 \dot{\phi}) \hat{\phi} + \ddot{z} \hat{k} \end{aligned}$$

2.3 Diferenciales

$$\begin{aligned} \text{Línea:} \quad d\vec{l} &= d\rho \hat{\rho} + \rho d\phi \hat{\phi} + dz \hat{k} \\ \text{Superficie:} \quad d\vec{S} &= \rho d\phi dz \hat{\rho} + \rho d\rho dz \hat{\phi} + \rho d\rho d\phi \hat{k} \\ \text{Volumen:} \quad dV &= \rho d\rho d\phi dz \end{aligned}$$

2.4 Operadores

$$\begin{aligned} \text{Gradiente:} \quad \nabla \psi &= \frac{\partial \psi}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial \psi}{\partial \phi} \hat{\phi} + \frac{\partial \psi}{\partial z} \hat{k} \\ \text{Divergencia:} \quad \nabla \cdot \vec{F} &= \frac{1}{\rho} \frac{\partial(\rho F_{\rho})}{\partial \rho} + \frac{1}{\rho} \frac{\partial F_{\phi}}{\partial \phi} + \frac{\partial F_z}{\partial z} \\ \text{Rotor:} \quad \nabla \times \vec{F} &= \left[\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_{\phi}}{\partial z} \right] \hat{\rho} + \left[\frac{\partial F_{\rho}}{\partial z} - \frac{\partial F_z}{\partial \rho} \right] \hat{\phi} + \frac{1}{\rho} \left[\frac{\partial(\rho F_{\phi})}{\partial \rho} - \frac{\partial F_{\rho}}{\partial \phi} \right] \hat{k} \\ \text{Laplaciano:} \quad \nabla^2 \psi &= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2} \end{aligned}$$

3 Coordenadas Esféricas: (r, θ, ϕ)

3.1 Transformación de vectores unitarios

$$\hat{\mathbf{r}} = \sin \theta \cos \phi \hat{\mathbf{i}} + \sin \theta \sin \phi \hat{\mathbf{j}} + \cos \theta \hat{\mathbf{k}} \quad \hat{\boldsymbol{\theta}} = \cos \theta \cos \phi \hat{\mathbf{i}} + \cos \theta \sin \phi \hat{\mathbf{j}} - \sin \theta \hat{\mathbf{k}} \quad \hat{\boldsymbol{\phi}} = -\sin \phi \hat{\mathbf{i}} + \cos \phi \hat{\mathbf{j}}$$

3.2 Cinemática

Posición:	$\vec{r} = r\hat{\mathbf{r}}$
Velocidad:	$\vec{v} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} + r\dot{\phi}\sin\theta\hat{\boldsymbol{\phi}}$
Aceleración:	$\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{\boldsymbol{\phi}}$
Aceleración:	$\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\dot{\phi}\sin^2\theta\right)\hat{\boldsymbol{\phi}}$

3.3 Diferenciales

Línea	$d\vec{l} = dr\hat{\mathbf{r}} + r d\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
Superficie	$d\vec{S} = r^2\sin\theta d\theta d\phi\hat{\mathbf{r}} + r\sin\theta dr d\phi\hat{\boldsymbol{\theta}} + r dr d\theta\hat{\boldsymbol{\phi}}$
Volumen	$dV = r^2\sin\theta dr d\theta d\phi$

3.4 Operadores

Gradiente:	$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\hat{\boldsymbol{\phi}}$
Divergencia:	$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial(r^2 F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$
Rotor:	$\nabla \times \vec{F} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta F_\phi)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial(r F_\phi)}{\partial r}\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial(r F_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$
Laplaciano:	$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$