Auxiliar 2

Coordenadas esféricas

P1 Como $\dot{\theta} = \omega_0$ constante

$$\Rightarrow \int_0^{\theta} d\theta = \int_0^t \omega_0 \, dt$$

$$\Leftrightarrow \theta(t) = \omega_0 t$$

a) Ahora, como $\vec{v}$ y $\vec{a}$ tiene primeras y segundas derivadas, derivamos cada variable

- $r = R \Rightarrow \dot{r} = \ddot{r} = 0$
- $\theta(t) = \omega_0 t,\ \dot{\theta} = \omega_0 \Rightarrow \ddot{\theta} = 0$
- $\phi = N\theta \Rightarrow \dot{\phi} = N\dot{\theta} = N\omega_0 \Rightarrow \ddot{\phi} = 0$

reemplazamos dejando todo en función de θ (posición)

- $\vec{v} = \cancel{0}\hat{r} + R\omega_0 \hat{\theta} + RN\omega_0 \sin\theta \, \hat{\phi}$
- $\vec{a} = (\cancel{0} - R\omega_0^2 - RN^2\omega_0^2 \sin^2\theta)\hat{r}$

$$+ (\cancel{0} + \cancel{0} - RN^2\omega_0^2 \sin\theta\cos\theta)\hat{\theta}$$

$$+ (\cancel{0} + \cancel{0} + 2RN\omega_0^2 \cos\theta)\hat{\phi}$$

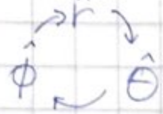
b) El radio de curvatura se calcula como

$$\rho_c = \frac{v^3}{\|\vec{v} \times \vec{a}\|}$$

El módulo de $\vec{v}$ sería $\|\vec{v}\| = \sqrt{R^2\omega_0^2 + R^2N^2\omega_0^2 \sin^2\theta}$

Y el producto cruz

$$\vec{v} \times \vec{a} = \left[R\omega_0 \hat{\theta} + RN\omega_0 \sin\theta \hat{\phi} \right] \times \left[(-R\omega_0^2 - RN^2\omega_0^2 \sin^2\theta)\hat{r} \right.$$



$$- RN^2\omega_0^2 \sin\theta\cos\theta \hat{\theta}$$

$$\left. + 2RN\omega_0^2 \cos\theta \hat{\phi} \right]$$

$$= R\omega_0 (R\omega_0^2 + RN^2\omega_0^2 \sin^2\theta)\hat{\phi}$$

$$+ 2R^2 N \omega_0^3 \cos\theta \hat{r}$$

$$+ (-R^2 N \omega_0^3 \sin\theta - R^2 N^3 \omega_0^3 \sin^3\theta)\hat{\theta}$$

$$+ R^2 N^3 \omega_0^3 \sin^2\theta \cos\theta \hat{r}$$

$$= (2R^2 N \omega_0^3 \cos\theta + R^2 N^3 \omega_0^3 \sin^2\theta \cos\theta)\hat{r}$$

$$- (R^2 N \omega_0^3 \sin\theta + R^2 N^3 \omega_0^3 \sin^3\theta)\hat{\theta}$$

$$+ (R^2 \omega_0^3 + R^2 N^2 \omega_0^3 \sin^2\theta)\hat{\phi}$$

$$\Rightarrow \|\vec{v} \times \vec{a}\| = \left[4R^4 N^2 \omega_0^6 \cos^2\theta + 4R^4 N^4 \omega_0^6 \sin^2\theta \cos^2\theta + R^4 N^6 \omega_0^6 \sin^4\theta \cos^2\theta \right.$$

$$+ R^4 N^2 \omega_0^6 \sin^2\theta + 2R^4 N^4 \omega_0^6 \sin^4\theta + R^4 N^6 \omega_0^6 \sin^6\theta$$

$$\left. + R^4 \omega_0^4 + 2R^4 N^2 \omega_0^6 \sin^2\theta \right]^{1/2}$$

$$\underset{\theta=\pi/2}{\Rightarrow} \ldots = R^2 \omega_0^3 (1+N^2)^{3/2}$$ → lo hice con Mathematica

Recomendación: Evaluar $\theta = \pi/2$ antes

Reemplazando

$$\rho_c = \frac{R^3 \omega_0^3 (1+N^2)^{3/2}}{R^2 \omega_0^3 (1+N^2)^{3/2}} = R$$

c) Para calcular la distancia recorrida, se ocupa la fórmula

$$d = \int_{t_0}^{t_f} \|\vec{v}\| \, dt$$

Además, como $\dot{\theta} = \omega_0 \Rightarrow \theta(t) = \omega_0 t$, la partícula se demora

$$\theta(t_f) = \pi = \omega_0 t_f$$

$$\Rightarrow t_f = \frac{\pi}{\omega_0}$$

Reemplazando

$$\Rightarrow d = \int_0^{\pi/\omega_0} (R^2\omega_0^2 + R^2N^2\omega_0^2 \sin^2\theta)^{1/2} \, dt$$

como $\theta = \omega_0 t$

$$= \int_0^{\pi/\omega_0} R\omega_0 (1 + N^2 \sin^2 \omega_0 t)^{1/2} \, dt$$

$\boxed{P_2}$ Tenemos

- $\dot{r} = c \Rightarrow r = ct$ y $\ddot{r} = 0$
- $\phi = \frac{2\pi}{R} r = \frac{2\pi}{R} ct \Rightarrow \dot{\phi} = \frac{2\pi c}{R} \Rightarrow \ddot{\phi} = 0$
- $\theta = \pi/4 \Rightarrow \dot{\theta} = \ddot{\theta} = 0$

Reemplazamos en la fórmula de la velocidad esférica

$$\vec{v} = c\hat{r} + c^2 t \frac{2\pi}{R} \sin\theta \hat{\phi} =$$

$$= c\hat{r} + \frac{\sqrt{2}\, c^2 t \pi \hat{\phi}}{R}$$

$$\Rightarrow \|\vec{v}\| = \left(c^2 + \frac{2c^4 t^2 \pi^2}{R^2}\right)^{1/2}$$

Veamos cuándo $\|\vec{v}(t^*)\| = 3c$

$$\Rightarrow \cancel{c}\left(1 + \frac{2c^2 t^{*2} \pi^2}{R^2}\right)^{1/2} = 3\cancel{c}$$

$$\Rightarrow \frac{2c^2 t^{*2} \pi^2}{R^2} = 8 \Rightarrow t^* = \frac{2R}{\pi c} \Rightarrow r^* = \frac{2R}{\pi}$$

b) Ocupando la misma fórmula que antes

$$d = \int_0^{t^*} c\left(1 + \frac{2c^2\pi^2 t^2}{R^2}\right)^{1/2} dt$$

c) Necesitamos expresar la aceleración

$$\vec{a} = -ct\,\frac{4\pi^2 c^2}{R^2}\,\frac{1}{2}\,\hat{r} - ct\,\frac{4\pi^2 c^2}{R^2}\,\frac{1}{2}\,\hat{\theta} + 2c\,\frac{2\pi c}{R}\,\frac{\sqrt{2}}{2}\,\hat{\phi}$$

$$\Rightarrow -\frac{4\pi c^2}{R}\hat{r} - \frac{4\pi c^2}{R}\hat{\theta} + \frac{2\pi c^2 \sqrt{2}}{R}\hat{\phi} = \vec{a}(t^*)$$

Hacemos el producto cruz

$$\hat{\phi} \curvearrowleft \hat{r} \curvearrowright \hat{\theta}$$

$$\vec{v} \times \vec{a} = (c\hat{r} + 2\sqrt{2}c\hat{\phi}) \times \left(-\frac{4\pi c^2}{R}\hat{r} - \frac{4\pi c^2}{R}\hat{\theta} + \frac{2\sqrt{2}\pi c^2}{R}\hat{\phi}\right)$$

$$= -\frac{4\pi c^3}{R}\hat{\phi} - \frac{2\sqrt{2}\pi c^3}{R}\hat{\theta}$$

$$- \frac{8\sqrt{2}\pi c^3}{R}\hat{\theta} + \frac{8\sqrt{2}\pi c^3}{R}\hat{r}$$

$$= \frac{8\sqrt{2}\pi c^3}{R}\hat{r} - \frac{10\sqrt{2}\pi c^3}{R}\hat{\theta} - \frac{4\pi c^3}{R}\hat{\phi}$$

$$\Rightarrow \|\vec{v} \times \vec{a}\| = \frac{\pi c^3}{R}(128 + 200 + 16)^{1/2}$$

$$= \frac{\pi c^3}{R}\sqrt{344} = \frac{2\pi c^3}{R}\sqrt{86}$$

Reemplazamos en la fórmula

$$\rho_c = \frac{\|\vec{v}\|^3}{\|\vec{v} \times \vec{a}\|} = \frac{27c^3 R}{2\pi c^3 \sqrt{86}}$$