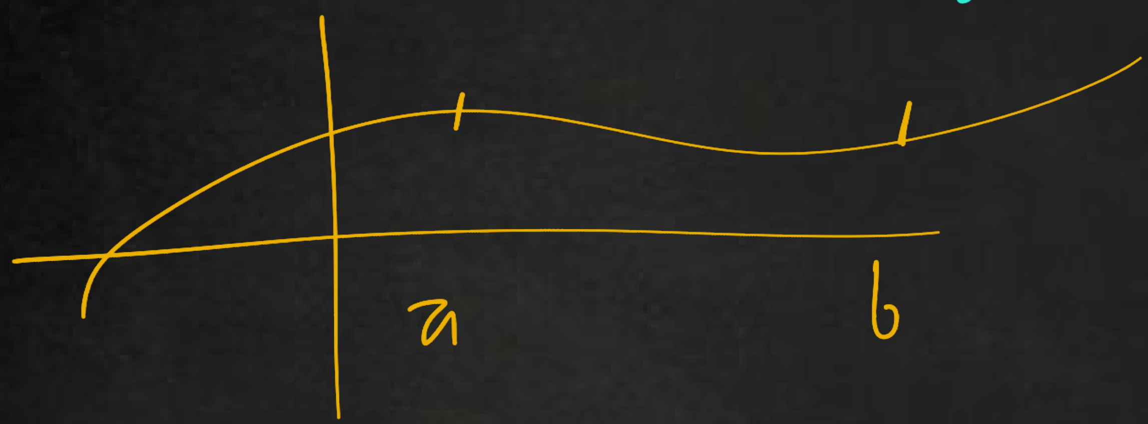
Longitud de curva.

$$\mathcal{L}_a^b(f) = \int_a^b \sqrt{1+|f'(x)|^2}\,dx$$

$f \in C^1 \Leftrightarrow$ Es una vez diferenciable



P6 L1 a) Sea $f:[0,+\infty) \to \mathbb{R}$

$f(0)=0$ y $\mathcal{L}_0^x f(x) = x^2+2x - f(x)$

Determine $f(x)$. 96 Página.

$$\mathcal{L}_0^x(f) = \int_0^x \sqrt{1+|f'(x)|^2}\,dx = x^2+2x-f(x) \quad /()'$$

TFC - General

Recuerdo $\left(\int_{u(x)}^{v(x)} f(x)\,dx\right)' = f(v(x))v'(x) - f(u(x))u'(x)$.

$V(x) = x \qquad V'(x) = 1$

$u(x) = 0 \qquad u'(x) = 0$

$$\underbrace{\sqrt{1+|f'(x)|^2}}_{f(v(x))} \cdot \underbrace{1}_{v'(x)} - \underbrace{\sqrt{1+|f'(0)|^2}}_{f(u(x))} \cdot \underbrace{0'}_{u'(x)} = 2x + 2 - f'(x)$$

Dado $f(x) \in C^1$ linealidad

$$\sqrt{1+|f'(x)|^2} = (\overset{a}{\widehat{2x}} + \overset{b}{\widehat{2}} - \overset{c}{\widehat{f'(x)}}) / ()^2 \text{ derivada.}$$

$(a + b + c)^2$

$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$$1 + \cancel{|f'(x)|^2} = 4x^2 + 4 + \cancel{f'(x)^2} + 8x - 4xf'(x) - 4f'(x)$$

$$\Rightarrow 4f'(x)[x+1] = 4x^2 + 3 + 8x \quad , \; x \neq -1$$

$+1-1$

$f : [0, +\infty) \to \mathbb{R}$

$$f'(x) = \frac{4x^2 + 3 + 8x}{4x + 4}$$

$$f'(x) = \frac{4x+4}{4x+4} + \frac{4x^2 + 4x - 1}{4x+4}$$

$$f'(x) = 1 + \frac{\cancel{4}x\cancel{(x+1)}}{\cancel{4}\cancel{(x+1)}} - \frac{1}{4x+4}$$

$$f'(x) = 1 + x - \frac{1}{4x+4} \Big/ \int dx$$

$$f(x) = x + \frac{x^2}{2} - \frac{1}{4}\ln(|x+1|) + C \quad C\in\mathbb{R}$$

Yo recuerdo $f(0) = 0$ por enunciado

$$f(0) = 0 + \frac{0^2}{2} - \frac{1}{4}\ln(0+1) + C$$

$$\|$$

$$0 =: C$$

$$\Rightarrow \quad f(x) = x + \frac{x^2}{2} - \frac{1}{4}\ln(x+1) \;//$$

P11) Para $r>0$ se define la región R por

$$R := \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq r^2,\ x^2+y^2-2ry \geq 0,\ y \geq 0\}$$

Calcule superficie de sólido engendrado al rotar la región R entorno a OX.

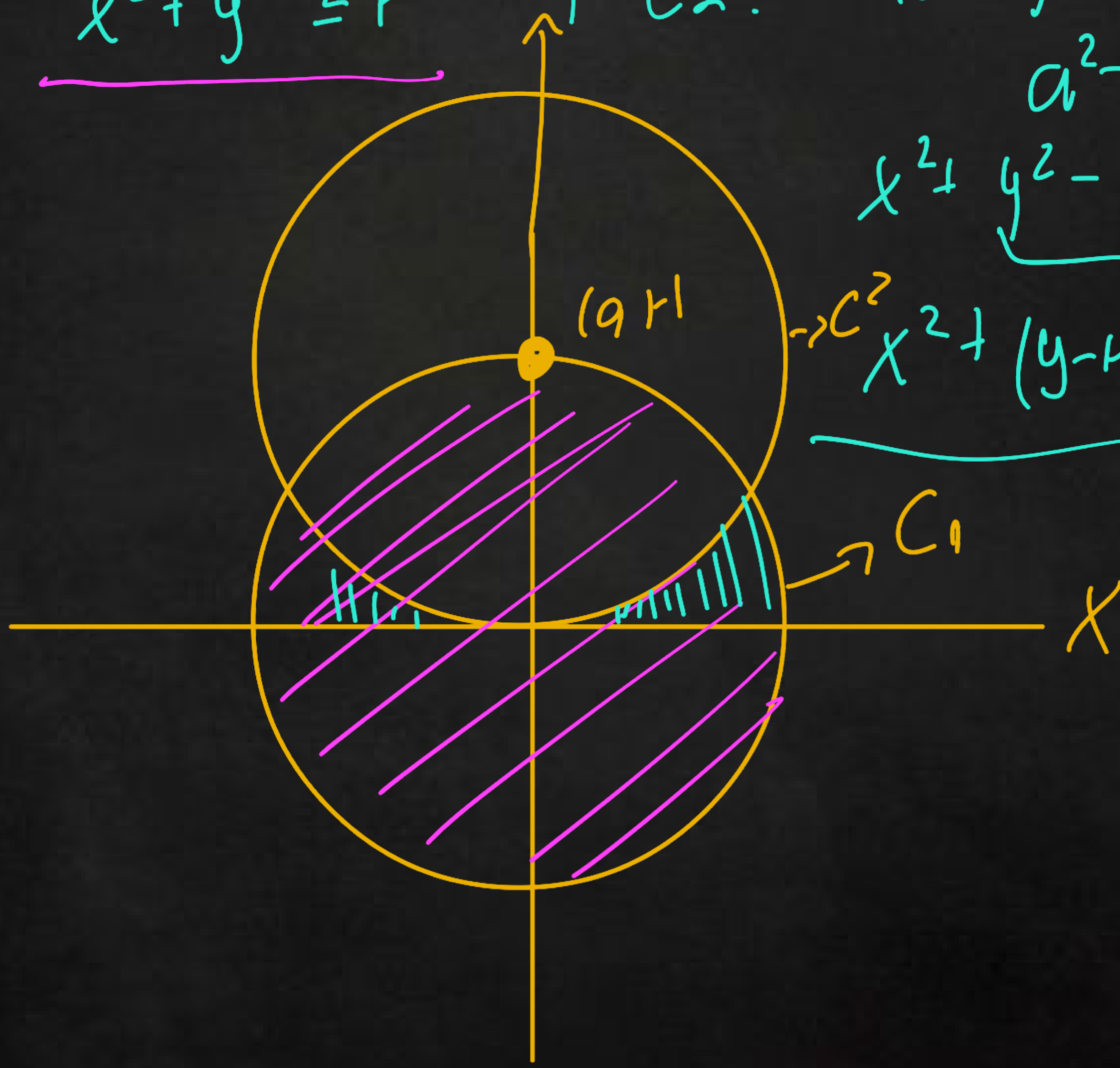
Completación cuadrado

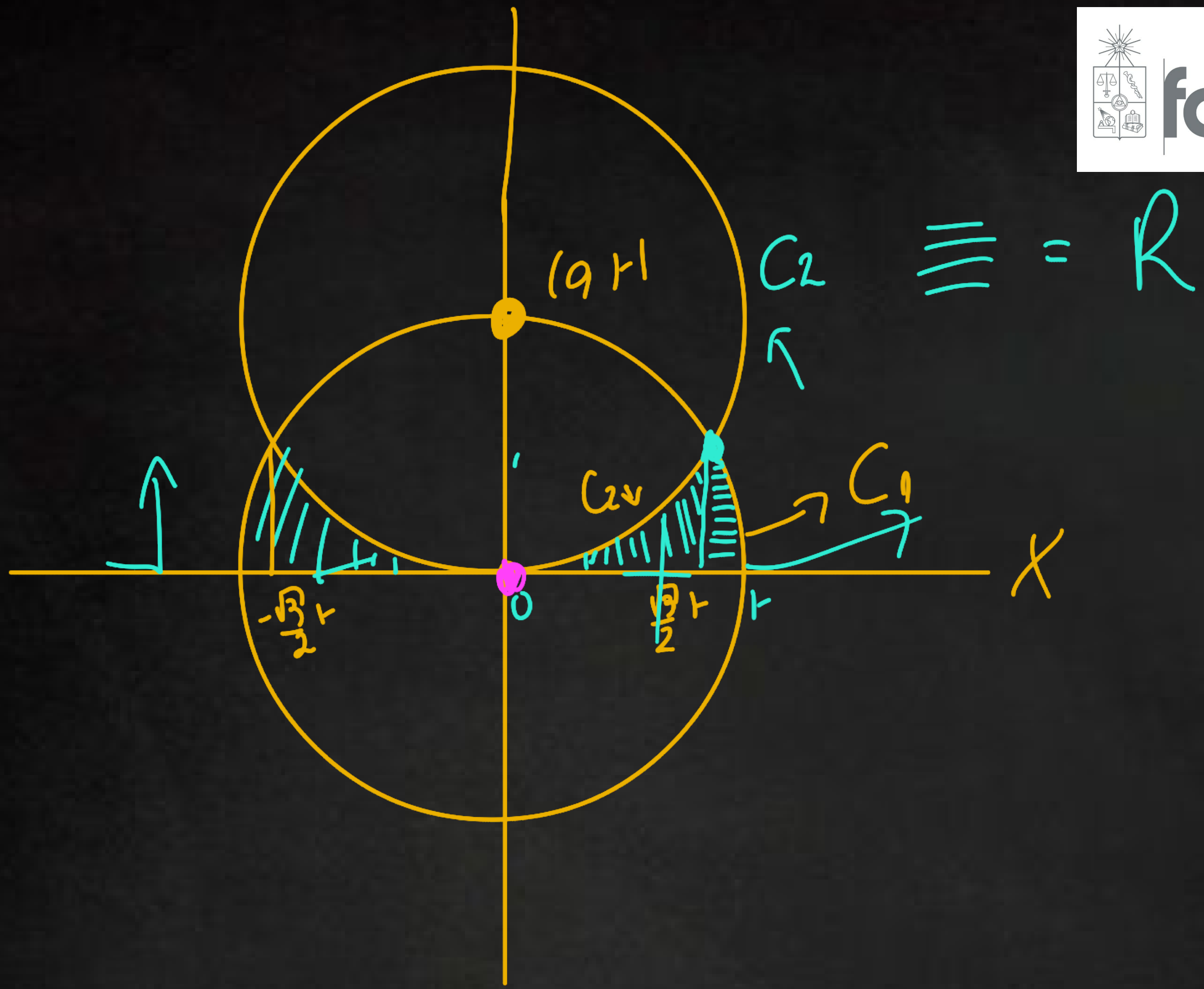
C_1: $\underline{x^2+y^2 \leq r^2}$ | C_2: $x^2+y^2-2ry \geq 0 \,/+r^2$

$$a^2-2ab+b^2$$

$$\underbrace{x^2+y^2-2ry+r^2}_{} \geq r^2$$

$\to C^2$ $\underline{x^2+(y-r)^2 \geq r^2}$





Recuerdo

Sea $f \in C^1$, área manto generado por rotación en torno OX f entre a y b

$$S_{OX} = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$

Entonces trabajaré solo de $x \in [0, r]$ pués la función es simétrica

Necesito encontrar $C_1 \cap C_2 = \{x_0\}$

$$\int_0^r \ldots = \int_0^{x_0} \ldots + \int_{x_0}^{r} \ldots$$

Borde de C_1

1) $x^2+y^2=r^2$

$y_1 = \pm\sqrt{r^2-x^2}$

Borde de C_2

2) $x^2+(y-r)^2=r^2$

$y_2 = \pm\sqrt{r^2-x^2}+r$

1) (+) 2(+)

$$\cancel{\sqrt{r^2-x^2}} = \cancel{\sqrt{r^2-x^2}} + r^2$$

$$0 = r^2 \Rightarrow r=0$$

1) (+) 2(−)

$$\sqrt{r^2-x^2} = -\sqrt{r^2-x^2}+r$$

$$2\sqrt{r^2-x^2} = r \quad /()^2$$

$$4r^2-4x^2=r^2$$

$$3r^2=4x^2$$

$$\frac{3r^2}{4}=x^2 \Rightarrow \boxed{x=\pm\frac{\sqrt{3}}{2}r}$$

$$\boxed{S_{Ox} = \int_a^b 2\pi |f(x)| \sqrt{1 + |f'(x)|^2}\, dx}$$

$$S_{Ox} = 2\Bigg[2\pi \underbrace{\int_0^{\frac{\sqrt{3}}{2}r} \underbrace{\left(r - \sqrt{r^2 - x^2}\right)}_{f(x)} \underbrace{\left(\sqrt{1 + \left|\left(r - \sqrt{r^2-x^2}\right)'\right|^2}\right)}_{\sqrt{1+|f'(x)|^2}} dx}_{I_1}$$

$$+ 2\pi \underbrace{\int_{\frac{\sqrt{3}}{2}r}^{r} \left(\sqrt{r^2-x^2}\right)\left(\sqrt{1 + \left|\left(\sqrt{r^2-x^2}\right)'\right|^2}\right) dx}_{I_2}\Bigg]$$

$(*)$
$$\left(r - \sqrt{r^2 - x^2}\right)' = -\frac{1}{2\sqrt{r^2-x^2}} \cdot -2x = \frac{x}{\sqrt{r^2-x^2}} \qquad I_1$$

$(\forall)$
$$\left(\sqrt{r^2-x^2}\right)' = \frac{1}{2\sqrt{r^2-x^2}} \cdot -2x = \frac{-x}{\sqrt{r^2-x^2}} \qquad I_2$$

$\boxed{I_1}$

$$\int_0^{\frac{\sqrt{3}}{2}r} \underbrace{\left(r - \sqrt{r^2 - x^2}\right)}_{f(x)} \underbrace{\left(\sqrt{1 + \left|\left(r - \sqrt{r^2-x^2}\right)'\right|^2}\right)}_{\sqrt{1+|f'(x)|^2}} dx \qquad (*)$$

$$= \int_0^{\frac{\sqrt{3}}{2}r} \left(r - \sqrt{r^2-x^2}\right)\left(\sqrt{1 + \left(\frac{x}{\sqrt{r^2-x^2}}\right)^2}\right) dx \qquad \frac{r^2-x^2}{r^2-x^2} = 1$$

$$= \int_0^{\frac{\sqrt{3}}{2}r} \left(r - \sqrt{r^2-x^2}\right)\left(\sqrt{\frac{r^2 \cancel{-x^2} + \cancel{x^2}}{r^2-x^2}}\right) dx$$

$$= \int_0^{\frac{\sqrt{3}}{2}r} \left(r - \sqrt{r^2-x^2}\right)\frac{r}{\sqrt{r^2-x^2}}\, dx = \int_0^{\frac{\sqrt{3}}{2}r} \left(\frac{r^2}{\sqrt{r^2-x^2}} - r\right) dx$$

$$= r^2 \int_0^{\frac{\sqrt{3}}{2}r} \frac{1}{\sqrt{r^2 - x^2}} - r \int_0^{\frac{\sqrt{3}}{2}r} dx$$

$$= r^2 \int_0^{\frac{\sqrt{3}}{2}r} \frac{1}{\sqrt{r^2\left(1 - \frac{x^2}{r^2}\right)}} dx - \frac{\sqrt{3}r^2}{2}$$

$$= r \int_0^{\frac{\sqrt{3}}{2}r} \frac{1}{\sqrt{1 - \frac{x^2}{r^2}}} dx - \frac{\sqrt{3}}{2} r^2$$

$$u = \frac{x}{r} \quad du = \frac{dx}{r} \quad \Bigg| \quad \begin{matrix} x = 0 \\ u = 0 \end{matrix} \qquad \begin{matrix} x = \frac{\sqrt{3}}{2} r \\ u = \frac{\sqrt{3}}{2} \end{matrix}$$

$$= r^2 \int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1 - u^2}} du - \frac{\sqrt{3}}{2} r^2$$

$\rightarrow$ Primitiva $\arcsen(x)$

$$I_1 = r^2 \arcsen(u) \Big|_0^{\frac{\sqrt{3}}{2}} - \frac{\sqrt{3}}{2} r^2$$

$$\underbrace{r^2 \arcsen\left(\frac{\sqrt{3}}{2}\right)}_{\frac{\pi}{3}} - \cancel{\arcsen(0)} \rightarrow 0 - \frac{\sqrt{3}}{2} r^2$$

$$I_1 = r^2 \frac{\pi}{3} - \frac{\sqrt{3}}{2} r^2$$

$$I_2 = \int_{\frac{\sqrt{3}}{2}r}^{r} \sqrt{r^2-x^2}\,\underbrace{\left(\sqrt{1+\left|\left(\sqrt{r^2-x^2}\right)'\right|^2}\right)}_{(**)}\,dr$$

$(**)$

$$\# \quad \sqrt{1+\left|\frac{1\cdot -\cancel{2}x}{\cancel{2}\sqrt{r^2-x^2}}\right|^2} = \sqrt{\frac{r^2-x^2}{r^2-x^2}+\frac{x^2}{r^2-x^2}}$$

$$= \underline{\frac{r}{\sqrt{r^2-x^2}}}$$

$$I_2 = \int_{\frac{\sqrt{3}}{2}r}^{r} \cancel{\sqrt{r^2-x^2}}\cdot\frac{r}{\cancel{\sqrt{r^2-x^2}}} = r\left(r-\frac{\sqrt{3}}{2}r\right) = \underline{I_2}$$

$$S_{ox} = 2\left[2\pi I_1 + 2\pi I_2\right]$$

$$= 2\left[2\pi\frac{r^2\pi}{3} - \frac{2\pi r^2\sqrt{3}}{2} + 2\pi r^2 - 2\pi\frac{\sqrt{3}}{2}r^2\right.$$

$$= 4\pi r^2\left(\frac{\pi}{3}+1-\sqrt{3}\right)$$

<u>4,0 pto</u>

Propuesto $f(x) = \cosh(x)$

$$= \frac{e^x + e^{-x}}{2}$$

a) Largo curva en $[0,1]$ Foro (1)

b) Área manto en Revolución OX Foro (2)

$$\Omega = \{(x,y) \in \mathbb{R}^2 : x \in [0,1], 0 \le y \le f(x)\}$$

(Sox)

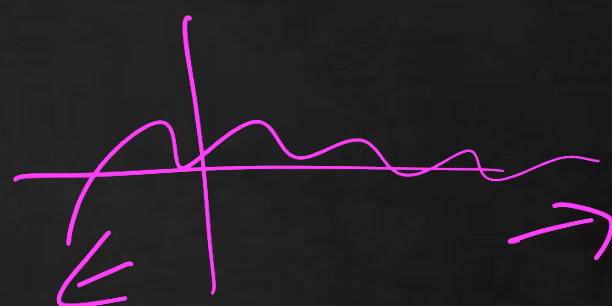
Integral Impropia

$$\int_{-\infty}^{\infty} \frac{1+x}{1+x^2}\,dx = \lim_{M\to\infty} \underbrace{\int_{-M}^{M} \frac{1+x}{1+x^2}\,dx}_{I}$$

$\lim I$

Especies Impropias

1° $$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{M\to+\infty} \int_{-M}^{M} f(x)\,dx$$



$$\left.\begin{array}{l} f(x): A \to B \\ \quad\downarrow \\ (a,+\infty) \quad (-\infty,a) \quad (-\infty,+\infty) \end{array}\right\} \text{Dominio no acotado}$$

2da Especie

Aditividad

$$\int_0^{40} \frac{1}{1-x}\,dx = \int_0^{1^-} \frac{1}{1-x}\,dx + \int_{1^+}^{40} \frac{1}{1-x}\,dx$$

$\frac{1}{1-x}$ se indetermina en $x=1$

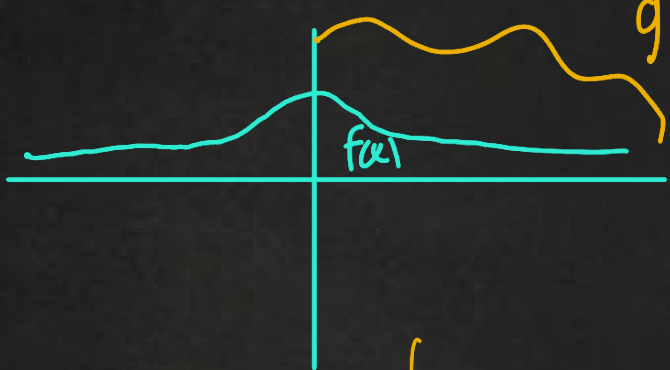
$\Rightarrow$ es integral 2da especie $x\in[0,40]$

3ra $\int_0^{+\infty} \frac{1}{1-x}\,dx$

Criterio Cociente

$$\lim \frac{f(x)}{g(x)} = L, \quad \left.\begin{array}{l} L>0 \\ \neq 0 \end{array}\right\} \text{ Converge}$$

1) Mayorar



g(x) → converge

→ converge

2) $\int |f|\,dx \Rightarrow \int f\,dx$

Converge

Convergencia

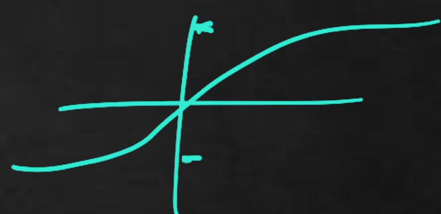
Ingeniería Matemática
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS
UNIVERSIDAD DE CHILE

Verifique que diverge

$$\pi = \int_{-M}^{M} \frac{1+x}{1+x^2} \overset{(?)}{=} \int_{-\infty}^{+\infty} \frac{1+x}{1+x^2}\,dx$$

$$= \int_{-M}^{M} \frac{1}{1+x^2} + \int_{-M}^{M} \frac{x}{1+x^2}$$

$$\int_{-M}^{M} \frac{1+x}{1+x^2} = \int_{-M}^{M} \frac{1}{1+x^2} + \cancel{\int_{-M}^{M} \frac{x}{x^2+1}} \to 0$$



$$= \int_{-M}^{M} \frac{1}{1+x^2} = \operatorname{arctg}(M) - \operatorname{arctg}(-M)$$

$$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$

$$\int_{-\infty}^{+\infty} \frac{1+x}{1+x^2} = \int_{-\infty}^{+\infty} \frac{1}{1+x^2} + \int_{-\infty}^{+\infty} \frac{x}{1+x^2} = \lim_{M\to\infty} \int_{-M}^{M} \left(\frac{1}{1+x^2} + \frac{x}{1+x^2}\right)$$

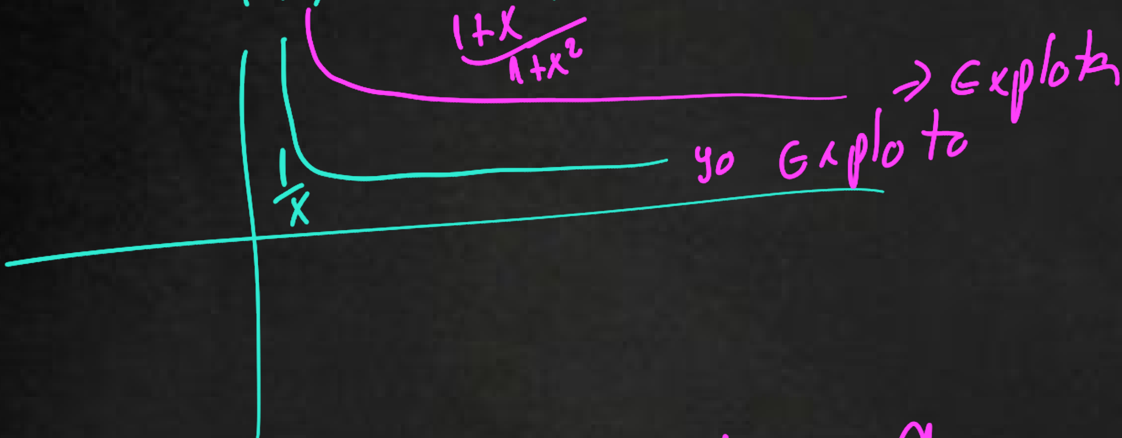
Diverge

$$\int_{-\infty}^{+\infty} \frac{1+x}{1+x^2}\,dx = \int_{-\infty}^{1} \frac{1+x}{1+x^2}\,dx + \int_{1}^{+\infty} \frac{1+x}{1+x^2}\,dx$$

todo Converge = converge + converge.

$$\frac{1+x}{1+x^2} \approx \frac{x}{x^2} = \frac{1}{x}$$

Diverge !



Criterio de Cociente $\frac{a_n}{b_n}$

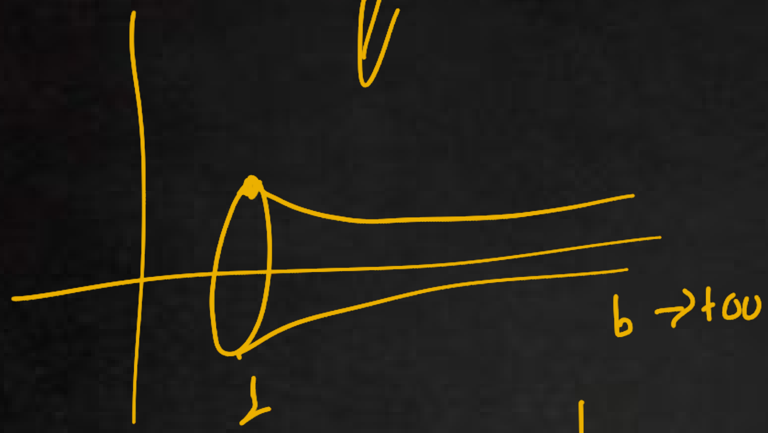
$$\lim_{x\to\infty} \frac{\frac{1+x}{1+x^2}}{1/x} = \lim_{x\to+\infty} \frac{(x+x^2)}{1+x^2}\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}+1}{\frac{1}{x^2}+1} = 1 > 0$$

$$\Rightarrow \quad L = 1 > 0 \quad \int_{1}^{+\infty} \frac{1+x}{1+x^2} \text{ tiene } \text{diverge}$$

$\Uparrow$

mismo comportamiento $\int_{1}^{+\infty} \frac{1}{x}\,dx$ diverge

$\frac{1}{x}$ $\quad x \in [1, b]$ Volumen finito



$$A = S_{ox} = 2\pi \int_1^b \frac{1}{x}\sqrt{1 + \left(\frac{1}{x}\right)'^2}\, dx$$

$$\left(-\frac{1}{x^2}\right)^2 = \frac{1}{x^4}$$

$$= \int_1^b \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\, dx$$

Criterio de comparación por cota

$A = S_{ox}$

↓

$$A = S_{0x} = 2\pi \lim_{b\to+\infty} \int_1^b \frac{1}{x}\sqrt{1+\overbrace{\frac{1}{x^4}}^{(+)}}\,dx$$

$$\geq 2\pi \lim_{b\to+\infty} \int_1^b \frac{1}{x}\sqrt{1+0}\,dx$$

$$\geq 2\pi \lim_{b\to+\infty} \int_1^b \frac{1}{x}\,dx$$

$$= 2\pi \lim_{b\to+\infty} \ln(b) - \cancel{\ln(1)}^{\,0}$$

$$= 2\pi \cdot \infty = +\infty \leftarrow$$

$\Rightarrow A = S_{0x}$ diverge.

$\lim_{n\to+\infty} \left(\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}\right)$

TAREA

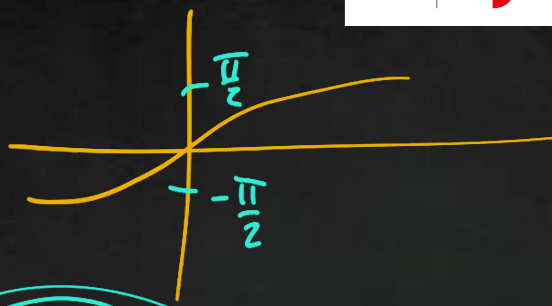
Rieman

$\exp(\ln(x)) = x$

$$y = \lim \left(\frac{n!}{n^n}\right)^{\frac{1}{n}} \quad / \ln(n)$$

.... $\int_0^1 \ln(x)\,dx$ 2da especie

$$\# \int_0^{\infty} \frac{\operatorname{arctg}(x)}{x^{3/2}}$$



$\downarrow$ Mixta

$$= \underbrace{\int_{0^+}^{1} \frac{\operatorname{arctg}(x)}{x^{3/2}}}_{2da} + \underbrace{\int_{1}^{+\infty} \frac{\operatorname{arctg}(x)}{x^{3/2}}}_{1ra}$$

$\frac{1}{x^{\alpha}}$	1ra	$\alpha > 1$	Converge
		$\alpha < 1$	diverge
	2da	$\alpha > 1$	diverge
		$\alpha < 1$	Converge

$$\int_{0^+}^{1} \frac{\operatorname{arctg}(x)}{x^{3/2}}\,dx \quad (2da) \quad , \quad \frac{1}{x^{1/2}} \qquad L = \lim_{x \to 0} \frac{\frac{\operatorname{arctg}(x)}{x^{3/2}}}{1/x^{1/2}} = 1 \neq 0$$

$$\int_{1}^{+\infty} \frac{\operatorname{arctg}(x)}{x^{3/2}} \quad (1ra) \quad , \quad \frac{1}{x^{3/2}} \qquad L = \lim_{x \to \infty} \frac{\frac{\operatorname{arctg}(x)}{x^{3/2}}}{1/x^{3/2}} = \frac{\pi}{2} \neq 0$$