

# **801 ejercicios resueltos de integral indefinida**

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## ABREVIATURAS DE USO FRECUENTE

|                     |                                |
|---------------------|--------------------------------|
| $e$ :               | Base de logaritmos neperianos. |
| $\ell\eta$ :        | Logaritmo natural o neperiano. |
| $\log$ :            | Logaritmo vulgar o de briggs.  |
| $\text{sen}$ :      | Seno.                          |
| $\arcsen$ :         | Arco seno.                     |
| $\cos$ :            | Coseno.                        |
| $\arccos$ :         | Arco coseno.                   |
| $\arccos$ :         | Arco coseno.                   |
| $\tau g$ :          | Tangente.                      |
| $\arctg$ :          | Arco tangente.                 |
| $\text{co}\tau g$ : | Cotangente.                    |
| $\text{arccotg}$ :  | Arco cotangente.               |
| $\sec$ :            | Secante.                       |
| $\text{arcsec}$ :   | Arco secante.                  |
| $\text{cosec}$ :    | Cosecante.                     |
| $\text{arcsec}$ :   | Arco cosecante.                |
| $\exp$ :            | Exponencial.                   |
| $dx$ :              | Diferencial de x.              |
| $ x $ :             | Valor absoluto de x.           |
| m.c.m:              | Mínimo común múltiplo.         |

## IDENTIFICACIONES USUALES

|                                      |                                 |
|--------------------------------------|---------------------------------|
| $\text{sen}^n x = (\text{sen } x)^n$ | $\text{sen}^{-1} x = \arcsen x$ |
| $\ell\eta^n x = (\ell\eta x)^n$      | $\log^n x = (\log x)^n$         |
| $\log x = \log  x $                  |                                 |

## IDENTIDADES ALGEBRAICAS

|                                                          |                                                                |
|----------------------------------------------------------|----------------------------------------------------------------|
| 1. Sean a, b: bases; m, n números naturales.             |                                                                |
| $a^m a^n = a^{m+n}$                                      | $(a^m)^n = a^{mn}$                                             |
| $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$                    | $(ab)^n = a^n b^n$                                             |
| $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ | $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ |
| $a^{-n} = \frac{1}{a^n}$                                 | $a^0 = 1, a \neq 0$                                            |

2. Sean a, b, c: bases; m, n números naturales

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 + b^3$$

$$(a \pm b)^4 = a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^{2n} - b^{2n} = (a^n + b^n)(a^n - b^n)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab \pm b^2)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)$$

3. Sean b, n, x, y, z: números naturales

$$\log(xyz) = \log_b x + \log_b y + \log_b z$$

$$\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$$

$$\log_b x^n = n \log_b x$$

$$\log_b \sqrt[n]{x} = \frac{1}{n} \log_b x$$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b e = 1$$

$$\log_b \exp x = x = x$$

$$\log_b e^x = x$$

$$e^{\log_b x} = x$$

$$\exp(\log_b x) = x$$

## IDENTIDADES TRIGONOMETRICAS

1.

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cos \theta \csc \theta = \cot \theta$$

$$\cos \theta \tan \theta = \sin \theta$$

2.

(a)

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

**(b)**

$$\begin{aligned}\cos(\alpha + \beta) &= \cos \alpha \cos \beta - \operatorname{sen} \alpha \operatorname{sen} \beta & \cos \frac{\alpha}{2} &= \pm \sqrt{\frac{1 + \cos \alpha}{2}} \\ \cos^2 \alpha &= \frac{1 + \cos 2\alpha}{2} & \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \operatorname{sen} \alpha \operatorname{sen} \beta \\ \cos 2\alpha &= \cos^2 \alpha - \operatorname{sen}^2 \alpha = 1 - 2\operatorname{sen}^2 \alpha = 2\cos^2 \alpha - 1\end{aligned}$$

**(c)**

$$\begin{aligned}\tau g(\alpha + \beta) &= \frac{\tau g \alpha + \tau g \beta}{1 - \tau g \alpha \tau g \beta} & \tau g 2\alpha &= \frac{2\tau g \alpha}{1 - \tau g^2 \alpha} \\ \tau g^2 \alpha &= \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} & \tau g(\alpha - \beta) &= \frac{\tau g \alpha - \tau g \beta}{1 + \tau g \alpha \tau g \beta} \\ \tau g \frac{\alpha}{2} &= \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} = \frac{\operatorname{sen} \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\operatorname{sen} \alpha}\end{aligned}$$

**(d)**

$$\begin{aligned}\operatorname{sen} \alpha \cos \beta &= \frac{1}{2} [\operatorname{sen}(\alpha + \beta) + \operatorname{sen}(\alpha - \beta)] & \cos \alpha \operatorname{sen} \beta &= \frac{1}{2} [\operatorname{sen}(\alpha + \beta) - \operatorname{sen}(\alpha - \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)] & \operatorname{sen} \alpha \operatorname{sen} \beta &= -\frac{1}{2} [\cos(\alpha + \beta) - \cos(\alpha - \beta)] \\ \operatorname{sen} \alpha + \operatorname{sen} \beta &= 2 \operatorname{sen} \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} & \operatorname{sen} \alpha - \operatorname{sen} \beta &= 2 \cos \frac{\alpha + \beta}{2} \operatorname{sen} \frac{\alpha - \beta}{2} \\ \cos \alpha + \cos \beta &= 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2} & \cos \alpha - \cos \beta &= -2 \operatorname{sen} \frac{\alpha + \beta}{2} \operatorname{sen} \frac{\alpha - \beta}{2}\end{aligned}$$

**(e)**

$$\begin{aligned}\arcsen(\operatorname{sen} x) &= x & \arccos(\cos x) &= x \\ \operatorname{arc} \tau g(\tau g x) &= x & \operatorname{arc} \operatorname{co} \tau g(\operatorname{co} \tau g x) &= x \\ \operatorname{arc} \sec(\sec x) &= x & \operatorname{arc} \operatorname{cosec}(\operatorname{cosec} x) &= x\end{aligned}$$

## FORMULAS FUNDAMENTALES

### Diferenciales

- 1.-  $du = \frac{du}{u} dx$
- 2.-  $d(au) = a du$
- 3.-  $d(u + v) = du + dv$
- 4.-  $d(u^n) = nu^{n-1} du$
- 5.-  $d(\ell \eta u) = \frac{du}{u}$
- 6.-  $d(e^u) = e^u du$
- 7.-  $d(a^u) = a^u \ell \eta a du$
- 8.-  $d(\text{sen } u) = \cos u du$
- 9.-  $d(\cos u) = -\text{sen } u du$
- 10.-  $d(\tau gu) = \sec^2 u du$
- 11.-  $d(\text{co } \tau gu) = -\text{cosec}^2 u du$
- 12.-  $d(\sec u) = \sec u \tau gu du$
- 13.-  $d(\text{co sec } u) = -\text{co sec } u \text{ co } \tau gu du$
- 14.-  $d(\arcsen u) = \frac{du}{\sqrt{1-u^2}}$
- 15.-  $d(\arccos u) = \frac{-du}{\sqrt{1-u^2}}$
- 16.-  $d(\arctan u) = \frac{du}{1+u^2}$
- 17.-  $d(\text{arccot } \tau gu) = \frac{-du}{1+u^2}$
- 18.-  $d(\text{arcsec } u) = \frac{du}{u\sqrt{u^2-1}}$
- 19.-  $d(\text{arccosec } u) = \frac{-du}{u\sqrt{u^2-1}}$

### Integrales

- 1.-  $\int du = u + c$
- 2.-  $\int a du = a \int du$
- 3.-  $\int (du + dv) = \int du + \int dv$
- 4.-  $\int u^n du = \frac{u^{n+1}}{n+1} + c (n \neq -1)$
- 5.-  $\int \frac{du}{u} = \ell \eta |u| + c$
- 6.-  $\int e^u du = e^u + c$
- 7.-  $\int a^u du = \frac{a^u}{\ell \eta a} + c$
- 8.-  $\int \cos u du = \text{sen } u + c$
- 9.-  $\int \text{sen } u du = -\cos u + c$
- 10.-  $\int \sec^2 u du = \tau gu + c$
- 11.-  $\int \text{cosec}^2 u du = -\text{co } \tau gu + c$
- 12.-  $\int \sec u \tau gu du = \sec u + c$
- 13.-  $\int \text{co sec } u \text{ co } \tau gu du = -\text{co sec } u + c$
- 14.-  $\int \frac{du}{\sqrt{1-u^2}} = \arcsen u + c$
- 15.-  $\int \frac{du}{\sqrt{1-u^2}} = -\arccos u + c$
- 16.-  $\int \frac{du}{1+u^2} = \arctan u + c$
- 17.-  $\int \frac{du}{1+u^2} = -\text{arccot } \tau gu + c$
- 18.-  $\int \frac{du}{u\sqrt{u^2-1}} = \begin{cases} \text{arcsec } u + c; u > 0 \\ -\text{arcsec } u + c; u < 0 \end{cases}$
- 19.-  $\int \frac{-du}{u\sqrt{u^2-1}} = \begin{cases} -\text{arccosec } u + c; u > 0 \\ \text{arccosec } u + c; u < 0 \end{cases}$

## OTRAS INTEGRALES INMEDIATAS

$$1.- \int \tau g u du = \begin{cases} \ell \eta |\sec u| + c \\ -\ell \eta |\cos u| + c \end{cases}$$

$$2.- \int \co \tau g u du = \ell \eta |\sec u| + c$$

$$3.- \int \sec u du = \begin{cases} \ell \eta |\sec u + \tau g u| + c \\ \ell \eta \left| \tau g u \left( \frac{u}{2} + \frac{\pi}{4} \right) \right| + c \end{cases}$$

$$4.- \int \co \sec u du = \ell \eta |\co \sec u - \co \tau g u| + c$$

$$5.- \int \sec u du = \cos hu + c$$

$$6.- \int \cos hu du = \sec hu + c$$

$$7.- \int \tau g hu du = \ell \eta |\cos hu| + c$$

$$8.- \int \co \tau g hu du = \ell \eta |\sec hu| + c$$

$$9.- \int \sec hu du = \arcc \tau gh(\sec hu) + c$$

$$10.- \int \co \sec hu du = -\arcc \co \tau gh(\cos hu) + c$$

$$11.- \int \frac{du}{\sqrt{a^2 - u^2}} = \begin{cases} \arcsen \frac{u}{a} + c \\ -\arcsen \frac{u}{a} + c \end{cases}$$

$$12.- \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ell \eta \left| u + \sqrt{u^2 \pm a^2} \right| + c$$

$$13.- \int \frac{du}{u^2 + a^2} = \begin{cases} \frac{1}{a} \arcc \tau g \frac{u}{a} + c \\ \frac{1}{a} \arcc \co \tau g \frac{u}{a} + c \end{cases}$$

$$14.- \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ell \eta \left| \frac{u-a}{u+a} \right| + c$$

$$15.- \int \frac{du}{u\sqrt{a^2 \pm u^2}} = \frac{1}{a} \ell \eta \left| \frac{u}{a + \sqrt{a^2 \pm u^2}} \right| + c$$

$$16.- \int \frac{du}{u\sqrt{u^2 - a^2}} = \begin{cases} \frac{1}{a} \arccos \frac{u}{a} + c \\ \frac{1}{a} \arccsec \frac{u}{a} + c \end{cases}$$

$$17.- \int \sqrt{u^2 \pm a^2} du = \frac{u}{2} \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ell \eta \left| u + \sqrt{u^2 \pm a^2} \right| + c$$

$$18.- \int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsen \frac{u}{a} + c$$

$$19.- \int e^{au} \sec bu du = \frac{e^{au} (a \sec bu - b \cos bu)}{a^2 + b^2} + c$$

$$20.- \int e^{au} \cos bu du = \frac{e^{au} (a \cos bu + b \sec bu)}{a^2 + b^2} + c$$

Realmente, algunas de estas integrales no son estrictamente inmediatas; tal como se verá mas adelante y donde se desarrollan varias de ellas.



# CAPITULO 1

## INTEGRALES ELEMENTALES

El Propósito de este capítulo, antes de conocer y practicar las técnicas propiamente tales; es familiarizarse con aquellas integrales para las cuales basta una transformación algebraica elemental.

## EJERCICIOS DESARROLLADOS

**1.1.- Encontrar:**  $\int e^{\ell \eta x^2} x dx$

Solución.- Se sabe que:  $e^{\ell \eta x^2} = x^2$

Por lo tanto:  $\int e^{\ell \eta x^2} x dx = \int x^2 x dx = \int x^3 dx = \frac{x^4}{4} + c$

**Respuesta:**  $\int e^{\ell \eta x^2} x dx = \frac{x^4}{4} + c$ ,      Fórmula utilizada:  $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$

**1.2.- Encontrar:**  $\int 3a^7 x^6 dx$

Solución.-

$$\int 3a^7 x^6 dx = 3a^7 \int x^6 dx = 3a^7 \frac{x^7}{7} + c$$

**Respuesta:**  $\int 3a^7 x^6 dx = 3a^7 \frac{x^7}{7} + c$ ,      Fórmula utilizada: del ejercicio anterior.

**1.3.- Encontrar:**  $\int (3x^2 + 2x + 1) dx$

Solución.-

$$\begin{aligned} \int (3x^2 + 2x + 1) dx &= \int (3x^2 + 2x + 1) dx = \int 3x^2 dx + \int 2x dx + \int dx \\ &= 3 \int x^2 dx + 2 \int x dx + \int dx = \cancel{3} \frac{x^3}{\cancel{3}} + \cancel{2} \frac{x^2}{\cancel{2}} + x + c = x^3 + x^2 + x + c \end{aligned}$$

**Respuesta:**  $\int (3x^2 + 2x + 1) dx = x^3 + x^2 + x + c$

**1.4.- Encontrar:**  $\int x(x+a)(x+b) dx$

Solución.-

$$\begin{aligned} \int x(x+a)(x+b) dx &= \int x[x^2 + (a+b)x + ab] dx = \int [x^3 + (a+b)x^2 + abx] dx \\ &= \int x^3 dx + \int (a+b)x^2 dx + \int abx dx = \int x^3 dx + (a+b) \int x^2 dx + ab \int x dx \\ &= \frac{x^4}{4} + (a+b) \frac{x^3}{3} + ab \frac{x^2}{2} + c \end{aligned}$$

**Respuesta:**  $\int x(x+a)(x+b)dx = \frac{x^4}{4} + \frac{(a+b)x^3}{3} + \frac{abx^2}{2} + c$

**1.5.- Encontrar:**  $\int (a+bx^3)^2 dx$

Solución.-

$$\begin{aligned}\int (a+bx^3)^2 dx &= \int (a^2 + 2abx^3 + b^2x^6)dx = \int a^2 dx + \int 2abx^3 dx + \int b^2x^6 dx \\ &= a^2 \int dx + 2ab \int x^3 dx + b^2 \int x^6 dx = a^2x + 2ab \frac{x^4}{4} + b^2 \frac{x^7}{7} + c\end{aligned}$$

**Respuesta:**  $\int (a+bx^3)^2 dx = a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7} + c$

**1.6.- Encontrar:**  $\int \sqrt{2px} dx$

Solución.-

$$\int \sqrt{2px} dx = \int \sqrt{2p} x^{\frac{1}{2}} dx = \sqrt{2p} \int x^{\frac{1}{2}} dx = \sqrt{2p} \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2\sqrt{2p}x^{\frac{3}{2}}}{3} + c$$

**Respuesta:**  $\int \sqrt{2px} dx = \frac{2\sqrt{2p}x\sqrt{x}}{3} + c$

**1.7.-Encontrar:**  $\int \frac{dx}{\sqrt[n]{x}}$

Solución.-

$$\int \frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}} dx = \frac{x^{-\frac{1}{n}+1}}{-\frac{1}{n}+1} + c = \frac{x^{\frac{-1+n}{n}}}{\frac{-1+n}{n}} + c = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt[n]{x}} = \frac{nx^{\frac{-1+n}{n}}}{n-1} + c$

**1.8.- Encontrar:**  $\int (nx)^{\frac{1-n}{n}} dx$

Solución.-

$$\begin{aligned}\int (nx)^{\frac{1-n}{n}} dx &= \int n^{\frac{1-n}{n}} x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1-n}{n}} dx = n^{\frac{1-n}{n}} \int x^{\frac{1}{n}-1} dx \\ &= n^{\frac{1-n}{n}} \frac{x^{\frac{1}{n}-1+1}}{\frac{1}{n}-1+1} + c = n^{\frac{1-n}{n}} \frac{x^{\frac{1}{n}}}{\frac{1}{n}} + c = n^{\frac{1-n}{n}} nx^{\frac{1}{n}} + c = n^{\frac{1-n}{n}+1} x^{\frac{1}{n}} + c = n^{\frac{1-n+n}{n}} x^{\frac{1}{n}} + c = n^{\frac{1}{n}} x^{\frac{1}{n}} + c\end{aligned}$$

**Respuesta:**  $\int (nx)^{\frac{1-n}{n}} dx = \sqrt[n]{nx} + c$

**1.9.- Encontrar:**  $\int (a^{\frac{2}{3}} - x^{\frac{2}{3}})^3 dx$

Solución.-

$$\int (a^{\frac{2}{3}} - x^{\frac{2}{3}})^3 dx = \int \left[ \left(a^{\frac{2}{3}}\right)^3 - 3\left(a^{\frac{2}{3}}\right)^2 x^{\frac{2}{3}} + 3a^{\frac{2}{3}} \left(x^{\frac{2}{3}}\right)^2 - \left(x^{\frac{2}{3}}\right)^3 \right] dx$$

$$\begin{aligned}
&= \int (a^2 - 3a^{\frac{4}{3}}x^{\frac{2}{3}} + 3a^{\frac{2}{3}}x^{\frac{4}{3}} - x^2)dx = \int a^2 dx - \int 3a^{\frac{4}{3}}x^{\frac{2}{3}}dx + \int 3a^{\frac{2}{3}}x^{\frac{4}{3}}dx - \int x^2 dx \\
&= a^2 \int dx - 3a^{\frac{4}{3}} \int x^{\frac{2}{3}}dx + 3a^{\frac{2}{3}} \int x^{\frac{4}{3}}dx - \int x^2 dx = a^2 x - 3a^{\frac{4}{3}} \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 3a^{\frac{2}{3}} \frac{x^{\frac{7}{3}}}{\frac{7}{3}} - \frac{x^3}{3} + c \\
&= a^2 x - \frac{9a^{\frac{4}{3}}x^{\frac{5}{3}}}{5} + \frac{9a^{\frac{2}{3}}x^{\frac{7}{3}}}{7} - \frac{x^3}{3} + c
\end{aligned}$$

**Respuesta:**  $\int (a^{\frac{2}{3}} - x^{\frac{2}{3}})^3 dx = a^2 x - \frac{9a^{\frac{4}{3}}x^{\frac{5}{3}}}{5} + \frac{9a^{\frac{2}{3}}x^{\frac{7}{3}}}{7} - \frac{x^3}{3} + c$

**1.10.- Encontrar:**  $\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx$

Solución.-

$$\begin{aligned}
&\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx = \int (x\sqrt{x} - (\sqrt{x})^2 + \sqrt{x} + \cancel{x} - \cancel{\sqrt{x}} + 1)dx \\
&= \int (x\sqrt{x}+1)dx = \int (xx^{\frac{1}{2}}+1)dx = \int (x^{\frac{3}{2}}+1)dx = \int x^{\frac{3}{2}}dx + \int dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + x + c = \frac{2x^{\frac{5}{2}}}{5} + x + c
\end{aligned}$$

**Respuesta:**  $\int (\sqrt{x}+1)(x-\sqrt{x}+1)dx = \frac{2x^{\frac{5}{2}}}{5} + x + c$

**1.11.- Encontrar:**  $\int \frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}}$

Solución.-

$$\begin{aligned}
&\int \frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}} = \int \frac{(x^4-x^2-2)dx}{x^{\frac{2}{3}}} = \int \frac{x^4}{x^{\frac{2}{3}}}dx - \int \frac{x^2}{x^{\frac{2}{3}}}dx - \int \frac{2}{x^{\frac{2}{3}}}dx \\
&= \int x^{\frac{10}{3}}dx - \int x^{\frac{4}{3}}dx - 2 \int x^{-\frac{2}{3}}dx = \frac{x^{\frac{10}{3}+1}}{\frac{10}{3}+1} - \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} - 2 \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} = \frac{x^{\frac{13}{3}}}{\frac{13}{3}} - \frac{x^{\frac{7}{3}}}{\frac{7}{3}} - 2 \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c \\
&= 3 \frac{x^{\frac{13}{3}}}{13} - 3 \frac{x^{\frac{7}{3}}}{7} - 6x^{\frac{1}{3}} + c = 3 \frac{\sqrt[3]{x^{13}}}{13} - 3 \frac{\sqrt[3]{x^7}}{7} - 6\sqrt[3]{x} + c = 3 \frac{x^4 \sqrt[3]{x}}{13} - 3 \frac{x^2 \sqrt[3]{x}}{7} - 6\sqrt[3]{x} + c
\end{aligned}$$

**Respuesta:**  $\int \frac{(x^2+1)(x^2-2)dx}{\sqrt[3]{x^2}} = \left( \frac{3x^4}{13} - \frac{3x^2}{7} - 6 \right) \sqrt[3]{x} + c$

**1.12.- Encontrar:**  $\int \frac{(x^m-x^n)^2}{\sqrt{x}}dx$

Solución.-

$$\begin{aligned}
&\int \frac{(x^m-x^n)^2}{\sqrt{x}}dx = \int \frac{(x^{2m}-2x^m x^n+x^{2n})}{\sqrt{x}}dx = \int \frac{(x^{2m}-2x^m x^n+x^{2n})}{x^{1/2}}dx \\
&= \int (x^{2m-1/2}-2x^{m+n-1/2}+x^{2n-1/2})dx = \frac{x^{2m-1/2+1}}{2m-1/2+1} - \frac{2x^{m+n+1/2}}{m+n+1/2} + \frac{x^{2n+1/2}}{2n+1/2} + c \\
&= \frac{x^{\frac{4m+1}{2}}}{\frac{4m+1}{2}} - \frac{2x^{\frac{2m+2n+1}{2}}}{\frac{2m+2n+1}{2}} + \frac{x^{\frac{4n+1}{2}}}{\frac{4n+1}{2}} + c = \frac{2x^{\frac{4m+1}{2}}}{4m+1} - \frac{4x^{\frac{2m+2n+1}{2}}}{2m+2n+1} + \frac{2x^{\frac{4n+1}{2}}}{4n+1} + c
\end{aligned}$$

$$= \frac{2x^{2m}\sqrt{x}}{4m+1} - \frac{4x^{m+n}\sqrt{x}}{2m+2n+1} + \frac{2x^{2n}\sqrt{x}}{4n+1} + c$$

**Respuesta:**  $\int \frac{(x^m - x^n)^2}{\sqrt{x}} dx = \sqrt{x} \left( \frac{2x^{2m}}{4m+1} - \frac{4x^{m+n}}{2m+2n+1} + \frac{2x^{2n}}{4n+1} \right) + c$

**1.13.- Encontrar:**  $\int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx$

Solución.-

$$\begin{aligned} \int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx &= \int \frac{a^2 - 4a\sqrt{ax} + 6xa - 4x\sqrt{ax} + x^2}{\sqrt{ax}} dx \\ &= \int \frac{a^2}{(ax)^{1/2}} dx - \int \frac{4a\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{6ax}{(ax)^{1/2}} dx - \int \frac{4x\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{x^2}{(ax)^{1/2}} dx \\ &= \int a^2 a^{-1/2} x^{-1/2} dx - \int 4a dx + \int 6a a^{-1/2} x x^{-1/2} dx - \int 4x dx + \int a^{-1/2} x^2 x^{-1/2} dx \\ &= a^{3/2} \int x^{-1/2} dx - 4a \int dx + 6a^{1/2} \int x^{1/2} dx - 4 \int x dx + a^{-1/2} \int x^{3/2} dx \\ &= a^{3/2} \frac{x^{-1/2+1}}{\frac{-1}{2}+1} - 4ax + 6a^{1/2} \frac{x^{1/2+1}}{\frac{1}{2}+1} - 4 \frac{x^{1+1}}{1+1} + a^{-1/2} \frac{x^{3/2+1}}{\frac{3}{2}+1} + c \\ &= a^{3/2} \frac{x^{1/2}}{\frac{1}{2}} - 4ax + 6a^{1/2} \frac{x^{3/2}}{\frac{3}{2}} - 4 \frac{x^2}{2} + a^{-1/2} \frac{x^{5/2}}{\frac{5}{2}} + c \\ &= 2a^{3/2} x^{1/2} - 4ax + 4a^{1/2} x^{3/2} - 2x^2 + 2a^{-1/2} \frac{x^{5/2}}{5} + c \end{aligned}$$

**Respuesta:**  $\int \frac{(\sqrt{a} - \sqrt{x})^4}{\sqrt{ax}} dx = 2a^{3/2} x^{1/2} - 4ax + 4a^{1/2} x^{3/2} - 2x^2 + \frac{2x^3}{5\sqrt{xa}} + c$

**1.14.- Encontrar:**  $\int \frac{dx}{x^2 - 10}$

Solución.-

Sea:  $a = \sqrt{10}$ , Luego:  $\int \frac{dx}{x^2 - 10} = \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ell \eta \left| \frac{x-a}{x+a} \right| + c$

$$= \frac{1}{2\sqrt{10}} \ell \eta \left| \frac{x - \sqrt{10}}{x + \sqrt{10}} \right| + c = \frac{\sqrt{10}}{20} \ell \eta \left| \frac{x - \sqrt{10}}{x + \sqrt{10}} \right| + c$$

**Respuesta:**  $\int \frac{dx}{x^2 - 10} = \frac{\sqrt{10}}{20} \ell \eta \left| \frac{x - \sqrt{10}}{x + \sqrt{10}} \right| + c$

**1.15.- Encontrar:**  $\int \frac{dx}{x^2 + 7}$

Solución.- Sea:  $a = \sqrt{7}$ , Luego:  $\int \frac{dx}{x^2 + 7} = \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + c$

$$\frac{1}{\sqrt{7}} \operatorname{arctg} \frac{x}{\sqrt{7}} + c = \frac{\sqrt{7}}{7} \operatorname{arctg} \frac{\sqrt{7}x}{a} + c$$

**Respuesta:**  $\int \frac{dx}{x^2+7} = \frac{\sqrt{7}}{7} \operatorname{arctg} \frac{\sqrt{7}x}{a} + c$

**1.16.- Encontrar:**  $\int \sqrt{\frac{dx}{4+x^2}}$

Solución.-

Sea:  $a = 2$ , Luego:  $\int \frac{dx}{\sqrt{4+x^2}} = \int \frac{dx}{\sqrt{a^2+x^2}} = \ell \eta \left| x + \sqrt{a^2+x^2} \right| + c$   
 $= \ell \eta \left| x + \sqrt{4+x^2} \right| + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{4+x^2}} = \ell \eta \left| x + \sqrt{4+x^2} \right| + c$

**1.17.- Encontrar:**  $\int \frac{dx}{\sqrt{8-x^2}}$

Solución.-

Sea:  $a = \sqrt{8}$ , Luego:  $\int \frac{dx}{\sqrt{8-x^2}} = \int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{arcsen} \frac{x}{a} + c$   
 $= \operatorname{arcsen} \frac{x}{\sqrt{8}} + c = \operatorname{arcsen} \frac{x}{2\sqrt{2}} + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{8-x^2}} = \operatorname{arcsen} \frac{\sqrt{2}x}{4} + c$

**1.18.- Encontrar:**  $\int \frac{dy}{x^2+9}$

Solución.-

La expresión:  $\frac{1}{x^2+9}$  actúa como constante, luego:

$$\int \frac{dy}{x^2+9} = \frac{1}{x^2+9} \int dy = \frac{1}{x^2+9} y + c = \frac{y}{x^2+9} + c$$

**Respuesta:**  $\int \frac{dy}{x^2+9} = \frac{y}{x^2+9} + c$

**1.19.- Encontrar:**  $\int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx$

Solución.-

$$\begin{aligned} \int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx &= \int \sqrt{\frac{2+x^2}{4-x^4}} dx - \int \sqrt{\frac{2-x^2}{4-x^4}} dx \\ &= \int \sqrt{\frac{\cancel{2}+x^2}{(2-x^2)(\cancel{2}+x^2)}} dx - \int \sqrt{\frac{\cancel{2}-x^2}{(\cancel{2}-x^2)(2+x^2)}} dx = \int \frac{dx}{\sqrt{2-x^2}} - \int \frac{dx}{\sqrt{2+x^2}} \end{aligned}$$

Sea:  $a = \sqrt{2}$ , Luego:  $\int \frac{dx}{\sqrt{a^2 - x^2}} - \int \frac{dx}{\sqrt{a^2 + x^2}} = \arcsen \frac{x}{a} - \ell \eta \left| x + \sqrt{a^2 + x^2} \right| + c$

$$= \arcsen \frac{x}{\sqrt{2}} - \ell \eta \left| x + \sqrt{(\sqrt{2})^2 + x^2} \right| + c = \arcsen \frac{x}{\sqrt{2}} - \ell \eta \left| x + \sqrt{2 + x^2} \right| + c$$

**Respuesta:**  $\int \frac{\sqrt{2+x^2} - \sqrt{2-x^2}}{\sqrt{4-x^4}} dx = \arcsen \frac{x}{\sqrt{2}} - \ell \eta \left| x + \sqrt{2+x^2} \right| + c$

**1.20.- Encontrar:**  $\int \tau g^2 x dx$

Solución.-

$$\int \tau g^2 x dx = \int (\sec^2 x - 1) dx = \int \sec^2 x dx - \int dx = \tau g x - x + c$$

**Respuesta:**  $\int \tau g^2 x dx = \tau g x - x + c$

**1.21.- Encontrar:**  $\int \text{co } \tau g^2 x dx$

Solución.-

$$\int \text{co } \tau g^2 x dx = \int (\cos ec^2 x - 1) dx = \int \cos ec^2 x dx - \int dx = -\text{co } \tau g x - x + c$$

**Respuesta:**  $\int \text{co } \tau g^2 x dx = -\text{co } \tau g x - x + c$

**1.22.- Encontrar:**  $\int \frac{dx}{2x^2 + 4}$

Solución.-

$$\int \frac{dx}{2x^2 + 4} = \int \frac{dx}{2(x^2 + 2)} = \frac{1}{2} \int \frac{dx}{x^2 + 2} = \frac{1}{2} \frac{1}{\sqrt{2}} \arctan g \frac{x}{\sqrt{2}} + c = \frac{\sqrt{2}}{4} \arctan g \frac{\sqrt{2}x}{2} + c$$

**Respuesta:**  $\int \frac{dx}{2x^2 + 4} = \frac{\sqrt{2}}{4} \arctan g \frac{\sqrt{2}x}{2} + c$

**1.23.- Encontrar:**  $\int \frac{dx}{7x^2 - 8}$

Solución.-

$$\int \frac{dx}{7x^2 - 8} = \int \frac{dx}{7(x^2 - \frac{8}{7})} = \int \frac{dx}{7 \left[ x^2 - \left( \sqrt{\frac{8}{7}} \right)^2 \right]} = \frac{1}{7} \int \frac{dx}{\left[ x^2 - \left( \sqrt{\frac{8}{7}} \right)^2 \right]}$$

$$= \frac{1}{7} \frac{1}{2 \left( \sqrt{\frac{8}{7}} \right)} \ell \eta \left| \frac{x - \sqrt{\frac{8}{7}}}{x + \sqrt{\frac{8}{7}}} \right| + c = \frac{1}{14 \frac{\sqrt{8}}{\sqrt{7}}} \ell \eta \left| \frac{x - \sqrt{\frac{8}{7}}}{x + \sqrt{\frac{8}{7}}} \right| + c = \frac{\sqrt{7}}{14\sqrt{8}} \ell \eta \left| \frac{\sqrt{7}x - \sqrt{8}}{\sqrt{7}x + \sqrt{8}} \right| + c$$

$$= \frac{1}{4\sqrt{14}} \ell \eta \left| \frac{\sqrt{7}x - 2\sqrt{2}}{\sqrt{7}x + 2\sqrt{2}} \right| + c = \frac{\sqrt{14}}{56} \ell \eta \left| \frac{\sqrt{7}x - 2\sqrt{2}}{\sqrt{7}x + 2\sqrt{2}} \right| + c$$

**Respuesta:**  $\int \frac{dx}{7x^2 - 8} = \frac{\sqrt{14}}{56} \ell \eta \left| \frac{\sqrt{7}x - 2\sqrt{2}}{\sqrt{7}x + 2\sqrt{2}} \right| + c$

**1.24.- Encontrar:**  $\int \frac{x^2 dx}{x^2 + 3}$

Solución.-

$$\int \frac{x^2 dx}{x^2 + 3} = \int \left(1 - \frac{3}{x^2 + 3}\right) dx = \int dx - 3 \int \frac{dx}{x^2 + 3} = \int dx - 3 \int \frac{dx}{x^2 + (\sqrt{3})^2}$$

$$= x - 3 \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + c = x - \sqrt{3} \operatorname{arctg} \frac{\sqrt{3}x}{3} + c$$

**Respuesta:**  $\int \frac{x^2 dx}{x^2 + 3} = x - \sqrt{3} \operatorname{arctg} \frac{\sqrt{3}x}{3} + c$

**1.25.- Encontrar:**  $\int \frac{dx}{\sqrt{7+8x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7+8x^2}} = \int \frac{dx}{\sqrt{(\sqrt{8}x)^2 + (\sqrt{7})^2}} = \frac{1}{\sqrt{8}} \ell \eta \left| \sqrt{8}x + \sqrt{7+8x^2} \right| + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt{7+8x^2}} = \frac{\sqrt{2}}{4} \ell \eta \left| \sqrt{8}x + \sqrt{7+8x^2} \right| + c$

**1.26.- Encontrar:**  $\int \frac{dx}{\sqrt{7-5x^2}}$

Solución.-

$$\int \frac{dx}{\sqrt{7-5x^2}} = \int \frac{dx}{\sqrt{(\sqrt{7})^2 - (\sqrt{5}x)^2}} = \frac{1}{\sqrt{5}} \operatorname{arcsen} x \frac{\sqrt{5}}{\sqrt{7}} + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt{7-5x^2}} = \frac{\sqrt{5}}{5} \operatorname{arcsen} \frac{\sqrt{35}x}{7} + c$

**1.27.- Encontrar:**  $\int \frac{(a^x - b^x)^2 dx}{a^x b^x}$

Solución.-

$$\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \int \frac{(a^{2x} - 2a^x b^x + b^{2x})}{a^x b^x} dx = \int \frac{a^{2x}}{a^x b^x} dx - \int \frac{2a^x b^x}{a^x b^x} dx + \int \frac{b^{2x}}{a^x b^x} dx$$

$$= \int \frac{a^x}{b^x} dx - \int 2 dx + \int \frac{b^x}{a^x} dx = \int \left(\frac{a}{b}\right)^x dx - 2 \int dx + \int \left(\frac{b}{a}\right)^x dx = \frac{(a/b)^x}{\ell \eta \frac{a}{b}} - 2x + \frac{(b/a)^x}{\ell \eta \frac{b}{a}} + c$$

$$= \frac{(a/b)^x}{\ell \eta a - \ell \eta b} - 2x + \frac{(b/a)^x}{\ell \eta b - \ell \eta a} + c = \frac{(a/b)^x}{\ell \eta a - \ell \eta b} - 2x - \frac{(b/a)^x}{\ell \eta a - \ell \eta b} + c$$

$$= \frac{\left(\frac{a^x}{b^x} - \frac{b^x}{a^x}\right)}{\ell \eta a - \ell \eta b} - 2x + c$$

**Respuesta:**  $\int \frac{(a^x - b^x)^2 dx}{a^x b^x} = \frac{\left(\frac{a^{2x} - b^{2x}}{a^x b^x}\right)}{\ell \eta a - \ell \eta b} - 2x + c$

**1.28.- Encontrar:**  $\int \operatorname{sen}^2 \frac{x}{2} dx$

Solución.-

$$\begin{aligned} \int \operatorname{sen}^2 \frac{x}{2} dx &= \int \frac{1 - \cos \cancel{\frac{x}{2}}}{2} dx = \int \frac{1 - \cos x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx \\ &= \frac{x}{2} - \frac{\operatorname{sen} x}{2} + c \end{aligned}$$

**Respuesta:**  $\int \operatorname{sen}^2 \frac{x}{2} dx = \frac{x}{2} - \frac{\operatorname{sen} x}{2} + c$

**1.29.- Encontrar:**  $\int \frac{dx}{(a+b) + (a-b)x^2}; (0 < b < a)$

Solución.-

$$\begin{aligned} \text{Sea: } c^2 &= a+b, \quad d^2 = a-b, ; \text{ luego } \int \frac{dx}{(a+b) + (a-b)x^2} = \int \frac{dx}{c^2 + d^2 x^2} \\ \int \frac{dx}{d^2 \left( \frac{c^2}{d^2} + x^2 \right)} &= \frac{1}{d^2} \int \frac{dx}{\left( \frac{c}{d} \right)^2 + x^2} = \frac{1}{d^2} \frac{1}{\cancel{\frac{c}{d}}} \operatorname{arctg} \frac{x}{\cancel{c/d}} + c = \frac{1}{cd} \operatorname{arctg} \frac{dx}{c} + c \\ &= \frac{1}{\sqrt{a+b}\sqrt{a-b}} \operatorname{arctg} \frac{\sqrt{a-b}x}{\sqrt{a+b}} + c = \frac{1}{\sqrt{a^2-b^2}} \operatorname{arctg} \sqrt{\frac{a-b}{a+b}} x + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{(a+b) + (a-b)x^2} = \frac{1}{\sqrt{a^2-b^2}} \operatorname{arctg} \sqrt{\frac{a-b}{a+b}} x + c$

**1.30.-Encontrar:**  $\int \frac{dx}{(a+b) - (a-b)x^2}; (0 < b < a)$

Solución.-

$$\begin{aligned} \text{Sea: } c^2 &= a+b, \quad d^2 = a-b, \text{ Luego: } \int \frac{dx}{(a+b) - (a-b)x^2} = \int \frac{dx}{c^2 - d^2 x^2} \\ &= \int \frac{dx}{d^2 \left( \frac{c^2}{d^2} - x^2 \right)} = \frac{1}{d^2} \int \frac{dx}{\left( \frac{c}{d} \right)^2 - x^2} = -\frac{1}{d^2} \frac{1}{\cancel{\frac{2c}{d}}} \ell \eta \left| \frac{x - \cancel{c/d}}{x + \cancel{c/d}} \right| + c = -\frac{1}{2cd} \ell \eta \left| \frac{dx - c}{dx + c} \right| + c \\ &= -\frac{1}{2\sqrt{a^2-b^2}} \ell \eta \left| \frac{\sqrt{a-b}x - \sqrt{a+b}}{\sqrt{a-b}x + \sqrt{a+b}} \right| + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{(a+b) - (a-b)x^2} = -\frac{1}{2\sqrt{a^2-b^2}} \ell \eta \left| \frac{\sqrt{a-b}x - \sqrt{a+b}}{\sqrt{a-b}x + \sqrt{a+b}} \right| + c$

**1.31.- Encontrar:**  $\int \left[ (a^{2x})^0 - 1 \right] dx$

Solución.-



$$\int \left[ \left( a^{2x} \right)^0 - 1 \right] dx = \int (a^0 - 1) dx = \int (1 - 1) dx = \int dx - \int dx = \int 0 dx = c$$

**Respuesta:**  $\int \left[ \left( a^{2x} \right)^0 - 1 \right] dx = c$

## EJERCICIOS PROPUESTOS

Mediante el uso del álgebra elemental, o algunas identidades trigonométricas, transformar en integrales de fácil solución, las integrales que se presentan a continuación.

**1.32.-**  $\int 3x^5 dx$

**1.35.-**  $\int \cos^2 \frac{x}{2} dx$

**1.38.-**  $\int \frac{1 + \frac{\sqrt{x}}{2}}{1 + \frac{\sqrt{x}}{3}} dy$

**1.41.-**  $\int \frac{dx}{\sqrt{x^2 + 5}}$

**1.44.-**  $\int (\sec^2 x + \cos^2 x - 1) dx$

**1.47.-**  $\int \frac{dx}{x^2 - 12}$

**1.50.-**  $\int \frac{dx}{\sqrt{x^2 + 12}}$

**1.53.-**  $\int \frac{dx}{x\sqrt{12 - x^2}}$

**1.56.-**  $\int \frac{dx}{\sqrt{2x^2 - 8}}$

**1.59.-**  $\int \sqrt{x^2 + 10} dx$

**1.62.-**  $\int \sqrt{1 - \sec^2 x} dx$

**1.65.-**  $\int (2^0 - 3^0)^n dx$

**1.68.-**  $\int \sqrt{\frac{3}{4} - x^2} dx$

**1.71.-**  $\int \frac{dx}{x\sqrt{3 - x^2}}$

**1.74.-**  $\int \sec^{3x} \theta dy$

**1.77.-**  $\int e^{\ell \eta x^2} dx$

**1.80.-**  $\int \sqrt{x^2 - 11} dx$

**1.33.-**  $\int (1 + e)^x dx$

**1.36.-**  $\int (1 + \sqrt{x})^3 dx$

**1.39.-**  $\int \frac{dx}{\sqrt{5 - x^2}}$

**1.42.-**  $\int \frac{dx}{x^2 + 5}$

**1.45.-**  $\int \sqrt{x}(1 - \sqrt{x}) dx$

**1.48.-**  $\int \frac{dx}{x^2 + 12}$

**1.51.-**  $\int \frac{dx}{\sqrt{12 - x^2}}$

**1.54.-**  $\int \frac{dx}{x\sqrt{12 + x^2}}$

**1.57.-**  $\int \frac{dx}{\sqrt{2x^2 + 8}}$

**1.60.-**  $\int \sqrt{10 - x^2} dx$

**1.63.-**  $\int \sqrt{1 - \cos^2 x} dx$

**1.66.-**  $\int \left( \tau g x - \frac{\sec x}{\cos x} \right) dx$

**1.69.-**  $\int \sqrt{x^2 - \frac{3}{4}} dx$

**1.72.-**  $\int \frac{dx}{x\sqrt{x^2 - 3}}$

**1.75.-**  $\int \ell \eta |u| dx$

**1.78.-**  $\int \frac{\sqrt{x} - \sqrt{2}}{\sqrt{2x}} dx$

**1.81.-**  $\int \sqrt{x^2 + 11} dx$

**1.34.-**  $\int (1 + \tau g x) dx$

**1.37.-**  $\int (1 + \sqrt{x})^0 dx$

**1.40.-**  $\int \frac{dx}{\sqrt{x^2 - 5}}$

**1.43.-**  $\int \frac{dx}{x^2 - 5}$

**1.46.-**  $\int (\tau g^2 x + 1) dx$

**1.49.-**  $\int \frac{dx}{\sqrt{x^2 - 12}}$

**1.52.-**  $\int \frac{dx}{x\sqrt{x^2 - 12}}$

**1.55.-**  $\int \frac{dx}{\sqrt{8 - 2x^2}}$

**1.58.-**  $\int \sqrt{x^2 - 10} dx$

**1.61.-**  $\int \frac{1 - \cos^2 x}{\sec^2 x} dx$

**1.64.-**  $\int (2^x - 3^x)^0 dx$

**1.67.-**  $\int \frac{dx}{3^{-x}}$

**1.70.-**  $\int \sqrt{x^2 + \frac{3}{4}} dx$

**1.73.-**  $\int \frac{dx}{x\sqrt{x^2 + 3}}$

**1.76.-**  $\int \exp(\ell \eta x) dx$

**1.79.-**  $\int \sqrt{11 - x^2} dx$

**1.82.-**  $\int \ell \eta (e^{\sqrt{x}}) dx$

$$\begin{array}{lll}
1.83.- \int \left[ \frac{1+\sqrt{x}+\sqrt{x^3}}{1-\sqrt{x}} \right] dx & 1.84.- \int (\tau g^2 x + \sec^2 x - 1) dx & 1.85.- \int \frac{dx}{\sqrt{3x^2-1}} \\
1.86.- \int (\cos \tau g \theta - \operatorname{sen} \theta) dx & 1.87.- \int \frac{dx}{\sqrt{1+3x^2}} & 1.88.- \int \frac{dx}{\sqrt{1-3x^2}} \\
1.89.- \int \frac{dx}{1+3x^2} & 1.90.- \int \frac{dx}{3x^2+4} & 1.91.- \int \frac{dx}{3x^2-1} \\
1.92.- \int \frac{dx}{x\sqrt{3x^2-1}} & 1.93.- \int \frac{dx}{x\sqrt{1+3x^2}} & 1.94.- \int \frac{dx}{x\sqrt{1-3x^2}} \\
1.95.- \int \sqrt{1-3x^2} dx & 1.96.- \int \sqrt{1+3x^2} dx & 1.97.- \int \sqrt{3x^2-1} dx \\
1.98.- \int (3x^2-1) dx & 1.99.- \int (3x^2-1)^0 dx & 1.100.- \int (3x^2-1)^n du \\
1.101.- \int \exp(\ell \eta^{\frac{\sqrt{x}}{3}}) dx & 1.102.- \int \ell \eta (e^{\frac{2x-1}{2}}) dx & 1.103.- \int (e^2 + e + 1)^x dx \\
1.104.- \int \left( \frac{1+\tau g^2 x}{\sec^2 x} - 1 \right) dx & 1.105.- \int \exp(\ell \eta |1+x|) dx & 1.106.- \int \sqrt{27-x^2} dx \\
1.107.- \int \sqrt{x^2-27} dx & 1.108.- \int \sqrt{x^2+27} dx & 1.109.- \int \frac{dx}{3x\sqrt{x^2-1}} \\
1.110.- \int \frac{dx}{2x\sqrt{1-x^2}} & 1.111.- \int \frac{dx}{5x\sqrt{x^2+1}} & 1.112.- \int \frac{dx}{3x\sqrt{9-x^2}} \\
1.113.- \int \frac{dx}{4x\sqrt{x^2+16}} & 1.114.- \int \frac{dx}{5x\sqrt{x^2-25}} & 1.115.- \int \frac{(1-\sqrt{x})^2}{x^2} dx \\
1.116.- \int (1+\sqrt{x}+x)^2 dx & 1.117.- \int (1-\sqrt{x}+x)^2 dx & 1.118.- \int (1+x)^4 dx \\
1.119.- \int e^{\ell \eta \left| \frac{1-\cos x}{2} \right|} dx & 1.120.- \int \exp \ell \eta \left( \frac{1+x^2}{x^2} \right) dx & 1.121.- \int \ell \eta e^{\frac{1-\operatorname{sen} x}{3}} dx \\
1.122.- \int (1+\sqrt{x-3x})^0 dx & 1.123.- \int \ell \eta e^{\frac{(1+x)^2}{2}} dx &
\end{array}$$

## RESPUESTAS

$$1.32.- \int 3x^5 dx = 3 \int x^5 dx = \frac{3x^{5+1}}{5+1} + c = 3 \frac{x^6}{6} + c = \frac{x^6}{2} + c$$

$$1.33.- \int (1+e)^x dx$$

$$\text{Sea: } a = 1+e, \text{ Luego: } \int (1+e)^x dx = \int a^x dx = \frac{a^x}{\ell \eta a} + c = \frac{(1+e)^x}{\ell \eta (1+e)} + c$$

$$1.34.- \int (1+\tau g x) dx = \int dx + \int \tau g x dx = x + \ell \eta |\sec x| + c$$

$$1.35.- \int \cos^2 \frac{x}{2} dx = \int \frac{1+\cos x}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos x dx = \frac{1}{2} x + \frac{1}{2} \operatorname{sen} x + c$$

$$\begin{aligned}
 1.36.- \int (1+\sqrt{x})^3 dx &= \int (1+3\sqrt{x}+3(\cancel{\sqrt{x^2}})+\sqrt{x^3})dx = \int dx+3\sqrt{x}+3\int xdx+\int x^{\frac{3}{2}}dx \\
 &= x+2x^{\frac{3}{2}}+3\frac{x^2}{2}+\frac{2}{5}x^{\frac{5}{2}}+c = x+2x\sqrt{x}+3\frac{x^2}{2}+\frac{2}{5}x^2\sqrt{x}+c
 \end{aligned}$$

$$1.37.- \int (1+\sqrt{x})^0 dx = \int dx = x+c$$

$$1.38.- \int \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} dy = \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} \int dy = \frac{1+\frac{\sqrt{x}}{2}}{1+\frac{\sqrt{x}}{3}} y+c$$

$$1.39.- \int \frac{dx}{\sqrt{5-x^2}}$$

$$\text{Sea: } a = \sqrt{5}, \text{ Luego: } \int \frac{dx}{\sqrt{5-x^2}} = \int \frac{dx}{\sqrt{(\sqrt{5})^2-x^2}} = \arcsen \frac{x}{\sqrt{5}} + c = \arcsen \frac{\sqrt{5}x}{5} + c$$

$$1.40.- \int \frac{dx}{\sqrt{x^2-5}} = \int \frac{dx}{\sqrt{x^2-(\sqrt{5})^2}} = \ell \eta \left| x+\sqrt{x^2-5} \right| + c$$

$$1.41.- \int \frac{dx}{\sqrt{x^2+5}} = \int \frac{dx}{\sqrt{x^2+(\sqrt{5})^2}} = \ell \eta \left| x+\sqrt{x^2+5} \right| + c$$

$$1.42.- \int \frac{dx}{x^2+5}$$

$$\begin{aligned}
 \text{Sea: } a = \sqrt{5}, \text{ Luego: } \int \frac{dx}{x^2+(\sqrt{5})^2} &= \frac{1}{\sqrt{5}} \arctan \frac{x}{\sqrt{5}} + c \\
 &= \frac{\sqrt{5}}{5} \arctan \frac{\sqrt{5}x}{5} + c
 \end{aligned}$$

$$1.43.- \int \frac{dx}{x^2-5} = \int \frac{dx}{x^2-(\sqrt{5})^2} = \frac{1}{2\sqrt{5}} \ell \eta \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| + c = \frac{\sqrt{5}}{10} \ell \eta \left| \frac{x-\sqrt{5}}{x+\sqrt{5}} \right| + c$$

$$1.44.- \int (\sec^2 x + \cos^2 x - 1) dx = \int (1-1) dx = \int 0 dx = c$$

$$1.45.- \int \sqrt{x}(1-\sqrt{x}) dx = \int (\sqrt{x}-x) dx = \int \sqrt{x} dx - \int x dx = \frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{2} + c$$

$$1.46.- \int (\tau g^2 x + 1) dx = \int \sec^2 x dx = \tau g x + c$$

$$\begin{aligned}
 1.47.- \int \frac{dx}{x^2-12} &= \int \frac{dx}{x^2-(\sqrt{12})^2} = \frac{1}{2\sqrt{12}} \ell \eta \left| \frac{x-\sqrt{12}}{x+\sqrt{12}} \right| + c = \frac{1}{4\sqrt{3}} \ell \eta \left| \frac{x-2\sqrt{3}}{x+2\sqrt{3}} \right| + c \\
 &= \frac{\sqrt{3}}{12} \ell \eta \left| \frac{x-2\sqrt{3}}{x+2\sqrt{3}} \right| + c
 \end{aligned}$$

$$1.48.- \int \frac{dx}{x^2+12}$$

$$\text{Sea: } a = \sqrt{12}, \text{ Luego: } \int \frac{dx}{x^2+(\sqrt{12})^2} = \frac{1}{\sqrt{12}} \arctan \frac{x}{\sqrt{12}} + c$$

$$= \frac{1}{2\sqrt{3}} \operatorname{arc} \tau g \frac{x}{2\sqrt{3}} + c = \frac{\sqrt{3}}{6} \operatorname{arc} \tau g \frac{\sqrt{3}x}{6} + c$$

$$\mathbf{1.49.-} \int \frac{dx}{\sqrt{x^2-12}} = \int \frac{dx}{\sqrt{x^2-(\sqrt{12})^2}} = \ell \eta \left| x + \sqrt{x^2-12} \right| + c$$

$$\mathbf{1.50.-} \int \frac{dx}{\sqrt{x^2+12}} = \int \frac{dx}{\sqrt{x^2+(\sqrt{12})^2}} = \ell \eta \left| x + \sqrt{x^2+12} \right| + c$$

$$\mathbf{1.51.-} \int \frac{dx}{\sqrt{12-x^2}}$$

$$\text{Sea: } a = \sqrt{12} \quad , \text{Luego: } \int \frac{dx}{\sqrt{12-x^2}} = \int \frac{dx}{\sqrt{(\sqrt{12})^2-x^2}}$$

$$= \operatorname{arcsen} \frac{x}{\sqrt{12}} + c = \operatorname{arcsen} \frac{x}{2\sqrt{3}} + c = \operatorname{arcsen} \frac{\sqrt{3}x}{6} + c$$

$$\mathbf{1.52.-} \int \frac{dx}{x\sqrt{x^2-12}} = \int \frac{dx}{x\sqrt{x^2-(\sqrt{12})^2}} = \frac{1}{\sqrt{12}} \operatorname{arcsec} \frac{x}{\sqrt{12}} + c = \frac{1}{2\sqrt{3}} \operatorname{arcsec} \frac{x}{2\sqrt{3}} + c$$

$$= \frac{\sqrt{3}}{6} \operatorname{arcsec} \frac{\sqrt{3}x}{6} + c$$

$$\mathbf{1.53.-} \int \frac{dx}{x\sqrt{12-x^2}} = \int \frac{dx}{x\sqrt{(\sqrt{12})^2-x^2}} = \frac{1}{\sqrt{12}} \ell \eta \left| \frac{x}{\sqrt{12} + \sqrt{12-x^2}} \right| + c$$

$$= \frac{\sqrt{3}}{6} \ell \eta \left| \frac{x}{\sqrt{12} + \sqrt{12-x^2}} \right| + c$$

$$\mathbf{1.54.-} \int \frac{dx}{x\sqrt{12+x^2}} = \frac{\sqrt{3}}{6} \ell \eta \left| \frac{x}{\sqrt{12} + \sqrt{12+x^2}} \right| + c$$

$$\mathbf{1.55.-} \int \frac{dx}{\sqrt{8-2x^2}} = \int \frac{dx}{\sqrt{2(4-x^2)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{4-x^2}} = \frac{1}{\sqrt{2}} \operatorname{arcsen} \frac{x}{2} + c = \frac{\sqrt{2}}{2} \operatorname{arcsen} \frac{x}{2} + c$$

$$\mathbf{1.56.-} \int \frac{dx}{\sqrt{2x^2-8}} = \int \frac{dx}{\sqrt{2(x^2-4)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2-4}} = \frac{1}{\sqrt{2}} \ell \eta \left| x + \sqrt{x^2-4} \right| + c$$

$$= \frac{\sqrt{2}}{2} \ell \eta \left| x + \sqrt{x^2-4} \right| + c$$

$$\mathbf{1.57.-} \int \frac{dx}{\sqrt{2x^2+8}} = \int \frac{dx}{\sqrt{2(x^2+4)}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2+4}} = \frac{1}{\sqrt{2}} \ell \eta \left| x + \sqrt{x^2+4} \right| + c$$

$$= \frac{\sqrt{2}}{2} \ell \eta \left| x + \sqrt{x^2+4} \right| + c$$

$$\mathbf{1.58.-} \int \sqrt{x^2-10} dx = \int \sqrt{x^2-(\sqrt{10})^2} dx = \frac{x}{2} \sqrt{x^2-10} - \frac{10}{2} \ell \eta \left| x + \sqrt{x^2-10} \right| + c$$

$$= \frac{x}{2} \sqrt{x^2 - 10} - 5\ell\eta \left| x + \sqrt{x^2 - 10} \right| + c$$

$$\mathbf{1.59.-} \int \sqrt{x^2 + 10} dx = \frac{x}{2} \sqrt{x^2 + 10} + 5\ell\eta \left| x + \sqrt{x^2 + 10} \right| + c$$

$$\mathbf{1.60.-} \int \sqrt{10 - x^2} dx = \int \sqrt{(\sqrt{10})^2 - x^2} dx = \frac{x}{2} \sqrt{10 - x^2} + \frac{10}{2} \arcsen \frac{x}{\sqrt{10}} + c$$

$$= \frac{x}{2} \sqrt{10 - x^2} + 5 \arcsen \frac{\sqrt{10}x}{10} + c$$

$$\mathbf{1.61.-} \int \frac{1 - \cos^2 x}{\sen^2 x} dx = \int \frac{\sen^2 x}{\sen^2 x} dx = \int dx = x + c$$

$$\mathbf{1.62.-} \int \sqrt{1 - \sen^2 x} dx = \int \sqrt{\cos^2 x} dx = \int \cos x dx = \sen x + c$$

$$\mathbf{1.63.-} \int \sqrt{1 - \cos^2 x} dx = \int \sqrt{\sen^2 x} dx = \int \sen x dx = -\cos x + c$$

$$\mathbf{1.64.-} \int (2^x - 3^x)^0 dx = \int dx = x + c$$

$$\mathbf{1.65.-} \int (2^0 - 3^0)^n dx = \int (0)^n dx = \int 0 dx = c$$

$$\mathbf{1.66.-} \int \left( \tau gx - \frac{\sen x}{\cos x} \right) dx = \int (\tau gx - \tau gx) dx = \int 0 dx = c$$

$$\mathbf{1.67.-} \int \frac{dx}{3^{-x}} = \int 3^x dx = \frac{3^x}{\ell\eta 3} + c$$

$$\mathbf{1.68.-} \int \sqrt{\frac{3}{4} - x^2} dx = \int \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 - x^2} dx = \frac{x}{2} \sqrt{\frac{3}{4} - x^2} + \frac{3/4}{2} \arcsen \frac{x}{\sqrt{3}/2} + c$$

$$= \frac{x}{2} \sqrt{\frac{3}{4} - x^2} + \frac{3}{8} \arcsen \frac{2x}{\sqrt{3}} + c$$

$$\mathbf{1.69.-} \int \sqrt{x^2 - \frac{3}{4}} dx = \int \sqrt{x^2 - \left(\frac{\sqrt{3}}{2}\right)^2} dx = \frac{x}{2} \sqrt{x^2 - \frac{3}{4}} - \frac{3/4}{2} \ell\eta \left| x + \sqrt{x^2 - \frac{3}{4}} \right| + c$$

$$= \frac{x}{2} \sqrt{x^2 - \frac{3}{4}} - \frac{3}{8} \ell\eta \left| x + \sqrt{x^2 - \frac{3}{4}} \right| + c$$

$$\mathbf{1.70.-} \int \sqrt{x^2 + \frac{3}{4}} dx = \int \sqrt{x^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx = \frac{x}{2} \sqrt{x^2 + \frac{3}{4}} + \frac{3}{8} \ell\eta \left| x + \sqrt{x^2 + \frac{3}{4}} \right| + c$$

$$\mathbf{1.71.-} \int \frac{dx}{x\sqrt{3 - x^2}} = \int \frac{dx}{x\sqrt{(\sqrt{3})^2 - x^2}} = \frac{1}{\sqrt{3}} \ell\eta \left| \frac{x}{\sqrt{3} + \sqrt{3 - x^2}} \right| + c$$

$$= \frac{\sqrt{3}}{3} \ell\eta \left| \frac{x}{\sqrt{3} + \sqrt{3 - x^2}} \right| + c$$

$$\mathbf{1.72.-} \int \frac{dx}{x\sqrt{x^2 - 3}} = \frac{1}{\sqrt{3}} \operatorname{arcsec} \frac{x}{\sqrt{3}} + c = \frac{\sqrt{3}}{3} \operatorname{arcsec} \frac{\sqrt{3}x}{3} + c$$

$$\mathbf{1.73.-} \int \frac{dx}{x\sqrt{x^2 + 3}} = \frac{\sqrt{3}}{3} \ell\eta \left| \frac{x}{\sqrt{3} + \sqrt{x^2 + 3}} \right| + c$$

$$1.74.- \int (\operatorname{sen}^{3x} \theta) dy = \operatorname{sen}^{3x} \theta \int dy = (\operatorname{sen}^{3x} \theta) y + c$$

$$1.75.- \int \ell \eta |u| dx = \ell \eta |u| \int dx = \ell \eta |u| x + c$$

$$1.76.- \int \exp(\ell \eta x) dx = \int x dx = \frac{x^2}{2} + c$$

$$1.77.- \int e^{\ell \eta x^2} dx = \int x^2 dx = \frac{x^3}{3} + c$$

$$1.78.- \int \frac{\sqrt{x} - \sqrt{2}}{\sqrt{2x}} dx = \int \frac{\sqrt{x}}{\sqrt{2x}} dx - \int \frac{\sqrt{2}}{\sqrt{2x}} dx = \int \sqrt{\frac{x}{2x}} dx - \int \sqrt{\frac{2}{2x}} dx = \frac{1}{\sqrt{2}} \int dx - \int \frac{1}{\sqrt{x}} dx = \\ = \frac{1}{\sqrt{2}} \int dx - \int x^{-\frac{1}{2}} dx = \frac{1}{\sqrt{2}} x - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{\sqrt{2}}{2} x - 2x^{\frac{1}{2}} + c$$

$$1.79.- \int \sqrt{11-x^2} dx = \frac{x}{2} \sqrt{11-x^2} + \frac{11}{2} \arcsen \frac{x}{\sqrt{11}} + c = \frac{x}{2} \sqrt{11-x^2} + \frac{11}{2} \arcsen \frac{\sqrt{11}x}{11} + c$$

$$1.80.- \int \sqrt{x^2-11} dx = \frac{x}{2} \sqrt{x^2-11} - \frac{11}{2} \ell \eta \left| x + \sqrt{x^2-11} \right| + c$$

$$1.81.- \int \sqrt{x^2+11} dx = \frac{x}{2} \sqrt{x^2+11} + \frac{11}{2} \ell \eta \left| x + \sqrt{x^2+11} \right| + c$$

$$1.82.- \int \ell \eta (e^{\sqrt{x}}) dx = \int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3} x \sqrt{x} + c$$

$$1.83.- \int \left[ \frac{1 + \sqrt{x} + \sqrt{x^3}}{1 - \sqrt{x}} \right] dx = \int dx = x + c$$

$$1.84.- \int (\tau g^2 x + \sec^2 x - 1) dx = \int 0 dx = c$$

$$1.85.- \int \frac{dx}{\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}\sqrt{(x^2-\frac{1}{3})}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{(x^2-\frac{1}{3})}} = \frac{1}{\sqrt{3}} \ell \eta \left| x + \sqrt{(x^2-\frac{1}{3})} \right| + c \\ = \frac{\sqrt{3}}{3} \ell \eta \left| x + \sqrt{(x^2-\frac{1}{3})} \right| + c$$

$$1.86.- \int (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) dx = (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) \int dx = (\operatorname{co} \tau g \theta - \operatorname{sen} \theta) x + c$$

$$1.87.- \int \frac{dx}{\sqrt{1+3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{\frac{1}{3}+x^2}} = \frac{\sqrt{3}}{3} \ell \eta \left| x + \sqrt{\frac{1}{3}+x^2} \right| + c$$

$$1.88.- \int \frac{dx}{\sqrt{1-3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{\frac{1}{3}-x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\frac{1}{3}-x^2}} = \frac{1}{\sqrt{3}} \arcsen \frac{x}{\frac{1}{\sqrt{3}}} + c \\ = \frac{\sqrt{3}}{3} \arcsen \sqrt{3}x + c$$

$$1.89.- \int \frac{dx}{1+3x^2} = \int \frac{dx}{3(\frac{1}{3}+x^2)} = \frac{1}{3} \int \frac{dx}{\frac{1}{3}+x^2} = \frac{1}{3} \frac{1}{\frac{1}{\sqrt{3}}} \operatorname{arc} \tau g \frac{x}{\frac{1}{\sqrt{3}}} + c = \frac{\sqrt{3}}{3} \operatorname{arc} \tau g \sqrt{3}x + c$$

$$1.90.- \int \frac{dx}{3x^2+4} = \frac{1}{3} \int \frac{dx}{x^2+\frac{4}{3}} = \frac{1}{3} \frac{1}{\frac{2}{\sqrt{3}}} \operatorname{arctg} \frac{x}{\frac{2}{\sqrt{3}}} + c = \frac{\sqrt{3}}{6} \operatorname{arctg} \frac{\sqrt{3}x}{2} + c$$

$$1.91.- \int \frac{dx}{3x^2-1} = \frac{1}{3} \int \frac{dx}{x^2-\frac{1}{3}} = \frac{1}{3} \frac{1}{2\frac{1}{\sqrt{3}}} \ell \eta \left| \frac{x-\frac{1}{\sqrt{3}}}{x+\frac{1}{\sqrt{3}}} \right| + c = \frac{\sqrt{3}}{6} \ell \eta \left| \frac{\sqrt{3}x-1}{\sqrt{3}x+1} \right| + c$$

$$1.92.- \int \frac{dx}{x\sqrt{3x^2-1}} = \int \frac{dx}{\sqrt{3}x\sqrt{x^2-\frac{1}{3}}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{x^2-\frac{1}{3}}} = \frac{1}{\cancel{\sqrt{3}}} \frac{1}{\cancel{1/\sqrt{3}}} \operatorname{arcsec} \frac{x}{\cancel{1/\sqrt{3}}} + c$$

$$= \operatorname{arcsec} \sqrt{3}x + c$$

$$1.93.- \int \frac{dx}{x\sqrt{1+3x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{\frac{1}{3}+x^2}} = \frac{1}{\cancel{\sqrt{3}}} \frac{1}{\cancel{1/\sqrt{3}}} \ell \eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}} \right| + c$$

$$= \ell \eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}+x^2}} \right| + c$$

$$1.94.- \int \frac{dx}{x\sqrt{1-3x^2}} = \frac{1}{\sqrt{3}} \int \frac{dx}{x\sqrt{\frac{1}{3}-x^2}} = \ell \eta \left| \frac{x}{\frac{1}{\sqrt{3}}+\sqrt{\frac{1}{3}-x^2}} \right| + c$$

$$1.95.- \int \sqrt{1-3x^2} dx = \sqrt{3} \int \sqrt{\frac{1}{3}-x^2} dx = \sqrt{3} \left[ \frac{x}{2} \sqrt{\frac{1}{3}-x^2} + \frac{1}{2} \operatorname{arcsen} \frac{x}{\frac{1}{\sqrt{3}}} \right] + c$$

$$= \sqrt{3} \left[ \frac{x}{2} \sqrt{\frac{1}{3}-x^2} + \frac{1}{6} \operatorname{arcsen} \sqrt{3}x \right] + c$$

$$1.96.- \int \sqrt{1+3x^2} dx = \sqrt{3} \int \sqrt{\frac{1}{3}+x^2} dx = \sqrt{3} \left[ \frac{x}{2} \sqrt{\frac{1}{3}+x^2} + \frac{1}{2} \ell \eta \left| x + \sqrt{\frac{1}{3}+x^2} \right| \right] + c$$

$$= \sqrt{3} \left[ \frac{x}{2} \sqrt{\frac{1}{3}+x^2} + \frac{1}{6} \ell \eta \left| x + \sqrt{\frac{1}{3}+x^2} \right| \right] + c$$

$$1.97.- \int \sqrt{3x^2-1} dx = \sqrt{3} \int \sqrt{x^2-\frac{1}{3}} dx = \sqrt{3} \left[ \frac{x}{2} \sqrt{x^2-\frac{1}{3}} - \frac{1}{6} \ell \eta \left| x + \sqrt{x^2-\frac{1}{3}} \right| \right] + c$$

$$1.98.- \int (3x^2-1) dx = 3 \int x^2 dx - \int dx = x^3 - x + c$$

$$1.99.- \int (3x^2-1)^0 dx = \int dx = x + c$$

$$1.100.- \int (3x^2-1)^n du = (3x^2-1)^n \int du = (3x^2-1)^n u + c$$

$$1.101.- \int \exp(\ell \eta \frac{\sqrt{x}}{3}) dx = \int \frac{\sqrt{x}}{3} dx = \frac{1}{3} \int x^{\frac{1}{2}} dx = \frac{1}{3} \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{9} x^{\frac{3}{2}} + c$$

$$1.102.- \int \ell \eta (e^{\frac{2x-1}{2}}) dx = \int \frac{2x-1}{2} dx = \int x dx - \frac{1}{2} \int dx = \frac{x^2}{2} - \frac{1}{2} x + c$$

$$1.103.- \int (e^2 + e + 1)^x dx$$

Sea:  $a = (e^2 + e + 1)$ , Luego:  $\int a^x dx = \frac{a^x}{\ell \eta a} + c = \frac{(e^2 + e - 1)^x}{\ell \eta (e^2 + e - 1)} + c$

**1.104.-**  $\int \left( \frac{1 + \tau g^2 x}{\sec^2 x} - 1 \right) dx = \int (1 - 1) dx = \int 0 dx = c$

**1.105.-**  $\int \exp(\ell \eta |1 + x|) dx = \int (1 + x) dx = \int dx + \int x dx = x + \frac{x^2}{2} + c$

**1.106.-**  $\int \sqrt{27 - x^2} dx = \frac{x}{2} \sqrt{27 - x^2} + \frac{27}{2} \operatorname{arcsen} \frac{x}{3\sqrt{3}} + c$

**1.107.-**  $\int \sqrt{x^2 - 27} dx = \frac{x}{2} \sqrt{x^2 - 27} - \frac{27}{2} \ell \eta \left| x + \sqrt{x^2 - 27} \right| + c$

**1.108.-**  $\int \sqrt{x^2 + 27} dx = \frac{x}{2} \sqrt{x^2 + 27} + \frac{27}{2} \ell \eta \left| x + \sqrt{x^2 + 27} \right| + c$

**1.109.-**  $\int \frac{dx}{3x\sqrt{x^2 - 1}} = \frac{1}{3} \int \frac{dx}{x\sqrt{x^2 - 1}} = \frac{1}{3} \operatorname{arcsec} x + c$

**1.110.-**  $\int \frac{dx}{2x\sqrt{1 - x^2}} = \frac{1}{2} \int \frac{dx}{x\sqrt{1 - x^2}} = \frac{1}{2} \ell \eta \left| \frac{x}{1 + \sqrt{1 - x^2}} \right| + c$

**1.111.-**  $\int \frac{dx}{5x\sqrt{x^2 + 1}} = \frac{1}{5} \int \frac{dx}{x\sqrt{x^2 + 1}} = \frac{1}{5} \ell \eta \left| \frac{x}{1 + \sqrt{x^2 + 1}} \right| + c$

**1.112.-**  $\int \frac{dx}{3x\sqrt{9 - x^2}} = \frac{1}{3} \int \frac{dx}{x\sqrt{9 - x^2}} = \frac{1}{3} \frac{1}{3} \ell \eta \left| \frac{x}{3 + \sqrt{9 - x^2}} \right| + c = \frac{1}{9} \ell \eta \left| \frac{x}{3 + \sqrt{9 - x^2}} \right| + c$

**1.113.-**  $\int \frac{dx}{4x\sqrt{x^2 + 16}} = \frac{1}{4} \int \frac{dx}{x\sqrt{x^2 + 16}} = \frac{1}{4} \frac{1}{4} \ell \eta \left| \frac{x}{4 + \sqrt{x^2 + 16}} \right| + c$

$= \frac{1}{16} \ell \eta \left| \frac{x}{4 + \sqrt{x^2 + 16}} \right| + c$

**1.114.-**  $\int \frac{dx}{5x\sqrt{x^2 - 25}} = \frac{1}{5} \int \frac{dx}{x\sqrt{x^2 - 25}} = \frac{1}{5} \frac{1}{5} \operatorname{arcsec} \frac{x}{5} + c = \frac{1}{25} \operatorname{arcsec} \frac{x}{5} + c$

**1.115.-**  $\int \frac{(1 - \sqrt{x})^2}{x^2} dx = \int \frac{1 - 2\sqrt{x} + x}{x^2} dx = \int (x^{-2} - 2x^{-3/2} + x^{-1}) dx$

$= \int x^{-2} dx - \int 2x^{-3/2} dx + \int x^{-1} dx = -x^{-1} - 2 \frac{x^{-1/2}}{-1/2} + \ell \eta |x| + c = -x^{-1} - 2 \frac{x^{-1/2}}{-1/2} + \ell \eta |x| + c$

$= -x^{-1} + 4x^{-1/2} + \ell \eta |x| + c = -\frac{1}{x} + \frac{4}{\sqrt{x}} + \ell \eta |x| + c$

**1.116.-**  $\int (1 + \sqrt{x} + x)^2 dx = (1 + x + x^2 + 2\sqrt{x} + 2x + 2x^{3/2}) dx$

$= \int (1 + 2x^{1/2} + 3x + 2x^{3/2} + x^2) dx = \int dx + 2 \int x^{1/2} dx + 3 \int x dx + 2 \int x^{3/2} dx + \int x^2 dx$

$x + \frac{2x^{3/2}}{3/2} + 3 \frac{x^2}{2} + 2 \frac{x^{5/2}}{5/2} + \frac{x^3}{3} + c = x + \frac{4x^{3/2}}{3} + 3 \frac{x^2}{2} + 4 \frac{x^{5/2}}{5} + \frac{x^3}{3} + c$



$$\begin{aligned}
\mathbf{1.117.-} \int (1 - \sqrt{x} + x)^2 dx &= \int (1 + x + x^2 - 2\sqrt{x} + 2x - 2x^{3/2}) dx \\
&= \int (1 - 2x^{1/2} + 3x - 2x^{3/2} + x^2) dx = x - \frac{4x^{3/2}}{3} + 3\frac{x^2}{2} - 4\frac{x^{5/2}}{5} + \frac{x^3}{3} + c
\end{aligned}$$

$$\begin{aligned}
\mathbf{1.118.-} \int (1+x)^4 dx &= \int (1+4x+6x^2+4x^3+x^4) dx \\
&= \int dx + 4\int x dx + 6\int x^2 dx + 4\int x^3 dx + \int x^4 dx = x + 2x^2 + 2x^3 + x^4 + \frac{1}{5}x^5 + c
\end{aligned}$$

$$\mathbf{1.119.-} \int e^{\ell \eta^{\left| \frac{1-\cos x}{2} \right|}} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx = \frac{1}{2} x - \frac{1}{2} \sin x + c$$

$$\mathbf{1.120.-} \int \exp \ell \eta \left( \frac{1+x^2}{x^2} \right) dx = \int \frac{1+x^2}{x^2} dx = \int \frac{1}{x^2} dx + \int dx = \int x^{-2} dx + \int dx = -\frac{1}{x} + x + c$$

$$\mathbf{1.121.-} \int \ell \eta e^{\frac{1-\sin x}{3}} dx = \int \frac{1-\sin x}{3} dx = \frac{1}{3} \int dx - \frac{1}{3} \int \sin x dx = \frac{1}{3} x + \frac{1}{3} \cos x + c$$

$$\mathbf{1.122.-} \int (1 + \sqrt{x-3x})^0 dx = \int dx = x + c$$

$$\begin{aligned}
\mathbf{1.123.-} \int \ell \eta e^{\frac{(1+x)^2}{2}} dx &= \int \frac{(1+x)^2}{2} dx = \int \frac{1+2x+x^2}{2} dx = \frac{1}{2} \int dx + \int x dx + \frac{1}{2} \int x^2 dx \\
&= \frac{1}{2} x + \frac{x^2}{2} + \frac{x^3}{6} + c
\end{aligned}$$

## CAPITULO 2

### INTEGRACION POR SUSTITUCION

A veces es conveniente hacer un cambio de variable, para transformar la integral dada en otra, de forma conocida. La técnica en cuestión recibe el nombre de método de sustitución.

### EJERCICIOS DESARROLLADOS

**2.1.-Encontrar:**  $\int \frac{e^{\ell \eta x} dx}{x^2 + 7}$

Solución.- Como:  $e^{\ell \eta x} = x$ , se tiene:  $\int \frac{e^{\ell \eta x} dx}{x^2 + 7} = \int \frac{x dx}{x^2 + 7}$

Sea la sustitución:  $u = x^2 + 7$ , donde:  $du = 2x dx$ , Dado que:  $\int \frac{x dx}{x^2 + 7} = \frac{1}{2} \int \frac{2x dx}{x^2 + 7}$ ,

Se tiene:  $\frac{1}{2} \int \frac{2x dx}{x^2 + 7} = \frac{1}{2} \int \frac{du}{u}$ , integral que es inmediata.

Luego:  $= \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ell \eta |u| + c = \frac{1}{2} \ell \eta |x^2 + 7| + c$

**Respuesta:**  $\int \frac{e^{\ell \eta x} dx}{x^2 + 7} = \frac{1}{2} \ell \eta |x^2 + 7| + c$

**2.2.-Encontrar:**  $\int \frac{e^{\ell \eta x^2} dx}{x^3 + 8}$

Solución.- Como:  $e^{\ell \eta x^2} = x^2$ , se tiene:  $\int \frac{e^{\ell \eta x^2} dx}{x^3 + 8} = \int \frac{x^2 dx}{x^3 + 8}$

Sea la sustitución:  $w = x^3 + 8$ , donde:  $dw = 3x^2 dx$ , Dado que:  $\int \frac{x^2 dx}{x^3 + 8} = \frac{1}{3} \int \frac{3x^2 dx}{x^3 + 8}$ ,

Se tiene:  $\frac{1}{3} \int \frac{3x^2 dx}{x^3 + 8} = \frac{1}{3} \int \frac{dw}{w}$  integral que es inmediata.

Luego:  $\frac{1}{3} \int \frac{dw}{w} = \frac{1}{3} \ell \eta |w| + c = \frac{1}{3} \ell \eta |x^3 + 8| + c$

**Respuesta:**  $\int \frac{e^{\ell \eta x^2} dx}{x^3 + 8} = \frac{1}{3} \ell \eta |x^3 + 8| + c$

**2.3.-Encontrar:**  $\int (x+2) \operatorname{sen}(x^2 + 4x - 6) dx$

Solución.- Sea la sustitución:  $u = x^2 + 4x - 6$ , donde:  $du = (2x + 4) dx$

Dado que:  $\int (x+2) \operatorname{sen}(x^2 + 4x - 6) dx = \frac{1}{2} \int (2x + 4) \operatorname{sen}(x^2 + 4x - 6) dx$ , se tiene:

$$= \frac{1}{2} \int (2x+4) \operatorname{sen}(x^2+4x-6) dx = \frac{1}{2} \int \operatorname{sen} u du, \text{ integral que es inmediata.}$$

$$\text{Luego: } = \frac{1}{2} \int \operatorname{sen} u du = \frac{1}{2} (-\cos u) + c = -\frac{1}{2} \cos u + c = -\frac{1}{2} \cos(x^2+4x-6) + c$$

$$\text{Respuesta: } \int (x+2) \operatorname{sen}(x^2+4x-6) dx = -\frac{1}{2} \cos(x^2+4x-6) + c$$

$$\text{2.4.-Encontrar: } \int x \operatorname{sen}(1-x^2) dx$$

Solución.-Sea la sustitución:  $w = 1 - x^2$ , donde:  $dw = -2x dx$

$$\text{Dado que: } \int x \operatorname{sen}(1-x^2) dx = -\frac{1}{2} \int (-2x) \operatorname{sen}(1-x^2) dx$$

$$\text{Se tiene que: } -\frac{1}{2} \int (-2x) \operatorname{sen}(1-x^2) dx = -\frac{1}{2} \int \operatorname{sen} w dw, \text{ integral que es inmediata.}$$

$$\text{Luego: } -\frac{1}{2} \int \operatorname{sen} w dw = -\frac{1}{2} (-\cos w) + c = \frac{1}{2} \cos w + c = \frac{1}{2} \cos(1-x^2) + c$$

$$\text{Respuesta: } \int x \operatorname{sen}(1-x^2) dx = \frac{1}{2} \cos(1-x^2) + c$$

$$\text{2.5.-Encontrar: } \int x \operatorname{co} \tau g(x^2+1) dx$$

Solución.-Sea la sustitución:  $u = x^2 + 1$ , donde:  $du = 2x dx$

$$\text{Dado que: } \int x \operatorname{co} \tau g(x^2+1) dx = \frac{1}{2} \int 2x \operatorname{co} \tau g(x^2+1) dx$$

$$\text{Se tiene que: } \frac{1}{2} \int 2x \operatorname{co} \tau g(x^2+1) dx = \frac{1}{2} \int \operatorname{co} \tau g u du, \text{ integral que es inmediata.}$$

$$\text{Luego: } \frac{1}{2} \int \operatorname{co} \tau g u du = \frac{1}{2} \ell \eta |\operatorname{sen} u| + c = \frac{1}{2} \ell \eta |\operatorname{sen}(x^2+1)| + c$$

$$\text{Respuesta: } \int x \operatorname{co} \tau g(x^2+1) dx = \frac{1}{2} \ell \eta |\operatorname{sen}(x^2+1)| + c$$

$$\text{2.6.-Encontrar: } \int \sqrt{1+y^4} y^3 dy$$

Solución.-Sea la sustitución:  $w = 1 + y^4$ , donde:  $dw = 4y^3 dy$

$$\text{Dado que: } \int \sqrt{1+y^4} y^3 dy = \frac{1}{4} \int (1+y^4)^{\frac{1}{2}} 4y^3 dy$$

$$\text{Se tiene que: } \frac{1}{4} \int (1+y^4)^{\frac{1}{2}} 4y^3 dy = \frac{1}{4} \int w^{\frac{1}{2}} dw, \text{ integral que es inmediata.}$$

$$\text{Luego: } \frac{1}{4} \int w^{\frac{1}{2}} dw = \frac{1}{4} \frac{w^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{1}{6} w^{\frac{3}{2}} + c = \frac{1}{6} (1+y^4)^{\frac{3}{2}} + c$$

$$\text{Respuesta: } \int \sqrt{1+y^4} y^3 dy = \frac{1}{6} (1+y^4)^{\frac{3}{2}} + c$$

$$\text{2.7.-Encontrar: } \int \frac{3t dt}{\sqrt[3]{t^2+3}}$$

Solución.-Sea la sustitución:  $u = t^2 + 3$ , donde:  $du = 2t dt$

Dado que:  $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{3}{2} \int \frac{2tdt}{(t^2+3)^{1/3}}$

Se tiene que:  $\frac{3}{2} \int \frac{2tdt}{(t^2+3)^{1/3}} = \frac{3}{2} \int \frac{du}{u^{1/3}}$ , integral que es inmediata

Luego:  $\frac{3}{2} \int \frac{du}{u^{1/3}} = \frac{3}{2} \int u^{-1/3} du = \frac{3}{2} \frac{u^{2/3}}{2/3} + c = \frac{9}{4} u^{2/3} + c = \frac{9}{4} (t^2+3)^{2/3} + c$

**Respuesta:**  $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{9}{4} (t^2+3)^{2/3} + c$

**2.8.-Encontrar:**  $\int \frac{dx}{(a+bx)^{1/3}}$ , a y b constantes.

Solución.- Sea:  $w = a + bx$ , donde:  $dw = bdx$

Luego:  $\int \frac{dx}{(a+bx)^{1/3}} = \frac{1}{b} \int \frac{bdx}{(a+bx)^{1/3}} = \frac{1}{b} \int \frac{dw}{w^{1/3}} = \frac{1}{b} \int w^{-1/3} = \frac{1}{b} \frac{w^{2/3}}{2/3} + c = \frac{3}{2b} w^{2/3} + c$   
 $= \frac{3}{2b} (a+bx)^{2/3} + c$

**Respuesta:**  $\int \frac{dx}{(a+bx)^{1/3}} = \frac{3}{2b} (a+bx)^{2/3} + c$

**2.9.-Encontrar:**  $\int \frac{\sqrt{\arcsen x}}{1-x^2} dx$

Solución.-  $\int \frac{\sqrt{\arcsen x}}{1-x^2} dx = \int \sqrt{\arcsen x} \frac{dx}{\sqrt{1-x^2}}$ ,

Sea:  $u = \arcsen x$ , donde:  $du = \frac{dx}{\sqrt{1-x^2}}$

Luego:  $\int \sqrt{\arcsen x} \frac{dx}{\sqrt{1-x^2}} = \int u^{1/2} du = \frac{2}{3} u^{3/2} + c = \frac{2}{3} \sqrt{(\arcsen x)^3} + c$

**Respuesta:**  $\int \frac{\sqrt{\arcsen x}}{1-x^2} dx = \frac{2}{3} \sqrt{(\arcsen x)^3} + c$

**2.10.-Encontrar:**  $\int \frac{\arctg \frac{x}{2}}{4+x^2} dx$

Solución.- Sea:  $w = \arctg \frac{x}{2}$ , donde:  $dw = \frac{1}{1+(\frac{x}{2})^2} (\frac{1}{2}) dx = \frac{2dx}{4+x^2}$

Luego:  $\int \frac{\arctg \frac{x}{2}}{4+x^2} dx = \frac{1}{2} \int \arctg \left( \frac{x}{2} \right) \frac{2dx}{4+x^2} = \frac{1}{2} \int w dw = \frac{1}{4} w^2 + c = \frac{1}{4} \left( \arctg \frac{x}{2} \right)^2 + c$

**Respuesta:**  $\int \frac{\arctg \frac{x}{2}}{4+x^2} dx = \frac{1}{4} \left( \arctg \frac{x}{2} \right)^2 + c$

**2.11.-Encontrar:**  $\int \frac{x - \operatorname{arctg} 2x}{1+4x^2} dx$

Solución.-  $\int \frac{x - \operatorname{arctg} 2x}{1+4x^2} dx = \int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arctg} 2x}}{1+4x^2}$

Sea:  $u = 1+4x^2$ , donde:  $du = 8xdx$ ;  $w = \operatorname{arctg} 2x$ , donde:  $dw = \frac{2dx}{1+4x^2}$

Luego:  $\int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arctg} 2x}}{1+4x^2} = \frac{1}{8} \int \frac{8xdx}{1+4x^2} - \frac{1}{2} \int \sqrt{\operatorname{arctg} 2x} \frac{2dx}{1+4x^2}$   
 $= \frac{1}{8} \int \frac{du}{u} - \frac{1}{2} \int w^{\frac{1}{2}} dw = \frac{1}{8} \ell \eta |u| - \frac{1}{3} w^{\frac{3}{2}} + c = \frac{1}{8} \ell \eta |1+4x^2| - \frac{1}{3} (\operatorname{arctg} 2x)^{\frac{3}{2}} + c$

**Respuesta:**  $\int \frac{x - \operatorname{arctg} 2x}{1+4x^2} dx = \frac{1}{8} \ell \eta |1+4x^2| - \frac{1}{3} (\operatorname{arctg} 2x)^{\frac{3}{2}} + c$

**2.12.-Encontrar:**  $\int \frac{dx}{\sqrt{(1+x^2) \ell \eta |x + \sqrt{1+x^2}|}}$

Solución.-  $\int \frac{dx}{\sqrt{(1+x^2) \ell \eta |x + \sqrt{1+x^2}|}} = \int \frac{dx}{\sqrt{1+x^2} \sqrt{\ell \eta |x + \sqrt{1+x^2}|}}$

Sea:  $u = \ell \eta |x + \sqrt{1+x^2}|$ , donde:  $du = \frac{1}{x + \sqrt{1+x^2}} (1 + \frac{2x}{2\sqrt{1+x^2}}) \Rightarrow du = \frac{dx}{\sqrt{1+x^2}}$

Luego:  $\int \frac{dx}{\sqrt{1+x^2} \sqrt{\ell \eta |x + \sqrt{1+x^2}|}} = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + c = 2\sqrt{\ell \eta |x + \sqrt{1+x^2}|} + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{(1+x^2) \ell \eta |x + \sqrt{1+x^2}|}} = 2\sqrt{\ell \eta |x + \sqrt{1+x^2}|} + c$

**2.13.-Encontrar:**  $\int \frac{\operatorname{coseg}(\ell \eta x)}{x} dx$

Solución.- Sea:  $w = \ell \eta x$ , donde:  $dw = \frac{dx}{x}$

Luego:  $\int \frac{\operatorname{coseg}(\ell \eta x)}{x} dx = \int \operatorname{coseg} w dw = \ell \eta |\operatorname{sen} w| + c = \ell \eta |\operatorname{sen}(\ell \eta x)| + c$

**Respuesta:**  $\int \frac{\operatorname{coseg}(\ell \eta x)}{x} dx = \ell \eta |\operatorname{sen}(\ell \eta x)| + c$

**2.14.-Encontrar:**  $\int \frac{dx}{x(\ell \eta x)^3}$

Solución.- Sea:  $u = \ell \eta x$ , donde:  $du = \frac{dx}{x}$

Luego:  $\int \frac{dx}{x(\ell \eta x)^3} = \int \frac{du}{u^3} = \int u^{-3} du = \frac{u^{-2}}{-2} + c = \frac{1}{2u^2} + c = \frac{1}{2(\ell \eta x)^2} + c$

**Respuesta:**  $\int \frac{dx}{x(\ell \eta x)^3} = \frac{1}{2(\ell \eta x)^2} + c$

**2.15.-Encontrar:**  $\int \frac{e^{\sqrt{x}}}{x^3} dx$

Solución.- Sea:  $w = \frac{1}{x^2}$ , donde:  $dw = -\frac{2}{x^3} dx$

Luego:  $\int \frac{e^{\sqrt{x}}}{x^3} dx = -\frac{1}{2} \int e^{\sqrt{x}} \frac{-2dx}{x^3} = -\frac{1}{2} \int e^w dw = -\frac{1}{2} e^w + c = -\frac{1}{2} e^{\sqrt{x}} + c$

**Respuesta:**  $\int \frac{e^{\sqrt{x}}}{x^3} dx = -\frac{1}{2} e^{\sqrt{x}} + c$

**2.16.-Encontrar:**  $\int e^{-x^2+2} x dx$

Solución.- Sea:  $u = -x^2 + 2$ , donde:  $du = -2x dx$

Luego:  $\int e^{-x^2+2} x dx = -\frac{1}{2} \int e^{-x^2+2} (-2x dx) = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2+2} + c$

**Respuesta:**  $\int e^{-x^2+2} x dx = -\frac{1}{2} e^{-x^2+2} + c$

**2.17.-Encontrar:**  $\int x^2 e^{x^3} dx$

Solución.- Sea:  $w = x^3$ , donde:  $dw = 3x^2 dx$

Luego:  $\int x^2 e^{x^3} dx = \frac{1}{3} \int 3x^2 e^{x^3} dx = \frac{1}{3} \int e^w dw = \frac{1}{3} e^w + c$

**Respuesta:**  $\int x^2 e^{x^3} dx = \frac{1}{3} e^{x^3} + c$

**2.18.-Encontrar:**  $\int (e^x + 1)^2 e^x dx$

Solución.- Sea:  $u = e^x + 1$ , donde:  $du = e^x dx$

Luego:  $\int (e^x + 1)^2 e^x dx = \int u^2 du = \frac{u^3}{3} + c = \frac{(e^x + 1)^3}{3} + c$

**Respuesta:**  $\int (e^x + 1)^2 e^x dx = \frac{(e^x + 1)^3}{3} + c$

**2.19.-Encontrar:**  $\int \frac{e^x - 1}{e^x + 1} dx$

Solución.-  $\int \frac{e^x - 1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{e^x e^{-x}}{e^x + 1} dx$

$= \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{e^{-x}(e^x + 1)} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^x} dx$

Sea:  $u = e^x + 1$ , donde:  $du = e^x dx$ ;  $w = 1 + e^{-x}$ , donde:  $dw = -e^{-x} dx$

Luego:  $\int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^x} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{-e^{-x}}{1 + e^{-x}} dx = \int \frac{du}{u} + \int \frac{dw}{w}$

$$= \ell \eta |u| + c_1 + \ell \eta |w| + c_2 = \ell \eta |e^x + 1| + \ell \eta |1 + e^{-x}| + C = \ell \eta [|e^x + 1| |1 + e^{-x}|] + c$$

**Respuesta:**  $\int \frac{e^x - 1}{e^x + 1} dx = \ell \eta [|e^x + 1| (1 + e^{-x})] + c$ , otra respuesta seria:

$$\int \frac{e^x - 1}{e^x + 1} dx = \ell \eta |e^x + 1|^2 - x + c$$

**2.20.-Encontrar:**  $\int \frac{e^{2x} - 1}{e^{2x} + 3} dx$

Solución.-  $\int \frac{e^{2x} - 1}{e^{2x} + 3} dx = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^0}{e^{2x} + 3} dx$

$$= \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{2x} e^{-2x}}{e^{2x} + 3} dx = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{-2x}}{e^{-2x}(e^{2x} + 3)} dx = \int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{-2x}}{1 + 3e^{-2x}} dx$$

Sea:  $u = e^{2x} + 3$ , donde:  $du = 2e^{2x} dx$ ;  $w = 1 + 3e^{-2x}$ , donde:  $dw = -6e^{-2x} dx$

Luego:  $\int \frac{e^{2x}}{e^{2x} + 3} dx - \int \frac{e^{-2x}}{1 + 3e^{-2x}} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x} + 3} dx + \frac{1}{6} \int \frac{-6e^{-2x}}{1 + 3e^{-2x}} dx = \frac{1}{2} \int \frac{du}{u} + \frac{1}{6} \int \frac{dw}{w}$

$$\frac{1}{2} \ell \eta |u| + \frac{1}{6} \ell \eta |w| + c = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta |1 + 3e^{-2x}| + c = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta \left| 1 + \frac{3}{e^{2x}} \right| + c$$

$$= \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta \left| \frac{e^{2x} + 3}{e^{2x}} \right| + c = \frac{1}{2} \ell \eta |e^{2x} + 3| + \frac{1}{6} \ell \eta |e^{2x} + 3| - \frac{1}{6} \ell \eta e^{2x} + c$$

$$= \ell \eta (e^{2x} + 3)^{1/2} + \ell \eta (e^{2x} + 3)^{1/6} - \frac{1}{6} 2x + c = \ell \eta \left[ (e^{2x} + 3)^{1/2} (e^{2x} + 3)^{1/6} \right] - \frac{x}{3} + c$$

$$= \ell \eta (e^{2x} + 3)^{2/3} - \frac{x}{3} + c$$

**Respuesta:**  $\int \frac{e^{2x} - 1}{e^{2x} + 3} dx = \ell \eta (e^{2x} + 3)^{2/3} - \frac{x}{3} + c$

**2.22.-Encontrar:**  $\int \frac{x^2 + 1}{x - 1} dx$

Solución.- Cuando el grado del polinomio dividendo es MAYOR o IGUAL que el grado del polinomio divisor, es necesario efectuar previamente la división de polinomios. El resultado de la división dada es:

$$\frac{x^2 + 1}{x - 1} = (x + 1) + \frac{2}{x - 1}, \text{ Luego: } \int \frac{x^2 + 1}{x - 1} dx = \int \left( x + 1 + \frac{2}{x - 1} \right) dx = \int x dx + \int dx + 2 \int \frac{dx}{x - 1}$$

Sea  $u = x - 1$ , donde  $du = dx$

Luego:  $\int x dx + \int dx + 2 \int \frac{dx}{x - 1} = \int x dx + \int dx + 2 \int \frac{du}{u} = \frac{x^2}{2} + x + \ell \eta |x - 1| + c$

**Respuesta:**  $\int \frac{x^2 + 1}{x - 1} dx = \frac{x^2}{2} + x + \ell \eta |x - 1| + c$

**2.23.-Encontrar:**  $\int \frac{x + 2}{x + 1} dx$

Solución.-  $\frac{x+2}{x+1} = 1 + \frac{1}{x+1}$ , Luego:  $\int \frac{x+2}{x+1} dx = \int \left(1 + \frac{1}{x+1}\right) dx = \int dx + \int \frac{dx}{x+1}$

Sea  $u = x+1$ , donde  $du = dx$

$$\int dx + \int \frac{du}{u} = x + \ell \eta |u| + c = x + \ell \eta |x+1| + c$$

**Respuesta:**  $\int \frac{x+2}{x+1} dx = x + \ell \eta |x+1| + c$

**2.24.-Encontrar:**  $\int \tau g^5 x \sec^2 x dx$

Solución.- Sea:  $w = \tau g x$ , donde:  $dw = \sec^2 x$

$$\text{Luego: } \int \tau g^5 x \sec^2 x dx = \int (\tau g x)^5 \sec^2 x dx = \int w^5 dw = \frac{w^6}{6} + c = \frac{(\tau g x)^6}{6} + c = \frac{\tau g^6 x}{6} + c$$

**Respuesta:**  $\int \tau g^5 x \sec^2 x dx = \frac{\tau g^6 x}{6} + c$

**2.25.-Encontrar:**  $\int \text{sen } x \sec^2 x dx$

$$\text{Solución.- } \int \text{sen } x \sec^2 x dx = \int \text{sen } x \frac{1}{\cos^2 x} dx = \int \frac{\text{sen } x}{\cos^2 x} dx$$

Sea:  $u = \cos x$ , donde:  $du = -\text{sen } x$

$$\text{Luego: } \int \frac{\text{sen } x}{\cos^2 x} dx = -\int \frac{-\text{sen } x dx}{\cos^2 x} = -\int \frac{du}{u} = -\int u^{-2} du = -\frac{u^{-1}}{-1} + c = \frac{1}{u} + c = \frac{1}{\cos x} + c$$

**Respuesta:**  $\int \text{sen } x \sec^2 x dx = \sec x + c$

**2.26.-Encontrar:**  $\int \frac{\sec^2 3x dx}{1 + \tau g 3x}$

Solución.- Sea:  $u = 1 + \tau g 3x$ , donde:  $du = 3 \sec^2 3x dx$

$$\text{Luego: } \int \frac{\sec^2 3x dx}{1 + \tau g 3x} = \frac{1}{3} \int \frac{3 \sec^2 3x dx}{1 + \tau g 3x} = \frac{1}{3} \int \frac{du}{u} = \frac{1}{3} \ell \eta |u| + c = \frac{1}{3} \ell \eta |1 + \tau g 3x| + c$$

**Respuesta:**  $\int \frac{\sec^2 3x dx}{1 + \tau g 3x} = \frac{1}{3} \ell \eta |1 + \tau g 3x| + c$

**2.27.-Encontrar:**  $\int \text{sen}^3 x \cos x dx$

Solución.- Sea:  $w = \text{sen } x$ , donde:  $dw = \cos x dx$

$$\text{Luego: } \int \text{sen}^3 x \cos x dx = \int (\text{sen } x)^3 \cos x dx = \int w^3 dw = \int \frac{w^4}{4} + c = \int \frac{\text{sen}^4 x}{4} + c$$

**Respuesta:**  $\int \text{sen}^3 x \cos x dx = \int \frac{\text{sen}^4 x}{4} + c$

**2.28.-Encontrar:**  $\int \cos^4 x \text{sen } x dx$

Solución.- Sea:  $u = \cos x$ , donde:  $du = -\text{sen } x$

$$\text{Luego: } \int \cos^4 x \text{sen } x dx = \int (\cos x)^4 \text{sen } x dx = -\int (\cos x)^4 (-\text{sen } x) dx = -\int u^4 du$$



$$= -\frac{u^5}{5} + c = -\frac{\cos^5 x}{5} + c = -\frac{\cos^5 x}{5} + c$$

**Respuesta:**  $\int \cos^4 x \operatorname{sen} x dx = -\frac{\cos^5 x}{5} + c$

**2.29.-Encontrar:**  $\int \frac{\sec^5}{\cos ecx} dx$

Solución.-  $\int \frac{\sec^5}{\cos ecx} dx = \int \frac{\frac{1}{\cos^5 x}}{\frac{1}{\operatorname{sen} x}} dx = \int \frac{\operatorname{sen} x}{(\cos x)^5} dx$

Sea:  $w = \cos x$ , donde:  $dw = -\operatorname{sen} x dx$

Luego:  $\int \frac{\operatorname{sen} x}{(\cos x)^5} dx = -\int \frac{dw}{w^5} = -\int w^{-5} dw = -\frac{w^{-4}}{-4} + c = \frac{1}{4} \frac{1}{w^4} + c = \frac{1}{4 \cos^4 x} + c$   
 $= \frac{\sec^4 x}{4} + c$

**Respuesta:**  $\int \frac{\sec^5}{\cos ecx} dx = \frac{\sec^4 x}{4} + c$

**2.30.-Encontrar:**  $\int e^{\tau g 2x} \sec^2 2x dx$

Solución.- Sea:  $u = \tau g 2x$ , donde:  $du = 2 \sec^2 2x dx$

Luego:  $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2} \int e^{\tau g 2x} (2 \sec^2 2x dx) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + c = \frac{1}{2} e^{\tau g 2x} + c$

**Respuesta:**  $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2} e^{\tau g 2x} + c$

**2.31.-Encontrar:**  $\int \frac{2x-5}{3x^2-2} dx$

Solución.- Sea:  $w = 3x^2 - 2$ , donde:  $dw = 6x dx$

Luego:  $\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3} \int \frac{3(2x-5)}{3x^2-2} dx = \frac{1}{3} \int \frac{6x-15}{3x^2-2} dx = \frac{1}{3} \int \frac{6x dx}{3x^2-2} - \frac{15}{3} \int \frac{dx}{3x^2-2}$   
 $= \frac{1}{3} \int \frac{6x dx}{3x^2-2} - 5 \int \frac{dx}{3(x^2-\frac{2}{3})} = \frac{1}{3} \int \frac{6x dx}{3x^2-2} - \frac{5}{3} \int \frac{dx}{(x^2-\frac{2}{3})} = \frac{1}{3} \int \frac{6x dx}{3x^2-2} - \frac{5}{3} \int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2}$   
 $\frac{1}{3} \int \frac{dw}{w} - \frac{5}{3} \int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2} = \frac{1}{3} \ell \eta |w| + c_1 - \frac{5}{3} \int \frac{dx}{x^2-(\sqrt{\frac{2}{3}})^2}$ ; Sea:  $v = x$ , donde:  $dv = dx$

Además:  $a = \sqrt{\frac{2}{3}}$ ; se tiene:  $\frac{1}{3} \ell \eta |w| + c_1 - \frac{5}{3} \int \frac{dv}{v^2 - a^2}$

$= \frac{1}{3} \ell \eta |3x^2 - 2| + c_1 - \frac{5}{3} \frac{1}{2a} \ell \eta \left| \frac{v-a}{v+a} \right| + c_2 = \frac{1}{3} \ell \eta |3x^2 - 2| - \frac{5}{3} \left[ \frac{1}{2\sqrt{\frac{2}{3}}} \ell \eta \left| \frac{x-\sqrt{\frac{2}{3}}}{x+\sqrt{\frac{2}{3}}} \right| \right] + C$   
 $= \frac{1}{3} \ell \eta |3x^2 - 2| - \frac{5}{\sqrt{32}\sqrt{2}} \ell \eta \left| \frac{\sqrt{3}x - \sqrt{2}}{\sqrt{3}x + \sqrt{2}} \right| + C = \frac{1}{3} \ell \eta |3x^2 - 2| - \frac{5}{2\sqrt{6}} \ell \eta \left| \frac{\sqrt{3}x - \sqrt{2}}{\sqrt{3}x + \sqrt{2}} \right| + C$

**Respuesta:**  $\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3} \ell \eta |3x^2-2| - \frac{5}{2\sqrt{6}} \ell \eta \left| \frac{\sqrt{3x}-\sqrt{2}}{\sqrt{3x}+\sqrt{2}} \right| + C$

**2.32.-Encontrar:**  $\int \frac{dx}{x\sqrt{4-9\ell\eta^2x}}$

Solución.-  $\int \frac{dx}{x\sqrt{4-9\ell\eta^2x}} = \int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}}$

Sea:  $u = 3\ell\eta x$ , donde:  $du = \frac{3dx}{x}$

Luego:  $\int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3} \int \frac{3dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3} \int \frac{du}{\sqrt{2^2-u^2}} = \frac{1}{3} \arcsen \frac{u}{2} + c$   
 $= \frac{1}{3} \arcsen \frac{3\ell\eta x}{2} + c = \frac{1}{3} \arcsen \ell \eta |x|^{\frac{3}{2}} + c$

**Respuesta:**  $\int \frac{dx}{x\sqrt{4-9\ell\eta^2x}} = \frac{1}{3} \arcsen \ell \eta |x|^{\frac{3}{2}} + c$

**2.33.-Encontrar:**  $\int \frac{dx}{\sqrt{e^x-1}}$

Solución.- Sea:  $u = \sqrt{e^x-1}$ , donde:  $du = \frac{e^x dx}{2\sqrt{e^x-1}}$ ; Tal que:  $e^x = u^2 + 1$

Luego:  $\int \frac{dx}{\sqrt{e^x-1}} = \int \frac{2du}{u^2+1} = 2 \int \frac{du}{u^2+1} = 2 \operatorname{arctg} u + c = 2 \operatorname{arctg} \sqrt{e^x-1} + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{e^x-1}} = 2 \operatorname{arctg} \sqrt{e^x-1} + c$

**2.34.-Encontrar:**  $\int \frac{x^2+2x+2}{x+1} dx$

Solución.-  $\int \frac{x^2+2x+2}{x+1} dx = \int \frac{(x^2+2x+1)+1}{x+1} dx = \int \frac{(x+1)^2+1}{x+1} dx = \int \frac{(x+1)^2+1}{x+1} dx$   
 $= \int (x+1 + \frac{1}{x+1}) dx = \int x dx + \int dx + \int \frac{dx}{x+1}$ , Sea:  $w = x+1$ , donde:  $dw = dx$

Luego:  $\int x dx + \int dx + \int \frac{dx}{x+1} = \int x dx + \int dx + \int \frac{dw}{w} = \frac{x^2}{2} + x + \ell \eta |w| + c$   
 $= \frac{x^2}{2} + x + \ell \eta |x+1| + c$

**Respuesta:**  $\int \frac{x^2+2x+2}{x+1} dx = \frac{x^2}{2} + x + \ell \eta |x+1| + c$

**2.35.-Encontrar:**  $\int \frac{e^{2x}}{\sqrt{e^x+1}} dx$

Solución.- Sea:  $u = e^x + 1$ , donde:  $du = e^x dx$

$$\begin{aligned}\text{Luego: } \int \frac{e^{2x}}{\sqrt{e^x+1}} dx &= \int \frac{u-1}{u^{1/2}} du = \int (u^{1/2} - u^{-1/2}) du = \int u^{1/2} du - \int u^{-1/2} du = \frac{u^{3/2}}{3/2} - \frac{u^{-1/2}}{-1/2} + c \\ &= \frac{u^{3/2}}{3/2} - \frac{u^{-1/2}}{-1/2} + c = \frac{2}{3} u^{3/2} - \frac{1}{2} u^{1/2} + c = \frac{2}{3} \sqrt{(e^x+1)^3} - 2\sqrt{e^x+1} + c\end{aligned}$$

$$\text{Respuesta: } \int \frac{e^{2x}}{\sqrt{e^x+1}} dx = \frac{2}{3} \sqrt{(e^x+1)^3} - 2\sqrt{e^x+1} + c$$

$$\text{2.36.-Encontrar: } \int \frac{\ell \eta 2x}{\ell \eta 4x} \frac{dx}{x}$$

$$\begin{aligned}\text{Solución.- Sea: } u &= \ell \eta 4x, \text{ donde: } du = \frac{dx}{x}; \text{ además: } \ell \eta 4x = (2 \times 2x) = \ell \eta 2 + \ell \eta 2x \\ \Rightarrow u &= \ell \eta 2 + \ell \eta 2x \Rightarrow \ell \eta 2x = u - \ell \eta 2\end{aligned}$$

$$\begin{aligned}\text{Luego: } \int \frac{\ell \eta 2x}{\ell \eta 4x} \frac{dx}{x} &= \int \frac{u - \ell \eta 2}{u} du = \int du - \int \frac{\ell \eta 2}{u} du = \int du - \ell \eta 2 \int \frac{du}{u} = u - \ell \eta 2 |u| + c \\ &= \ell \eta 4x - \ell \eta 2 [\ell \eta (\ell \eta 4x)] + c\end{aligned}$$

$$\text{Respuesta: } \int \frac{\ell \eta 2x}{\ell \eta 4x} \frac{dx}{x} = \ell \eta 4x - \ell \eta 2 [\ell \eta (\ell \eta 4x)] + c$$

$$\text{2.37.-Encontrar: } \int x(3x+1)^7 dx$$

$$\text{Solución.- Sea: } w = 3x+1, \text{ donde: } dw = 3dx; \text{ además: } w-1 = 3x \Rightarrow x = \frac{w-1}{3}$$

$$\begin{aligned}\text{Luego: } \int x(3x+1)^7 dx &= \int \frac{w-1}{3} w^7 \frac{dw}{3} = \frac{1}{9} \int (w-1)w^7 dw = \frac{1}{9} \int (w^8 - w^7) dw \\ &= \frac{1}{9} \int w^8 dw - \frac{1}{9} \int w^7 dw = \frac{1}{9} \frac{w^9}{9} - \frac{1}{9} \frac{w^8}{8} + c = \frac{1}{81} w^9 - \frac{1}{72} w^8 + c \\ &= \frac{1}{81} (3x+1)^9 - \frac{1}{72} (3x+1)^8 + c\end{aligned}$$

$$\text{Respuesta: } \int x(3x+1)^7 dx = \frac{(3x+1)^9}{81} - \frac{(3x+1)^8}{72} + c$$

$$\text{2.38.-Encontrar: } \int \frac{x^2-5x+6}{x^2+4} dx$$

$$\text{Solución.- } \frac{x^2-5x+6}{x^2+4} dx = 1 + \frac{2-5x}{x^2+4}$$

$$\text{Luego: } \int \frac{x^2-5x+6}{x^2+4} dx = \int (1 + \frac{2-5x}{x^2+4}) dx = \int dx + 2 \int \frac{dx}{x^2+4} - 5 \int \frac{xdx}{x^2+4}$$

$$\text{Sea: } u = x^2+4, \text{ donde: } du = 2xdx; \text{ Entonces:}$$

$$= x + \arctan g \frac{x}{2} - \frac{5}{2} \int \frac{du}{u} = x + \arctan g \frac{x}{2} - \frac{5}{2} \ell \eta |u| + c = x + \arctan g \frac{x}{2} - \frac{5}{2} \ell \eta |x^2+4| + c$$

$$\text{Respuesta: } \int \frac{x^2-5x+6}{x^2+4} dx = x + \arctan g \frac{x}{2} - \frac{5}{2} \ell \eta |x^2+4| + c$$

## EJERCICIOS PROPUESTOS

Usando Esencialmente la técnica de integración por sustitución, encontrar las siguientes integrales:

$$2.39.- \int 3^x e^x dx$$

$$2.42.- \int \frac{1-3x}{3+2x} dx$$

$$2.45.- \int \frac{3t^2+3}{t-1} dt$$

$$2.48.- \int \left( a + \frac{b}{x-a} \right)^2 dx$$

$$2.51.- \int \sqrt{a-bx} dx$$

$$2.54.- \int \frac{dx}{3x^2+5}$$

$$2.57.- \int \frac{6t-15}{3t^2-2} dt$$

$$2.60.- \int \frac{x dx}{x^2-5}$$

$$2.63.- \int \frac{x dx}{\sqrt{a^4-x^4}}$$

$$2.66.- \int \frac{x - \sqrt{\arcsen t} \operatorname{tg} 3x}{1+9x^2} dx$$

$$2.69.- \int \frac{dt}{\sqrt{(9+9t^2)\ell\eta|t+\sqrt{1+t^2}|}}$$

$$2.72.- \int (e^t - e^{-t}) dt$$

$$2.75.- \int \frac{a^{2x}-1}{\sqrt{a^x}} dx$$

$$2.78.- \int x^{7x^2} dx$$

$$2.81.- \int (e^{\frac{x}{a}} + 1)^{\frac{1}{2}} e^{\frac{x}{a}} dx$$

$$2.84.- \int \frac{e^{-bx}}{1-e^{-2bx}} dx$$

$$2.87.- \int \operatorname{sen}(a+bx) dx$$

$$2.90.- \int (\cos ax + \operatorname{sen} ax)^2 dx$$

$$2.40.- \int \frac{adx}{a-x}$$

$$2.43.- \int \frac{x dx}{a+bx}$$

$$2.46.- \int \frac{x^2+5x+7}{x+3} dx$$

$$2.49.- \int \frac{x}{(x+1)^2} dx$$

$$2.52.- \int \frac{x dx}{\sqrt{x^2+1}}$$

$$2.55.- \int \frac{x^3 dx}{a^2-x^2}$$

$$2.58.- \int \frac{3-2x}{5x^2+7} dx$$

$$2.61.- \int \frac{x dx}{2x^2+3}$$

$$2.64.- \int \frac{x^2 dx}{1+x^6}$$

$$2.67.- \int \sqrt{\arcsen t} dt$$

$$2.70.- \int ae^{-mx} dx$$

$$2.73.- \int e^{-(x^2+1)} x dx$$

$$2.76.- \int \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$2.79.- \int \frac{e^t dt}{e^t-1}$$

$$2.82.- \int \frac{dx}{2^x+3}$$

$$2.85.- \int \frac{e^t dt}{\sqrt{1-e^{2t}}}$$

$$2.88.- \int \cos \sqrt{x} \frac{dx}{\sqrt{x}}$$

$$2.91.- \int \operatorname{sen}^2 x dx$$

$$2.41.- \int \frac{4t+6}{2t+1} dt$$

$$2.44.- \int \frac{ax-b}{\alpha x+\beta} dx$$

$$2.47.- \int \frac{x^4+x^2+1}{x-1} dx$$

$$2.50.- \int \frac{b dy}{\sqrt{1-y}}$$

$$2.53.- \int \frac{\sqrt{x} + \ell \eta x}{x} dx$$

$$2.56.- \int \frac{y^2-5y+6}{y^2+4} dy$$

$$2.59.- \int \frac{3x+1}{\sqrt{5x^2+1}} dx$$

$$2.62.- \int \frac{ax+b}{a^2x^2+b^2} dx$$

$$2.65.- \int \frac{x^2 dx}{\sqrt{x^6-1}}$$

$$2.68.- \int \frac{\arcsen t \operatorname{tg}(\frac{x}{3})}{9+x^2} dx$$

$$2.71.- \int 4^{2-3x} dx$$

$$2.74.- \int (e^{\frac{x}{a}} - e^{-\frac{x}{a}})^2 dx$$

$$2.77.- \int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}}$$

$$2.80.- \int e^x \sqrt{a-be^x} dx$$

$$2.83.- \int \frac{a^x dx}{1+a^{2x}}; a > 0$$

$$2.86.- \int \cos \frac{x}{\sqrt{2}} dx$$

$$2.89.- \int \operatorname{sen}(\ell \eta x) \frac{dx}{x}$$

$$2.92.- \int \cos^2 x dx$$

$$2.93.- \int \sec^2(ax+b)dx$$

$$2.96.- \int \frac{dx}{3\cos(5x-\frac{\pi}{4})}$$

$$2.99.- \int \cot g \frac{x}{a-b} dx$$

$$2.102.- \int \left( \frac{1}{\operatorname{sen} x \sqrt{2}} - 1 \right)^2 dx$$

$$2.105.- \int t \operatorname{sen}(1-2t^2) dt$$

$$2.108.- \int \frac{\operatorname{sen} x \cos x}{\sqrt{\cos^2 x - \operatorname{sen}^2 x}} dx$$

$$2.111.- \int t \cot g(2t^2-3) dt$$

$$2.114.- \int \sqrt{1+3\cos^2 x} \operatorname{sen} 2x dx$$

$$2.117.- \int \frac{(\cos ax + \operatorname{sen} ax)^2}{\operatorname{sen} ax} dx$$

$$2.120.- \int \frac{x^3-1}{x^4-4x+1} dx$$

$$2.123.- \int \frac{\tau g 3x - \cot g 3x}{\operatorname{sen} 3x} dx$$

$$2.126.- \int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x - 2}}$$

$$2.129.- \int \frac{x^2}{\sqrt{x^3+1}} dx$$

$$2.132.- \int \frac{\sec^2 x dx}{\sqrt{4-\tau g^2 x}}$$

$$2.135.- \int \tau g \sqrt{x-1} \frac{dx}{\sqrt{x-1}}$$

$$2.138.- \int \frac{e^{\arctan x} + x \ell \eta (1+x^2) + 1}{1+x^2} dx$$

$$2.141.- \int \frac{(1-\operatorname{sen} \frac{x}{\sqrt{2}})^2}{\operatorname{sen} \frac{x}{\sqrt{2}}} dx$$

$$2.144.- \int \frac{d\theta}{\operatorname{sen} a\theta \cos a\theta}$$

$$2.94.- \int \cos \tau g^2 ax dx$$

$$2.97.- \int \frac{dx}{\operatorname{sen}(ax+b)}$$

$$2.100.- \int \tau g \sqrt{x} \frac{dx}{\sqrt{x}}$$

$$2.103.- \int \frac{dx}{\operatorname{sen} x \cos x}$$

$$2.106.- \int \frac{\operatorname{sen} 3x}{3+\cos 3x} dx$$

$$2.109.- \int \frac{\sqrt{\tau g x}}{\cos^2 x} dx$$

$$2.112.- \int \frac{x^3 dx}{x^8+5}$$

$$2.115.- \int x^5 \sqrt{5-x^2} dx$$

$$2.118.- \int \frac{x^3-1}{x+1} dx$$

$$2.121.- \int x e^{-x^2} dx$$

$$2.124.- \int \frac{dx}{\sqrt{e^x}}$$

$$2.127.- \int \frac{dx}{x \ell \eta^2 x}$$

$$2.130.- \int \frac{x dx}{\sqrt{1-x^4}}$$

$$2.133.- \int \frac{dx}{\cos \frac{x}{a}}$$

$$2.136.- \int \frac{x dx}{\operatorname{sen} x^2}$$

$$2.139.- \int \frac{x^2 dx}{x^2-2}$$

$$2.142.- \int \frac{5-3x}{\sqrt{4-3x^2}} dx$$

$$2.145.- \int \frac{e^s}{\sqrt{e^{2s}-2}} ds$$

$$2.95.- \int \frac{dx}{\operatorname{sen} \frac{x}{a}}$$

$$2.98.- \int \frac{x dx}{\cos^2 x^2}$$

$$2.101.- \int \frac{dx}{\tau g \frac{x}{5}}$$

$$2.104.- \int \frac{\cos ax}{\operatorname{sen}^5 ax} dx$$

$$2.107.- \int \tau g^3 \frac{x}{3} \sec^2 \frac{x}{3} dx$$

$$2.110.- \int \cos \frac{x}{a} \operatorname{sen} \frac{x}{a} dx$$

$$2.113.- \int \operatorname{sen}^3 6x \cos 6x dx$$

$$2.116.- \int \frac{1+\operatorname{sen} 3x}{\cos^2 3x} dx$$

$$2.119.- \int \frac{\operatorname{cosec}^2 3x dx}{b-a \cot g 3x}$$

$$2.122.- \int \frac{3-\sqrt{2+3x^2}}{2+3x^2} dx$$

$$2.125.- \int \frac{1+\operatorname{sen} x}{x+\cos x} dx$$

$$2.128.- \int a^{\operatorname{sen} x} \cos x dx$$

$$2.131.- \int \tau g^2 ax dx$$

$$2.134.- \int \frac{\sqrt[3]{1+\ell \eta x}}{x} dx$$

$$2.137.- \int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x} dx$$

$$2.140.- \int e^{\operatorname{sen}^2 x} \operatorname{sen} 2x dx$$

$$2.143.- \int \frac{ds}{e^s+1}$$

$$2.146.- \int \operatorname{sen}(\frac{2\pi t}{T} + \varphi_0) dt$$

$$2.147.- \int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}} dx$$

$$2.150.- \int \frac{\operatorname{sen} x \cos x}{\sqrt{2-\operatorname{sen}^4 x}} dx$$

$$2.153.- \int \frac{\arcsen x + x}{\sqrt{1-x^2}} dx$$

$$2.156.- \int \frac{\ell \eta (x + \sqrt{x^2 + 1})}{x^2 + 1} dx$$

$$2.159.- \int \frac{(\arcsen x)^2}{\sqrt{1-x^2}} dx$$

$$2.162.- \int \frac{2t^2 - 10t + 12}{t^2 + 4} dt$$

$$2.148.- \int \frac{dx}{x(4 - \ell \eta^2 x)}$$

$$2.151.- \int \frac{\sec x \operatorname{tg} x}{\sqrt{\sec^2 x + 1}} dx$$

$$2.154.- \int \frac{x dx}{\sqrt{x+1}}$$

$$2.157.- \int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}} dx$$

$$2.150.- \int e^{x+e^x} dx$$

$$2.163.- \int \frac{e^t - e^{-t}}{e^t + e^{-t}} dt$$

$$2.149.- \int e^{-\operatorname{tg} x} \sec^2 x dx$$

$$2.152.- \int \frac{dt}{\operatorname{sen}^2 t \cos^2 t}$$

$$2.155.- \int x(5x^2 - 3)^7 dx$$

$$2.158.- \int \frac{\cos x dx}{\sqrt{1 + \operatorname{sen}^2 x}}$$

$$2.161.- \int t(4t + 1)^7 dt$$

## RESPUESTAS

$$2.39.- \int 3^x e^x dx,$$

$$\text{Sea: } u = x, du = dx, a = 3e$$

$$\int (3e)^x dx = \int (a)^u du = \frac{a^u}{\ell \eta a} + c = \frac{(3e)^x}{\ell \eta (3e)} + c = \frac{(3e)^x}{\ell \eta 3 \ell \eta e} + c = \frac{3^x e^x}{\ell \eta 3 + \ell \eta e} + c = \frac{3^x e^x}{\ell \eta 3 + 1} + c$$

$$2.40.- \int \frac{adx}{a-x},$$

$$\text{Sea: } u = a - x, du = -dx$$

$$\int \frac{adx}{a-x} = -a \int \frac{du}{u} = -a \ell \eta |u| + c = -a \ell \eta |a-x| + c$$

$$2.41.- \int \frac{4t+6}{2t+1} dt,$$

$$\text{Sea: } u = 2t+1, du = 2dt; \quad \frac{2t+3}{2t+1} = 1 + \frac{2}{2t+1}$$

$$\int \frac{4t+6}{2t+1} dt = 2 \int \left( 1 + \frac{2}{2t+1} \right) dt = 2 \int dt + 2 \int \frac{2}{2t+1} dt = 2 \int dt + 2 \int \frac{du}{u} = 2t + 2 \ell \eta |u| + c = 2t + 2 \ell \eta |2t+1| + c$$

$$2.42.- \int \frac{1-3x}{3+2x} dx,$$

$$\text{Sea: } u = 3+2x, du = 2dx; \quad \frac{1-3x}{3+2x} = -\frac{3}{2} + \frac{11/2}{2x+3}$$

$$\int \frac{1-3x}{3+2x} dx = \int \left( -\frac{3}{2} + \frac{11/2}{2x+3} \right) dx = -\frac{3}{2} \int dx + \frac{11}{4} \int \frac{dx}{2x+3} = -\frac{3}{2} \int dx + \frac{11}{4} \int \frac{du}{u} = -\frac{3}{2} x + \frac{11}{4} \ell \eta |2x+3| + c$$

$$2.43.- \int \frac{xdx}{a+bx},$$

$$\text{Sea: } u = a+bx, du = bdx; \quad \frac{x}{a+bx} = \frac{1}{b} - \frac{a/b}{a+bx}$$

$$\int \frac{xdx}{a+bx} = \frac{1}{b} \int dx - \frac{a}{b} \int \frac{dx}{a+bx} = \frac{1}{b} \int dx - \frac{a}{b^2} \int \frac{du}{u} = \frac{1}{b} x - \frac{a}{b^2} \ell \eta |u| + c = \frac{x}{b} - \frac{a}{b^2} \ell \eta |a+bx| + c$$

$$\mathbf{2.44.-} \int \frac{ax-b}{\alpha x+\beta} dx, \quad \text{Sea: } u = \alpha x + \beta, du = \alpha dx; \quad \frac{ax-b}{\alpha x+\beta} = \frac{a}{\alpha} - \frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x}$$

$$\begin{aligned} \int \frac{ax-b}{\alpha x+\beta} dx &= \int \left( \frac{a}{\alpha} - \frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x} \right) dx = \int \frac{a}{\alpha} dx - \int \frac{\frac{\alpha\beta+\alpha b}{\alpha}}{\alpha x+\beta} dx = \frac{a}{\alpha} \int dx - \frac{a\beta+\alpha b}{\alpha} \int \frac{dx}{a\beta+\alpha b} \\ &= \frac{a}{\alpha} \int dx - \frac{a\beta+\alpha b}{\alpha^2} \int \frac{du}{u} = \frac{a}{\alpha} x - \frac{a\beta+\alpha b}{\alpha^2} \ell \eta |u| + c = \frac{a}{\alpha} x - \frac{a\beta+\alpha b}{\alpha^2} \ell \eta |\ell x + \beta| + c \end{aligned}$$

$$\mathbf{2.45.-} \int \frac{3t^2+3}{t-1} dt, \quad \text{Sea: } u = t-1, du = dt; \quad \frac{t^2+1}{t-1} = t+1 + \frac{2}{t-1}$$

$$\begin{aligned} \int \frac{3t^2+3}{t-1} dt &= 3 \int \left( t+1 + \frac{2}{t-1} \right) dt = 3 \int t dt + 3 \int dt + 3 \int \frac{2}{t-1} dt = \frac{3}{2} t^2 + 3t + 6 \ell \eta |u| + c \\ &= \frac{3}{2} t^2 + 3t + 6 \ell \eta |t-1| + c \end{aligned}$$

$$\mathbf{2.46.-} \int \frac{x^2+5x+7}{x+3} dx, \quad \text{Sea: } u = t-1, du = t+1; \quad \frac{x^2+5x+7}{x+3} = x+2 + \frac{1}{x+3}$$

$$\begin{aligned} \int \frac{x^2+5x+7}{x+3} dx &= \int \left( x+2 + \frac{1}{x+3} \right) dx = \int x dx + 2 \int dx + \int \frac{1}{x+3} dx = \frac{x^2}{2} + 2x + \ell \eta |u| + c \\ &= \frac{x^2}{2} + 2x + \ell \eta |u| + c = \frac{x^2}{2} + 2x + \ell \eta |x+3| + c \end{aligned}$$

$$\mathbf{2.47.-} \int \frac{x^4+x^2+1}{x-1} dx, \quad \text{Sea: } u = x-1, du = dx;$$

$$\begin{aligned} \int \frac{x^4+x^2+1}{x-1} dx &= \int \left( x^3 + x^2 + 2x + 2 + \frac{3}{x-1} \right) dx = \int x^3 dx + \int x^2 dx + 2 \int dx + 3 \int \frac{dx}{x-1} \\ &= \frac{x^4}{4} + \frac{x^3}{3} + x^2 + 2 + 3 \ell \eta |u| + c = \frac{x^4}{4} + \frac{x^3}{3} + x^2 + 2x + 3 \ell \eta |x-1| + c \end{aligned}$$

$$\mathbf{2.48.-} \int \left( a + \frac{b}{x-a} \right)^2 dx, \quad \text{Sea: } u = x-a, du = dx$$

$$\begin{aligned} \int \left( a + \frac{b}{x-a} \right)^2 dx &= \int \left( a^2 + \frac{2ab}{x-a} + \frac{b^2}{(x-a)^2} \right) dx = a^2 \int dx + 2ab \int \frac{dx}{x-a} + b^2 \int \frac{dx}{(x-a)^2} \\ &= a^2 \int dx + 2ab \int \frac{du}{u} + b^2 \int \frac{du}{u^2} = a^2 x + 2ab \ell \eta |u| + b^2 \frac{u^{-1}}{-1} + c = a^2 x + 2ab \ell \eta |x-a| - \frac{b^2}{x-a} + c \quad \mathbf{2.} \end{aligned}$$

$$\mathbf{49.-} \int \frac{x}{(x+1)^2} dx, \quad \text{Sea: } u = x+1, du = dx$$

$$\int \frac{x}{(x+1)^2} dx = \int \frac{(x+1)-1}{(x+1)^2} dx = \int \frac{x+1}{(x+1)^2} dx - \int \frac{dx}{(x+1)^2} = \int \frac{dx}{u} - \int \frac{dx}{u^2} = \ell \eta |u| - \frac{u^{-1}}{-1} + c$$

$$= \ell \eta |x+1| + \frac{1}{x+1} + c$$

$$\mathbf{2.50.} \int \frac{b dy}{\sqrt{1-y}}, \quad \text{Sea: } u = 1-y, du = -dy$$

$$\int \frac{b dy}{\sqrt{1-y}} = -b \int \frac{du}{\sqrt{u}} = -b \int u^{-1/2} du = -2bu^{1/2} + c = -2b(1-y)^{1/2} + c$$

$$\mathbf{2.51.} \int \sqrt{a-bx} dx, \quad \text{Sea: } u = a-bx, du = -b dx$$

$$\int \sqrt{a-bx} dx = -\frac{1}{b} \int u^{1/2} du = -\frac{1}{b} \frac{u^{3/2}}{3/2} + c = -\frac{2}{3b} u^{3/2} + c = -\frac{3}{2b} (a-bx)^{3/2} + c$$

$$\mathbf{2.52.} \int \frac{x dx}{\sqrt{x^2+1}}, \quad \text{Sea: } u = x^2+1, du = 2x dx$$

$$\int \frac{x dx}{\sqrt{x^2+1}} = \frac{1}{2} \int \frac{du}{\sqrt{u}} = \frac{1}{2} \int u^{-1/2} du = \frac{1}{2} \frac{u^{1/2}}{1/2} + c = (x^2+1)^{1/2} + c$$

$$\mathbf{2.53.} \int \frac{\sqrt{x} + \ell \eta x}{x} dx, \quad \text{Sea: } u = \ell \eta x, du = \frac{dx}{x}$$

$$\begin{aligned} \int \frac{\sqrt{x} + \ell \eta x}{x} dx &= \int x^{-1/2} dx + \int \frac{\ell \eta x}{x} dx = \int x^{-1/2} dx + \int u du = \frac{x^{1/2}}{1/2} + \frac{u^2}{2} + c \\ &= 2\sqrt{x} + \frac{\ell \eta^2 x}{2} + c \end{aligned}$$

$$\mathbf{2.54.} \int \frac{dx}{3x^2+5}, \quad \text{Sea: } u^2 = 3x^2, u = \sqrt{3}x, du = \sqrt{3}dx; a^2 = 5; a = \sqrt{5}$$

$$\int \frac{dx}{3x^2+5} = \frac{1}{\sqrt{3}} \int \frac{du}{u^2+a^2} = \frac{1}{\sqrt{3}} \frac{1}{a} \operatorname{arctg} \frac{u}{a} + c = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{5}} \operatorname{arctg} \frac{\sqrt{3}x}{\sqrt{5}} + c = \frac{\sqrt{15}}{15} \operatorname{arctg} \sqrt{\frac{3x}{5}} + c$$

$$\mathbf{2.55.} \int \frac{x^3 dx}{a^2-x^2}, \quad \text{Sea: } u = x^2-a^2, du = 2x dx$$

$$\begin{aligned} \int \frac{x^3 dx}{a^2-x^2} &= -\int x dx - \int \frac{a^2 x dx}{x^2-a^2} = -\int x dx - a^2 \int \frac{x dx}{x^2-a^2} = -\int x dx - \frac{a^2}{2} \int \frac{du}{u} \\ &= -\frac{x^2}{2} - \frac{a^2}{2} \ell \eta |u| + c = -\frac{x^2}{2} - \frac{a^2}{2} \ell \eta |x^2-a^2| + c \end{aligned}$$

$$\mathbf{2.56.} \int \frac{y^2-5y+6}{y^2+4} dy, \quad \text{Sea: } u = y^2+4, du = 2y dy$$

$$\begin{aligned} \int \frac{y^2-5y+6}{y^2+4} dy &= \int \left(1 + \frac{-5y+2}{y^2+4}\right) dy = \int dy + \int \frac{-5y+2}{y^2+4} dy = \int dy - 5 \int \frac{y dy}{y^2+4} + 2 \int \frac{dy}{y^2+2^2} \\ &= y - \frac{5}{2} \ell \eta |u| + \frac{1}{2} \operatorname{arctg} \frac{y}{2} + c = y - \frac{5}{2} \ell \eta |y^2+4| + \operatorname{arctg} \frac{y}{2} + c \end{aligned}$$

$$\mathbf{2.57.} \int \frac{6t-15}{3t^2-2} dt, \quad \text{Sea: } u = 3t^2-2, du = 6t dt; w = \sqrt{3}t, dw = \sqrt{3} dt$$



$$\begin{aligned}
\int \frac{6t-15}{3t^2-2} dt &= 6 \int \frac{tdt}{3t^2-2} - 15 \int \frac{dt}{3t^2-2} = 6 \int \frac{tdt}{3t^2-2} - 15 \int \frac{dt}{(\sqrt{3t})^2 - (\sqrt{2})^2} \\
&= \int \frac{du}{u} - \frac{15}{\sqrt{3}} \int \frac{dw}{w^2 - (\sqrt{2})^2} = \ell \eta |u| - \frac{15\sqrt{3}}{3} \frac{1}{2\sqrt{2}} \ell \eta \left| \frac{w-\sqrt{2}}{w+\sqrt{2}} \right| + c \\
&= \ell \eta |3t^2-2| - \frac{5\sqrt{6}}{4} \ell \eta \left| \frac{t\sqrt{3}-\sqrt{2}}{t\sqrt{3}+\sqrt{2}} \right| + c
\end{aligned}$$

**2.58.-**  $\int \frac{3-2x}{5x^2+7} dx$ , Sea:  $u = 5x^2 + 7, du = 10xdx; w = \sqrt{5}x, dw = \sqrt{5}dx$

$$\begin{aligned}
\int \frac{3-2x}{5x^2+7} dx &= 3 \int \frac{dx}{5x^2+7} - 2 \int \frac{dx}{5x^2+7} = 3 \int \frac{dx}{(\sqrt{5}x)^2 + (\sqrt{7})^2} - \frac{2}{10} \int \frac{du}{u} \\
&= \frac{3}{\sqrt{5}} \int \frac{dw}{w^2 + (\sqrt{7})^2} - \frac{1}{5} \int \frac{du}{u} = \frac{3}{\sqrt{5}} \frac{1}{\sqrt{7}} \operatorname{arc} \tau g \frac{x\sqrt{5}}{\sqrt{7}} - \frac{1}{5} \ell \eta |u| + c \\
&= \frac{3\sqrt{35}}{35} \operatorname{arc} \tau g x \sqrt{\frac{5}{7}} - \frac{1}{5} \ell \eta |5x^2+7| + c
\end{aligned}$$

**2.59.-**  $\int \frac{3x+1}{\sqrt{5x^2+1}} dx$ , Sea:  $u = 5x^2 + 1, du = 10xdx; w = x\sqrt{5}, dw = \sqrt{5}dx$

$$\begin{aligned}
\int \frac{3x+1}{\sqrt{5x^2+1}} dx &= 3 \int \frac{xdx}{\sqrt{5x^2+1}} + \int \frac{dx}{\sqrt{(x\sqrt{5})^2 + 1^2}} = 3 \int \frac{xdx}{\sqrt{5x^2+1}} + \int \frac{dw}{\sqrt{(x\sqrt{5})^2 + 1^2}} \\
&= \frac{3}{10} \int \frac{du}{\sqrt{u}} + \frac{1}{\sqrt{5}} \int \frac{dw}{\sqrt{w^2 + 1^2}} = \frac{3}{10} \frac{u^{1/2}}{1/2} + \frac{1}{\sqrt{5}} \ell \eta |w + \sqrt{w^2 + 1}| + c \\
&= \frac{3}{5} \sqrt{5x^2+1} + \frac{1}{\sqrt{5}} \ell \eta |x\sqrt{5} + \sqrt{5x^2+1}| + c
\end{aligned}$$

**2.60.-**  $\int \frac{xdx}{x^2-5}$ , Sea:  $u = x^2 + 5, du = 2xdx$

$$\int \frac{xdx}{x^2-5} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ell \eta |u| + c = \frac{1}{2} \ell \eta |x^2-5| + c$$

**2.61.-**  $\int \frac{xdx}{2x^2+3}$ , Sea:  $u = 2x^2 + 3, du = 4xdx$

$$\int \frac{xdx}{2x^2+3} = \frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ell \eta |u| + c = \frac{1}{4} \ell \eta |2x^2+3| + c$$

**2.62.-**  $\int \frac{ax+b}{a^2x^2+b^2} dx$ , Sea:  $u = a^2x^2 + b^2, du = 2a^2xdx; w = ax, dw = adx$

$$\begin{aligned}
\int \frac{ax+b}{a^2x^2+b^2} dx &= a \int \frac{xdx}{a^2x^2+b^2} + b \int \frac{dx}{a^2x^2+b^2} = \frac{a}{2a^2} \int \frac{du}{u} + \frac{b}{a} \int \frac{dw}{w^2+b^2} \\
&= \frac{1}{2} \ell \eta |u| + \frac{b}{a} \frac{1}{b} \operatorname{arc} \tau g \frac{w}{b} + c = \frac{1}{2} \ell \eta |a^2x^2+b^2| + \frac{1}{a} \operatorname{arc} \tau g \frac{ax}{b} + c
\end{aligned}$$

$$2.63.- \int \frac{xdx}{\sqrt{a^4 - x^4}}, \quad \text{Sea: } u = x^2, du = 2xdx$$

$$\int \frac{xdx}{\sqrt{a^4 - x^4}} = \int \frac{xdx}{\sqrt{(\sqrt{a^2})^2 - (\sqrt{x^2})^2}} = \frac{1}{2} \int \frac{du}{\sqrt{(\sqrt{a^2})^2 - u^2}} = \frac{1}{2} \arcsen \frac{u}{a^2} + c$$

$$= \frac{1}{2} \arcsen \frac{x^2}{a^2} + c$$

$$2.64.- \int \frac{x^2 dx}{1+x^6}, \quad \text{Sea: } u = x^3, du = 3x^2 dx$$

$$\int \frac{x^2 dx}{1+x^6} = \int \frac{x^2 dx}{1+(x^3)^2} = \frac{1}{3} \int \frac{du}{1+u^2} = \frac{1}{3} \arctan |u| + c = \frac{1}{3} \arctan x^3 + c$$

$$2.65.- \int \frac{x^2 dx}{\sqrt{x^6 - 1}}, \quad \text{Sea: } u = x^3, du = 3x^2 dx$$

$$\int \frac{x^2 dx}{\sqrt{x^6 - 1}} = \int \frac{x^2 dx}{\sqrt{(x^3)^2 - 1}} = \frac{1}{3} \int \frac{du}{\sqrt{u^2 - 1}} = \frac{1}{3} \ell \eta |u + \sqrt{u^2 - 1}| + c = \frac{1}{3} \ell \eta |x^3 + \sqrt{x^6 - 1}| + c$$

$$2.66.- \int \frac{x - \sqrt{\arctan 3x}}{1+9x^2} dx, \quad \text{Sea: } u = 1+9x^2, du = 18xdx; w = \arctan 3x, dw = \frac{3dx}{1+9x^2}$$

$$\int \frac{x - \sqrt{\arctan 3x}}{1+9x^2} dx = \int \frac{xdx}{1+9x^2} - \int \frac{\sqrt{\arctan 3x}}{1+9x^2} dx = \frac{1}{18} \int \frac{du}{u} - \frac{1}{3} \int w^{1/2} dw$$

$$= \frac{1}{18} \ell \eta |u| - \frac{1}{3} \frac{w^{3/2}}{3/2} + c = \frac{1}{18} \ell \eta |1+9x^2| - \frac{2(\arctan 3x)^{3/2}}{9} + c$$

$$2.67.- \int \sqrt{\frac{\arcsen t}{4-4t^2}} dt, \quad \text{Sea: } u = \arcsen t, du = \frac{dt}{\sqrt{1-t^2}}$$

$$\int \sqrt{\frac{\arcsen t}{4-4t^2}} dt = \frac{1}{2} \int \sqrt{\frac{\arcsen t}{1-t^2}} dt = \frac{1}{2} \int \frac{\sqrt{\arcsen t}}{\sqrt{1-t^2}} dt = \frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \frac{u^{3/2}}{3/2} + c = \frac{1}{3} u^{3/2} + c$$

$$= \frac{1}{3} \sqrt{(\arcsen t)^3} + c$$

$$2.68.- \int \frac{\arctan(\frac{x}{3})}{9+x^2} dx, \quad \text{Sea: } u = \arctan \frac{x}{3}, du = \frac{3dx}{9+x^2}$$

$$\int \frac{\arctan(\frac{x}{3})}{9+x^2} dx = \frac{1}{3} \int u du = \frac{1}{3} \frac{u^2}{2} + c = \frac{1}{6} u^2 + c = \frac{\arctan(\frac{x}{3})^2}{6} + c$$

$$2.69.- \int \frac{dt}{\sqrt{(9+9t^2) \ell \eta |t + \sqrt{1+t^2}|}}, \quad \text{Sea: } u = \ell \eta |t + \sqrt{1+t^2}|, du = \frac{dt}{\sqrt{1+t^2}}$$

$$= \frac{1}{3} \int \frac{dt}{\sqrt{(1+t^2) \ell \eta |t + \sqrt{1+t^2}|}} = \frac{1}{3} \int \frac{du}{\sqrt{u}} = \frac{1}{3} \frac{u^{1/2}}{1/2} + c = \frac{2}{3} \sqrt{u} + c = \frac{2}{3} \sqrt{\ell \eta |t + \sqrt{1+t^2}|} + c$$

$$2.70.- \int a e^{-mx} dx, \quad \text{Sea: } u = -mx, du = -mdx$$

$$\int a e^{-mx} dx = a \int e^{-mx} dx = -\frac{a}{m} \int e^u du = -\frac{a}{m} e^u + c = -\frac{a}{m} e^{-mx} + c$$

$$2.71.- \int 4^{2-3x} dx, \quad \text{Sea: } u = 2-3x, du = -3dx; a = 4$$

$$\int 4^{2-3x} dx = -\frac{1}{3} \int a^u du = -\frac{1}{3} \frac{a^u}{\ell \eta a} + c = -\frac{4^{2-3x}}{3\ell \eta 4} + c$$

$$2.72.- \int (e^t - e^{-t}) dt, \quad \text{Sea: } u = -t, du = -dt$$

$$\int (e^t - e^{-t}) dt = \int e^t dt - \int e^{-t} dt = \int e^t dt - \int e^u du = e^t + e^u + c = e^t + e^{-t} + c$$

$$2.73.- \int e^{-(x^2+1)} x dx, \quad \text{Sea: } u = -x^2 - 1, du = -2x dx$$

$$\int e^{-(x^2+1)} x dx = \int e^{-x^2-1} x dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-(x^2+1)} + c = -\frac{1}{2e^{x^2+1}} + c$$

$$2.74.- \int (e^{\frac{x}{a}} - e^{-\frac{x}{a}})^2 dx, \quad \text{Sea: } u = \frac{2x}{a}, du = \frac{2dx}{a}; w = -\frac{2x}{a}, dw = -\frac{2dx}{a}$$

$$\begin{aligned} \int (e^{\frac{x}{a}} - e^{-\frac{x}{a}})^2 dx &= \int (e^{\frac{2x}{a}} + 2e^{\frac{x}{a}} e^{-\frac{x}{a}} + e^{-\frac{2x}{a}}) dx = \int e^{\frac{2x}{a}} dx + 2 \int dx + \int e^{-\frac{2x}{a}} dx \\ &= \frac{a}{2} \int e^u du + 2 \int dx - \frac{a}{2} \int e^w dw = \frac{a}{2} e^u + 2x - \frac{a}{2} e^w + c = \frac{a}{2} e^{\frac{2x}{a}} + 2x - \frac{a}{2} e^{-\frac{2x}{a}} + c \end{aligned}$$

$$2.75.- \int \frac{a^{2x} - 1}{\sqrt{a^x}} dx, \quad \text{Sea: } u = -\frac{x}{2}, du = -\frac{dx}{2}; w = \frac{3x}{2}, dw = \frac{3dx}{2}$$

$$\begin{aligned} \int \frac{a^{2x} - 1}{\sqrt{a^x}} dx &= \int \frac{a^{2x} dx}{\sqrt{a^x}} - \int \frac{dx}{\sqrt{a^x}} = \int a^{2x-\frac{1}{2}} dx - \int a^{-\frac{1}{2}} dx = \int a^{\frac{3x}{2}} dx - \int a^{-\frac{1}{2}} dx \\ &= \frac{2}{3} \int a^w dw + 2 \int a^u du = \frac{2}{3} \frac{a^w}{\ell \eta a} + 2 \frac{a^u}{\ell \eta a} + c = \frac{2}{3} \frac{a^{\frac{3x}{2}}}{\ell \eta a} + 2 \frac{a^{-\frac{1}{2}}}{\ell \eta a} + c = \frac{2}{\ell \eta a} \left( \frac{a^{\frac{3x}{2}}}{3} + a^{-\frac{1}{2}} \right) + c \end{aligned}$$

$$2.76.- \int \frac{e^{\frac{1}{x}}}{x^2} dx, \quad \text{Sea: } u = \frac{1}{x}, du = -\frac{dx}{x^2}$$

$$\int \frac{e^{\frac{1}{x}}}{x^2} dx = -\int e^u du = -e^u + c = -e^{\frac{1}{x}} + c = -\sqrt[x]{e} + c$$

$$2.77.- \int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}}, \quad \text{Sea: } u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$$

$$\int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}} = 2 \int 5^u du = \frac{2 \times 5^u}{\ell \eta 5} + c = \frac{2 \times 5^{\sqrt{x}}}{\ell \eta 5} + c$$

$$2.78.- \int x 7^{x^2} dx, \quad \text{Sea: } u = x^2, du = 2x dx$$

$$\int x 7^{x^2} dx = \frac{1}{2} \int 7^u du = \frac{1}{2} \frac{7^u}{\ell \eta 7} + c = \frac{1}{2} \frac{7^{x^2}}{\ell \eta 7} + c$$

$$2.79.- \int \frac{e^t dt}{e^t - 1}, \quad \text{Sea: } u = e^t - 1, du = e^t dt$$

$$\int \frac{e^t dt}{e^t - 1} = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |e^t - 1| + c$$

**2.80.-**  $\int e^x \sqrt{a - be^x} dx$ , Sea:  $u = a - be^x$ ,  $du = -be^x dx$

$$\int e^x \sqrt{a - be^x} dx = -\frac{1}{b} \int \sqrt{u} du = -\frac{1}{b} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{3b} u^{\frac{3}{2}} + c = -\frac{2}{3b} (a - be^x)^{\frac{3}{2}} + c$$

**2.81.-**  $\int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx$ , Sea:  $u = e^{\frac{x}{a} + 1}$ ,  $du = \frac{e^{\frac{x}{a}}}{a} dx$

$$\int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx = \int \sqrt[3]{e^{\frac{x}{a}} + 1} e^{\frac{x}{a}} dx = a \int u^{\frac{1}{3}} du = \frac{au^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3a(e^{\frac{x}{a}} + 1)^{\frac{4}{3}}}{4} + c$$

**2.82.-**  $\int \frac{dx}{2^x + 3}$ , Sea:  $u = 2^x + 3$ ,  $du = 2^x \ell \eta 2 dx$

$$\begin{aligned} \int \frac{dx}{2^x + 3} &= \frac{1}{3} \int \frac{3dx}{2^x + 3} = \frac{1}{3} \int \frac{2^x + 3 - 2^x}{2^x + 3} dx = \frac{1}{3} \int \frac{2^x + 3}{2^x + 3} dx - \frac{1}{3} \int \frac{2^x}{2^x + 3} dx = \frac{1}{3} \int dx - \frac{1}{3} \int \frac{du}{u} \\ &= \frac{1}{3} x - \frac{1}{3} \ell \eta |u| + c = \frac{1}{3} x - \frac{1}{3\ell \eta 2} \ell \eta |u| + c = \frac{1}{3} x - \frac{\ell \eta |2^x + 3|}{3\ell \eta 2} + c \end{aligned}$$

**2.83.-**  $\int \frac{a^x dx}{1 + a^{2x}}$ , Sea:  $u = a^x$ ,  $du = a^x \ell \eta a dx$ ;  $a > 0$

$$\int \frac{a^x dx}{1 + a^{2x}} = \int \frac{a^x dx}{1 + (a^x)^2} = \frac{1}{\ell \eta a} \int \frac{du}{1 + u^2} = \frac{1}{\ell \eta a} \arctan u + c = \frac{1}{\ell \eta a} \arctan a^x + c$$

**2.84.-**  $\int \frac{e^{-bx}}{1 - e^{-2bx}} dx$ , Sea:  $u = e^{-bx}$ ,  $du = -be^{-bx} dx$

$$\begin{aligned} \int \frac{e^{-bx}}{1 - e^{-2bx}} dx &= \int \frac{e^{-bx}}{1 - (e^{-bx})^2} dx = -\frac{1}{b} \int \frac{du}{1 - u^2} = -\frac{1}{b} \int \frac{du}{(-1)(u^2 - 1)} = \frac{1}{2b} \ell \eta \left| \frac{u - 1}{u + 1} \right| + c \\ &= \frac{1}{2b} \ell \eta \left| \frac{e^{-bx} - 1}{e^{-bx} + 1} \right| + c. \end{aligned}$$

**2.85.-**  $\int \frac{e^t dt}{\sqrt{1 - e^{2t}}}$ , Sea:  $u = e^t$ ,  $du = e^t dt$

$$\int \frac{e^t dt}{\sqrt{1 - e^{2t}}} = \int \frac{e^t dt}{\sqrt{1 - (e^t)^2}} = \int \frac{du}{\sqrt{1 - u^2}} = \arcsen u + c = \arcsen e^t + c$$

**2.86.-**  $\int \cos \frac{x}{\sqrt{2}} dx$ , Sea:  $u = \frac{x}{\sqrt{2}}$ ,  $du = \frac{dx}{\sqrt{2}}$

$$\int \cos \frac{x}{\sqrt{2}} dx = \sqrt{2} \int \cos u du = \sqrt{2} \sen u + c = \sqrt{2} \sen \frac{x}{\sqrt{2}} + c$$

**2.87.-**  $\int \sen(a + bx) dx$ , Sea:  $u = a + bx$ ,  $du = b dx$

$$\int \sen(a + bx) dx = \frac{1}{b} \int \sen u du = -\frac{1}{b} \cos u + c = -\frac{1}{b} \cos(a + bx) + c$$

$$\mathbf{2.88.-} \int \cos \sqrt{x} \frac{dx}{\sqrt{x}}, \quad \text{Sea: } u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$$

$$\int \cos \sqrt{x} \frac{dx}{\sqrt{x}} = 2 \int \cos u du = 2 \operatorname{sen} u + c = 2 \operatorname{sen} \sqrt{x} + c$$

$$\mathbf{2.89.-} \int \operatorname{sen}(\ell \eta x) \frac{dx}{x}, \quad \text{Sea: } u = \ell \eta x, du = \frac{dx}{x}$$

$$\int \operatorname{sen}(\ell \eta x) \frac{dx}{x} = \int \operatorname{sen} u du = -\cos u + c = -\cos \ell \eta x + c$$

$$\mathbf{2.90.-} \int (\cos ax + \operatorname{sen} ax)^2 dx, \quad \text{Sea: } u = 2ax, du = 2adx$$

$$\begin{aligned} \int (\cos ax + \operatorname{sen} ax)^2 dx &= \int (\cos^2 ax + 2 \cos ax \operatorname{sen} ax + \operatorname{sen}^2 ax) dx \\ &= \int (1 + 2 \cos ax \operatorname{sen} ax) dx = \int dx + 2 \int \cos ax \operatorname{sen} ax dx = \int dx + \int \operatorname{sen} 2ax dx \\ &= x - \frac{1}{2a} \cos 2ax + c \end{aligned}$$

$$\mathbf{2.91.-} \int \operatorname{sen}^2 x dx, \quad \text{Sea: } u = 2x, du = 2dx$$

$$\begin{aligned} \int \operatorname{sen}^2 x dx &= \int \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx = \frac{1}{2} \int dx - \frac{1}{4} \int \cos u du = \frac{1}{2} x - \frac{1}{4} \operatorname{sen} u + c \\ &= \frac{1}{2} x - \frac{1}{4} \operatorname{sen} 2x + c \end{aligned}$$

$$\mathbf{2.92.-} \int \cos^2 x dx, \quad \text{Sea: } u = 2x, du = 2dx$$

$$\begin{aligned} \int \cos^2 x dx &= \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x dx = \frac{1}{2} \int dx + \frac{1}{4} \int \cos u du = \frac{1}{2} x + \frac{1}{4} \operatorname{sen} u + c \\ &= \frac{1}{2} x + \frac{1}{4} \operatorname{sen} 2x + c \end{aligned}$$

$$\mathbf{2.93.-} \int \sec^2(ax+b) dx, \quad \text{Sea: } u = ax+b, du = adx$$

$$\int \sec^2(ax+b) dx = \frac{1}{a} \int \sec^2 u du = \frac{1}{a} \tau g u + c = \frac{1}{a} \tau g(ax+b) + c$$

$$\mathbf{2.94.-} \int \cot \tau g^2 ax dx, \quad \text{Sea: } u = ax, du = adx$$

$$\begin{aligned} \int \cot \tau g^2 ax dx &= \frac{1}{a} \int \cot \tau g^2 u du = \frac{1}{a} \int (\operatorname{cosec}^2 u - 1) du = \frac{1}{a} \int \operatorname{cosec}^2 u du - \frac{1}{a} \int du \\ &= -\frac{\cot \tau g u}{a} - \frac{u}{a} + c = -\frac{\cot \tau g ax}{a} - \frac{x}{a} + c = -\frac{\cot \tau g ax}{a} - x + c \end{aligned}$$

$$\mathbf{2.95.-} \int \frac{dx}{\operatorname{sen} \frac{x}{a}}, \quad \text{Sea: } u = \frac{x}{a}, du = \frac{dx}{a}$$

$$\begin{aligned} \int \frac{dx}{\operatorname{sen} \frac{x}{a}} &= \int \operatorname{cosec} \frac{x}{a} dx = a \int \operatorname{cosec} u du = a \ell \eta |\operatorname{cosec} u - \cot \tau g u| + c \\ &= a \ell \eta |\operatorname{cosec} \frac{x}{a} - \cot \tau g \frac{x}{a}| + c \end{aligned}$$

$$\mathbf{2.96.-} \int \frac{dx}{3\cos(5x - \frac{\pi}{4})}, \quad \text{Sea: } u = 5x - \frac{\pi}{4}, du = 5dx$$

$$\begin{aligned} \int \frac{dx}{3\cos(5x - \frac{\pi}{4})} &= \frac{1}{3} \int \sec(5x - \frac{\pi}{4}) dx = \frac{1}{15} \int \sec u du = \frac{1}{15} \ell \eta |\sec u + \tau gu| + c \\ &= \frac{1}{15} \ell \eta |\sec(5x - \frac{\pi}{4}) + \tau g(5x - \frac{\pi}{4})| + c \end{aligned}$$

$$\mathbf{2.97.-} \int \frac{dx}{\operatorname{sen}(ax+b)}, \quad \text{Sea: } u = ax+b, du = a dx$$

$$\begin{aligned} \int \frac{dx}{\operatorname{sen}(ax+b)} &= \int \operatorname{cosec}(ax+b) dx = \frac{1}{a} \int \operatorname{cosec} u du = \frac{1}{a} \ell \eta |\operatorname{cosec} u - \cot \tau gu| + c \\ &= \frac{1}{a} \ell \eta |\operatorname{cosec}(ax+b) - \cot \tau g(ax+b)| + c \end{aligned}$$

$$\mathbf{2.98.-} \int \frac{x dx}{\cos^2 x^2}, \quad \text{Sea: } u = x^2, du = 2x dx$$

$$\int \frac{x dx}{\cos^2 x^2} = \int x \sec^2 x^2 dx = \frac{1}{2} \int \sec^2 u du = \frac{1}{2} \tau gu + c = \frac{1}{2} \tau g x^2 + c$$

$$\mathbf{2.99.-} \int \cot \tau g \frac{x}{a-b} dx, \quad \text{Sea: } u = \frac{x}{a-b}, du = \frac{dx}{a-b}$$

$$\int \cot \tau g \frac{x}{a-b} dx = (a-b) \int \cot \tau g u du = (a-b) \ell \eta |\operatorname{sen} u| + c = (a-b) \ell \eta \left| \operatorname{sen} \frac{x}{a-b} \right| + c$$

$$\mathbf{2.100.-} \int \tau g \sqrt{x} \frac{dx}{\sqrt{x}}, \quad \text{Sea: } u = \sqrt{x}, du = \frac{dx}{2\sqrt{x}}$$

$$\int \tau g \sqrt{x} \frac{dx}{\sqrt{x}} = 2 \int \tau g u du = 2 \ell \eta |\sec u| + c = 2 \ell \eta |\sec \sqrt{x}| + c$$

$$\mathbf{2.101.-} \int \frac{dx}{\tau g \frac{x}{5}}, \quad \text{Sea: } u = \frac{x}{5}, du = \frac{dx}{5}$$

$$\int \frac{dx}{\tau g \frac{x}{5}} = \int \cot \tau g \frac{x}{5} dx = 5 \int \cot \tau g u du = 5 \ell \eta |\operatorname{sen} u| + c = 5 \ell \eta \left| \operatorname{sen} \frac{x}{5} \right| + c$$

$$\mathbf{2.102.-} \int \left( \frac{1}{\operatorname{sen} x \sqrt{2}} - 1 \right)^2 dx, \quad \text{Sea: } u = x\sqrt{2}, du = \sqrt{2} dx$$

$$\begin{aligned} \int \left( \frac{1}{\operatorname{sen} x \sqrt{2}} - 1 \right)^2 dx &= \int (\operatorname{cosec} x \sqrt{2} - 1)^2 dx = \int (\operatorname{cosec}^2 x \sqrt{2} - 2 \operatorname{cosec} x \sqrt{2} + 1) dx \\ &= \int \operatorname{cosec}^2 x \sqrt{2} dx - 2 \int \operatorname{cosec} x \sqrt{2} dx + \int dx = \frac{1}{\sqrt{2}} \int \operatorname{cosec}^2 u du - \frac{2}{\sqrt{2}} \int \operatorname{cosec} u du + \int dx \\ &= -\frac{1}{\sqrt{2}} \cot \tau gu - \sqrt{2} \ell \eta |\operatorname{cosec} u - \cot \tau gu| + x + c \\ &= -\frac{1}{\sqrt{2}} \cot \tau g x \sqrt{2} - \sqrt{2} \ell \eta |\operatorname{cosec} x \sqrt{2} - \cot \tau g x \sqrt{2}| + x + c \end{aligned}$$

**2.103.-**  $\int \frac{dx}{\text{sen } x \cos x}$ , Sea:  $u = 2x, du = 2dx$

$$\int \frac{dx}{\text{sen } x \cos x} = \int \frac{dx}{\frac{1}{2} \text{sen } 2x} = 2 \int \cos \text{ec } 2x dx = \int \cos \text{ec } u du = \ell \eta |\cos \text{ec } u - \cot \tau gu| + c$$

$$= \ell \eta |\cos \text{ec } 2x - \cot \tau g 2x| + c$$

**2.104.-**  $\int \frac{\cos ax}{\text{sen}^5 ax} dx$ , Sea:  $u = \text{sen } ax, du = a \cos ax dx$

$$\int \frac{\cos ax}{\text{sen}^5 ax} dx = \frac{1}{a} \int \frac{du}{u^5} = \frac{1}{a} \frac{u^{-4}}{-4} + c = -\frac{u^{-4}}{4a} + c = -\frac{\text{sen}^{-4} ax}{4a} + c = -\frac{1}{4a \text{sen}^4 ax} + c$$

**2.105.-**  $\int t \text{sen}(1-2t^2) dt$ , Sea:  $u = 1-2t^2, du = -4t dt$

$$\int t \text{sen}(1-2t^2) dt = -\frac{1}{4} \int \text{sen } u du = \frac{1}{4} \cos u + c = \frac{1}{4} \cos(1-2t^2) + c$$

**2.106.-**  $\int \frac{\text{sen } 3x}{3 + \cos 3x} dx$ , Sea:  $u = 3 + \cos 3x, du = -3 \text{sen } 3x dx$

$$\int \frac{\text{sen } 3x}{3 + \cos 3x} dx = -\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ell \eta |u| + c = -\frac{1}{3} \ell \eta |3 + \cos 3x| + c$$

**2.107.-**  $\int \tau g^{\frac{x}{3}} \sec^2 \frac{x}{3} dx$ , Sea:  $u = \tau g(\frac{x}{3}), du = \frac{1}{3} \sec^2(\frac{x}{3}) dx$

$$\int \tau g^{\frac{x}{3}} \sec^2 \frac{x}{3} dx = 3 \int u^3 du = \frac{3u^4}{4} + c = \frac{3\tau g^4(\frac{x}{3})}{4} + c$$

**2.108.-**  $\int \frac{\text{sen } x \cos x}{\sqrt{\cos^2 x - \text{sen}^2 x}} dx$ , Sea:  $u = \cos 2x, du = 2 \text{sen } 2x dx$

$$\int \frac{\text{sen } x \cos x}{\sqrt{\cos^2 x - \text{sen}^2 x}} dx = \int \frac{\text{sen } x \cos x}{\sqrt{\cos 2x}} dx = \frac{1}{4} \int \frac{\text{sen } 2x}{\sqrt{\cos 2x}} = \frac{1}{4} \int \frac{du}{\sqrt{u}} = \frac{1}{4} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{u^{\frac{1}{2}}}{2} + c$$

$$= \frac{\sqrt{\cos 2x}}{2} + c$$

**2.109.-**  $\int \frac{\sqrt{\tau gx}}{\cos^2 x} dx$ , Sea:  $u = \tau gx, du = \sec^2 x dx$

$$\int \frac{\sqrt{\tau gx}}{\cos^2 x} dx = \int \sqrt{\tau gx} \sec^2 x dx = \int u^{\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3} u^{\frac{3}{2}} + c = \frac{2}{3} \tau g^{\frac{3}{2}} x + c$$

**2.110.-**  $\int \cos^{\frac{x}{a}} \text{sen}^{\frac{x}{a}} dx$ , Sea:  $u = 2x/a, du = 2dx$

$$\int \cos^{\frac{x}{a}} \text{sen}^{\frac{x}{a}} dx = \frac{1}{2} \int \text{sen}^{\frac{2x}{a}} dx = \frac{a}{4} \int \text{sen } u du = -\frac{a}{4} \cos u + c = -\frac{a}{4} \cos \frac{2x}{a} + c$$

**2.111.-**  $\int t \cot \tau g(2t^2 - 3) dt$ , Sea:  $u = 2t^2 - 3, du = 4t dt$

$$\int t \cot \tau g(2t^2 - 3) dt = \frac{1}{4} \int \cot \tau g u du = \frac{1}{4} \ell \eta |\text{sen } u| + c = \frac{1}{4} \ell \eta |\text{sen}(2t^2 - 3)| + c$$

$$\mathbf{2.112.-} \int \frac{x^3 dx}{x^8 + 5}, \quad \text{Sea: } u = x^4, du = 4x^3 dx$$

$$\int \frac{x^3 dx}{x^8 + 5} = \int \frac{x^3 dx}{(x^4)^2 + (\sqrt{5})^2} = \frac{1}{4} \int \frac{du}{u^2 + (\sqrt{5})^2} = \frac{1}{4} \frac{1}{\sqrt{5}} \operatorname{arctg} \frac{u}{\sqrt{5}} + c = \frac{\sqrt{5}}{20} \operatorname{arctg} \frac{x^4}{\sqrt{5}} + c$$

$$\mathbf{2.113.-} \int \operatorname{sen}^3 6x \cos 6x dx, \quad \text{Sea: } u = \operatorname{sen} 6x, du = 6 \cos 6x dx$$

$$\int \operatorname{sen}^3 6x \cos 6x dx = \frac{1}{6} \int u^3 du = \frac{1}{6} \frac{u^4}{4} + c = \frac{u^4}{24} + c = \frac{\operatorname{sen}^4 6x}{24} + c$$

$$\mathbf{2.114.-} \int \sqrt{1 + 3 \cos^2 x} \operatorname{sen} 2x dx, \quad \text{Sea: } u = \frac{5 + 3 \cos 2x}{2}, du = -3 \operatorname{sen} 2x dx$$

$$\int \sqrt{1 + 3 \cos^2 x} \operatorname{sen} 2x dx = \int \sqrt{1 + 3 \left( \frac{1 + \cos 2x}{2} \right)} \operatorname{sen} 2x dx = \int \sqrt{1 + \frac{3 + 3 \cos 2x}{2}} \operatorname{sen} 2x dx$$

$$= \int \sqrt{\frac{5 + 3 \cos 2x}{2}} \operatorname{sen} 2x dx = -\frac{1}{3} \int u^{1/2} du = -\frac{1}{3} \frac{u^{3/2}}{3/2} + c = -\frac{2}{9} u^{3/2} + c$$

$$= -\frac{2}{9} \left( \frac{5 + 3 \cos 2x}{2} \right)^{3/2} + c$$

$$\mathbf{2.115.-} \int x \sqrt[5]{5 - x^2} dx, \quad \text{Sea: } u = 5 - x^2, du = -2x dx$$

$$\int x \sqrt[5]{5 - x^2} dx = -\frac{1}{2} \int u^{1/5} du = -\frac{1}{2} \frac{u^{6/5}}{6/5} + c = -\frac{5}{12} u^{6/5} + c = -\frac{5(5 - x^2)^{6/5}}{12} + c$$

$$\mathbf{2.116.-} \int \frac{1 + \operatorname{sen} 3x}{\cos^2 3x} dx, \quad \text{Sea: } u = \operatorname{sen} 3x, du = 3 dx; w = \cos u, dw = -\operatorname{sen} u du$$

$$\begin{aligned} \int \frac{1 + \operatorname{sen} 3x}{\cos^2 3x} dx &= \int \frac{dx}{\cos^2 3x} + \int \frac{\operatorname{sen} 3x}{\cos^2 3x} dx = \frac{1}{3} \int \sec^2 u du + \frac{1}{3} \int \frac{\operatorname{sen} u}{\cos^2 u} du \\ &= \frac{1}{3} \int \sec^2 u du - \frac{1}{3} \int \frac{dw}{w^2} = \frac{1}{3} \operatorname{tg} u + \frac{1}{3w} + c = \frac{1}{3} \operatorname{tg} u + \frac{1}{3 \cos u} + c = \frac{1}{3} \operatorname{tg} 3x + \frac{1}{3 \cos 3x} + c \end{aligned}$$

$$\mathbf{2.117.-} \int \frac{(\cos ax + \operatorname{sen} ax)^2}{\operatorname{sen} ax} dx, \quad \text{Sea: } u = ax, du = a dx$$

$$\int \frac{(\cos ax + \operatorname{sen} ax)^2}{\operatorname{sen} ax} dx = \int \frac{\cos^2 ax + 2 \cos ax \operatorname{sen} ax + \operatorname{sen}^2 ax}{\operatorname{sen} ax} dx$$

$$= \int \frac{\cos^2 ax}{\operatorname{sen} ax} dx + 2 \int \frac{\cos ax \cancel{\operatorname{sen} ax}}{\cancel{\operatorname{sen} ax}} dx + \int \frac{\cancel{\operatorname{sen}^2 ax}}{\cancel{\operatorname{sen} ax}} dx$$

$$= \int \frac{1 - \operatorname{sen}^2 ax}{\operatorname{sen} ax} dx + 2 \int \cos ax dx + \int \operatorname{sen} ax dx$$

$$= \int \frac{dx}{\operatorname{sen} ax} + 2 \int \cos ax dx$$

$$= \int \operatorname{cosec} ax dx + 2 \int \cos ax dx = \frac{1}{a} \int \operatorname{cosec} u du + \frac{2}{a} \int \cos u du$$



$$= \frac{1}{a} \ell \eta |\cos ec u - \co \tau g u| + \frac{2}{a} \text{sen } u + c = \frac{1}{a} \ell \eta |\cos ec ax - \co \tau g ax| + \frac{2}{a} \text{sen } ax + c$$

**2.118.-**  $\int \frac{x^3-1}{x+1} dx$ , Sea:  $u = x+1, du = dx$

$$\int \frac{x^3-1}{x+1} dx = \int (x^2 - x + 1 - \frac{2}{x+1}) dx = \int x^2 dx - \int x dx + \int dx - \int \frac{2}{x+1} dx$$

$$= \int x^2 dx - \int x dx + \int dx - 2 \int \frac{du}{u} = \frac{x^3}{3} - \frac{x^2}{2} + x - 2 \ell \eta |x+1| + c$$

**2.119.-**  $\int \frac{\cos ec^2 3x dx}{b - a \co \tau g 3x}$ , Sea:  $u = b - a \co \tau g 3x, du = 3a \cos ec^2 3x dx$

$$\int \frac{\cos ec^2 3x dx}{b - a \co \tau g 3x} = \frac{1}{3a} \int \frac{du}{u} = \frac{1}{3a} \ell \eta |u| + c = \frac{1}{3a} \ell \eta |b - a \co \tau g 3x| + c$$

**2.120.-**  $\int \frac{x^3-1}{x^4-4x+1} dx$ , Sea:  $u = x^4 - 4x + 1, du = (4x^3 - 4) dx$

$$\int \frac{x^3-1}{x^4-4x+1} dx = \frac{1}{4} \int \frac{(4x^3-4) dx}{x^4-4x+1} = \frac{1}{4} \int \frac{du}{u} = \frac{1}{4} \ell \eta |u| + c = \frac{1}{4} \ell \eta |x^4 - 4x + 1| + c$$

**2.121.-**  $\int x e^{-x^2} dx$ , Sea:  $u = -x^2, du = -2x dx$

$$\int x e^{-x^2} dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2} + c$$

**2.122.-**  $\int \frac{3 - \sqrt{2+3x^2}}{2+3x^2} dx$ , Sea:  $u = x\sqrt{3}, du = \sqrt{3} dx; a = \sqrt{2}$

$$\int \frac{3 - \sqrt{2+3x^2}}{2+3x^2} dx = 3 \int \frac{dx}{(\sqrt{2})^2 + (\sqrt{3}x)^2} - \int \frac{(2+3x^2)^{1/2}}{2+3x^2} dx$$

$$\frac{3}{\sqrt{3}} \int \frac{\sqrt{3} dx}{(\sqrt{2})^2 + (\sqrt{3}x)^2} - \int \frac{(2+3x^2)^{1/2}}{2+3x^2} dx = \frac{3}{\sqrt{3}} \int \frac{\sqrt{3} dx}{(\sqrt{2})^2 + (\sqrt{3}x)^2} - \int (2+3x^2)^{-1/2} dx$$

$$= \frac{3}{\sqrt{3}} \int \frac{du}{(a)^2 + (u)^2} - \int (2+3x^2)^{-1/2} dx = \sqrt{3} \int \frac{du}{(a)^2 + (u)^2} - \int \frac{dx}{\sqrt{(\sqrt{2})^2 + (x\sqrt{3})^2}}$$

$$= \sqrt{3} \int \frac{du}{(a)^2 + (u)^2} - \frac{1}{\sqrt{3}} \int \frac{du}{\sqrt{a^2 + u^2}} = \frac{\sqrt{3}}{a} \text{arc } \tau g \frac{u}{a} - \frac{1}{\sqrt{3}} \ell \eta |u + \sqrt{a^2 + u^2}| + c$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \text{arc } \tau g \frac{x\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{3}}{3} \ell \eta |x\sqrt{3} + \sqrt{2+3x^2}| + c$$

**2.123.-**  $\int \frac{\tau g 3x - \co \tau g 3x}{\text{sen } 3x} dx$ , Sea:  $u = 3x, du = 3 dx; w = \text{sen } u, dw = \cos u du$

$$\int \frac{\tau g 3x - \co \tau g 3x}{\text{sen } 3x} dx = \int \frac{\frac{\text{sen } 3x}{\cos 3x} - \frac{\cos 3x}{\text{sen } 3x}}{\text{sen } 3x} dx = \int \frac{dx}{\cos 3x} - \int \frac{\cos 3x}{\text{sen}^2 3x} dx$$

$$= \int \sec 3x dx - \int \frac{\cos 3x}{\sec^2 3x} dx = \frac{1}{3} \int \sec u du - \frac{1}{3} \int \frac{\cos u}{\sec^2 u} du = \frac{1}{3} \int \sec u du - \frac{1}{3} \int \frac{dw}{w^2}$$

$$= \frac{1}{3} \ell \eta |\sec u + \tau g u| - \frac{1}{3} \frac{w^{-1}}{-1} + c = \frac{1}{3} \ell \eta |\sec 3x + \tau g 3x| + \frac{1}{3 \sec 3x} + c$$

$$\mathbf{2.124.-} \int \frac{dx}{\sqrt{e^x}}, \quad \text{Sea: } u = -\frac{x}{2}, du = -\frac{dx}{2}$$

$$\int \frac{dx}{\sqrt{e^x}} = \int \frac{dx}{(e^x)^{1/2}} = \int e^{-x/2} dx = -2 \int e^u du = -2e^u + c = -2e^{-x/2} + c = \frac{-2}{e^{x/2}} + c = \frac{-2}{\sqrt{e^x}} + c$$

$$\mathbf{2.125.-} \int \frac{1 + \sec x}{x + \cos x} dx, \quad \text{Sea: } u = x + \cos x, du = (1 - \sec x) dx$$

$$\int \frac{1 + \sec x}{x + \cos x} dx = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |x + \cos x| + c$$

$$\mathbf{2.126.-} \int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x - 2}}, \quad \text{Sea: } u = \tau g x, du = \sec^2 x dx$$

$$\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x - 2}} = \int \frac{du}{\sqrt{u^2 - 2}} = \ell \eta |u + \sqrt{u^2 - 2}| + c = \ell \eta |\tau g x + \sqrt{\tau g x^2 - 2}| + c$$

$$\mathbf{2.127.-} \int \frac{dx}{x \ell \eta^2 x}, \quad \text{Sea: } u = \ell \eta x, du = \frac{dx}{2}$$

$$\int \frac{dx}{x \ell \eta^2 x} = \int \frac{dx}{x (\ell \eta x)^2} = \int \frac{du}{u^2} = \frac{u^{-1}}{-1} + c = -\frac{1}{u} + c = -\frac{1}{\ell \eta |x|} + c$$

$$\mathbf{2.128.-} \int a^{\sec x} \cos x dx, \quad \text{Sea: } u = \sec x, du = \cos x dx$$

$$\int a^{\sec x} \cos x dx = \int a^u du = \frac{a^u}{\ell \eta a} + c = \frac{a^{\sec x}}{\ell \eta a} + c$$

$$\mathbf{2.129.-} \int \frac{x^2}{\sqrt{x^3 + 1}} dx, \quad \text{Sea: } u = x^3 + 1, du = 3x^2 dx$$

$$\int \frac{x^2 dx}{\sqrt{x^3 + 1}} = \int \frac{x^2 dx}{(x^3 + 1)^{1/2}} = \frac{1}{3} \int \frac{du}{u^{1/2}} = \frac{1}{3} \frac{u^{1/2}}{1/2} + c = \frac{u^{1/2}}{2} + c = \frac{(x^3 + 1)^{1/2}}{2} + c = \frac{\sqrt{(x^3 + 1)^2}}{2} + c$$

$$\mathbf{2.130.-} \int \frac{xdx}{\sqrt{1 - x^4}}, \quad \text{Sea: } u = x^2, du = 2xdx$$

$$\int \frac{xdx}{\sqrt{1 - x^4}} = \int \frac{xdx}{\sqrt{1 - (x^2)^2}} = \frac{1}{2} \int \frac{2xdx}{\sqrt{1 - (x^2)^2}} = \frac{1}{2} \int \frac{2xdx}{\sqrt{1 - (u)^2}} = \frac{1}{2} \arcsen u + c$$

$$= \frac{1}{2} \arcsen x^2 + c$$

$$\mathbf{2.131.-} \int \tau g^2 ax dx, \quad \text{Sea: } u = ax, du = adx$$

$$\int \tau g^2 ax dx = \int (\sec^2 ax - 1) dx = \int \sec^2 ax dx - \int dx = \frac{1}{a} \int \sec^2 u du - \int dx = \frac{1}{a} \tau gu - x + c$$

$$= \frac{1}{a} \tau g ax - x + c$$

**2.132.-**  $\int \frac{\sec^2 x dx}{\sqrt{4 - \tau g^2 x}}$ , Sea:  $u = \tau gx, du = \sec^2 x dx$

$$\int \frac{\sec^2 x dx}{\sqrt{4 - \tau g^2 x}} = \int \frac{du}{\sqrt{2^2 - u^2}} = \arcsen \frac{u}{2} + c = \arcsen \frac{\tau gx}{2} + c$$

**2.133.-**  $\int \frac{dx}{\cos x/a}$ , Sea:  $u = x/a, du = dx/a$

$$\int \frac{dx}{\cos x/a} = \int \sec x/a dx = a \int \sec u du = a \ell \eta |\sec u + \tau gu| + c = a \ell \eta |\sec x/a + \tau g x/a| + c$$

**2.134.-**  $\int \frac{\sqrt[3]{1 + \ell \eta x}}{x} dx$ , Sea:  $u = 1 + \ell \eta x, du = \frac{dx}{x}$

$$\int \frac{\sqrt[3]{1 + \ell \eta x}}{x} dx = \int u^{1/3} du = \frac{u^{4/3}}{4/3} + c = \frac{3u^{4/3}}{4} + c = \frac{3(1 + \ell \eta x)^{4/3}}{4} + c$$

**2.135.-**  $\int \tau g \sqrt{x-1} \frac{dx}{\sqrt{x-1}}$ , Sea:  $u = \sqrt{x-1}, du = \frac{dx}{2\sqrt{x-1}}$

$$\int \tau g \sqrt{x-1} \frac{dx}{\sqrt{x-1}} = 2 \int \tau gu \frac{du}{u} = 2 \ell \eta |\sec \sqrt{x-1}| + c = -2 \ell \eta |\cos \sqrt{x-1}| + c$$

**2.136.-**  $\int \frac{x dx}{\sec x^2}$ , Sea:  $u = x^2, du = 2x dx$

$$\int \frac{x dx}{\sec x^2} = \frac{1}{2} \int \frac{du}{\sec u} = \frac{1}{2} \int \cos u du = \frac{1}{2} \ell \eta |\cos u - \co \tau gu| + c$$

$$= \frac{1}{2} \ell \eta |\cos x^2 - \co \tau g x^2| + c$$

**2.137.-**  $\int \frac{\sec x - \cos x}{\sec x + \cos x} dx$ , Sea:  $u = \sec x + \cos x, du = (\cos x - \sec x) dx$

$$\int \frac{\sec x - \cos x}{\sec x + \cos x} dx = - \int \frac{du}{u} = - \ell \eta |\sec x + \cos x| + c$$

**2.138.-**  $\int \frac{e^{\arctan x} + x \ell \eta (1+x^2) + 1}{1+x^2} dx$ , Sea:  $u = \arctan x, du = \frac{dx}{1+x^2}; w = \ell \eta (1+x^2) dx, dw = \frac{2x dx}{1+x^2}$

$$\int \frac{e^{\arctan x} + x \ell \eta (1+x^2) + 1}{1+x^2} dx = \int \frac{e^{\arctan x} dx}{1+x^2} + \int \frac{x \ell \eta (1+x^2) dx}{1+x^2} + \int \frac{dx}{1+x^2}$$

$$= \int e^u du + \frac{1}{2} \int w dw + \int \frac{dx}{1+x^2} = e^u + \frac{1}{2} \frac{w^2}{2} + \arctan x + c = e^u + \frac{\ell \eta^2 (1+x^2)}{4} + \arctan x + c$$

**2.139.-**  $\int \frac{x^2 dx}{x^2 - 2}$ ,

$$\int \frac{x^2 dx}{x^2 - 2} = \int \left(1 + \frac{2}{x^2 - 2}\right) dx = \int dx + 2 \int \frac{dx}{x^2 - 2} = x + 2 \frac{1}{2\sqrt{2}} \ell \eta \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| + c$$

$$= x + \frac{\sqrt{2}}{2} \ell \eta \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| + c$$

**2.140.-**  $\int e^{\text{sen}^2 x} \text{sen} 2x dx$ , Sea:  $u = \frac{1 - \cos 2x}{2}$ ,  $du = \text{sen} 2x dx$

$$\int e^{\text{sen}^2 x} \text{sen} 2x dx = \int e^{\frac{1 - \cos 2x}{2}} \text{sen} 2x dx = \int e^u du = e^u + c = e^{\text{sen}^2 x} + c$$

**2.141.-**  $\int \frac{(1 - \text{sen} \frac{x}{\sqrt{2}})^2}{\text{sen} \frac{x}{\sqrt{2}}} dx$ , Sea:  $u = \frac{x}{\sqrt{2}}$ ,  $du = \frac{dx}{\sqrt{2}}$

$$\int \frac{(1 - \text{sen} \frac{x}{\sqrt{2}})^2}{\text{sen} \frac{x}{\sqrt{2}}} dx = \int \left( \frac{1 - 2 \text{sen} \frac{x}{\sqrt{2}} + \text{sen}^2 \frac{x}{\sqrt{2}}}{\text{sen} \frac{x}{\sqrt{2}}} \right) dx = \int \text{cosec} \frac{x}{\sqrt{2}} dx - 2 \int dx + \int \text{sen} \frac{x}{\sqrt{2}} dx$$

$$= \sqrt{2} \int \text{cosec} u du - 2 \int dx + \sqrt{2} \int \text{sen} u du = \sqrt{2} \ell \eta |\text{cosec} u - \cot u| - 2x - \sqrt{2} \cos u + c$$

$$= \sqrt{2} \ell \eta |\text{cosec} \frac{x}{\sqrt{2}} - \cot \frac{x}{\sqrt{2}}| - 2x - \sqrt{2} \cos \frac{x}{\sqrt{2}} + c$$

**2.142.-**  $\int \frac{5 - 3x}{\sqrt{4 - 3x^2}} dx$ , Sea:  $u = x\sqrt{3}$ ,  $du = \sqrt{3} dx$ ;  $w = 4 - 3x^2$ ,  $dw = -6x dx$

$$\int \frac{5 - 3x}{\sqrt{4 - 3x^2}} dx = 5 \int \frac{dx}{\sqrt{4 - 3x^2}} - 3 \int \frac{x dx}{\sqrt{4 - 3x^2}} = 5 \int \frac{dx}{\sqrt{4 - (x\sqrt{3})^2}} - 3 \int \frac{x dx}{\sqrt{4 - 3x^2}}$$

$$= \frac{5}{\sqrt{3}} \int \frac{du}{\sqrt{2^2 - u^2}} + \frac{3}{6} \int \frac{dw}{\sqrt{w}} = \frac{5}{\sqrt{3}} \arcsen \frac{u}{2} + \frac{1}{2} \frac{w^{1/2}}{1/2} + c = \frac{5\sqrt{3}}{3} \arcsen \frac{x\sqrt{3}}{2} + \sqrt{4 - 3x^2} + c$$

**2.143.-**  $\int \frac{ds}{e^s + 1}$ , Sea:  $u = 1 + e^{-s}$ ,  $du = -e^{-s} ds$

$$\int \frac{ds}{e^s + 1} = \int \frac{e^{-s} ds}{e^{-s} + 1} = - \int \frac{du}{u} = -\ell \eta |u| + c = -\ell \eta |e^{-s} + 1| + c$$

**2.144.-**  $\int \frac{d\theta}{\text{sen} a\theta \cos a\theta}$ , Sea:  $u = 2a\theta$ ,  $du = 2ad\theta$

$$\int \frac{d\theta}{\text{sen} a\theta \cos a\theta} = \int \frac{d\theta}{\frac{1}{2} \text{sen} 2a\theta} = 2 \int \text{cosec} 2a\theta d\theta = \frac{2}{2a} \int \text{cosec} u du$$

$$= \frac{1}{a} \ell \eta |\text{cosec} u - \cot u| + c = \frac{1}{a} \ell \eta |\text{cosec} 2a\theta - \cot 2a\theta| + c$$

**2.145.-**  $\int \frac{e^s}{\sqrt{e^{2s} - 2}} ds$ , Sea:  $u = e^s$ ,  $du = e^s ds$

$$\int \frac{e^s}{\sqrt{e^{2s} - 2}} ds = \int \frac{e^s}{\sqrt{(e^s)^2 - 2}} ds = - \int \frac{du}{\sqrt{u^2 - 2}} = \ell \eta |u + \sqrt{u^2 - 2}| + c$$

$$= \ell \eta |e^s + \sqrt{(e^s)^2 - 2}| + c = \ell \eta |e^s + \sqrt{e^{2s} - 2}| + c$$

$$\mathbf{2.146.-} \int \operatorname{sen}\left(\frac{2\pi t}{T} + \varphi_0\right) dt, \quad \text{Sea: } u = \frac{2\pi t}{T} + \varphi_0, du = \frac{2\pi}{T} dt$$

$$\int \operatorname{sen}\left(\frac{2\pi t}{T} + \varphi_0\right) dt = \frac{T}{2\pi} \int \operatorname{sen} u du = -\frac{T}{2\pi} \cos u + c = -\frac{T}{2\pi} \cos\left(\frac{2\pi t}{T} + \varphi_0\right) + c$$

$$\mathbf{2.147.-} \int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}} dx, \quad \text{Sea: } u = \arccos \frac{x}{2}, du = -\frac{dx}{\sqrt{4-x^2}}$$

$$\int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}} dx = -\int u du = -\frac{u^2}{2} + c = -\frac{(\arccos \frac{x}{2})^2}{2} + c$$

$$\mathbf{2.148.-} \int \frac{dx}{x(4-\ell \eta^2 x)}, \quad \text{Sea: } u = \ell \eta x, du = \frac{dx}{x}$$

$$\int \frac{dx}{x(4-\ell \eta^2 x)} = \int \frac{dx}{x[2^2-(\ell \eta x)^2]} = \int \frac{du}{2^2-u^2} = \frac{1}{4} \ell \eta \left| \frac{2+u}{2-u} \right| + c = \frac{1}{4} \ell \eta \left| \frac{2+\ell \eta x}{2-\ell \eta x} \right| + c$$

$$\mathbf{2.149.-} \int e^{-\tau g x} \sec^2 x dx, \quad \text{Sea: } u = -\tau g x, du = -\sec^2 x dx$$

$$\int e^{-\tau g x} \sec^2 x dx = -\int e^u du = -e^u + c = -e^{-\tau g x} + c$$

$$\mathbf{2.150.-} \int \frac{\operatorname{sen} x \cos x}{\sqrt{2-\operatorname{sen}^4 x}} dx, \quad \text{Sea: } u = \operatorname{sen}^2 x, du = 2 \operatorname{sen} x \cos x dx$$

$$\int \frac{\operatorname{sen} x \cos x}{\sqrt{2-\operatorname{sen}^4 x}} dx = \int \frac{\operatorname{sen} x \cos x}{\sqrt{2-(\operatorname{sen}^2 x)^2}} dx = \frac{1}{2} \int \frac{du}{\sqrt{2-u^2}} = \frac{1}{2} \arcsen \frac{u}{\sqrt{2}} + c$$

$$= \frac{1}{2} \arcsen \frac{(\operatorname{sen}^2 x)}{\sqrt{2}} + c$$

$$\mathbf{2.151.-} \int \frac{\sec x \tau g x}{\sqrt{\sec^2 x + 1}} dx, \quad \text{Sea: } u = \sec x, du = \sec x \tau g x dx$$

$$\int \frac{\sec x \tau g x}{\sqrt{\sec^2 x + 1}} dx = \int \frac{du}{\sqrt{u^2 + 1}} = \ell \eta \left| u + \sqrt{u^2 + 1} \right| + c = \ell \eta \left| \sec x + \sqrt{\sec^2 x + 1} \right| + c$$

$$\mathbf{2.152.-} \int \frac{dt}{\operatorname{sen}^2 t \cos^2 t}, \quad \text{Sea: } u = 2t, du = 2dt$$

$$\int \frac{dt}{\operatorname{sen}^2 t \cos^2 t} = \int \frac{dt}{(\operatorname{sen} t \cos t)^2} = \int \frac{dt}{(1/2 \operatorname{sen} 2t)^2} = 4 \int \frac{dt}{\operatorname{sen}^2 2t} = 4 \int \operatorname{cosec}^2 2t dt$$

$$= 2 \int \operatorname{cosec}^2 u du = -2 \cot u + c = -2 \cot 2t + c$$

$$\mathbf{2.153.-} \int \frac{\arcsen x + x}{\sqrt{1-x^2}} dx, \quad \text{Sea: } u = \arcsen x, du = \frac{dx}{\sqrt{1-x^2}}; w = 1-x^2, dw = -2x dx$$

$$\int \frac{\arcsen x + x}{\sqrt{1-x^2}} dx = \int \frac{\arcsen x}{\sqrt{1-x^2}} dx + \int \frac{x}{\sqrt{1-x^2}} dx = \int u du - \frac{1}{2} \int \frac{dw}{\sqrt{w}} = \int u du - \frac{1}{2} \int w^{-1/2} dw$$

$$= \frac{u^2}{2} - \frac{1}{2} \frac{w^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{(\arcsen x)^2}{2} - \sqrt{1-x^2} + c$$

**2.154.-**  $\int \frac{xdx}{\sqrt{x+1}}$ , Sea:  $t = \sqrt{x+1} \Rightarrow x = t^2 - 1; dx = 2tdt$

$$\int \frac{xdx}{\sqrt{x+1}} = \int \frac{(t^2-1)2tdt}{t} = 2 \int (t^2-1)dt = 2\left(\frac{t^3}{3} - t\right) + c = \frac{2\sqrt{(x+1)^3}}{3} - 2\sqrt{x+1} + c$$

**2.155.-**  $\int x(5x^2-3)^7 dx$ , Sea:  $u = 5x^2-3, du = 10xdx$

$$\int x(5x^2-3)^7 dx = \frac{1}{10} \int u^7 du = \frac{1}{10} \frac{u^8}{8} + c = \frac{u^8}{80} + c = \frac{(5x^2-3)^8}{80} + c$$

**2.156.-**  $\int \sqrt{\frac{\ell \eta(x + \sqrt{x^2+1})}{x^2+1}} dx$ , Sea:  $u = \ell \eta(x + \sqrt{x^2+1}), du = \frac{dx}{\sqrt{x^2+1}}$

$$\int \sqrt{\frac{\ell \eta(x + \sqrt{x^2+1})}{x^2+1}} dx = \int \frac{\sqrt{\ell \eta(x + \sqrt{x^2+1})}}{\sqrt{x^2+1}} dx = \int \sqrt{u} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2\sqrt{[\ell \eta(x + \sqrt{x^2+1})]^3}}{3} + c$$

**2.157.-**  $\int \frac{\sen^3 x}{\sqrt{\cos x}} dx$ , Sea:  $u = \cos x, du = -\sen x dx$

$$\int \frac{\sen^3 x}{\sqrt{\cos x}} dx = \int \frac{\sen^2 x \sen x dx}{\sqrt{\cos x}} = \int \frac{(1-\cos^2 x) \sen x dx}{\sqrt{\cos x}} = \int \frac{\sen x dx}{\sqrt{\cos x}} - \int \frac{\cos^2 x \sen x dx}{\sqrt{\cos x}}$$

$$= \int \cos^{-\frac{1}{2}} x \sen x dx - \int \cos^{\frac{3}{2}} x \sen x dx = -\int u^{\frac{1}{2}} du + \int u^{\frac{3}{2}} du = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= -\frac{2u^{\frac{3}{2}}}{3} + \frac{2u^{\frac{5}{2}}}{5} + c = -\frac{2\cos x^{\frac{3}{2}}}{3} + \frac{2\cos x^{\frac{5}{2}}}{5} + c = -\frac{2\sqrt{\cos^3 x}}{3} + \frac{2\sqrt{\cos^5 x}}{5} + c$$

**2.158.-**  $\int \frac{\cos x dx}{\sqrt{1+\sen^2 x}}$ ,

Sea:  $t = \sqrt{1+\sen^2 x} \Rightarrow \sen^2 x = t^2 - 1; 2\sen x \cos x dx = 2tdt$

$$\int \frac{\cos x dx}{\sqrt{1+\sen^2 x}} = \int \frac{t}{t} = \int \frac{dt}{\sqrt{t^2-1}} = \ell \eta \left| \sqrt{1+\sen^2 x} + \sen x \right| + c$$

**2.159.-**  $\int \frac{(\arcsen x)^2}{\sqrt{1-x^2}} dx$ , Sea:  $u = \arcsen x, du = \frac{dx}{\sqrt{1-x^2}}$

$$\int \frac{(\arcsen x)^2}{\sqrt{1-x^2}} dx = \int u^2 du = \frac{u^3}{3} + c = \frac{(\arcsen x)^3}{3} + c$$

**2.150.-**  $\int e^{x+e^x} dx$ , Sea:  $u = e^{e^x}, du = e^{e^x} e^x dx$

$$\int e^{x+e^x} dx = \int e^x e^{e^x} dx = \int du = u + c = e^{e^x} + c$$

**2.161.-**  $\int t(4t+1)^7 dt$ , Sea:  $u = 4t+1 \Rightarrow t = \frac{u-1}{4}, du = 4dt$

$$\begin{aligned} \int t(4t+1)^7 dt &= \int \frac{u-1}{4} u^7 \frac{du}{4} = \frac{1}{16} \int (u-1)u^7 du = \frac{1}{16} \int (u^8 - u^7) du = \frac{1}{16} \frac{u^9}{9} - \frac{1}{16} \frac{u^8}{8} + c \\ &= \frac{(4t+1)^9}{144} - \frac{(4t+1)^8}{128} + c \end{aligned}$$

**2.162.-**  $\int \frac{2t^2-10t+12}{t^2+4} dt$ , Sea:  $u = t^2+4, du = 2t dt$

$$\begin{aligned} \int \frac{2t^2-10t+12}{t^2+4} dt &= 2 \int \frac{t^2-5t+6}{t^2+4} dt = 2 \int \left( 1 + \frac{2-5t}{t^2+4} \right) dt = 2 \int dt + 4 \int \frac{dt}{t^2+4} - 10 \int \frac{t dt}{t^2+4} \\ &= 2 \int dt + 4 \int \frac{dt}{t^2+4} - 5 \int \frac{du}{u} = 2t + 2 \operatorname{arctg} \frac{t}{2} - 5 \ell \eta |u| + c = 2t + 2 \operatorname{arctg} \frac{t}{2} - 5 \ell \eta |t^2+4| + c \end{aligned}$$

**2.163.-**  $\int \frac{e^t - e^{-t}}{e^t + e^{-t}} dt$ ,

Sea:  $u = e^{2t} + 1, du = 2e^{2t} dt; w = 1 + e^{-2t}, dw = -2e^{-2t} dt$

$$\begin{aligned} \int \frac{e^t - e^{-t}}{e^t + e^{-t}} dt &= \int \frac{e^t dt}{e^t + e^{-t}} - \int \frac{e^{-t} dt}{e^t + e^{-t}} = \int \frac{e^{2t} dt}{e^{2t} + 1} - \int \frac{e^{-2t} dt}{1 + e^{-2t}} = \frac{1}{2} \int \frac{du}{u} + \frac{1}{2} \int \frac{dw}{w} \\ &= \frac{1}{2} (\ell \eta |u| + \ell \eta |w|) + c = \frac{1}{2} \ell \eta |uw| + c = \frac{1}{2} \ell \eta (e^{2t} + 1)(1 + e^{-2t}) + c \end{aligned}$$

## CAPITULO 3

### INTEGRACION DE FUNCIONES TRIGONOMETRICAS

En esta parte, serán consideradas las integrales trigonométricas de la forma:

i)  $\int \operatorname{sen}^m u \cos^n u du$

ii)  $\int \operatorname{tg}^m u \sec^n u du$

iii)  $\int \operatorname{cosec}^m u \cos^n u du$

O bien, formas trigonométricas reducibles a algunos de los casos ya señalados.

### EJERCICIOS DESARROLLADOS

**3.1.-Encontrar:**  $\int \cos^2 x dx$

Solución.-  $\cos^2 x = \frac{1 + \cos 2x}{2}$

Luego:  $\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x dx = \frac{x}{2} + \frac{1}{4} \operatorname{sen} 2x + c$ ,

Como:  $\int \cosh x dx = \frac{1}{h} \operatorname{senh} x + c$

**Respuesta:**  $\int \cos^2 x dx = \frac{1}{2} x + \frac{1}{4} \operatorname{sen} 2x + c$

**3.2.-Encontrar:**  $\int \cos^4 \frac{1}{2} x dx$

Solución.-  $\cos^2 \frac{1}{2} x = \frac{1 + \cos x}{2}$

Luego:  $\int \cos^4 \frac{1}{2} x dx = \int (\cos^2 \frac{1}{2} x)^2 dx = \int \left( \frac{1 + \cos x}{2} \right)^2 dx = \frac{1}{4} \int (1 + 2 \cos x + \cos^2 x) dx$

$= \frac{1}{4} \int dx + \frac{1}{2} \int \cos x dx + \frac{1}{4} \int \cos^2 x dx$ , como:  $\int \cos^2 x dx = \frac{1}{2} x + \frac{1}{4} \operatorname{sen} 2x + c$

$= \frac{1}{4} \int dx + \frac{1}{2} \int \cos x dx + \frac{1}{4} \int \cos^2 x dx = \frac{1}{4} x + \frac{1}{2} \operatorname{sen} x + \frac{1}{4} \left( \frac{1}{2} x + \frac{1}{4} \operatorname{sen} 2x \right) + c$

$= \frac{1}{4} x + \frac{1}{2} \operatorname{sen} x + \frac{1}{8} x + \frac{1}{16} \operatorname{sen} 2x + c = \frac{3}{8} x + \frac{1}{2} \operatorname{sen} x + \frac{1}{16} \operatorname{sen} 2x + c$

**Respuesta:**  $\int \cos^4 \frac{1}{2} x dx = \frac{3}{8} x + \frac{1}{2} \operatorname{sen} x + \frac{1}{16} \operatorname{sen} 2x + c$

**3.3.-Encontrar:**  $\int \cos^3 x dx$

Solución.-  $\int \cos^3 x dx = \int \cos x \cos^2 x dx$ , como:  $\cos^2 x = 1 - \operatorname{sen}^2 x$



$$= \int \cos x \cos^2 x dx = \int \cos x (1 - \sin^2 x) dx = \int \cos x dx - \int \cos x \sin^2 x dx$$

Sea:  $u = \sin x, du = \cos x dx$

$$= \int \cos x dx - \int \cos x \sin^2 x dx = \int \cos x dx - \int u^2 du = \sin x - \frac{u^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c$$

**Respuesta:**  $\int \cos^3 x dx = \sin x - \frac{\sin^3 x}{3} + c$

**3.4.-Encontrar:**  $\int \sin x^3 4x dx$

Solución.-  $\int \sin x^3 4x dx = \int \sin 4x \sin^2 4x dx$ , como:  $\sin^2 4x = 1 - \cos^2 4x$

$$= \int \sin 4x \sin^2 4x dx = \int \sin 4x (1 - \cos^2 4x) dx = \int \sin 4x dx - \int \sin 4x (\cos 4x)^2 dx$$

Sea:  $u = \cos 4x, du = -4 \sin 4x dx$

$$= \int \sin 4x dx + \frac{1}{4} \int u^2 du = -\frac{1}{4} \cos 4x + \frac{1}{4} \frac{u^3}{3} + c = -\frac{\cos 4x}{4} + \frac{\cos^3 4x}{12} + c$$

**Respuesta:**  $\int \sin x^3 4x dx = -\frac{\cos 4x}{4} + \frac{\cos^3 4x}{12} + c$

**3.5.-Encontrar:**  $\int \sin^2 x \cos^3 x dx$

Solución.-  $\int \sin^2 x \cos^3 x dx = \int \sin^2 x \cos^2 x \cos x dx = \int \sin^2 x (1 - \sin^2 x) \cos x dx$

$$= \int \sin^2 x \cos x dx - \int \sin^4 x \cos x dx; \quad \text{Sea: } u = \sin x, du = \cos x dx$$

$$= \int u^2 du - \int u^4 du = \frac{u^3}{3} - \frac{u^5}{5} + c = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$$

**Respuesta:**  $\int \sin^2 x \cos^3 x dx = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

**3.6.-Encontrar:**  $\int \sin^3 x \cos^2 x dx$

Solución.-  $\int \sin^3 x \cos^2 x dx = \int \sin^2 x \sin x \cos^2 x dx = \int (1 - \cos^2 x) \sin x \cos^2 x dx$

$$= \int (1 - \cos^2 x) \sin x \cos^2 x dx = \int \sin x \cos^2 x dx - \int \sin x \cos^4 x dx$$

Sea:  $u = \cos x, du = -\sin x dx$

$$= \int \sin x \cos^2 x dx - \int \sin x \cos^4 x dx = -\int u^2 du + \int u^4 du = -\frac{u^3}{3} + \frac{u^5}{5} + c$$

$$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$$

**Respuesta:**  $\int \sin^3 x \cos^2 x dx = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$

**3.7.-Encontrar:**  $\int \sin^2 x \cos^5 x dx$

Solución.-  $\int \sin^2 x \cos^5 x dx = \int \sin^2 x (\cos^2 x)^2 \cos x dx = \int \sin^2 x (1 - \sin^2 x)^2 \cos x dx$

$$= \int \sin^2 x (1 - 2\sin^2 x + \sin^4 x) \cos x dx$$

$$= \int (\operatorname{sen} x)^2 \cos x dx - 2 \int (\operatorname{sen} x)^4 \cos x dx + \int (\operatorname{sen} x)^6 \cos x dx$$

Sea:  $u = \operatorname{sen} x, du = \cos x dx$

$$= \int u^2 du - 2 \int u^4 du + \int u^6 du = \frac{u^3}{3} - 2 \frac{u^5}{5} + \frac{u^7}{7} + c = \frac{\operatorname{sen}^3 x}{3} - 2 \frac{\operatorname{sen}^5 x}{5} + \frac{\operatorname{sen}^7 x}{7} + c$$

**Respuesta:**  $\int \operatorname{sen}^2 x \cos^5 x dx = \frac{\operatorname{sen}^3 x}{3} - 2 \frac{\operatorname{sen}^5 x}{5} + \frac{\operatorname{sen}^7 x}{7} + c$

**3.8.-Encontrar:**  $\int \operatorname{sen}^3 x \cos^3 x dx$

Solución.-  $\int \operatorname{sen}^3 x \cos^3 x dx = \int (\operatorname{sen} x \cos x)^3 dx$ ; como:  $\operatorname{sen} 2x = 2 \operatorname{sen} x \cos x$ ,

Se tiene que:  $\operatorname{sen} x \cos x = \frac{\operatorname{sen} 2x}{2}$ ; Luego:

$$= \int (\operatorname{sen} x \cos x)^3 dx = \int \left( \frac{\operatorname{sen} 2x}{2} \right)^3 dx = \frac{1}{8} \int \operatorname{sen}^3 2x dx = \frac{1}{8} \int \operatorname{sen} 2x \operatorname{sen}^2 2x dx$$

$$= \frac{1}{8} \int \operatorname{sen} 2x (1 - \cos^2 2x) dx = \frac{1}{8} \int \operatorname{sen} 2x dx - \frac{1}{8} \int \operatorname{sen} 2x (\cos 2x)^2 dx$$

Sea:  $u = \cos 2x, du = -2 \operatorname{sen} 2x dx$

$$= \frac{1}{8} \int \operatorname{sen} 2x dx + \frac{1}{16} \int -2 \operatorname{sen} 2x (\cos 2x)^2 dx = \frac{1}{8} \int \operatorname{sen} 2x dx + \frac{1}{16} \int u^2 du$$

$$= -\frac{1}{16} \cos 2x + \frac{1}{16} \frac{u^3}{3} + c = -\frac{1}{16} \cos 2x + \frac{\cos^3 2x}{48} + c$$

**Respuesta:**  $\int \operatorname{sen}^3 x \cos^3 x dx = -\frac{1}{16} \cos 2x + \frac{\cos^3 2x}{48} + c$

**3.9.-Encontrar:**  $\int \operatorname{sen}^4 x \cos^4 x dx$

Solución.-  $\int \operatorname{sen}^4 x \cos^4 x dx = \int (\operatorname{sen} x \cos x)^4 dx = \int \left( \frac{\operatorname{sen} 2x}{2} \right)^4 dx = \frac{1}{16} \int \operatorname{sen}^4 2x dx$

$$= \frac{1}{16} \int (\operatorname{sen}^2 2x)^2 dx = \frac{1}{16} \int \left( \frac{1 - \cos 4x}{2} \right)^2 dx = \frac{1}{16 \times 4} \int (1 - \cos 4x)^2 dx$$

$$= \frac{1}{64} \int (1 - 2 \cos 4x + \cos^2 4x) dx = \frac{1}{64} \int dx - \frac{1}{32} \int \cos 4x dx + \frac{1}{64} \int \cos^2 4x dx$$

$$= \frac{1}{64} \int dx - \frac{1}{32} \int \cos 4x dx + \frac{1}{64} \int \frac{1 + \cos 8x}{2} dx$$

$$= \frac{1}{64} \int dx - \frac{1}{32} \int \cos 4x dx + \frac{1}{128} \int dx + \frac{1}{128} \int \cos 8x dx$$

$$= \frac{1}{64} x - \frac{1}{128} \operatorname{sen} 4x + \frac{1}{128} x + \frac{1}{1024} \operatorname{sen} 8x + c = \frac{3x}{128} - \frac{\operatorname{sen} 4x}{128} + \frac{\operatorname{sen} 8x}{1024} + c$$

**Respuesta:**  $\int \operatorname{sen}^4 x \cos^4 x dx = \frac{1}{128} \left( 3x - \operatorname{sen} 4x + \frac{\operatorname{sen} 8x}{8} \right) + c$

**3.10.-Encontrar:**  $\int x(\cos^3 x^2 - \operatorname{sen}^3 x^2) dx$ ; Sea:  $u = x^2, du = 2x dx$

$$\begin{aligned}
\int x(\cos^3 x^2 - \operatorname{sen}^3 x^2) dx &= \frac{1}{2} \int 2x(\cos^3 x^2 - \operatorname{sen}^3 x^2) dx = \frac{1}{2} \int (\cos^3 u - \operatorname{sen}^3 u) du \\
&= \frac{1}{2} \int \cos^3 u du - \frac{1}{2} \int \operatorname{sen}^3 u du = \frac{1}{2} \int \cos u \cos^2 u du - \frac{1}{2} \int \operatorname{sen} u \operatorname{sen}^2 u du \\
&= \frac{1}{2} \int \cos u (1 - \operatorname{sen}^2 u) du - \frac{1}{2} \int \operatorname{sen} u (1 - \cos^2 u) du \\
&= \frac{1}{2} \int \cos u du - \frac{1}{2} \int \cos u \operatorname{sen}^2 u du - \frac{1}{2} \int \operatorname{sen} u du + \frac{1}{2} \int \operatorname{sen} u \cos^2 u du
\end{aligned}$$

Sea:  $w = \operatorname{sen} u$ ,  $dw = \cos u du$ ;  $z = \cos u$ ,  $dz = -\operatorname{sen} u du$

$$\begin{aligned}
&= \frac{1}{2} \int \cos u du - \frac{1}{2} \int w^2 dw - \frac{1}{2} \int \operatorname{sen} u du - \frac{1}{2} \int z^2 dz = \frac{1}{2} \operatorname{sen} u - \frac{1}{2} \frac{w^3}{3} + \frac{1}{2} \cos u - \frac{1}{2} \frac{z^3}{3} + c \\
&= \frac{\operatorname{sen} u}{2} - \frac{\operatorname{sen}^3 u}{6} + \frac{\cos u}{2} - \frac{\cos^3 u}{6} + c = \frac{1}{2} (\operatorname{sen} u + \cos u) - \frac{1}{6} (\operatorname{sen}^3 u + \cos^3 u) + c
\end{aligned}$$

Dado que:  $\operatorname{sen}^3 u + \cos^3 u = (\operatorname{sen} u + \cos u)(\operatorname{sen}^2 u - \operatorname{sen} u \cos u + \cos^2 u)$

O bien:  $\operatorname{sen}^3 u + \cos^3 u = (\operatorname{sen} u + \cos u)(1 - \operatorname{sen} u \cos u)$ ; Lo que equivale a:

$$\begin{aligned}
&= \frac{1}{2} (\operatorname{sen} u + \cos u) - \frac{1}{6} (\operatorname{sen} u + \cos u)(1 - \operatorname{sen} u \cos u) + c \\
&= \frac{1}{2} (\operatorname{sen} u + \cos u) - \frac{1}{6} (\operatorname{sen} u + \cos u) \left(1 - \frac{2 \operatorname{sen} u \cos u}{2}\right) + c \\
&= \frac{1}{2} (\operatorname{sen} u + \cos u) - \frac{1}{6} (\operatorname{sen} u + \cos u) \left(1 - \frac{\operatorname{sen} 2u}{2}\right) + c \\
&= \frac{1}{2} (\operatorname{sen} u + \cos u) - \frac{1}{6} (\operatorname{sen} u + \cos u) \frac{1}{2} (2 - \operatorname{sen} 2u) + c \\
&= \frac{1}{12} (\operatorname{sen} u + \cos u) (6 - (2 - \operatorname{sen} 2u)) + c = \frac{1}{12} (\operatorname{sen} u + \cos u) (4 + \operatorname{sen} 2u) + c \\
&= \frac{1}{12} (\operatorname{sen} x^2 + \cos x^2) (4 + \operatorname{sen} 2x^2) + c
\end{aligned}$$

**Respuesta:**  $\int x(\cos^3 x^2 - \operatorname{sen}^3 x^2) dx = \frac{1}{12} (\operatorname{sen} x^2 + \cos x^2) (4 + \operatorname{sen} 2x^2) + c$

**3.11.-Encontrar:**  $\int \operatorname{sen} 2x \cos 4x dx$

Solución.-  $\operatorname{sen} \alpha \cos \beta = \frac{1}{2} [\operatorname{sen}(\alpha - \beta) + \operatorname{sen}(\alpha + \beta)]$ ; Se tiene que:

$$\begin{aligned}
\operatorname{sen} 2x \cos 4x &= \frac{1}{2} [\operatorname{sen}(2x - 4x) + \operatorname{sen}(2x + 4x)] = \frac{1}{2} [\operatorname{sen}(-2x) + \operatorname{sen}(6x)] \\
&= \frac{1}{2} [-\operatorname{sen} 2x + \operatorname{sen} 6x], \text{ Luego: } \int \operatorname{sen} 2x \cos 4x dx = \int \frac{1}{2} (-\operatorname{sen} 2x + \operatorname{sen} 6x) dx \\
&= -\frac{1}{2} \int \operatorname{sen} 2x dx + \frac{1}{2} \int \operatorname{sen} 6x dx = \frac{1}{4} \cos 2x - \frac{1}{12} \cos 6x + c
\end{aligned}$$

**Respuesta:**  $\int \operatorname{sen} 2x \cos 4x dx = \frac{1}{4} \cos 2x - \frac{1}{12} \cos 6x + c$

**3.12.-Encontrar:**  $\int \cos 3x \cos 2x dx$ 

Solución.-  $\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$ ; Se tiene que:

$$\begin{aligned}\cos 3x \cos 2x &= \frac{1}{2} [\cos(3x - 2x) + \cos(3x + 2x)] = \frac{1}{2} [\cos x + \cos 5x], \text{ Luego:} \\ &= \int \cos 3x \cos 2x dx = \int \frac{1}{2} [\cos x + \cos 5x] dx = \frac{1}{2} \int \cos x dx + \frac{1}{2} \int \cos 5x dx \\ &= \frac{1}{2} \sin x + \frac{1}{10} \sin 5x + c\end{aligned}$$

**Respuesta:**  $\int \cos 3x \cos 2x dx = \frac{1}{2} \sin x + \frac{1}{10} \sin 5x + c$

**3.13.-Encontrar:**  $\int \sin 5x \sin x dx$ 

Solución.-  $\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$ ; Se tiene que:

$$\begin{aligned}\sin 5x \sin x &= \frac{1}{2} [\cos(5x - x) - \cos(5x + x)] = \frac{1}{2} [\cos 4x - \cos 6x]; \text{ Luego:} \\ &= \int \sin 5x \sin x dx = \int \frac{1}{2} [\cos 4x - \cos 6x] = \frac{1}{2} \int \cos 4x dx - \frac{1}{2} \int \cos 6x dx \\ &= \frac{1}{8} \sin 4x - \frac{1}{12} \sin 6x + c\end{aligned}$$

**Respuesta:**  $\int \sin 5x \sin x dx = \frac{1}{8} \sin 4x - \frac{1}{12} \sin 6x + c$

**3.14.-Encontrar:**  $\int \tau g^4 x dx$ 

Solución.-  $\int \tau g^4 x dx = \int \tau g^2 x \tau g^2 x dx$ ; como:  $\tau g^2 = \sec^2 x - 1$ ; Luego:

$$\begin{aligned}&= \int \tau g^2 x \tau g^2 x dx = \int \tau g^2 x (\sec^2 x - 1) dx = \int \tau g^2 x \sec^2 x dx - \int \tau g^2 x dx \\ &= \int (\tau g x)^2 \sec^2 x dx - \int \frac{\sin^2 x}{\cos^2 x} dx = \int (\tau g x)^2 \sec^2 x dx - \int \frac{1 - \cos^2 x}{\cos^2 x} dx \\ &= \int (\tau g x)^2 \sec^2 x dx - \int \sec^2 x dx + \int dx; \quad \text{Sea: } w = \tau g x, dw = \sec^2 x dx \\ &= \int w^2 dw - \int \sec^2 x + \int dx = \frac{w^3}{3} - \tau g x + x + c = \frac{\tau g^3}{3} - \tau g x + x + c\end{aligned}$$

**Respuesta:**  $\int \tau g^4 x dx = \frac{\tau g^3}{3} - \tau g x + x + c$

**3.15.-Encontrar:**  $\int \sec^6 x dx$ 

Solución.-  $\int \sec^6 x dx = \int (\sec^2 x)^2 \sec^2 x dx$ ; como:  $\sec^2 x dx = 1 + \tau g^2 x$

$$\begin{aligned}&= \int (\sec^2 x)^2 \sec^2 x dx = \int (1 + \tau g^2 x)^2 \sec^2 x dx = \int (1 + 2\tau g^2 x + \tau g^4 x) \sec^2 x dx \\ &= \int \sec^2 x dx + 2 \int (\tau g x)^2 \sec^2 x dx + \int (\tau g x)^4 \sec^2 x dx; \quad \text{Sea: } u = \tau g x, du = \sec^2 x dx\end{aligned}$$

$$= \int \sec^2 x dx + 2 \int u^2 du + \int u^4 du = \tau g x + \frac{2}{3} u^3 + \frac{1}{5} u^5 + c = \tau g x + \frac{2}{3} \tau g^3 x + \frac{1}{5} \tau g^5 x + c$$

**Respuesta:**  $\int \sec^6 x dx = \tau g x + \frac{2}{3} \tau g^3 x + \frac{1}{5} \tau g^5 x + c$

**3.16.-Encontrar:**  $\int \tau g^3 2x dx$

Solución.-

$$\int \tau g^3 2x dx = \int \tau g 2x \tau g^2 2x dx = \int \tau g 2x (\sec^2 2x - 1) dx = \int \tau g 2x \sec^2 2x dx - \int \tau g 2x dx$$

Sea:  $u = \tau g 2x, du = 2 \sec^2 2x dx$ ; Luego:

$$= \frac{1}{2} \int u du - \int \tau g 2x dx = \frac{1}{2} \frac{u^2}{2} - \frac{1}{2} \ell \eta |\sec 2x| + c = \frac{\tau g^2 2x}{4} - \frac{1}{2} \ell \eta \left| \frac{1}{\cos 2x} \right| + c$$

**Respuesta:**  $\int \tau g^3 2x dx = \frac{\tau g^2 2x}{4} - \frac{1}{2} \ell \eta \left| \frac{1}{\cos 2x} \right| + c$

**3.17.-Encontrar:**  $\int \tau g^2 5x dx$

Solución.-  $\int \tau g^2 5x dx = \int (\sec^2 5x - 1) dx = \int \sec^2 5x dx - \int dx = \frac{1}{5} \tau g 5x - x + c$

**Respuesta:**  $\int \tau g^2 5x dx = \frac{1}{5} \tau g 5x - x + c$

**3.18.-Encontrar:**  $\int \tau g^3 3x \sec 3x dx$

Solución.-  $\int \tau g^3 3x \sec 3x dx = \int \tau g^2 3x \tau g 3x \sec 3x dx = \int (\sec^2 3x - 1) \tau g 3x \sec 3x dx$

$$= \int (\sec 3x)^2 \tau g 3x \sec 3x dx - \int \tau g 3x \sec 3x dx; \text{ Sea: } u = \sec 3x, du = 3 \sec 3x \tau g 3x dx$$

Luego:  $\frac{1}{3} \int u^2 du - \frac{1}{3} \int 3 \tau g 3x \sec 3x dx$ ; como:  $d(\sec 3x) = 3 \tau g 3x \sec 3x dx$ , se admite:

$$\frac{1}{3} \int u^2 du - \frac{1}{3} \int d(\sec 3x) = \frac{1}{9} u^3 - \frac{1}{3} \sec 3x + c = \frac{1}{9} \sec^3 3x - \frac{1}{3} \sec 3x + c$$

**Respuesta:**  $\int \tau g^3 3x \sec 3x dx = \frac{1}{9} \sec^3 3x - \frac{1}{3} \sec 3x + c$

**3.19.-Encontrar:**  $\int \tau g^{\frac{3}{2}} x \sec^4 x dx$

Solución.-  $\int \tau g^{\frac{3}{2}} x \sec^4 x dx = \int \tau g^{\frac{3}{2}} x (\sec^2 x) \sec^2 x dx = \int \tau g^{\frac{3}{2}} x (1 + \tau g^2 x) \sec^2 x dx$

$$= \int (\tau g x)^{\frac{3}{2}} \sec^2 x dx + \int (\tau g x)^{\frac{5}{2}} \sec^2 x dx; \text{ Sea: } u = \tau g x, du = \sec^2 x dx$$

$$\text{Luego: } \int u^{\frac{3}{2}} du + \int u^{\frac{5}{2}} du = \frac{2}{5} u^{\frac{5}{2}} + \frac{2}{9} u^{\frac{7}{2}} + c = \frac{2}{5} \tau g^{\frac{5}{2}} x + \frac{2}{9} \tau g^{\frac{7}{2}} x + c$$

**Respuesta:**  $\int \tau g^{\frac{3}{2}} x \sec^4 x dx = \frac{2}{5} \tau g^{\frac{5}{2}} x + \frac{2}{9} \tau g^{\frac{7}{2}} x + c$

**3.20.-Encontrar:**  $\int \tau g^4 x \sec^4 x dx$

Solución.-  $\int \tau g^4 x (\sec^2 x) \sec^2 x dx = \int \tau g^4 x (1 + \tau g^2 x) \sec^2 x dx$

$$= \int (\tau g x)^4 \sec^2 x dx + \int (\tau g x)^6 \sec^2 x dx; \text{ Sea: } u = \tau g x, du = \sec^2 x dx$$

$$\text{Luego: } \int u^4 du + \int u^6 du = \frac{u^5}{5} + \frac{u^7}{7} + c = \frac{\tau g^5 x}{5} + \frac{\tau g^7 x}{7} + c$$

$$\text{Respuesta: } \int \tau g^4 x \sec^4 x dx = \frac{\tau g^5 x}{5} + \frac{\tau g^7 x}{7} + c$$

$$\text{3.21.-Encontrar: } \int \cot g^3 x \operatorname{cosec}^4 x dx$$

$$\text{Solución.-} \int \cot g^3 x \operatorname{cosec}^4 x dx = \int \cot g^3 x (\operatorname{cosec}^2 x) \operatorname{cosec}^2 x dx$$

$$\text{Como: } \operatorname{cosec}^2 x = 1 + \cot g^2 x; \quad \text{Luego:}$$

$$\int \cot g^3 x (1 + \cot g^2 x) \operatorname{cosec}^2 x dx = \int \cot g^3 x \operatorname{cosec}^2 x dx + \int \cot g^5 x \operatorname{cosec}^2 x dx$$

$$\text{Sea: } u = \cot g x, du = -\operatorname{cosec}^2 x dx,$$

$$\text{Luego: } -\int u^3 du - \int u^5 du = -\frac{u^4}{4} - \frac{u^6}{6} + c = -\frac{\cot g^4 x}{4} - \frac{\cot g^6 x}{6} + c$$

$$\text{Respuesta: } \int \cot g^3 x \operatorname{cosec}^4 x dx = -\frac{\cot g^4 x}{4} - \frac{\cot g^6 x}{6} + c$$

$$\text{3.22.-Encontrar: } \int \cot g 3x \operatorname{cosec}^4 3x dx$$

$$\text{Solución.-} \int \cot g 3x \operatorname{cosec}^4 3x dx = \int \cot g 3x (\operatorname{cosec}^2 3x) \operatorname{cosec}^2 3x dx$$

$$\int \cot g 3x (1 + \cot g^2 3x) \operatorname{cosec}^2 3x dx = \int \cot g 3x \operatorname{cosec}^2 3x dx + \int \cot g^3 3x \operatorname{cosec}^2 3x dx$$

$$\text{Sea: } u = \cot g 3x, du = -3 \operatorname{cosec}^2 3x dx; \quad \text{Luego:}$$

$$-\frac{1}{3} \int u du - \frac{1}{3} \int u^3 du = -\frac{u^2}{6} - \frac{u^4}{12} + c = -\frac{\cot g^2 3x}{6} - \frac{\cot g^4 3x}{12} + c$$

$$\text{Respuesta: } \int \cot g 3x \operatorname{cosec}^4 3x dx = -\frac{\cot g^2 3x}{6} - \frac{\cot g^4 3x}{12} + c$$

$$\text{3.23.-Encontrar: } \int \operatorname{cosec}^4 2x dx$$

$$\text{Solución.-} \int \operatorname{cosec}^2 2x \operatorname{cosec}^2 2x dx = \int (1 + \cot g^2 2x) \operatorname{cosec}^2 2x dx$$

$$\int \operatorname{cosec}^2 2x dx + \int \cot g^2 2x \operatorname{cosec}^2 2x dx; \quad \text{Sea: } u = \cot g 2x, du = -\operatorname{cosec}^2 2x dx$$

$$\text{Luego: } \int \operatorname{cosec}^2 2x dx - \frac{1}{2} \int u^2 du = -\frac{1}{2} \cot g 2x - \frac{u^3}{3} + c = -\frac{\cot g 2x}{2} - \frac{\cot g^3 2x}{6} + c$$

$$\text{Respuesta: } \int \operatorname{cosec}^4 2x dx = -\frac{\cot g 2x}{2} - \frac{\cot g^3 2x}{6} + c$$

$$\text{3.24.-Encontrar: } \int \cot g^3 x \operatorname{cosec}^3 x dx$$

$$\text{Solución.-} \int \cot g^3 x \operatorname{cosec}^3 x dx = \int \cot g^2 x \operatorname{cosec}^2 x \cot g x \operatorname{cosec} x dx$$

$$\text{Como: } \cot g^2 x = \operatorname{cosec}^2 x - 1; \quad \text{Luego: } \int (\operatorname{cosec}^2 x - 1) \operatorname{cosec}^2 x \cot g x \operatorname{cosec} x dx$$

$$= \int (\operatorname{cosec}^4 x \cot g x \operatorname{cosec} x dx - \int \operatorname{cosec}^2 x \cot g x \operatorname{cosec} x dx$$

$$\text{Sea: } u = \operatorname{cosec} x, du = -\operatorname{cosec} x \cot g x dx;$$

Entonces:  $-\int u^4 du + \int u^2 du = -\frac{u^5}{5} + \frac{u^3}{3} + c = -\frac{\cos ec^5 x}{5} + \frac{\cos ec^3 x}{3} + c$

**Respuesta:**  $\int \cot g^3 x \operatorname{cosec}^3 x dx = -\frac{\cos ec^5 x}{5} + \frac{\cos ec^3 x}{3} + c$

**3.25.-Encontrar:**  $\int \cot g^3 x dx$

Solución.-  $\int \cot g^3 x dx = \int \cot g^2 x \cot g x dx = \int (\cos ec^2 x - 1) \cot g x dx$   
 $= \int \cos ec^2 x \cot g x dx - \int \cot g x dx$ ; Sea:  $u = \cot g x, du = -\cos ec^2 x dx$

Luego:  $-\int u du - \int \cot g x dx = -\frac{u^2}{2} - \ell \eta |\operatorname{sen} x| + c = -\frac{\cot g^2 x}{2} - \ell \eta |\operatorname{sen} x| + c$

**Respuesta:**  $\int \cot g^3 x dx = -\frac{\cot g^2 x}{2} - \ell \eta |\operatorname{sen} x| + c$

## EJERCICIOS PROPUESTOS

Usando esencialmente el mecanismo tratado, encontrar las siguientes integrales:

**3.26.-**  $\int \tau g^2 5x dx$

**3.27.-**  $\int \operatorname{sen} x \cos x dx$

**3.28.-**  $\int \frac{dx}{\sec 2x}$

**3.29.-**  $\int \frac{\cos 2x}{\cos x} dx$

**3.30.-**  $\int \sqrt{\cos x} \operatorname{sen}^3 x dx$

**3.31.-**  $\int \tau g^{\frac{x}{3}} \sec^2 \frac{x}{3} dx$

**3.32.-**  $\int \tau g^3 4x \sec 4x dx$

**3.33.-**  $\int \operatorname{sen}^2 \frac{x}{6} dx$

**3.34.-**  $\int \frac{\operatorname{sen} 2x}{\operatorname{sen} x} dx$

**3.35.-**  $\int (\sec x + \cos ec x)^2 dx$

**3.36.-**  $\int \sec^{\frac{x}{4}} \tau g^{\frac{x}{4}} dx$

**3.37.-**  $\int \tau g^4 2x \sec^4 2x dx$

**3.38.-**  $\int \operatorname{sen} 8x \operatorname{sen} 3x dx$

**3.39.-**  $\int \cos 4x \cos 5x dx$

**3.40.-**  $\int \operatorname{sen} 2x \cos 3x dx$

**3.41.-**  $\int \left( \frac{\sec x}{\tau g x} \right)^4 dx$

**3.42.-**  $\int \frac{\cos^3 x}{\operatorname{sen}^4 x} dx$

**3.43.-**  $\int \cos ec^4 3x dx$

**3.44.-**  $\int (\tau g^{\frac{x}{3}} + \tau g^{\frac{4}{3}} \frac{x}{3}) dx$

**3.45.-**  $\int \cot g^3 \frac{x}{3} dx$

**3.46.-**  $\int \cot g^4 \frac{x}{6} dx$

**3.47.-**  $\int \frac{dx}{\operatorname{sen}^5 x \cos x}$

**3.48.-**  $\int \frac{\cos^2 x}{\operatorname{sen}^6 x} dx$

**3.49.-**  $\int \frac{dx}{\operatorname{sen}^2 x \cos^4 x}$

**3.50.-**  $\int \frac{dx}{\cos^6 4x}$

**3.51.-**  $\int \frac{\cos^3 x}{1 - \operatorname{sen} x} dx$

**3.52.-**  $\int \cos^{\frac{x}{7}} dx$

**3.53.-**  $\int \operatorname{sen}^5 \frac{x}{2} dx$

**3.54.-**  $\int \sqrt{1 - \cos x} dx$

**3.55.-**  $\int \frac{dx}{\cos ec^4 \frac{x}{3}}$

**3.56.-**  $\int \operatorname{sen}^3 \frac{x}{2} \cos^5 \frac{x}{2} dx$

**3.57.-**  $\int \operatorname{sen}^2 x \cos^2 x dx$

**3.58.-**  $\int \operatorname{sen}^4 x \cos^2 x dx$

**3.59.-**  $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$

**3.60.-**  $\int \frac{\cos^3 x}{\sqrt{\operatorname{sen} x}} dx$

**3.61.-**  $\int \operatorname{sen}^3 2x dx$

**3.62.-**  $\int \operatorname{sen}^2 2x \cos^2 2x dx$

**3.63.-**  $\int \cos^4 x dx$

**3.64.-**  $\int \tau g^4 x \sec^2 x dx$

$$\begin{array}{lll}
\mathbf{3.65.-} \int \tau g^3 x \sec x dx & \mathbf{3.66.-} \int \sec^6 a \theta d\theta & \mathbf{3.67.-} \int \sec x dx \\
\mathbf{3.68.-} \int \cos \tau g^2 2x \sec^2 2x dx & \mathbf{3.69.-} \int \frac{\sec^3 x}{\cos^2 x} dx & \mathbf{3.70.-} \int \sec^4 3x \tau g 3x dx \\
\mathbf{3.71.-} \int \sec^n x \tau g x dx; (n \neq 0) & \mathbf{3.72.-} \int \frac{\cos^3 x}{\sec^2 x} dx & \mathbf{3.73.-} \int \frac{dx}{\sec^4 x} \\
\mathbf{3.74.-} \int \tau g^n x \sec^2 x dx; (n \neq -1) & \mathbf{3.75.-} \int \sec^6 x dx & \mathbf{3.76.-} \int \sec^4 a x dx \\
\mathbf{3.77.-} \int \sec^n x \cos x dx; (n \neq -1) & \mathbf{3.78.-} \int \cos \tau g^n a x dx & \mathbf{3.79.-} \int \cos \tau g^4 3x dx \\
\mathbf{3.80.-} \int \cos x^n \sec x dx; (n \neq -1) & \mathbf{3.81.-} \int \tau g^n x dx & \mathbf{3.82.-} \int \tau g^4 x dx \\
\mathbf{3.83.-} \int \cos^{2n+1} x dx & & 
\end{array}$$

## RESPUESTAS

$$\begin{aligned}
\mathbf{3.26.-} \int \tau g^2 5x dx &= \int (\sec^2 5x - 1) dx = \int \sec^2 5x dx + \int dx = \frac{1}{5} \tau g^5 x - x + c \\
\mathbf{3.27.-} \int \sec x \cos x dx &= \frac{1}{2} \int 2 \sec x \cos x dx = \frac{1}{2} \int \sec 2x dx = -\frac{1}{4} \cos 2x + c \\
\mathbf{3.28.-} \int \frac{dx}{\sec 2x} &= \int \cos 2x dx = \frac{1}{2} \sec 2x + c \\
\mathbf{3.29.-} \int \frac{\cos 2x}{\cos x} dx &= \int \frac{\cos^2 x - \sec^2 x}{\cos x} dx = \int \frac{\cos^2 x}{\cos x} dx - \int \frac{\sec^2 x}{\cos x} dx \\
&= \int \cos x dx - \int \frac{1 - \cos^2 x}{\cos x} dx = \int \cos x dx - \int \frac{dx}{\cos x} + \int \cos x dx = 2 \int \cos x dx - \int \sec x dx \\
&= 2 \sec x - \ell \eta |\sec x + \tau g x| + c \\
\mathbf{3.30.-} \int \sqrt{\cos x} \sec^3 x dx &= \int \sqrt{\cos x} \sec^2 x \sec x dx = \int \sqrt{\cos x} (1 - \cos^2 x) \sec x dx \\
&= \int \sqrt{\cos x} \sec x dx - \int \sqrt{\cos x} \cos^2 x \sec x dx = \int \cos^{1/2} x \sec x dx - \int \cos^{5/2} x \sec x dx \\
\text{Sea: } u &= \cos x, du = -\sec x dx; \text{ Luego: } -\int u^{1/2} du + \int u^{5/2} du = -\frac{2}{3} u^{3/2} + \frac{2}{7} u^{7/2} + c \\
&= -\frac{2}{3} \cos^{3/2} x + \frac{2}{7} \cos^{7/2} x + c = -\frac{2}{3} \sqrt{\cos^3 x} + \frac{2}{7} \sqrt{\cos^7 x} + c \\
&= -\frac{2}{3} \cos x \sqrt{\cos x} + \frac{2}{7} \cos^3 x \sqrt{\cos x} + c \\
\mathbf{3.31.-} \int \tau g^2 \frac{x}{3} \sec^2 \frac{x}{3} dx &= \int (\tau g \frac{x}{3})^2 \sec^2 \frac{x}{3} dx; \text{ Sea: } u = \tau g \frac{x}{3}, du = \frac{1}{3} \sec^2 \frac{x}{3} dx \\
3 \int (\tau g \frac{x}{3})^2 \frac{1}{3} \sec^2 \frac{x}{3} dx &= 3 \int u^2 du = u^3 + c = \tau g^3 \frac{x}{3} + c \\
\mathbf{3.32.-} \int \tau g^3 4x \sec 4x dx &= \int (\tau g^2 4x) \tau g 4x \sec 4x dx = \int (\sec^2 4x - 1) \tau g 4x \sec 4x dx \\
&= \int \sec^2 4x \tau g 4x \sec 4x dx - \int \tau g 4x \sec 4x dx; \text{ Sea: } u = \sec 4x, du = 4 \sec 4x \tau g 4x dx
\end{aligned}$$



$$= \frac{1}{4} \int u^2 du - \frac{1}{4} \int du = \frac{1}{4} \frac{u^3}{3} - \frac{1}{4} u + c = \frac{\sec^3 4x}{12} - \frac{\sec 4x}{4} + c$$

$$\begin{aligned} \mathbf{3.33.-} \int \sec^2 \frac{x}{6} dx &= \int \frac{1 - \cos 2 \frac{x}{6}}{2} dx = \int \frac{1 - \cos \frac{x}{3}}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos \frac{x}{3} dx \\ &= \frac{1}{2} x - \frac{3}{2} \sec \frac{x}{3} + c \end{aligned}$$

$$\mathbf{3.34.-} \int \frac{\sec 2x}{\sec x} dx = \int \frac{2 \cancel{\sec x} \cos x}{\cancel{\sec x}} dx = 2 \int \cos x dx = 2 \sec x + c$$

$$\begin{aligned} \mathbf{3.35.-} \int (\sec x + \sec x)^2 dx &= \int (\sec^2 x + 2 \sec x \sec x + \sec^2 x) dx \\ &= \int \sec^2 x dx + 2 \int \sec x \sec x dx + \int \sec^2 x dx = \int \sec^2 x dx + 2 \int \frac{1}{\cos x} \frac{1}{\sec x} dx + \int \sec^2 x dx \\ &= \int \sec^2 x dx + 2 \times 2 \int \frac{dx}{2 \cos x \sec x} + \int \sec^2 x dx = \int \sec^2 x dx + 4 \int \frac{dx}{\sec 2x} + \int \sec^2 x dx \\ &= \int \sec^2 x dx + 4 \int \sec 2x dx + \int \sec^2 x dx \\ &= \tau g x + \frac{4}{2} \ell \eta |\cos 2x - \cot g 2x| - \cot g x + c \\ &= \tau g x + 2 \ell \eta |\cos 2x - \cot g 2x| - \cot g x + c \end{aligned}$$

$$\mathbf{3.36.-} \int \sec^3 \frac{x}{4} \tau g \frac{x}{4} dx = \int (\sec^2 \frac{x}{4}) \sec \frac{x}{4} \tau g \frac{x}{4} dx$$

$$\text{Sea: } u = \sec \frac{x}{4}, du = \frac{1}{4} \sec \frac{x}{4} \tau g \frac{x}{4} dx,$$

$$\text{Luego: } 4 \int u^2 du = 4 \frac{u^3}{3} + c = \frac{4 \sec^3 \frac{x}{4}}{3} + c$$

$$\begin{aligned} \mathbf{3.37.-} \int \tau g^4 2x \sec^4 2x dx &= \int \tau g^4 2x (\sec^2 2x) \sec^2 2x dx = \int \tau g^4 2x (1 + \tau g^2 2x) \sec^2 2x dx \\ &= \int (\tau g 2x)^4 \sec^2 2x dx + \int (\tau g 2x)^6 \sec^2 2x dx \end{aligned}$$

$$\text{Sea: } u = \tau g 2x, du = 2 \sec^2 2x dx, \quad \text{Luego:}$$

$$\begin{aligned} &= \frac{1}{2} \int (\tau g 2x)^4 2 \sec^2 2x dx + \frac{1}{2} \int (\tau g 2x)^6 2 \sec^2 2x dx = \frac{1}{2} \int u^4 du + \frac{1}{2} \int u^6 du \\ &= \frac{1}{2} \frac{u^5}{5} + \frac{1}{2} \frac{u^7}{7} + c = \frac{\tau g^5 2x}{10} + \frac{\tau g^7 2x}{14} + c \end{aligned}$$

$$\mathbf{3.38.-} \int \sec 8x \sec 3x dx$$

$$\text{Considerando: } \sec \alpha \sec \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\text{Luego: } \sec 8x \sec 3x = \frac{1}{2} (\cos 5x - \cos 11x); \text{ Se tiene:}$$

$$= \frac{1}{2} \int (\cos 5x - \cos 11x) dx = \frac{1}{2} \int \cos 5x dx - \frac{1}{2} \int \cos 11x dx = \frac{\sec 5x}{10} - \frac{\sec 11x}{22} + c$$

$$\mathbf{3.39.-} \int \cos 4x \cos 5x dx$$

$$\text{Considerando: } \cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

Luego:  $\cos 4x \cos 5x = \frac{1}{2}(\cos(-x) + \cos 9x)$ ;

Como:  $\cos(-x) = \cos x \Rightarrow \frac{1}{2}(\cos x + \cos 9x)$ ; entonces:

$$\begin{aligned}\int \cos 4x \cos 5x dx &= \frac{1}{2} \int (\cos x + \cos 9x) dx = \frac{1}{2} \int \cos x dx + \frac{1}{2} \int \cos 9x dx \\ &= \frac{\operatorname{sen} x}{2} + \frac{\operatorname{sen} 9x}{18} + c\end{aligned}$$

**3.40.-**  $\int \operatorname{sen} 2x \cos 3x dx$

Considerando:  $\operatorname{sen} \alpha \cos \beta = \frac{1}{2}[\operatorname{sen}(\alpha - \beta) + \operatorname{sen}(\alpha + \beta)]$

Luego:  $\operatorname{sen} 2x \cos 3x = \frac{1}{2}[\operatorname{sen}(-x) + \operatorname{sen} 5x]$

Como:  $\operatorname{sen}(-x) = -\operatorname{sen} x \Rightarrow \frac{1}{2}(-\operatorname{sen} x + \operatorname{sen} 5x)$ ; entonces:

$$\begin{aligned}\int \operatorname{sen} 2x \cos 3x dx &= \frac{1}{2} \int (-\operatorname{sen} x + \operatorname{sen} 5x) dx = -\frac{1}{2} \int \operatorname{sen} x dx + \frac{1}{2} \int \operatorname{sen} 5x dx \\ &= \frac{1}{2} \cos x - \frac{1}{10} \cos 5x + c\end{aligned}$$

**3.41.-**  $\int \left( \frac{\sec x}{\tau g x} \right)^4 dx = \int \left( \frac{\frac{1}{\cos x}}{\frac{\operatorname{sen} x}{\cos x}} \right)^4 dx = \int \left( \frac{1}{\operatorname{sen} x} \right)^4 dx = \int \cos ec^4 x dx = \int \cos ec^2 x \cos ec^2 x dx$

Sea:  $u = \cos \tau g x, du = -\cos ec^2 x dx$

Luego:  $\int \cos ec^2 x dx - \int u^2 du = -\cos \tau g x - \frac{u^3}{3} + c = -\cos \tau g x - \frac{\cos \tau g^3 x}{3} + c$

**3.42.-**  $\int \frac{\cos^3 x}{\operatorname{sen}^4 x} dx = \int \frac{\cos^3 x}{\operatorname{sen}^3 x \operatorname{sen} x} dx = \int \cos \tau g^3 x \cos ec x dx$

$$\begin{aligned}&= \int (\cos \tau g^2 x) \cos \tau g x \cos ec x dx = \int (\cos ec^2 x - 1) \cos \tau g x \cos ec x dx = \\ &= \int \cos ec^2 x \cos \tau g x \cos ec x dx - \int \cos \tau g x \cos ec x dx\end{aligned}$$

Sea:  $u = \cos ec x, du = -\cos ec x \cos \tau g x dx$

Luego:  $-\int u^2 du + \int du = -\frac{u^3}{3} + u + c = -\frac{\cos ec^3 x}{3} + \cos ec x + c$

**3.43.-**  $\int \cos ec^4 3x dx = \int (\cos ec^2 3x) \cos ec^2 3x dx = \int (1 + \cos \tau g^2 3x) \cos ec^2 3x dx$

$$= \int \cos ec^2 3x dx + \int \cos \tau g^2 3x \cos ec^2 3x dx$$

Sea:  $u = \cos \tau g 3x, du = -3 \cos ec^2 3x dx$

Luego:  $\int \cos ec^2 3x dx - \frac{1}{3} \int u^2 du = -\frac{1}{3} \cos \tau g 3x - \frac{1}{9} u^3 + c = -\frac{\cos \tau g 3x}{3} - \frac{\cos \tau g^3 3x}{9} + c$

$$\begin{aligned}
\mathbf{3.44.-} \int (\tau g^3 \frac{x}{3} + \tau g^4 \frac{x}{3}) dx &= \int \tau g^3 \frac{x}{3} dx + \int \tau g^4 \frac{x}{3} dx = \int (\tau g^2 \frac{x}{3}) \tau g \frac{x}{3} dx + \int (\tau g^2 \frac{x}{3}) \tau g^2 \frac{x}{3} dx \\
&= \int (\sec^2 \frac{x}{3} - 1) \tau g \frac{x}{3} dx + \int (\sec^2 \frac{x}{3} - 1) \tau g^2 \frac{x}{3} dx \\
&= \int \sec^2 \frac{x}{3} \tau g \frac{x}{3} dx - \int \tau g \frac{x}{3} dx + \int (\sec^2 \frac{x}{3}) \tau g^2 \frac{x}{3} dx - \int \tau g^2 \frac{x}{3} dx \\
&= \int \sec^2 \frac{x}{3} \tau g \frac{x}{3} dx - \int \tau g \frac{x}{3} dx + \int (\sec^2 \frac{x}{3}) \tau g^2 \frac{x}{3} dx - \int (\sec^2 \frac{x}{3} - 1) dx \\
&= \int \sec^2 \frac{x}{3} \tau g \frac{x}{3} dx - \int \tau g \frac{x}{3} dx + \int (\sec^2 \frac{x}{3}) \tau g^2 \frac{x}{3} dx - \int \sec^2 \frac{x}{3} dx + \int dx
\end{aligned}$$

$$\text{Sea: } u = \tau g \frac{x}{3}, du = \frac{1}{3} \sec^2 \frac{x}{3} dx$$

$$\begin{aligned}
\text{Luego: } 3 \int u du - \int \tau g \frac{x}{3} dx + 3 \int u^2 du - \int \sec^2 \frac{x}{3} dx + \int dx \\
= \frac{3}{2} u^2 - 3 \ell \eta |\sec \frac{x}{3}| + u^3 - 3 \tau g \frac{x}{3} + x + c = \frac{3}{2} \tau g^2 \frac{x}{3} - 3 \ell \eta |\sec \frac{x}{3}| + \tau g^3 \frac{x}{3} - 3 \tau g \frac{x}{3} + x + c
\end{aligned}$$

$$\begin{aligned}
\mathbf{3.45.-} \int \cos \tau g^3 \frac{x}{3} dx &= \int (\cos \tau g^2 \frac{x}{3}) \cos \tau g \frac{x}{3} dx = \int (\cos ec^2 \frac{x}{3} - 1) \cos \tau g \frac{x}{3} dx \\
&= \int \cos ec^2 \frac{x}{3} \cos \tau g \frac{x}{3} dx - \int \cos \tau g \frac{x}{3} dx; \quad \text{Sea: } u = \cos ec \frac{x}{3}, du = -\frac{1}{3} \cos ec \frac{x}{3} \cos \tau g \frac{x}{3} dx
\end{aligned}$$

$$\begin{aligned}
\text{Luego: } -3 \int (\cos ec \frac{x}{3}) (-\frac{1}{3} \cos ec \frac{x}{3} \cos \tau g \frac{x}{3}) dx - \int \cos \tau g \frac{x}{3} dx &= -3 \int u du - \int \cos \tau g \frac{x}{3} dx \\
&= \frac{-3u^2}{2} - 3 \ell \eta |\sen \frac{x}{3}| + c = \frac{-3 \cos ec^2 \frac{x}{3}}{2} - 3 \ell \eta |\sen \frac{x}{3}| + c
\end{aligned}$$

$$\begin{aligned}
\mathbf{3.46.-} \int \cos \tau g^4 \frac{x}{6} dx &= \int (\cos \tau g^2 \frac{x}{6}) \cos \tau g^2 \frac{x}{6} dx = \int (\cos ec^2 \frac{x}{6} - 1) \cos \tau g^2 \frac{x}{6} dx \\
&= \int \cos ec^2 \frac{x}{6} \cos \tau g^2 \frac{x}{6} dx - \int \cos \tau g^2 \frac{x}{6} dx = \int \cos ec^2 \frac{x}{6} \cos \tau g^2 \frac{x}{6} dx - \int (\cos ec^2 \frac{x}{6} - 1) dx \\
&= \int \cos ec^2 \frac{x}{6} \cos \tau g^2 \frac{x}{6} dx - \int \cos ec^2 \frac{x}{6} dx + \int dx
\end{aligned}$$

$$\text{Sea: } u = \cos \tau g \frac{x}{6}, du = -\frac{1}{6} \cos ec^2 \frac{x}{6} dx$$

$$\begin{aligned}
\text{Luego: } -6 \int u^2 du - \int \cos ec^2 \frac{x}{6} dx + \int dx &= -2u^3 + 6 \cos \tau g \frac{x}{6} + x + c \\
&= -2 \cos \tau g^3 \frac{x}{6} + 6 \cos \tau g \frac{x}{6} + x + c
\end{aligned}$$

$$\mathbf{3.47.-} \int \frac{dx}{\sen^5 x \cos x}; \quad \text{Como: } \sen^2 x + \cos^2 x = 1,$$

$$\begin{aligned}
\text{Luego: } \int \frac{\sen^2 x + \cos^2 x}{\sen^5 x \cos x} dx &= \int \frac{dx}{\sen^3 x \cos x} + \int \frac{\cos x dx}{\sen^5 x} \\
&= \int \frac{\sen^2 x + \cos^2 x}{\sen^3 x \cos x} dx + \int \frac{\cos x dx}{\sen^5 x} = \int \frac{dx}{\sen x \cos x} + \int \frac{\cos x dx}{\sen^3 x} + \int \frac{\cos x dx}{\sen^5 x} \\
&= \int \frac{dx}{\sen x \cos x} + \int (\sen x)^{-3} \cos x dx + \int (\sen x)^{-5} \cos x dx \\
&= \int \frac{dx}{\sen 2x} + \int (\sen x)^{-3} \cos x dx + \int (\sen x)^{-5} \cos x dx \\
&= 2 \int \cos ec 2x dx + \int (\sen x)^{-3} \cos x dx + \int (\sen x)^{-5} \cos x dx (*)
\end{aligned}$$

Sea:  $u = \operatorname{sen} x$ ,  $du = \cos x dx$ ,      Luego:

$$\begin{aligned}
 (*) &= 2 \int \operatorname{cosec} 2x dx + \int u^{-3} du + \int u^{-5} du = \ell \eta |\operatorname{cosec} 2x - \operatorname{cotg} 2x| - \frac{1}{2u^2} - \frac{1}{4u^4} + c \\
 &= \ell \eta |\operatorname{cosec} 2x - \operatorname{cotg} 2x| - \frac{1}{2 \operatorname{sen}^2 x} - \frac{1}{4 \operatorname{sen}^4 x} + c \\
 &= \ell \eta |\operatorname{cosec} 2x - \operatorname{cotg} 2x| - \frac{\operatorname{cosec}^2 x}{2} - \frac{\operatorname{cosec}^4 x}{4} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{3.48.} \quad &\int \frac{\cos^2 x}{\operatorname{sen}^6 x} dx = \int \frac{\cos^2 x}{\operatorname{sen}^2 x \operatorname{sen}^4 x} dx = \int \operatorname{cotg}^2 x \operatorname{cosec}^4 x dx \\
 &= \int \operatorname{cotg}^2 x (\operatorname{cosec}^2 x) \operatorname{cosec}^2 x dx = \int \operatorname{cotg}^2 x (1 + \operatorname{cotg}^2 x) \operatorname{cosec}^2 x dx \\
 &= \int \operatorname{cotg}^2 x \operatorname{cosec}^2 x dx + \int \operatorname{cotg}^4 x \operatorname{cosec}^2 x dx
 \end{aligned}$$

Sea:  $u = \operatorname{cotg} x$ ,  $du = -\operatorname{cosec}^2 x dx$ ,

$$\text{Luego:} \quad -\int u^2 du - \int u^4 du = -\frac{u^3}{3} - \frac{u^5}{5} + c = -\frac{\operatorname{cotg}^3 x}{3} - \frac{\operatorname{cotg}^5 x}{5} + c$$

$$\begin{aligned}
 \text{3.49.} \quad &\int \frac{dx}{\operatorname{sen}^2 x \cos^4 x} = \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^2 x \cos^4 x} dx = \int \frac{dx}{\cos^4 x} + \int \frac{dx}{\operatorname{sen}^2 x \cos^2 x} \\
 &= \int \sec^4 x dx + \int \frac{dx}{(\operatorname{sen} x \cos x)^2} = \int \sec^4 x dx + \int \frac{dx}{\left(\frac{\operatorname{sen} 2x}{2}\right)^2} = \int \sec^4 x dx + 4 \int \frac{dx}{\operatorname{sen}^2 2x} \\
 &= \int \sec^4 x dx + 4 \int \operatorname{cosec}^2 2x dx = \int \sec^2 x \sec^2 x dx + 4 \int \operatorname{cosec}^2 2x dx \\
 &= \int (1 + \operatorname{tg}^2 x) \sec^2 x dx + 4 \int \operatorname{cosec}^2 2x dx = \int \sec^2 x dx + \int \operatorname{tg}^2 x \sec^2 x dx + 4 \int \operatorname{cosec}^2 2x dx
 \end{aligned}$$

Sea:  $u = \operatorname{tg} x$ ,  $du = \sec^2 x dx$ ,

$$\begin{aligned}
 \text{Luego:} \quad &\int \sec^2 x dx + \int u^2 du + 4 \int \operatorname{cosec}^2 2x dx = \operatorname{tg} x + \frac{u^3}{3} - 2 \operatorname{cotg} 2x + c \\
 &= \operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} - 2 \operatorname{cotg} 2x + c
 \end{aligned}$$

$$\begin{aligned}
 \text{3.50.} \quad &\int \frac{dx}{\cos^6 4x} = \int \sec^6 4x dx = \int (\sec^2 4x)^2 \sec^2 4x dx = \int (1 + \operatorname{tg}^2 4x)^2 \sec^2 4x dx \\
 &= \int (1 + 2 \operatorname{tg}^2 4x + \operatorname{tg}^4 4x) \sec^2 4x dx \\
 &= \int \sec^2 4x dx + 2 \int (\operatorname{tg} 4x)^2 \sec^2 4x dx + \int (\operatorname{tg} 4x)^4 \sec^2 4x dx
 \end{aligned}$$

Sea:  $u = \operatorname{tg} 4x$ ,  $du = 4 \sec^2 4x dx$ ,      Luego:

$$\int \sec^2 4x dx + \frac{1}{2} \int u^2 du + \frac{1}{4} \int u^4 du = \frac{\operatorname{tg} 4x}{4} + \frac{1}{2} \frac{u^3}{3} + \frac{1}{4} \frac{u^5}{5} + c = \frac{\operatorname{tg} 4x}{4} + \frac{\operatorname{tg}^3 4x}{6} + \frac{\operatorname{tg}^5 4x}{20} + c$$

$$\begin{aligned}
 \text{3.51.} \quad &\int \frac{\cos^3 x}{1 - \operatorname{sen} x} dx = \int \frac{\cos^3 x (1 + \operatorname{sen} x)}{1 - \operatorname{sen}^2 x} dx = \int \frac{\cancel{\cos^2 x} x (1 + \operatorname{sen} x)}{\cancel{\cos^2 x}} dx \\
 &= \int \cos x (1 + \operatorname{sen} x) dx = \int \cos x dx + \int \cos x \operatorname{sen} x dx = \int \cos x dx + \frac{1}{2} \int \operatorname{sen} 2x dx
 \end{aligned}$$

$$= \operatorname{sen} x - \frac{1}{4} \cos 2x + c$$

$$\begin{aligned} \mathbf{3.52.-} \int \cos^3 \frac{x}{7} dx &= \int (\cos^2 \frac{x}{7}) \cos \frac{x}{7} dx = \int (1 - \operatorname{sen}^2 \frac{x}{7}) \cos \frac{x}{7} dx \\ &= \int \cos \frac{x}{7} dx - \int \operatorname{sen}^2 \frac{x}{7} \cos \frac{x}{7} dx \end{aligned}$$

$$\text{Sea: } u = \operatorname{sen} \frac{x}{7}, du = \frac{1}{7} \cos \frac{x}{7} dx$$

$$\text{Luego: } = \int \cos \frac{x}{7} dx - 7 \int u^2 du = 7 \operatorname{sen} \frac{x}{7} - \frac{7u^3}{3} + c = 7 \operatorname{sen} \frac{x}{7} - \frac{7}{3} \operatorname{sen}^3 \frac{x}{7} + c$$

$$\begin{aligned} \mathbf{3.53.-} \int \operatorname{sen}^5 \frac{x}{2} dx &= \int (\operatorname{sen}^2 \frac{x}{2})^2 \operatorname{sen} \frac{x}{2} dx = \int (1 - \cos^2 \frac{x}{2})^2 \operatorname{sen} \frac{x}{2} dx \\ &= \int (1 - 2 \cos^2 \frac{x}{2} + \cos^4 \frac{x}{2}) \operatorname{sen} \frac{x}{2} dx = \int \operatorname{sen} \frac{x}{2} dx - 2 \int \cos^2 \frac{x}{2} \operatorname{sen} \frac{x}{2} dx + \int \cos^4 \frac{x}{2} \operatorname{sen} \frac{x}{2} dx \end{aligned}$$

$$\text{Sea: } u = \cos \frac{x}{2}, du = -\frac{1}{2} \operatorname{sen} \frac{x}{2} dx, \quad \text{Luego:}$$

$$\begin{aligned} &= \int \operatorname{sen} \frac{x}{2} dx + 4 \int u^2 du - 2 \int u^4 du = -2 \cos \frac{x}{2} + \frac{4u^3}{3} - \frac{2u^5}{5} + c \\ &= -2 \cos \frac{x}{2} + \frac{4 \cos^3 \frac{x}{2}}{3} - \frac{2 \cos^5 \frac{x}{2}}{5} + c \end{aligned}$$

$$\mathbf{3.54.-} \int \sqrt{1 - \cos x} dx$$

$$\text{Considerando: } \operatorname{sen}^2 \alpha = \frac{1 - \cos 2\alpha}{2}, \text{ y } 2\alpha = x$$

$$\text{Se tiene: } \operatorname{sen}^2 \frac{x}{2} = \frac{1 - \cos x}{2}; \quad \text{además: } 1 - \cos x = 2 \operatorname{sen}^2 \frac{x}{2}$$

$$\text{Luego: } \int \sqrt{2 \operatorname{sen}^2 \frac{x}{2}} dx = \sqrt{2} \int \operatorname{sen} \frac{x}{2} dx = -2\sqrt{2} \cos \frac{x}{2} + c$$

$$\begin{aligned} \mathbf{3.55.-} \int \frac{dx}{\cos^4 \frac{x}{3}} &= \int \operatorname{sen}^4 \frac{x}{3} dx = \int (\operatorname{sen}^2 \frac{x}{3})^2 dx = \int \left( \frac{1 - \cos \frac{2x}{3}}{2} \right)^2 dx \\ &= \frac{1}{4} \int (1 - 2 \cos \frac{2x}{3} + \cos^2 \frac{2x}{3}) dx = \frac{1}{4} \int dx - \frac{1}{2} \int \cos \frac{2x}{3} dx + \frac{1}{4} \int \cos^2 \frac{2x}{3} dx \\ &= \frac{1}{4} \int dx - \frac{1}{2} \int \cos \frac{2x}{3} dx + \frac{1}{4} \int \frac{1 + \cos \frac{4x}{3}}{2} dx = \frac{1}{4} \int dx - \frac{1}{2} \int \cos \frac{2x}{3} dx + \frac{1}{8} \int (1 + \cos \frac{4x}{3}) dx \\ &= \frac{1}{4} \int dx - \frac{1}{2} \int \cos \frac{2x}{3} dx + \frac{1}{8} \int dx + \frac{1}{8} \int \cos \frac{4x}{3} dx = \frac{3}{8} \int dx - \frac{1}{2} \int \cos \frac{2x}{3} dx + \frac{1}{8} \int \cos \frac{4x}{3} dx \\ &= \frac{3}{8} x - \frac{1}{2} \frac{3}{2} \operatorname{sen} \frac{2x}{3} + \frac{1}{8} \frac{3}{4} \operatorname{sen} \frac{4x}{3} + c = \frac{3}{8} x - \frac{3 \operatorname{sen} \frac{2x}{3}}{4} + \frac{3 \operatorname{sen} \frac{4x}{3}}{32} + c \end{aligned}$$

$$\begin{aligned} \mathbf{3.56.-} \int \operatorname{sen}^3 \frac{x}{2} \cos^5 \frac{x}{2} dx &= \int \operatorname{sen} \frac{x}{2} \operatorname{sen}^2 \frac{x}{2} \cos^5 \frac{x}{2} dx = \int \operatorname{sen} \frac{x}{2} (1 - \cos^2 \frac{x}{2}) \cos^5 \frac{x}{2} dx \\ &= \int \operatorname{sen} \frac{x}{2} \cos^5 \frac{x}{2} dx - \int \cos^7 \frac{x}{2} \operatorname{sen} \frac{x}{2} dx \end{aligned}$$

$$\text{Sea: } u = \cos \frac{x}{2}, du = -\frac{1}{2} \operatorname{sen} \frac{x}{2} dx$$

$$\text{Luego: } -2 \int u^5 du + 2 \int u^7 du = -\frac{2u^6}{6} + \frac{2u^8}{8} + c = -\frac{u^6}{3} + \frac{u^8}{4} + c = -\frac{\cos^6 \frac{x}{2}}{3} + \frac{\cos^8 \frac{x}{2}}{4} + c$$

$$\begin{aligned} \mathbf{3.57.-} \int \operatorname{sen}^2 x \cos^2 x dx &= \int (\operatorname{sen} x \cos x)^2 dx = \int \left( \frac{\operatorname{sen} 2x}{2} \right)^2 dx = \frac{1}{4} \int \operatorname{sen}^2 2x dx \\ &= \frac{1}{4} \int \frac{1 - \cos 4x}{2} dx = \frac{1}{8} \int (1 - \cos 4x) dx = \frac{1}{8} \int dx - \frac{1}{8} \int \cos 4x dx = \frac{x}{8} - \frac{1}{32} \operatorname{sen} 4x + c \end{aligned}$$

$$\begin{aligned} \mathbf{3.58.-} \int \operatorname{sen}^4 x \cos^2 x dx &= \int (\operatorname{sen}^2 x \cos^2 x) \operatorname{sen}^2 x dx = \int (\operatorname{sen} x \cos x)^2 \operatorname{sen}^2 x dx \\ &= \int \left( \frac{\operatorname{sen} 2x}{2} \right)^2 \left( \frac{1 - \cos 2x}{2} \right) dx = \frac{1}{4} \int \operatorname{sen}^2 2x \left( \frac{1 - \cos 2x}{2} \right) dx \\ &= \frac{1}{8} \int \operatorname{sen}^2 2x dx - \frac{1}{8} \int \operatorname{sen}^2 2x \cos 2x dx = \frac{1}{8} \int \frac{1 - \cos 4x}{2} dx - \frac{1}{8} \int \operatorname{sen}^2 2x \cos 2x dx \\ &= \frac{1}{16} \int dx - \frac{1}{16} \int \cos 4x dx - \frac{1}{8} \int \operatorname{sen}^2 2x \cos 2x dx (*) \end{aligned}$$

Sea:  $u = \operatorname{sen} 2x$ ,  $du = 2 \cos 2x dx$ , luego:

$$\begin{aligned} (*) &= \frac{1}{16} \int dx - \frac{1}{16} \int \cos 4x dx - \frac{1}{16} \int u^2 du = \frac{1}{16} x - \frac{1}{64} \operatorname{sen} 4x - \frac{1}{16} \frac{u^3}{3} + c \\ &= \frac{1}{16} x - \frac{\operatorname{sen} 4x}{64} - \frac{\operatorname{sen}^3 2x}{48} + c \end{aligned}$$

$$\begin{aligned} \mathbf{3.59.-} \int \frac{1 - \cos 2x}{1 + \cos 2x} dx &= \int \frac{\frac{1 - \cos 2x}{2}}{\frac{1 + \cos 2x}{2}} dx = \int \frac{\operatorname{sen}^2 x}{\cos^2 x} dx = \int \tan^2 x dx = \int (\sec^2 x - 1) dx \\ &= \int \sec^2 x dx - \int dx = \tan x - x + c \end{aligned}$$

$$\begin{aligned} \mathbf{3.60.-} \int \frac{\cos^3 x}{\sqrt{\operatorname{sen} x}} dx &= \int (\operatorname{sen} x)^{-\frac{1}{2}} \cos^3 x dx = \int (\operatorname{sen} x)^{-\frac{1}{2}} \cos^2 x \cos x dx \\ &= \int (\operatorname{sen} x)^{-\frac{1}{2}} (1 - \operatorname{sen}^2 x) \cos x dx = \int (\operatorname{sen} x)^{-\frac{1}{2}} \cos x dx - \int \operatorname{sen}^{\frac{3}{2}} x \cos x dx (*) \end{aligned}$$

Sea:  $u = \operatorname{sen} x$ ,  $du = \cos x dx$ , luego:

$$(*) = \int u^{-\frac{1}{2}} du - \int u^{\frac{3}{2}} du = 2u^{\frac{1}{2}} - \frac{2\sqrt{\operatorname{sen}^5 x}}{5} + c$$

$$\begin{aligned} \mathbf{3.61.-} \int \operatorname{sen}^3 2x dx &= \int \operatorname{sen}^2 2x \operatorname{sen} 2x dx = \int (1 - \cos^2 2x) \operatorname{sen} 2x dx \\ &= \int \operatorname{sen} 2x dx - \int \cos^2 2x \operatorname{sen} 2x dx (*) \end{aligned}$$

Sea:  $u = \cos 2x$ ,  $du = -2 \operatorname{sen} 2x dx$ , luego:

$$\begin{aligned} (*) &= \int \operatorname{sen} 2x + \frac{1}{2} \int \frac{u^2}{2} du = -\frac{1}{2} \cos 2x + \frac{1}{2} \frac{u^3}{3} + c = -\frac{1}{2} \cos 2x + \frac{u^3}{6} + c \\ &= -\frac{1}{2} \cos 2x + \frac{(\cos^3 2x)}{6} + c \end{aligned}$$

$$\begin{aligned}
\mathbf{3.62.-} \int \sec^2 2x \cos^2 2x dx &= \int \left( \frac{1 - \cos 4x}{2} \right) \left( \frac{1 + \cos 4x}{2} \right) dx = \frac{1}{4} \int (1 - \cos^2 4x) dx \\
&= \frac{1}{4} \int dx - \frac{1}{4} \int \cos^2 4x dx = \frac{1}{4} \int dx - \frac{1}{4} \int \left( \frac{1 + \cos 8x}{2} \right) dx = \frac{1}{4} \int dx - \frac{1}{8} \int (1 + \cos 8x) dx \\
&= \frac{1}{4} \int dx - \frac{1}{8} \int dx - \frac{1}{8} \int \cos 8x dx = \frac{1}{8} \int dx - \frac{1}{8} \int \cos 8x dx = \frac{x}{8} - \frac{\sec 8x}{64} + c
\end{aligned}$$

$$\begin{aligned}
\mathbf{3.63.-} \int \cos^4 x dx &= \int (\cos^2 x)^2 dx = \int \left( \frac{1 + \cos 2x}{2} \right)^2 dx = \frac{1}{4} \int (1 + \cos 2x)^2 dx \\
&= \frac{1}{4} \int (1 + 2 \cos 2x + \cos^2 2x) dx = \frac{1}{4} \int dx + \frac{1}{2} \int \cos 2x dx + \frac{1}{4} \int \cos^2 2x dx \\
&= \frac{1}{4} \int dx + \frac{1}{2} \int \cos 2x dx + \frac{1}{4} \int \left( \frac{1 + \cos 4x}{2} \right) dx = \frac{1}{4} \int dx + \frac{1}{2} \int \cos 2x dx + \frac{1}{8} \int (1 + \cos 4x) dx \\
&= \frac{1}{4} \int dx + \frac{1}{2} \int \cos 2x dx + \frac{1}{8} \int dx + \frac{1}{8} \int \cos 4x dx = \frac{3}{8} \int dx + \frac{1}{2} \int \cos 2x dx + \frac{1}{8} \int \cos 4x dx \\
&= \frac{3}{8} x + \frac{1}{4} \sec 2x + \frac{1}{32} \sec 4x + c
\end{aligned}$$

$$\mathbf{3.64.-} \int \tau g^4 x \sec^2 x dx$$

$$\text{Sea: } u = \tau g x, du = \sec^2 x dx$$

$$\text{Luego: } \int u^4 du = \frac{u^5}{5} + c = \frac{\tau g^5 x}{5} + c$$

$$\begin{aligned}
\mathbf{3.65.-} \int \tau g^3 x \sec x dx &= \int \tau g^2 x \tau g x \sec x dx = \int (\sec^2 x - 1) \tau g x \sec x dx \\
&= \int (\sec^2 x) \tau g x \sec x dx - \int \tau g x \sec x dx
\end{aligned}$$

$$\text{Sea: } u = \sec x, du = \sec x \tau g x dx$$

$$\text{Luego: } \int u^2 du - \int du = \frac{u^3}{3} - u + c = \frac{\sec^3 x}{3} - \sec x + c$$

$$\begin{aligned}
\mathbf{3.66.-} \int \sec^6 a \theta d\theta &= \int \sec^4 a \theta \sec^2 a \theta d\theta = \int (\sec^2 a \theta)^2 \sec^2 a \theta d\theta \\
&= \int (1 + \tau g^2 a \theta)^2 \sec^2 a \theta d\theta = \int (1 + 2 \tau g^2 a \theta + \tau g^4 a \theta) \sec^2 a \theta d\theta \\
&= \int \sec^2 a \theta d\theta + 2 \int \tau g^2 a \theta \sec^2 a \theta d\theta + \int \tau g^4 a \theta \sec^2 a \theta d\theta
\end{aligned}$$

$$\text{Sea: } u = \tau g a \theta, du = a \sec^2 a \theta d\theta, \quad \text{Luego:}$$

$$\frac{1}{a} \int du + \frac{2}{a} \int u^2 du + \frac{1}{a} \int u^4 du = \frac{1}{a} \left[ u + \frac{2u^3}{3} + \frac{u^5}{5} \right] + c = \frac{1}{a} \left[ \tau g a \theta + \frac{2 \tau g^3 a \theta}{3} + \frac{\tau g^5 a \theta}{5} \right] + c$$

$$\mathbf{3.67.-} \int \sec x dx = \int \frac{\sec x (\tau g x + \sec x) dx}{\tau g x + \sec x} = \int \frac{\sec x \tau g x + \sec^2 x}{\tau g x + \sec x} dx$$

$$\text{Sea: } u = \sec x + \tau g x, du = (\sec x \tau g x + \sec^2 x) dx$$

$$\text{Luego: } \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |\sec x + \tau g x| + c$$

$$\mathbf{3.68.-} \int \cot g^2 2x \operatorname{cosec}^2 2x dx$$

$$\text{Sea: } u = \cot g 2x, du = -2 \operatorname{cosec}^2 2x dx$$

$$\text{Luego: } -\frac{1}{2} \int u^2 du = -\frac{u^3}{6} + c = -\frac{\cot g^3 2x}{6} + c$$

$$\mathbf{3.69.-} \int \frac{\operatorname{sen}^3 x}{\cos^2 x} dx = \int \frac{\operatorname{sen}^2 x \operatorname{sen} x dx}{\cos^2 x} = \int \frac{(1 - \cos^2 x) \operatorname{sen} x dx}{\cos^2 x} = \int \frac{\operatorname{sen} x dx}{\cos^2 x} - \int \operatorname{sen} x dx$$

$$\text{Sea: } u = \cos x, du = -\operatorname{sen} x dx,$$

$$\text{Luego: } -\int u^{-2} du - \int \operatorname{sen} x dx = \frac{1}{u} + \cos x + c = \frac{1}{\cos x} + \cos x + c = \sec x + \cos x + c$$

$$\mathbf{3.70.-} \int \sec^4 3x \operatorname{tg} 3x dx = \int \sec^3 3x (\sec 3x \operatorname{tg} 3x) dx$$

$$\text{Sea: } u = \sec 3x, du = 3 \sec 3x \operatorname{tg} 3x dx$$

$$\text{Luego: } \frac{1}{3} \int u^3 du = \frac{1}{3} \frac{u^4}{4} + c = \frac{u^4}{12} + c = \frac{\sec^4 3x}{12} + c$$

$$\mathbf{3.71.-} \int \sec^n x \operatorname{tg} x dx = \int \sec^{n-1} x (\sec x \operatorname{tg} x) dx$$

$$\text{Sea: } u = \sec x, du = \sec x \operatorname{tg} x dx, \quad \text{Luego:}$$

$$\int u^{n-1} du = \frac{u^n}{n} + c = \frac{\sec^n x}{n} + c, (n \neq 0)$$

$$\mathbf{3.72.-} \int \frac{\cos^3 x}{\operatorname{sen}^2 x} dx = \int \frac{\cos^2 x \cos x}{\operatorname{sen}^2 x} dx = \int \frac{(1 - \operatorname{sen}^2 x) \cos x}{\operatorname{sen}^2 x} dx = \int \frac{\cos x dx}{\operatorname{sen}^2 x} - \int \cos x dx$$

$$= -\frac{1}{\operatorname{sen} x} - \operatorname{sen} x + c$$

$$\mathbf{3.73.-} \int \frac{dx}{\operatorname{sen}^4 x} = \int \frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^4 x} dx = \int \frac{dx}{\operatorname{sen}^2 x} + \int \frac{\cos^2 x}{\operatorname{sen}^4 x} dx$$

$$= \int \operatorname{cosec}^2 x dx + \int \frac{\cos^2 x}{\operatorname{sen}^2 x \operatorname{sen}^2 x} dx = \int \operatorname{cosec}^2 x dx + \int \cot g^2 x \operatorname{cosec}^2 x dx$$

$$= -\cot g x - \frac{1}{3} \cot g^3 x + c$$

$$\mathbf{3.74.-} \int \operatorname{tg}^n x \sec^2 x dx; (n \neq -1)$$

$$\text{Sea: } u = \operatorname{tg} x, du = \sec^2 x dx$$

$$\text{Luego: } \int u^n du = \frac{u^{n+1}}{n+1} + c = \frac{\operatorname{tg}^{n+1} x}{n+1} + c, (n \neq -1)$$

$$\mathbf{3.75.-} \int \operatorname{sen}^6 x dx = \int (\operatorname{sen}^2 x)^3 dx = \int \left( \frac{1 - \cos 2x}{2} \right)^3 dx$$

$$= \frac{1}{8} \int (1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x) dx$$

$$= \frac{1}{8} \left[ \int dx - 3 \int \cos 2x dx + 3 \int \cos^2 2x dx - \int \cos^3 2x dx \right]$$



$$= \frac{5x}{16} - \frac{\operatorname{sen} 2x}{4} + \frac{3\operatorname{sen} 4x}{64} + \frac{\operatorname{sen}^3 2x}{48} + c$$

$$\mathbf{3.76.-} \int \operatorname{sen}^4 ax dx = \int (\operatorname{sen}^2 ax)^2 dx = \frac{1}{4} \int (1 - \cos 2ax)^2 dx$$

$$= \int (1 - 2\cos 2ax + \cos^2 2ax) dx = \frac{1}{4} \int dx - \frac{1}{2} \int \cos 2ax dx + \frac{1}{4} \int \cos^2 2ax dx$$

$$= \frac{1}{4} x - \frac{1}{4a} \operatorname{sen} 2ax + \frac{1}{4} \left( \frac{1}{2} x + \frac{1}{8a} \operatorname{sen} 4ax \right) + c = \frac{3}{8} x - \frac{1}{4a} \operatorname{sen} 2ax + \frac{1}{32a} \operatorname{sen} 4ax + c$$

$$\mathbf{3.77.-} \int \operatorname{sen}^n x \cos x dx = \frac{\operatorname{sen}^{n+1} x}{n+1} + c, (n \neq -1)$$

$$\mathbf{3.78.-} \int \operatorname{co} \tau g^n ax dx = \int \operatorname{co} \tau g^{n-2} ax \operatorname{co} \tau g^2 ax dx = \int \operatorname{co} \tau g^{n-2} ax (\operatorname{cosec}^2 ax - 1) dx$$

$$= \int \operatorname{co} \tau g^{n-2} ax \operatorname{cosec}^2 ax dx - \int \operatorname{co} \tau g^{n-2} ax dx = -\frac{1}{a} \frac{\operatorname{co} \tau g^{n-1} ax}{n-1} - \int \operatorname{co} \tau g^{n-2} ax dx$$

$$\mathbf{3.79.-} \int \operatorname{co} \tau g^4 3x dx, \text{ Haciendo uso del ejercicio anterior:}$$

$$= -\frac{\operatorname{co} \tau g^3 3x}{3 \times 3} - \int \operatorname{co} \tau g^2 3x dx = -\frac{\operatorname{co} \tau g^3 3x}{9} - \int (\operatorname{cosec}^2 3x - 1) dx$$

$$= -\frac{\operatorname{co} \tau g^3 3x}{9} - \int \operatorname{cosec}^2 3x dx + \int dx = -\frac{\operatorname{co} \tau g^3 3x}{9} - \int \operatorname{cosec}^2 3x dx + \int dx$$

$$= -\frac{\operatorname{co} \tau g^3 3x}{9} + \frac{\operatorname{co} \tau g 3x}{3} + x + c$$

$$\mathbf{3.80.-} \int \cos x^n \operatorname{sen} x dx = -\frac{\cos^{n+1} x}{n+1} + c; (n \neq -1)$$

$$\mathbf{3.81.-} \int \tau g^n x dx = \int \tau g^{n-2} x \tau g^2 x dx = \int \tau g^{n-2} x (\sec^2 x - 1) dx$$

$$= \int \tau g^{n-2} x \sec^2 x dx - \int \tau g^{n-2} x dx = \frac{\tau g^{n-1} x}{n-1} - \int \tau g^{n-2} x dx$$

$$\mathbf{3.82.-} \int \tau g^4 x dx = \frac{\tau g^3 x dx}{3} - \int \tau g^2 x dx = \frac{\tau g^3 x}{3} - \int (\sec^2 x - 1) dx$$

$$= \frac{\tau g^3 x}{3} - \int \sec^2 x dx - \int dx = \frac{\tau g^3 x}{3} - \tau g x + x + c$$

$$\mathbf{3.83.-} \int \cos^{2n+1} x dx = \int \cos^{2n} x \cos x dx = \int (\cos^2 x)^n \cos x dx = \int (1 - \operatorname{sen}^2 x)^n \cos x dx$$

Sea:  $u = \operatorname{sen} x, du = \cos x dx$ . El resultado se obtiene, evaluando  $(1 - u^2)^n$  por la fórmula del binomio de Newton y calculando cada sumando, cuyas integrales son del tipo:  $\int u^n du$ .

Las fórmulas provenientes de los ejercicios 3.78 y 3.81, se denominan **fórmulas de reducción** y su utilidad es obvia. Más adelante, en otros capítulos, usted deducirá nuevas fórmulas de reducción.

## CAPITULO 4

### INTEGRACION POR PARTES

Existe una variedad de integrales que se pueden desarrollar, usando la relación:  $\int u dv = uv - \int v du$ .

El problema es elegir  $u$  y  $dv$ , por lo cual es útil la siguiente identificación:

I: Función trigonométrica inversa.

L: Función logarítmica.

A: Función algebraica.

T: Función trigonométrica.

E: Función exponencial.

Se usa de la manera siguiente:

### EJERCICIOS DESARROLLADOS

**4.1.-Encontrar:**  $\int x \cos x dx$

Solución.- I L A T E

↓ ↓

$x \quad \cos x$

$$\begin{aligned} \therefore \quad u &= x & dv &= \cos x dx \\ du &= dx & v &= \sin x \end{aligned}$$

$$\therefore \int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + c$$

**Respuesta:**  $\int x \cos x dx = x \sin x + \cos x + c$

**4.2.-Encontrar:**  $\int x \sec^2 x dx$

Solución.- I L A T E

↓ ↓

$x \quad \sec^2 3x$

$$\begin{aligned} \therefore \quad u &= x & dv &= \sec^2 3x dx \\ du &= dx & v &= \frac{1}{3} \tan 3x \end{aligned}$$

$$\therefore \int x \sec^2 x dx = \frac{1}{3} x \tan 3x - \frac{1}{3} \int \tan 3x dx = \frac{x \tan 3x}{3} - \frac{1}{9} \ell \eta |\sec 3x| + c$$

**Respuesta:**  $\int x \sec^2 x dx = \frac{x \tan 3x}{3} - \frac{1}{9} \ell \eta |\sec 3x| + c$

**4.3.-Encontrar:**  $\int x^2 \sin x dx$

Solución.- I L A T E

↓ ↓

$x^2 \quad \sin x$

$$\begin{aligned}
\therefore \quad & u = x^2 & dv = \operatorname{sen} x dx \\
& du = 2x dx & v = -\cos x \\
\therefore \int x^2 \operatorname{sen} x dx &= -x^2 \cos x + 2 \int x \cos x dx, \text{ integrando por partes la segunda integral:} \\
\int x \cos x dx; & \quad u = x & dv = \cos x dx \\
& du = dx & v = \operatorname{sen} x \\
\therefore \int x^2 \operatorname{sen} x dx &= -x^2 \cos x + 2 \left[ x \operatorname{sen} x - \int \operatorname{sen} x dx \right] = -x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x + c
\end{aligned}$$

**Respuesta:**  $\int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x + c$

**4.4.-Encontrar:**  $\int (x^2 + 5x + 6) \cos 2x dx$

Solución.- I L A T E

$$\begin{aligned}
& \downarrow \quad \searrow \\
& x^2 + 5x + 6 \quad \cos 2x \\
\therefore \quad & u = x^2 + 5x + 6 & dv = \cos 2x dx \\
& du = (2x + 5) dx & v = \frac{1}{2} \operatorname{sen} 2x \\
\therefore \int (x^2 + 5x + 6) \cos 2x dx &= \frac{(x^2 + 5x + 6)}{2} \operatorname{sen} 2x - \frac{1}{2} \int (2x + 5) \operatorname{sen} 2x dx
\end{aligned}$$

Integrando por partes la segunda integral:

I L A T E

$$\begin{aligned}
& \swarrow \quad \searrow \\
& 2x + 5 \quad \operatorname{sen} 2x \\
\therefore \quad & u = 2x + 5 & dv = \operatorname{sen} 2x dx \\
& du = 2 dx & v = -\frac{1}{2} \cos 2x \\
\therefore \int (x^2 + 5x + 6) \cos 2x dx &= \frac{1}{2} \operatorname{sen} 2x (x^2 + 5x + 6) - \frac{1}{2} \left[ (2x + 5) \left(-\frac{1}{2} \cos 2x\right) + \int \cos 2x dx \right] \\
&= \frac{x^2 + 5x + 6}{2} \operatorname{sen} 2x + \frac{1}{4} \cos 2x (2x + 5) - \frac{1}{2} \int \cos 2x dx \\
&= \frac{x^2 + 5x + 6}{2} \operatorname{sen} 2x + \frac{2x + 5}{4} \cos 2x - \frac{1}{4} \operatorname{sen} 2x + c
\end{aligned}$$

**Respuesta:**  $\int (x^2 + 5x + 6) \cos 2x dx = \frac{x^2 + 5x + 6}{2} \operatorname{sen} 2x + \frac{2x + 5}{4} \cos 2x - \frac{1}{4} \operatorname{sen} 2x + c$

**Nota.-** Ya se habrá dado cuenta el lector, que la elección conveniente para el  $u$  y el  $dv$ , dependerá de la ubicación de los términos funcionales en la palabra ILATE. El de la izquierda corresponde al  $u$ , y el otro será el  $dv$ .

**4.5.-Encontrar:**  $\int \ell \eta x dx$

Solución.- I L A T E

$$\begin{aligned}
& \downarrow \quad \downarrow \\
& \ell \eta x \quad 1
\end{aligned}$$

$$\begin{aligned} u &= \ell \eta x & dv &= 1dx \\ \therefore du &= \frac{dx}{x} & v &= x \\ \therefore \int \ell \eta x dx &= x \ell \eta x - \int dx = x \ell \eta x - x + c = x(\ell \eta x - 1) + c \end{aligned}$$

**Respuesta:**  $\int \ell \eta x dx = x(\ell \eta x - 1) + c$

**4.6.-Encontrar:**  $\int \ell \eta(a^2 + x^2) dx$

Solución.- I L A T E

$$\begin{aligned} &\downarrow \searrow \\ &\ell \eta(a^2 + x^2) \quad 1 \\ u &= \ell \eta x & dv &= 1dx \\ \therefore du &= \frac{dx}{x} & v &= x \end{aligned}$$

$$\begin{aligned} \therefore \int \ell \eta(a^2 + x^2) dx &= x \ell \eta(a^2 + x^2) - \int \frac{2x^2 dx}{a^2 + x^2} = x \ell \eta(a^2 + x^2) - \int \left(2 - \frac{2a^2}{x^2 + a^2}\right) dx \\ &= x \ell \eta(a^2 + x^2) - 2 \int dx + 2a^2 \int \frac{dx}{x^2 + a^2} = x \ell \eta(a^2 + x^2) - 2x + \frac{2a^2}{a} \operatorname{arc} \tau g \frac{x}{a} + c \\ &= x \ell \eta(a^2 + x^2) - 2x + 2a \operatorname{arc} \tau g \frac{x}{a} + c \end{aligned}$$

**Respuesta:**  $\int \ell \eta(a^2 + x^2) dx = x \ell \eta(a^2 + x^2) - 2x + 2a \operatorname{arc} \tau g \frac{x}{a} + c$

**4.7.-Encontrar:**  $\int \ell \eta \left| x + \sqrt{x^2 - 1} \right| dx$

Solución.- I L A T E

$$\begin{aligned} &\downarrow \searrow \\ &\ell \eta \left| x + \sqrt{x^2 - 1} \right| \quad 1 & dv &= 1dx \\ & & v &= x \\ u &= \ell \eta \left| x + \sqrt{x^2 - 1} \right| \\ \therefore du &= \frac{1 + \frac{x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} dx \Rightarrow du = \frac{\frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} dx \Rightarrow du = \frac{dx}{\sqrt{x^2 - 1}} \end{aligned}$$

$$\therefore \int \ell \eta \left| x + \sqrt{x^2 - 1} \right| dx = x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - \int \frac{x dx}{\sqrt{x^2 - 1}}$$

Sea :  $w = x^2 - 1, dw = 2x dx$ .

$$\begin{aligned} \text{Luego: } x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - \frac{1}{2} \int (x^2 - 1)^{-\frac{1}{2}} 2x dx &= x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - \frac{1}{2} \int w^{-\frac{1}{2}} dw \\ &= x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - \frac{1}{2} \frac{w^{\frac{1}{2}}}{\frac{1}{2}} + c = x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - w^{\frac{1}{2}} + c = x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - \sqrt{x^2 - 1} + c \end{aligned}$$

**Respuesta:**  $\int \ell \eta \left| x + \sqrt{x^2 - 1} \right| dx = x \ell \eta \left| x + \sqrt{x^2 - 1} \right| - \sqrt{x^2 - 1} + c$

**4.8.-Encontrar:**  $\int \ell \eta^2 x dx$

Solución.- I L A T E

↓   ↓

$\ell \eta^2 x$    1

$$u = \ell \eta^2 x$$

$$dv = 1dx$$

∴

$$du = 2\ell \eta x \frac{1}{x} dx$$

$$v = x$$

$$\therefore \int \ell \eta^2 x dx = x \ell \eta^2 x - 2 \int \ell \eta x \frac{1}{x} dx = x \ell \eta^2 x - 2 \int \ell \eta dx$$

Por ejercicio 4.5, se tiene:  $\int \ell \eta dx = x(\ell \eta x - 1) + c$

$$\text{Luego: } \int \ell \eta^2 x dx = x \ell \eta^2 x - 2[x(\ell \eta x - 1) + c] = x \ell \eta^2 x - 2x(\ell \eta x - 1) + c$$

**Respuesta:**  $\int \ell \eta^2 x dx = x \ell \eta^2 x - 2x(\ell \eta x - 1) + c$

**4.9.-Encontrar:**  $\int \arctan x dx$

Solución.- I L A T E

↓   ↓

$\arctan x$    1

$$u = \arctan x$$

$$dv = 1dx$$

∴

$$du = \frac{dx}{1+x^2}$$

$$v = x$$

$$\therefore \int \arctan x dx = x \arctan x - \int \frac{x dx}{1+x^2}$$

Sea:  $w = 1+x^2, dw = 2x dx$

$$\text{Luego: } x \arctan x - \frac{1}{2} \int \frac{2x dx}{1+x^2} = x \arctan x - \frac{1}{2} \int \frac{dw}{w} = x \arctan x - \frac{1}{2} \ell \eta |w| + c$$

$$= x \arctan x - \frac{1}{2} \ell \eta |1+x^2| + c$$

**Respuesta:**  $\int \arctan x dx = x \arctan x - \frac{1}{2} \ell \eta |1+x^2| + c$

**4.10.-**  $\int x^2 \arctan x dx$

Solución.- I L A T E

↓   ↓

$\arctan x$     $x^2$

$$u = \arctan x$$

$$dv = x^2 dx$$

∴

$$du = \frac{dx}{1+x^2}$$

$$v = \frac{x^3}{3}$$

$$\therefore \int x^2 \arctan x dx = \frac{x^3}{3} \arctan x - \frac{1}{3} \int \frac{x^2 dx}{1+x^2} = \frac{x^3}{3} \arctan x - \frac{1}{3} \int \left(x - \frac{x}{x^2+1}\right) dx$$

$$= \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{3} \int x dx - \frac{1}{3} \int \frac{x}{x^2+1} dx$$

Por ejercicio 4.9, se tiene:  $\int \frac{x dx}{x^2+1} = \frac{1}{2} \ell \eta |x^2+1| + c$

$$\text{Luego: } \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{3} \int x dx + \frac{1}{6} \ell \eta |x^2+1| + c = \frac{x^3}{3} \operatorname{arctg} x - \frac{x^2}{6} + \frac{1}{6} \ell \eta |x^2+1| + c$$

$$\text{Respuesta: } \int x^2 \operatorname{arctg} x dx = \frac{x^3}{3} \operatorname{arctg} x - \frac{x^2}{6} + \frac{1}{6} \ell \eta |x^2+1| + c$$

**4.11.-Encontrar:**  $\int \arccos 2x dx$

Solución.- I L A T E

$\downarrow \quad \downarrow$   
 $\arccos 2x \quad 1$

$$u = \arccos 2x$$

$$\therefore \quad \begin{array}{ll} dv = 1 dx \\ v = x \end{array} \quad du = -\frac{2 dx}{\sqrt{1-4x^2}}$$

$$\therefore \int \arccos 2x dx = x \arccos 2x + 2 \int \frac{x dx}{\sqrt{1-4x^2}}$$

$$\text{Sea: } w = 1 - 4x^2, dw = -8x dx$$

$$\text{Luego: } x \arccos 2x - \frac{2}{8} \int \frac{-8x dx}{\sqrt{1-4x^2}} = x \arccos 2x - \frac{1}{4} \int w^{-1/2} dw = x \arccos 2x - \frac{1}{4} \frac{w^{1/2}}{1/2} + c$$

$$= x \arccos 2x - \frac{1}{2} \sqrt{1-4x^2} + c$$

$$\text{Respuesta: } \int \arccos 2x dx = x \arccos 2x - \frac{1}{2} \sqrt{1-4x^2} + c$$

**4.12.-Encontrar:**  $\int \frac{\arcsen \sqrt{x}}{\sqrt{x}} dx$

Solución.- I L A T E

$\downarrow \quad \searrow$   
 $\arcsen \sqrt{x} \quad 1$

$$u = \arcsen \sqrt{x}$$

$$\therefore \quad \begin{array}{ll} dv = x^{-1/2} dx \\ v = 2\sqrt{x} \end{array} \quad du = \frac{1}{\sqrt{1-x}} \frac{dx}{\sqrt{x}}$$

$$\therefore \int \arcsen \sqrt{x} x^{-1/2} dx = 2\sqrt{x} \arcsen \sqrt{x} - \int \frac{dx}{\sqrt{1-x}}$$

$$\text{Sea: } w = 1 - x, dw = -dx$$

$$\text{Luego: } 2\sqrt{x} \arcsen \sqrt{x} + \int \frac{-dx}{\sqrt{1-x}} = 2\sqrt{x} \arcsen \sqrt{x} + \int w^{-1/2} dw$$

$$= 2\sqrt{x} \arcsen \sqrt{x} + 2w^{1/2} + c = 2\sqrt{x} \arcsen \sqrt{x} + 2\sqrt{1-x} + c$$

**Respuesta:**  $\int \frac{\arcsen \sqrt{x}}{\sqrt{x}} dx = 2\sqrt{x} \arcsen \sqrt{x} + 2\sqrt{1-x} + c$

**4.13.-Encontrar:**  $\int x \arcsen 2x^2 dx$

Solución.- I L A T E

$$\begin{array}{ccc} & \downarrow & \searrow \\ & \arcsen 2x^2 & x \\ u = \arcsen 2x^2 & & dv = x dx \\ \therefore du = \frac{4x dx}{\sqrt{1-4x^4}} & & v = \frac{x^2}{2} \end{array}$$

$$\therefore \int x \arcsen 2x^2 dx = \frac{x^2}{2} \arcsen 2x^2 - 2 \int \frac{x^3 dx}{\sqrt{1-4x^4}}$$

Sea:  $w = 1 - 4x^4, dw = -16x^3 dx$

$$\begin{aligned} \text{Luego: } \frac{x^2}{2} \arcsen 2x^2 + \frac{2}{16} \int \frac{(-16x^3 dx)}{\sqrt{1-4x^4}} &= \frac{x^2}{2} \arcsen 2x^2 + \frac{1}{8} \int w^{-1/2} dw \\ &= \frac{x^2}{2} \arcsen 2x^2 + \frac{1}{8} \frac{w^{1/2}}{1/2} + c = \frac{x^2}{2} \arcsen 2x^2 + \frac{1}{4} w^{1/2} + c \\ &= \frac{x^2}{2} \arcsen 2x^2 + \frac{1}{4} \sqrt{1-4x^4} + c \end{aligned}$$

**Respuesta:**  $\int x \arcsen 2x^2 dx = \frac{x^2}{2} \arcsen 2x^2 + \frac{1}{4} \sqrt{1-4x^4} + c$

**4.14.-Encontrar:**  $\int x e^{x/a} dx$

Sea:  $w = \frac{x}{a}, dw = \frac{dx}{a}$

Luego:  $\int x e^{x/a} dx = a^2 \int \frac{x}{a} e^{x/a} \frac{dx}{a} = a^2 \int w e^w dw$ , integrando por partes se tiene:

Solución.- I L A T E

$$\begin{array}{ccc} & \downarrow & \downarrow \\ & w & e^w \\ \therefore u = w & & dv = e^w dw \\ du = dw & & v = e^w \end{array}$$

$$\begin{aligned} \therefore a^2 \int w e^w dw &= a^2 \left( w e^w - \int e^w dw \right) = a^2 \left( w e^w - e^w + c \right) = a^2 \left( w e^w - e^w \right) + c \\ &= a^2 \left( \frac{x}{a} e^{x/a} - e^{x/a} \right) + c = a^2 e^{x/a} \left( \frac{x}{a} - 1 \right) + c \end{aligned}$$

**Respuesta:**  $\int x e^{x/a} dx = a^2 e^{x/a} \left( \frac{x}{a} - 1 \right) + c$

**4.15.-Encontrar:**  $\int x^2 e^{-3x} dx$

Solución.- I L A T E

$$\begin{array}{ccc}
 & \downarrow & \downarrow \\
 & x^2 & e^{-3x} \\
 \therefore & u = x^2 & dv = e^{-3x} dx \\
 & du = 2x dx & v = -\frac{1}{3} e^{-3x}
 \end{array}$$

$\therefore \int x^2 e^{-3x} dx = -\frac{1}{3} x^2 e^{-3x} + \frac{2}{3} \int x e^{-3x} dx$ , integrando por partes la segunda integral:

I L A T E

$$\begin{array}{ccc}
 & \downarrow & \downarrow \\
 & x & e^{-3x} \\
 \therefore & u = x & dv = e^{-3x} dx \\
 & du = dx & v = -\frac{1}{3} e^{-3x}
 \end{array}$$

$$\begin{aligned}
 \therefore \int x^2 e^{-3x} dx &= -\frac{1}{3} x^2 e^{-3x} + \frac{2}{3} \left( -\frac{1}{3} x e^{-3x} + \frac{1}{3} \int e^{-3x} dx \right) = -\frac{x^2 e^{-3x}}{3} - \frac{2}{9} x e^{-3x} + \frac{2}{9} \int e^{-3x} dx \\
 &= -\frac{x^2 e^{-3x}}{3} - \frac{2}{9} x e^{-3x} - \frac{2}{27} e^{-3x} + c
 \end{aligned}$$

**Respuesta:**  $\int x^2 e^{-3x} dx = -\frac{e^{-3x}}{3} \left( x^2 + \frac{2}{3} x + \frac{2}{9} \right) + c$

**4.16.-Encontrar:**  $\int x^3 e^{-x^2} dx$

Solución.-  $\int x^3 e^{-x^2} dx = \int x^2 e^{-x^2} x dx$

Sea:  $w = -x^2$ ,  $dw = -2x dx$ , además:  $x^2 = -w$

Luego:  $\int x^2 e^{-x^2} x dx = -\frac{1}{2} \int x^2 e^{-x^2} x (-2x dx) = -\frac{1}{2} \int -w e^w dw = \frac{1}{2} \int w e^w dw$ , integrando por

Partes se tiene:

I L A T E

$$\begin{array}{ccc}
 & \downarrow & \downarrow \\
 & w & e^w \\
 \therefore & u = w & dv = e^w dw \\
 & du = dw & v = e^w
 \end{array}$$

$$\begin{aligned}
 \therefore \frac{1}{2} \int w e^w dw &= \frac{1}{2} \left( w e^w - \int e^w dw \right) = \frac{1}{2} w e^w - \frac{1}{2} \int e^w dw = \frac{1}{2} w e^w - \frac{1}{2} e^w + c \\
 &= -\frac{1}{2} x^2 e^{-x^2} - \frac{1}{2} e^{-x^2} + c = -\frac{1}{2} e^{-x^2} (x^2 + 1) + c
 \end{aligned}$$

**Respuesta:**  $\int x^3 e^{-x^2} dx = -\frac{1}{2} e^{-x^2} (x^2 + 1) + c$

**4.17.-Encontrar:**  $\int (x^2 - 2x + 5) e^{-x} dx$

Solución.- I L A T E

$\downarrow \quad \downarrow$



$$\begin{array}{l} x^2 - 2x + 5 \quad e^{-x} \\ \therefore u = x^2 - 2x + 5 \quad dv = e^{-x} dx \\ du = (2x - 2)dx \quad v = -e^{-x} \\ \therefore \int (x^2 - 2x + 5)e^{-x} dx = -e^{-x}(x^2 - 2x + 5) + \int (2x - 2)e^{-x} dx, \text{ integrando por partes la} \\ \text{segunda integral:} \end{array}$$

I L A T E

$$\begin{array}{l} \downarrow \quad \downarrow \\ 2x - 2 \quad e^{-x} \end{array}$$

$$\begin{array}{l} \therefore u = 2x - 2 \quad dv = e^{-x} dx \\ du = 2dx \quad v = -e^{-x} \\ \therefore \int (x^2 - 2x + 5)e^{-x} dx = -e^{-x}(x^2 - 2x + 5) + \left[ -e^{-x}(2x - 2) + 2 \int e^{-x} dx \right] \\ = -e^{-x}(x^2 - 2x + 5) - e^{-x}(2x - 2) + 2 \int e^{-x} dx = -e^{-x}(x^2 - 2x + 5) - e^{-x}(2x - 2) - 2e^{-x} + c \\ = -e^{-x}(x^2 - 2x + 5 + 2x - 2 + 2) + c = -e^{-x}(x^2 + 5) + c \end{array}$$

**Respuesta:**  $\int (x^2 - 2x + 5)e^{-x} dx = -e^{-x}(x^2 + 5) + c$

**4.18.-Encontrar:**  $\int e^{ax} \cos bx dx$

Solución.- I L A T E

$$\begin{array}{l} \swarrow \quad \downarrow \\ \cos bx \quad e^{ax} \end{array}$$

$$\begin{array}{l} u = \cos bx \quad dv = e^{ax} dx \\ \therefore du = -b \operatorname{sen} bx dx \quad v = \frac{1}{a} e^{ax} \\ \therefore \int e^{ax} \cos bx dx = \frac{e^{ax} \cos bx}{a} + \frac{b}{a} \int e^{ax} \operatorname{sen} bx dx, \text{ Nótese que la segunda integral es} \\ \text{semejante a la primera, salvo en la parte trigonométrica; integrando por partes la} \\ \text{segunda integral:} \end{array}$$

I L A T E

$$\begin{array}{l} \swarrow \quad \downarrow \\ \operatorname{sen} bx \quad e^{ax} \end{array}$$

$$\begin{array}{l} u = \operatorname{sen} bx \quad dv = e^{ax} dx \\ \therefore du = b \cos bx dx \quad v = \frac{1}{a} e^{ax} \\ \therefore = \frac{e^{ax} \cos bx}{a} + \frac{b}{a} \left( \frac{e^{ax} \operatorname{sen} bx}{a} - \frac{b}{a} \int e^{ax} \cos bx dx \right) \\ = \frac{e^{ax} \cos bx}{a} + \frac{be^{ax} \operatorname{sen} bx}{a^2} - \frac{b^2}{a^2} \int e^{ax} \cos bx dx, \text{ Nótese que:} \\ \int e^{ax} \cos bx dx = \frac{e^{ax} \cos bx}{a} + \frac{be^{ax} \operatorname{sen} bx}{a^2} - \frac{b^2}{a^2} \int e^{ax} \cos bx dx, \text{ la integral a encontrar} \\ \text{aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente:} \end{array}$$

$-\frac{b^2}{a^2}$ . Transponiendo éste término al primer miembro y dividiendo por el nuevo

coeficiente:  $1 + \frac{b^2}{a^2} = \frac{a^2 + b^2}{a^2}$ , se tiene:

$$\left(\frac{a^2 + b^2}{a^2}\right) \int e^{ax} \cos bx dx = \frac{ae^{ax} \cos bx + be^{ax} \operatorname{sen} bx}{a^2} + c$$

$$\int e^{ax} \cos bx dx = \frac{\cancel{a}e^{ax} \cos bx + be^{ax} \operatorname{sen} bx}{\left(\frac{a^2 + b^2}{\cancel{a}^2}\right)} + c = \frac{e^{ax}(a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2} + c$$

**Respuesta:**  $\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \operatorname{sen} bx)}{a^2 + b^2} + c$

**4.19.-Encontrar:**  $\int e^x \cos 2x dx$

Solución.- Este ejercicio es un caso particular del ejercicio anterior, donde:  $a=1$  y  $b=2$ . Invitamos al lector, resolverlo por partes, aún cuando la respuesta es inmediata.

**Respuesta:**  $\int e^x \cos 2x dx = \frac{e^x(\cos 2x + 2 \operatorname{sen} 2x)}{5} + c$

**4.20.-Encontrar:**  $\int e^{ax} \operatorname{sen} bx dx$

Solución.- I L A T E

$$\begin{array}{cc} \swarrow & \downarrow \\ \operatorname{sen} bx & e^{ax} \end{array}$$

$$\begin{array}{ll} \therefore u = \operatorname{sen} bx & dv = e^{ax} dx \\ du = b \cos bx dx & v = \frac{1}{a} e^{ax} \end{array}$$

$$\therefore \int e^{ax} \operatorname{sen} bx dx = \frac{e^{ax} \operatorname{sen} bx}{a} - \frac{b}{a} \int e^{ax} \cos bx dx, \text{ integrando por partes la segunda integral:}$$

I L A T E

$$\begin{array}{cc} \swarrow & \downarrow \\ \cos bx & e^{ax} \end{array}$$

$$\begin{array}{ll} \therefore u = \cos bx & dv = e^{ax} dx \\ du = -b \operatorname{sen} bx dx & v = \frac{1}{a} e^{ax} \end{array}$$

$$\begin{aligned} \therefore \int e^{ax} \operatorname{sen} bx dx &= \frac{e^{ax} \operatorname{sen} bx}{a} - \frac{b}{a} \left( \frac{e^{ax} \cos bx}{a} + \frac{b}{a} \int e^{ax} \operatorname{sen} bx dx \right) \\ &= \frac{e^{ax} \operatorname{sen} bx}{a} - \frac{be^{ax} \cos bx}{a^2} - \frac{b^2}{a^2} \int e^{ax} \operatorname{sen} bx dx, \end{aligned}$$

Como habrá notado el lector, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente:  $-\frac{b^2}{a^2}$ . Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente:  $1 + \frac{b^2}{a^2} = \frac{a^2 + b^2}{a^2}$ , se tiene:

$$\left(\frac{a^2 + b^2}{a^2}\right) \int e^{ax} \operatorname{sen} bx dx = \frac{ae^{ax} \operatorname{sen} bx - be^{ax} \cos bx}{a^2} + c$$

$$\int e^{ax} \operatorname{sen} bx dx = \frac{ae^{ax} \operatorname{sen} bx - be^{ax} \cos bx}{\cancel{a^2} \left(\frac{a^2 + b^2}{\cancel{a^2}}\right)} + c = \int e^{ax} \operatorname{sen} bx dx = \frac{e^{ax}(a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2} + c$$

**Respuesta:**  $\int e^{ax} \operatorname{sen} bx dx = \frac{e^{ax}(a \operatorname{sen} bx - b \cos bx)}{a^2 + b^2} + c$

#### 4.21.-Encontrar: $\int x\sqrt{1+x} dx$

Solución.- Cuando el integrando, está formado por el producto de funciones algebraicas, es necesario tomar como  $dv$ , la parte más fácil integrable y  $u$  como la parte más fácil derivable. Sin embargo, la opción de “más fácil” quedará a criterio del lector.

$$\begin{aligned} \therefore \quad u &= x & dv &= (1+x)^{1/2} dx \\ du &= dx & v &= \frac{2}{3}(1+x)^{3/2} \end{aligned}$$

$$\therefore \int x\sqrt{1+x} dx = \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3} \int (1+x)^{3/2} dx = \frac{2}{3}x(1+x)^{3/2} - \frac{2}{3} \frac{(1+x)^{5/2}}{5/2} + c$$

$$= \frac{2}{3}x(1+x)^{3/2} - \frac{4(1+x)^{5/2}}{15} + c$$

**Respuesta:**  $\int x\sqrt{1+x} dx = \frac{2}{3}x(1+x)^{3/2} - \frac{4(1+x)^{5/2}}{15} + c$

#### 4.22.-Encontrar: $\int \frac{x^2 dx}{\sqrt{1+x}}$

Solución.-  $\int \frac{x^2 dx}{\sqrt{1+x}} = \int x^2(1+x)^{-1/2} dx$

$$\begin{aligned} \therefore \quad u &= x^2 & dv &= (1+x)^{-1/2} dx \\ du &= 2x dx & v &= 2(1+x)^{1/2} \end{aligned}$$

$$\therefore \int \frac{x^2 dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - 4 \int x\sqrt{1+x} dx, \text{ integrando por partes la segunda integral:}$$

$$\begin{aligned} \therefore \quad u &= x & dv &= (1+x)^{\frac{1}{2}} dx \\ du &= dx & v &= \frac{2}{3}(1+x)^{\frac{3}{2}} \end{aligned}$$

$$\begin{aligned} \int \frac{x^2 dx}{\sqrt{1+x}} &= 2x^2 \sqrt{1+x} - 4 \left[ \frac{2}{3} x(1+x)^{\frac{3}{2}} - \frac{2}{3} \int (1+x)^{\frac{3}{2}} dx \right] \\ &= 2x^2 \sqrt{1+x} - \frac{8}{3} x(1+x)^{\frac{3}{2}} + \frac{8}{3} \frac{(1+x)^{\frac{5}{2}}}{\frac{5}{2}} + c = 2x^2 \sqrt{1+x} - \frac{8}{3} x(1+x)^{\frac{3}{2}} + \frac{16}{15} (1+x)^{\frac{5}{2}} + c \end{aligned}$$

**Respuesta:**  $\int \frac{x^2 dx}{\sqrt{1+x}} = 2x^2 \sqrt{1+x} - \frac{8}{3} x(1+x)^{\frac{3}{2}} + \frac{16}{15} (1+x)^{\frac{5}{2}} + c$

**4.23.-Encontrar:**  $\int \frac{x dx}{e^x}$

Solución.-  $\int \frac{x dx}{e^x} = \int x e^{-x} dx$

$$\begin{array}{cc} \text{I} & \text{L} & \text{A} & \text{T} & \text{E} \\ & \downarrow & & \downarrow & \\ & x & & e^{-x} & \end{array}$$

$$\begin{aligned} \therefore \quad u &= x & dv &= e^{-x} dx \\ du &= dx & v &= -e^{-x} \end{aligned}$$

$$\therefore \int x e^{-x} dx = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x} + c = e^{-x}(-x-1) + c = -e^{-x}(x+1) + c$$

**Respuesta:**  $\int \frac{x dx}{e^x} = -e^{-x}(x+1) + c$

**4.24.-Encontrar:**  $\int x^2 \ell \eta \sqrt{1-x} dx$

$$u = \ell \eta \sqrt{1-x}$$

$$dv = x^2 dx$$

Solución.-  $\therefore \quad du = \frac{1}{\sqrt{1-x}} \frac{1}{2} (1-x)^{-\frac{1}{2}} (-1) dx \Rightarrow du = \frac{-dx}{2(1-x)} \quad v = \frac{x^3}{3}$

$$\begin{aligned} \therefore \int x^2 \ell \eta \sqrt{1-x} dx &= \frac{x^3}{3} \ell \eta \sqrt{1-x} + \frac{1}{6} \int \frac{x^3}{1-x} dx = \frac{x^3}{3} \ell \eta \sqrt{1-x} - \frac{1}{6} \int \left( x^2 + x + 1 - \frac{1}{1-x} \right) dx \\ &= \frac{x^3}{3} \ell \eta \sqrt{1-x} - \frac{1}{6} \frac{x^3}{3} - \frac{1}{6} \frac{x^2}{2} - \frac{1}{6} x - \frac{1}{6} \ell \eta |1-x| + c \\ &= \frac{x^3}{3} \ell \eta \sqrt{1-x} - \frac{1}{6} \ell \eta |1-x| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c \end{aligned}$$

**Respuesta:**  $\int x^2 \ell \eta \sqrt{1-x} dx = \frac{x^3}{3} \ell \eta \sqrt{1-x} - \frac{1}{6} \ell \eta |1-x| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$

**4.25.-Encontrar:**  $\int x \operatorname{sen}^2 x dx$

Solución.-

$$\begin{aligned} \therefore \quad u &= x & dv &= \operatorname{sen}^2 x dx \\ du &= dx & v &= \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x \quad \left( v = \int \frac{1 - \cos 2x}{2} dx \right) \end{aligned}$$

$$\begin{aligned} \therefore \int x \operatorname{sen}^2 x dx &= \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{2} \int x dx + \frac{1}{4} \int \operatorname{sen} 2x dx \\ &= \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{4}x^2 - \frac{1}{8}\cos 2x + c = \frac{1}{4}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{8}\cos 2x + c \end{aligned}$$

$$\text{Respuesta: } \int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$$

**Otra solución.-**

$$\begin{aligned} \int x \operatorname{sen}^2 x dx &= \int x \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \int x dx - \frac{1}{2} \int x \cos 2x dx = \frac{1}{2} \frac{x^2}{2} - \frac{1}{2} \int x \cos 2x dx \\ &= \frac{x^2}{4} - \frac{1}{2} \int x \cos 2x dx; \text{ integrando por partes, la segunda integral:} \end{aligned}$$

$$\begin{aligned} \therefore \quad u &= x & dv &= \cos 2x dx \\ du &= dx & v &= \frac{1}{2}\operatorname{sen} 2x \end{aligned}$$

$$\begin{aligned} \int x \operatorname{sen}^2 x dx &= \frac{x^2}{4} - \frac{1}{2} \left( \frac{x}{2} \operatorname{sen} 2x - \frac{1}{2} \int \operatorname{sen} 2x dx \right) = \frac{x^2}{4} - \frac{x}{4} \operatorname{sen} 2x + \frac{1}{4} \int \operatorname{sen} 2x dx \\ &= \frac{x^2}{4} - \frac{x}{4} \operatorname{sen} 2x + \frac{1}{4} \left( -\frac{1}{2} \cos 2x \right) + c = \frac{x^2}{4} - \frac{x}{4} \operatorname{sen} 2x - \frac{\cos 2x}{8} + c \end{aligned}$$

$$\text{Respuesta: } \int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$$

$$\text{4.26.-Encontrar: } \int x(3x+1)^7 dx$$

**Solución.-**

$$\begin{aligned} \therefore \quad u &= x & dv &= (3x+1)^7 dx \\ du &= dx & v &= \frac{1}{24}(3x+1)^8 \quad \left( v = \int (3x+1)^7 dx \right) \end{aligned}$$

$$\begin{aligned} \therefore \int x(3x+1)^7 dx &= \frac{x}{24}(3x+1)^8 - \frac{1}{24} \int (3x+1)^8 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24} \frac{1}{3} \frac{(3x+1)^9}{9} + c \\ &= \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c \end{aligned}$$

$$\text{Respuesta: } \int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$$

## **EJERCICIOS PROPUESTOS**

Usando esencialmente el mecanismo presentado, encontrar las integrales siguientes:

|                                                                    |                                                             |                                                             |
|--------------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
| <b>4.27.-</b> $\int x(2x+5)^{10} dx$                               | <b>4.28.-</b> $\int \arcsen x dx$                           | <b>4.29.-</b> $\int x \sen x dx$                            |
| <b>4.30.-</b> $\int x \cos 3x dx$                                  | <b>4.31.-</b> $\int x 2^{-x} dx$                            | <b>4.32.-</b> $\int x^2 e^{3x} dx$                          |
| <b>4.33.-</b> $\int x^3 e^{-x/3} dx$                               | <b>4.34.-</b> $\int x \sen x \cos x dx$                     | <b>4.35.-</b> $\int x^2 \ell \eta x dx$                     |
| <b>4.36.-</b> $\int \frac{\ell \eta x}{x^3} dx$                    | <b>4.37.-</b> $\int \frac{\ell \eta x}{\sqrt{x}} dx$        | <b>4.38.-</b> $\int x \arc \tau g x dx$                     |
| <b>4.39.-</b> $\int x \arcsen x dx$                                | <b>4.40.-</b> $\int \frac{x dx}{\sen^2 x}$                  | <b>4.41.-</b> $\int e^x \sen x dx$                          |
| <b>4.42.-</b> $\int 3^x \cos x dx$                                 | <b>4.43.-</b> $\int \sen(\ell \eta x) dx$                   | <b>4.44.-</b> $\int (x^2 - 2x + 3) \ell \eta x dx$          |
| <b>4.45.-</b> $\int x \ell \eta \left  \frac{1-x}{1+x} \right  dx$ | <b>4.46.-</b> $\int \frac{\ell \eta^2 x}{x^2} dx$           | <b>4.47.-</b> $\int x^2 \arc \tau g 3x dx$                  |
| <b>4.48.-</b> $\int x(\arc \tau g x)^2 dx$                         | <b>4.49.-</b> $\int (\arcsen x)^2 dx$                       | <b>4.50.-</b> $\int \frac{\arcsen x}{x^2} dx$               |
| <b>4.51.-</b> $\int \frac{\arcsen \sqrt{x}}{\sqrt{1-x}} dx$        | <b>4.52.-</b> $\int \frac{\sen^2 x}{e^x} dx$                | <b>4.53.-</b> $\int \tau g^2 x \sec^3 x dx$                 |
| <b>4.54.-</b> $\int x^3 \ell \eta^2 x dx$                          | <b>4.55.-</b> $\int x \ell \eta (9 + x^2) dx$               | <b>4.56.-</b> $\int \arcsen \sqrt{x} dx$                    |
| <b>4.57.-</b> $\int x \arc \tau g (2x + 3) dx$                     | <b>4.58.-</b> $\int e^{\sqrt{x}} dx$                        | <b>4.59.-</b> $\int \cos^2(\ell \eta x) dx$                 |
| <b>4.60.-</b> $\int \frac{\ell \eta (\ell \eta x)}{x} dx$          | <b>4.61.-</b> $\int \ell \eta  x+1  dx$                     | <b>4.62.-</b> $\int x^2 e^x dx$                             |
| <b>4.63.-</b> $\int \cos^n x dx$                                   | <b>4.64.-</b> $\int \sen^n x dx$                            | <b>4.65.-</b> $\int x^m (\ell \eta x)^n dx$                 |
| <b>4.66.-</b> $\int x^3 (\ell \eta x)^2 dx$                        | <b>4.67.-</b> $\int x^n e^x dx$                             | <b>4.68.-</b> $\int x^3 e^x dx$                             |
| <b>4.69.-</b> $\int \sec^n x dx$                                   | <b>4.70.-</b> $\int \sec^3 x dx$                            | <b>4.71.-</b> $\int x \ell \eta x dx$                       |
| <b>4.72.-</b> $\int x^n \ell \eta  ax  dx, n \neq -1$              | <b>4.73.-</b> $\int \arcsen ax dx$                          | <b>4.74.-</b> $\int x \sen ax dx$                           |
| <b>4.75.-</b> $\int x^2 \cos ax dx$                                | <b>4.76.-</b> $\int x \sec^2 ax dx$                         | <b>4.77.-</b> $\int \cos(\ell \eta x) dx$                   |
| <b>4.78.-</b> $\int \ell \eta (9 + x^2) dx$                        | <b>4.79.-</b> $\int x \cos(2x + 1) dx$                      | <b>4.80.-</b> $\int x \arc \sec x dx$                       |
| <b>4.81.-</b> $\int \arc \sec \sqrt{x} dx$                         | <b>4.82.-</b> $\int \sqrt{a^2 - x^2} dx$                    | <b>4.83.-</b> $\int \ell \eta  1-x  dx$                     |
| <b>4.84.-</b> $\int \ell \eta (x^2 + 1) dx$                        | <b>4.85.-</b> $\int \arc \tau g \sqrt{x} dx$                | <b>4.86.-</b> $\int \frac{x \arcsen x}{\sqrt{1-x^2}} dx$    |
| <b>4.87.-</b> $\int x \arc \tau g \sqrt{x^2 - 1} dx$               | <b>4.88.-</b> $\int \frac{x \arc \tau g x}{(x^2 + 1)^2} dx$ | <b>4.89.-</b> $\int \arcsen x \frac{xdx}{\sqrt{(1-x^2)^3}}$ |
| <b>4.90.-</b> $\int x^2 \sqrt{1-x} dx$                             |                                                             |                                                             |

## RESPUESTAS

**4.27.-**  $\int x(2x+5)^{10} dx$

Solución.-

$$\begin{aligned}
& \therefore \quad u = x & dv = (2x+5)^{10} dx \\
& \quad du = dx & v = \frac{(2x+5)^{11}}{22} \\
& \int x(2x+5)^{10} dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{22} \int (2x+5)^{11} dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{44}(2x+5)^{12} + c \\
& = \frac{x}{22}(2x+5)^{11} - \frac{1}{528}(2x+5)^{12} + c
\end{aligned}$$

**4.28.-**  $\int \arcsen x dx$

Solución.-

$$\begin{aligned}
& u = \arcsen x & dv = dx \\
& \therefore \quad du = \frac{dx}{\sqrt{1-x^2}} & v = x \quad \text{Además: } w = 1-x^2, dw = -2x dx \\
& \int \arcsen x dx = x \arcsen x - \int \frac{x dx}{\sqrt{1-x^2}} = x \arcsen x + \frac{1}{2} \int \frac{dw}{w^{1/2}} = x \arcsen x + \sqrt{1-x^2} + c
\end{aligned}$$

**4.29.-**  $\int x \operatorname{sen} x dx$

Solución.-

$$\begin{aligned}
& \therefore \quad u = x & dv = \operatorname{sen} x dx \\
& \quad du = dx & v = -\cos x \\
& \int x \operatorname{sen} x dx = -x \cos x + \int \cos x dx = -x \cos x + \operatorname{sen} x + c
\end{aligned}$$

**4.30.-**  $\int x \cos 3x dx$

Solución.-

$$\begin{aligned}
& \therefore \quad u = x & dv = \cos 3x dx \\
& \quad du = dx & v = \frac{1}{3} \operatorname{sen} 3x \\
& \int x \cos 3x dx = \frac{x}{3} \operatorname{sen} 3x - \int \frac{1}{3} \operatorname{sen} 3x dx = \frac{x}{3} \operatorname{sen} 3x + \frac{\cos 3x}{9} + c
\end{aligned}$$

**4.31.-**  $\int x 2^{-x} dx$

Solución.-

$$\begin{aligned}
& \therefore \quad u = x & dv = 2^{-x} dx \\
& \quad du = dx & v = -\frac{2^{-x}}{\ell \eta 2} \\
& \int x 2^{-x} dx = -\frac{x 2^{-x}}{\ell \eta 2} + \frac{1}{\ell \eta 2} \int 2^{-x} dx = -\frac{x 2^{-x}}{\ell \eta 2} + \frac{1}{\ell \eta 2} \left( \frac{-2^{-x}}{\ell \eta 2} \right) + c = -\frac{x}{2^x \ell \eta 2} - \frac{1}{2^{-x} \ell \eta^2 2} + c
\end{aligned}$$

**4.32.-**  $\int x^2 e^{3x} dx$

Solución.-

$$\begin{aligned} \therefore \quad u &= x^2 & dv &= e^{3x} dx \\ du &= 2x dx & v &= \frac{1}{3} e^{3x} \end{aligned}$$

$\int x^2 e^{3x} dx = \frac{x^2}{3} e^{3x} - \frac{2}{3} \int x e^{3x} dx$ , integral la cual se desarrolla nuevamente por partes,

$$\text{esto es:} \quad \therefore \quad \begin{aligned} u &= x & dv &= e^{3x} dx \\ du &= dx & v &= \frac{1}{3} e^{3x} \end{aligned}$$

$$= \frac{x^2}{3} e^{3x} - \frac{2}{3} \left( \frac{x}{3} e^{3x} - \frac{1}{3} \int e^{3x} dx \right) = \frac{x^2}{3} e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{9} \int e^{3x} dx = \frac{x^2}{3} e^{3x} - \frac{2x}{9} e^{3x} + \frac{2}{27} e^{3x} + c$$

**4.33.-**  $\int x^3 e^{-x/3} dx$

Solución.-

$$\therefore \quad \begin{aligned} u &= x^3 & dv &= e^{-x/3} dx \\ du &= 3x^2 dx & v &= -3e^{-x/3} \end{aligned}$$

$\int x^3 e^{-x/3} dx = -3x^3 e^{-x/3} + 9 \int x^2 e^{-x/3} dx$ , integral la cual se desarrolla nuevamente por

$$\text{partes, esto es:} \quad \therefore \quad \begin{aligned} u &= x^2 & dv &= e^{-x/3} dx \\ du &= 2x dx & v &= -3e^{-x/3} \end{aligned}$$

$$= -3x^3 e^{-x/3} + 9 \left( -3x^2 e^{-x/3} + 6 \int x e^{-x/3} dx \right) = -3x^3 e^{-x/3} - 27x^2 e^{-x/3} + 54 \int x e^{-x/3} dx$$

, la nueva integral se desarrolla por partes, esto es:

$$\therefore \quad \begin{aligned} u &= x & dv &= e^{-x/3} dx \\ du &= dx & v &= -3e^{-x/3} \end{aligned}$$

$$\begin{aligned} &= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} + 54 \left( -3x e^{-x/3} + 3 \int e^{-x/3} dx \right) = -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} + 162(-3e^{-x/3}) + c \\ &= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} - \frac{486}{e^{x/3}} + c \end{aligned}$$

**4.34.-**  $\int x \operatorname{sen} x \cos x dx$

Solución.-

$$\therefore \quad \begin{aligned} u &= x & dv &= \operatorname{sen} 2x dx \\ du &= dx & v &= -\frac{\cos 2x}{2} \end{aligned}$$

$$\int x \operatorname{sen} x \cos x dx = \frac{1}{2} \int x \operatorname{sen} 2x dx = \frac{1}{2} \left( -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x dx \right)$$

$$= -\frac{x}{4} \cos 2x + \frac{1}{4} \int \cos 2x dx = -\frac{x}{4} \cos 2x + \frac{1}{8} \operatorname{sen} 2x + c$$

**4.35.-**  $\int x^2 \ell \eta x dx$

Solución.-



$$\begin{aligned}
 u &= \ell \eta x & dv &= x^2 dx \\
 \therefore du &= \frac{dx}{x} & v &= \frac{x^3}{3} \\
 \int x^2 \ell \eta x dx &= \frac{x^3 \ell \eta x}{3} - \frac{1}{3} \int x^2 dx = \frac{x^3 \ell \eta x}{3} - \frac{x^3}{9} + c
 \end{aligned}$$

**4.36.-**  $\int \frac{\ell \eta x}{x^3} dx$

Solución.-

$$\begin{aligned}
 u &= \ell \eta x & dv &= x^{-3} dx \\
 \therefore du &= \frac{dx}{x} & v &= -\frac{1}{2x^2} \\
 \int \frac{\ell \eta x}{x^3} dx &= \int x^{-3} \ell \eta x dx = -\frac{\ell \eta x}{2x^2} + \frac{1}{2} \int x^{-3} dx = -\frac{\ell \eta x}{2x^2} - \frac{1}{4x^2} + c
 \end{aligned}$$

**4.37.-**  $\int \frac{\ell \eta x}{\sqrt{x}} dx$

Solución.-

$$\begin{aligned}
 u &= \ell \eta x & dv &= x^{-1/2} dx \\
 \therefore du &= \frac{dx}{x} & v &= 2\sqrt{x} \\
 \int \frac{\ell \eta x}{\sqrt{x}} dx &= \int x^{-1/2} \ell \eta x dx = 2\sqrt{x} \ell \eta x - 2 \int x^{-1/2} dx = 2\sqrt{x} \ell \eta x - 4\sqrt{x} + c
 \end{aligned}$$

**4.38.-**  $\int x \arctan x dx$

Solución.-

$$\begin{aligned}
 u &= \arctan x & dv &= x dx \\
 \therefore du &= \frac{dx}{1+x^2} & v &= \frac{x^2}{2} \\
 \int x \arctan x dx &= \frac{x^2}{2} \arctan x - \frac{1}{2} \int \frac{x^2 dx}{1+x^2} = \frac{x^2}{2} \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx \\
 &= \frac{x^2}{2} \arctan x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2} = \frac{x^2}{2} \arctan x - \frac{1}{2} x + \frac{\arctan x}{2} + c
 \end{aligned}$$

**4.39.-**  $\int x \arcsen x dx$

Solución.-

$$\begin{aligned}
 u &= \arcsen x & dv &= x dx \\
 \therefore du &= \frac{dx}{\sqrt{1+x^2}} & v &= \frac{x^2}{2} \\
 \int x \arcsen x dx &= \frac{x^2}{2} \arcsen x - \frac{1}{2} \int \frac{x^2 dx}{\sqrt{1+x^2}}, \text{ integral para la cual se sugiere la} \\
 \text{sustitución siguiente: } \therefore & x = \sen \theta & dx &= \cos \theta d\theta
 \end{aligned}$$

$$\begin{aligned}
&= \frac{x^2}{2} \arcsen x - \frac{1}{2} \int \frac{\cancel{\text{sen}^2 \theta} \cos \theta d\theta}{\cancel{\cos \theta}} \\
&= \frac{x^2}{2} \arcsen x - \frac{1}{2} \int \left( \frac{1 - \cos 2\theta}{2} \right) d\theta = \frac{x^2}{2} \arcsen x - \frac{1}{4} \int d\theta + \frac{1}{4} \int \cos 2\theta d\theta \\
&= \frac{x^2}{2} \arcsen x - \frac{1}{4} \theta + \frac{1}{8} \text{sen } 2\theta + c = \frac{x^2}{2} \arcsen x - \frac{1}{4} \arcsen x + \frac{2 \text{sen } \theta \cos \theta}{8} + c
\end{aligned}$$

Como:  $\text{sen } \theta = x$ ,  $\cos \theta = \sqrt{1-x^2}$ ; luego:

$$= \frac{x^2}{2} \arcsen x - \frac{1}{4} \arcsen x + \frac{1}{4} x \sqrt{1-x^2} + c$$

**4.40.-**  $\int \frac{xdx}{\text{sen}^2 x}$

Solución.-

$$\begin{aligned}
\therefore \quad u &= x & dv &= \text{cosec}^2 x dx \\
du &= dx & v &= -\cot x
\end{aligned}$$

$$\int \frac{xdx}{\text{sen}^2 x} = \int x \text{cosec}^2 x dx = -x \cot x + \int \cot x dx = -x \cot x + \ell \eta |\text{sen } x| + c$$

**4.41.-**  $\int e^x \text{sen } x dx$

Solución.-

$$\begin{aligned}
\therefore \quad u &= \text{sen } x & dv &= e^x dx \\
du &= \cos x dx & v &= e^x
\end{aligned}$$

$\int e^x \text{sen } x dx = e^x \text{sen } x - \int e^x \cos x dx$ , integral la cual se desarrolla por partes, esto es:

$$\begin{aligned}
\therefore \quad u &= \cos x & dv &= e^x dx \\
du &= -\text{sen } x dx & v &= e^x
\end{aligned}$$

$$= e^x \text{sen } x - \left( e^x \cos x + \int e^x \text{sen } x dx \right) = e^x \text{sen } x - e^x \cos x - \int e^x \text{sen } x dx$$

Luego se tiene:  $\int e^x \text{sen } x dx = e^x \text{sen } x - e^x \cos x - \int e^x \text{sen } x dx$ , de donde es inmediato:

$$2 \int e^x \text{sen } x dx = e^x (\text{sen } x - \cos x) + c$$

$$\int e^x \text{sen } x dx = \frac{e^x}{2} (\text{sen } x - \cos x) + c$$

**4.42.-**  $\int 3^x \cos x dx$

Solución.-

$$\begin{aligned}
\therefore \quad u &= \cos x & dv &= 3^x dx \\
du &= -\text{sen } x dx & v &= \frac{3^x}{\ell \eta 3}
\end{aligned}$$

$\int 3^x \cos x dx = \cos x \frac{3^x}{\ell \eta 3} + \frac{1}{\ell \eta 3} \int 3^x \operatorname{sen} x dx$ , integral la cual se desarrolla por partes,

$$\begin{array}{ll} \text{esto es: } \therefore & u = \operatorname{sen} x \\ & du = \cos x dx \end{array} \quad \begin{array}{l} dv = 3^x dx \\ v = \frac{3^x}{\ell \eta 3} \end{array}$$

$$= \cos x \frac{3^x}{\ell \eta 3} + \frac{1}{\ell \eta 3} \left( \frac{3^x}{\ell \eta 3} \operatorname{sen} x - \frac{1}{\ell \eta 3} \int 3^x \cos x dx \right)$$

$$= \cos x \frac{3^x}{\ell \eta 3} + \frac{3^x \operatorname{sen} x}{\ell \eta^2 3} - \frac{1}{\ell \eta^2 3} \int 3^x \cos x dx, \text{ luego:}$$

$$= \int 3^x \cos x dx = \frac{3^x}{\ell \eta} \left( \cos x + \frac{\operatorname{sen} x}{\ell \eta 3} \right) - \frac{1}{\ell \eta^2 3} \int 3^x \cos x dx, \text{ de donde es inmediato:}$$

$$= \left( 1 + \frac{1}{\ell \eta^2 3} \right) \int 3^x \cos x dx = \frac{3^x}{\ell \eta 3} \left( \cos x + \frac{\operatorname{sen} x}{\ell \eta 3} \right) + c$$

$$= \left( \frac{\ell \eta^2 3 + 1}{\ell \eta^2 3} \right) \int 3^x \cos x dx = \frac{3^x}{\cancel{\ell \eta 3}} \left( \cos x + \frac{\operatorname{sen} x}{\ell \eta 3} \right) + c$$

$$= \int 3^x \cos x dx = \frac{3^x \ell \eta 3}{\ell \eta^2 3 + 1} \left( \cos x + \frac{\operatorname{sen} x}{\ell \eta 3} \right) + c$$

#### 4.43.- $\int \operatorname{sen}(\ell \eta x) dx$

Solución.-

$$\begin{array}{ll} u = \operatorname{sen}(\ell \eta x) & dv = dx \\ \therefore du = \frac{\cos(\ell \eta x)}{x} dx & v = x \end{array}$$

$\int \operatorname{sen}(\ell \eta x) dx = x \operatorname{sen}(\ell \eta x) - \int \cos(\ell \eta x) dx$ , integral la cual se desarrolla por partes, esto es:

$$\begin{array}{ll} u = \cos(\ell \eta x) & dv = dx \\ \therefore du = \frac{-\operatorname{sen}(\ell \eta x)}{x} dx & v = x \end{array}$$

$$= x \operatorname{sen}(\ell \eta x) - \left[ x \cos(\ell \eta x) + \int \operatorname{sen}(\ell \eta x) dx \right] = x \operatorname{sen}(\ell \eta x) - x \cos(\ell \eta x) - \int \operatorname{sen}(\ell \eta x) dx$$

Se tiene por tanto:

$$\int \operatorname{sen}(\ell \eta x) dx = x [\operatorname{sen}(\ell \eta x) - \cos(\ell \eta x)] - \int \operatorname{sen}(\ell \eta x) dx, \text{ de donde es inmediato:}$$

$$2 \int \operatorname{sen}(\ell \eta x) dx = x [\operatorname{sen}(\ell \eta x) - \cos(\ell \eta x)] + c \quad \int \operatorname{sen}(\ell \eta x) dx = \frac{x}{2} [\operatorname{sen}(\ell \eta x) - \cos(\ell \eta x)] + c$$

#### 4.44.- $\int (x^2 - 2x + 3) \ell \eta x dx$

Solución.-

$$\begin{aligned}
u &= \ell \eta x & dv &= (x^2 - 2x + 3)dx \\
\therefore du &= \frac{dx}{x} & v &= \frac{x^3}{3} - x^2 + 3x \\
\int (x^2 - 2x + 3) \ell \eta x dx &= \left( \frac{x^3}{3} - x^2 + 3x \right) \ell \eta x - \int \left( \frac{x^3}{3} - x^2 + 3 \right) dx \\
&= \left( \frac{x^3}{3} - x^2 + 3x \right) \ell \eta x - \int \frac{x^3}{3} dx - \int x dx + 3 \int dx = \left( \frac{x^3}{3} - x^2 + 3x \right) \ell \eta x - \frac{x^3}{9} - \frac{x^2}{2} + 3x + c
\end{aligned}$$

**4.45.-**  $\int x \ell \eta \left| \frac{1-x}{1+x} \right| dx$

Solución.-

$$\begin{aligned}
u &= \ell \eta \left| \frac{1-x}{1+x} \right| & dv &= x dx \\
\therefore du &= \frac{2dx}{x^2 - 1} & v &= \frac{x^2}{2} \\
\int x \ell \eta \left| \frac{1-x}{1+x} \right| dx &= \frac{x^2}{2} \ell \eta \left| \frac{1-x}{1+x} \right| - \int \frac{x^2 dx}{x^2 - 1} = \frac{x^2}{2} \ell \eta \left| \frac{1-x}{1+x} \right| - \int \left( 1 + \frac{1}{x^2 - 1} \right) dx \\
&= \frac{x^2}{2} \ell \eta \left| \frac{1-x}{1+x} \right| - \int dx - \int \frac{dx}{x^2 - 1} = \frac{x^2}{2} \ell \eta \left| \frac{1-x}{1+x} \right| - x - \frac{1}{2} \ell \eta \left| \frac{x-1}{x+1} \right| + c
\end{aligned}$$

**4.46.-**  $\int \frac{\ell \eta^2 x}{x^2} dx$

Solución.-

$$\begin{aligned}
u &= \ell \eta^2 x & dv &= x^{-2} dx \\
\therefore du &= \frac{2 \ell \eta x}{x} dx & v &= -\frac{1}{x} \\
\int \frac{\ell \eta^2 x}{x^2} dx &= -\frac{\ell \eta^2 x}{x} + 2 \int \frac{\ell \eta x}{x^2} dx = -\frac{\ell \eta^2 x}{x} + 2 \int x^{-2} \ell \eta x dx, \text{ integral la cual se desarrolla} \\
&\text{por partes, esto es:}
\end{aligned}$$

$$\begin{aligned}
u &= \ell \eta x & dv &= x^{-2} dx \\
\therefore du &= \frac{dx}{x} & v &= -\frac{1}{x} \\
&= -\frac{\ell \eta^2 x}{x} + 2 \left( -\frac{\ell \eta x}{x} + \int \frac{dx}{x^2} \right) = -\frac{\ell \eta^2 x}{x} - \frac{2 \ell \eta x}{x} + 2 \int \frac{dx}{x^2} = -\frac{\ell \eta^2 x}{x} - \frac{2 \ell \eta x}{x} - \frac{2}{x} + c
\end{aligned}$$

**4.47.-**  $\int x^2 \operatorname{arc} \tau g 3x dx$

Solución.-

$$\begin{aligned}
u &= \operatorname{arc} \tau g 3x & dv &= x^2 dx \\
\therefore du &= \frac{3dx}{1+9x^2} & v &= \frac{x^3}{3}
\end{aligned}$$

$$\begin{aligned}
\int x^2 \operatorname{arctg} 3x dx &= \frac{x^3}{3} \operatorname{arctg} 3x - \int \frac{x^3 dx}{1+9x^2} = \frac{x^3}{3} \operatorname{arctg} 3x - \frac{1}{9} \int \frac{x^3 dx}{1/9 + x^2} \\
&= \frac{x^3}{3} \operatorname{arctg} 3x - \frac{1}{9} \left[ \int \left( x - \frac{1/9 x}{x^2 + 1/9} \right) dx \right] = \frac{x^3}{3} \operatorname{arctg} 3x - \frac{1}{9} \frac{x^2}{2} + \frac{1}{81} \int \frac{xdx}{x^2 + 1/9} \\
&= \frac{x^3}{3} \operatorname{arctg} 3x - \frac{x^2}{18} + \frac{1}{162} \ell \eta \left| x^2 + \frac{1}{9} \right| + c
\end{aligned}$$

**4.48.-**  $\int x(\operatorname{arctg} x)^2 dx$

Solución.-

$$\begin{aligned}
u &= (\operatorname{arctg} x)^2 & dv &= x dx \\
\therefore du &= \frac{2 \operatorname{arctg} x dx}{1+x^2} & v &= \frac{x^2}{2} \\
\int x(\operatorname{arctg} x)^2 dx &= \frac{x^2}{2} (\operatorname{arctg} x)^2 - \int (\operatorname{arctg} x) \frac{x^2 dx}{1+x^2}, \text{ integral la cual se desarrolla por partes, esto es:}
\end{aligned}$$

$$\begin{aligned}
u &= \operatorname{arctg} x & dv &= \frac{x^2 dx}{1+x^2} \\
\therefore du &= \frac{dx}{1+x^2} & v &= x - \operatorname{arctg} x \\
&= \frac{(x \operatorname{arctg} x)^2}{2} - \left[ (x - \operatorname{arctg} x) \operatorname{arctg} x - \int (x - \operatorname{arctg} x) \frac{dx}{1+x^2} \right] \\
&= \frac{(x \operatorname{arctg} x)^2}{2} - x \operatorname{arctg} x + (\operatorname{arctg} x)^2 + \int \frac{xdx}{1+x^2} - \int \frac{\operatorname{arctg} x dx}{1+x^2} \\
&= \frac{(x \operatorname{arctg} x)^2}{2} - x \operatorname{arctg} x + (\operatorname{arctg} x)^2 + \frac{1}{2} \ell \eta (1+x^2) - \frac{(\operatorname{arctg} x)^2}{2} + c
\end{aligned}$$

**4.49.-**  $\int (\operatorname{arcsen} x)^2 dx$

Solución.-

$$\begin{aligned}
u &= (\operatorname{arcsen} x)^2 & dv &= dx \\
\therefore du &= \frac{2 \operatorname{arcsen} x dx}{\sqrt{1-x^2}} & v &= x \\
\int (\operatorname{arcsen} x)^2 dx &= x(\operatorname{arcsen} x)^2 - 2 \int \operatorname{arcsen} x \frac{xdx}{\sqrt{1-x^2}}, \text{ integral la cual se desarrolla por}
\end{aligned}$$

$$\begin{aligned}
u &= \operatorname{arcsen} x & dv &= \frac{xdx}{\sqrt{1-x^2}} \\
\text{partes, esto es: } \therefore du &= \frac{dx}{\sqrt{1-x^2}} & v &= -\sqrt{1-x^2}
\end{aligned}$$

$$\begin{aligned}
&= x(\operatorname{arcsen} x)^2 - 2 \left[ -\sqrt{1-x^2} \operatorname{arcsen} x + \int dx \right] \\
&= x(\operatorname{arcsen} x)^2 + 2\sqrt{1-x^2} \operatorname{arcsen} x - 2x + c
\end{aligned}$$

$$4.50.- \int \frac{\arcsen x}{x^2} dx$$

Solución.-

$$u = \arcsen x \quad dv = x^{-2} dx$$

$$\therefore \quad du = \frac{dx}{\sqrt{1-x^2}} \quad v = -\frac{1}{x}$$

$$\int \frac{\arcsen x}{x^2} dx = \int x^{-2} \arcsen x dx = -\frac{\arcsen x}{x} + \int \frac{dx}{x\sqrt{1-x^2}}$$

$$= -\frac{\arcsen x}{x} + \ell\eta \left| \frac{x}{1+\sqrt{1-x^2}} \right| + c$$

$$4.51.- \int \frac{\arcsen \sqrt{x}}{\sqrt{1-x}} dx$$

Solución.-

$$u = \arcsen \sqrt{x} \quad dv = \frac{dx}{\sqrt{1-x}}$$

$$\therefore \quad du = \frac{dx}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} \quad v = -2\sqrt{1-x}$$

$$\int \frac{\arcsen \sqrt{x}}{\sqrt{1-x}} dx = -2\sqrt{1-x} \arcsen \sqrt{x} + \int \frac{dx}{\sqrt{x}} = -2\sqrt{1-x} \arcsen \sqrt{x} + 2\sqrt{x} + c$$

$$4.52.- \int \frac{\sen^2 x}{e^x} dx$$

Solución.-

$$\therefore \quad u = \sen^2 x \quad dv = e^{-x} dx$$

$$du = 2\sen x \cos x \quad v = -e^{-x}$$

$$\int \frac{\sen^2 x}{e^x} dx = \int \sen^2 x e^{-x} dx = -e^{-x} \sen^2 x + 2 \int \sen x \cos x e^{-x} dx$$

$$= -e^{-x} \sen^2 x + \cancel{2} \int \frac{\sen 2x}{\cancel{2}} e^{-x} dx, \text{ * Integral la cual se desarrolla por partes, esto es:}$$

$$\therefore \quad u = \sen 2x \quad dv = e^{-x} dx$$

$$du = 2\cos 2x dx \quad v = -e^{-x}$$

$$= -e^{-x} \sen^2 x + 2 \int \cos 2x e^{-x} dx, \text{ Integral la cual se desarrolla por partes, esto es:}$$

$$\therefore \quad u = \cos 2x \quad dv = e^{-x} dx$$

$$du = -2\sen 2x dx \quad v = -e^{-x}$$

$$\int \sen 2x e^{-x} dx = -e^{-x} \sen 2x + 2 \left( -e^{-x} \cos 2x - 2 \int \sen 2x e^{-x} dx \right)$$

$$\int \sen 2x e^{-x} dx = -e^{-x} \sen 2x - 2e^{-x} \cos 2x - 4 \int \sen 2x e^{-x} dx, \text{ de donde:}$$

$$5 \int \sen 2x e^{-x} dx = -e^{-x} (\sen 2x + 2 \cos 2x) + c$$

$$\int \operatorname{sen} 2x e^{-x} dx = \frac{-e^{-x}}{5} (\operatorname{sen} 2x + 2 \cos 2x) + c, \text{ Sustituyendo en: } *$$

$$\int \frac{\operatorname{sen}^2 x dx}{e^x} = -e^{-x} \operatorname{sen}^2 x - \frac{2e^{-x}}{5} (\operatorname{sen} 2x + 2 \cos 2x) + c$$

$$4.53.- \int \tau g^2 x \sec^3 x dx = \int (\sec^2 x - 1) \sec^3 x dx = \int \sec^5 x dx (*) - \int \sec^3 x dx (**)$$

Solución.-

$$* \int \sec^5 x dx, \text{ Sea: } \begin{array}{ll} u = \sec^3 x & dv = \sec^2 x dx \\ du = 3 \sec^3 x \tau g x dx & v = \tau g x \end{array}$$

$$\int \sec^5 x dx = \int \sec^3 x \sec^2 x dx = \sec^3 x \tau g x - 3 \int \sec^3 x \tau g^2 x dx$$

$$** \int \sec^3 x dx, \text{ Sea: } \begin{array}{ll} u = \sec x & dv = \sec^2 x dx \\ du = \sec x \tau g x dx & v = \tau g x \end{array}$$

$$\int \sec^3 x dx = \int \sec x \sec^2 x dx = \sec x \tau g x - \int \sec x \tau g^2 x dx = \sec x \tau g x - \int \sec x (\sec^2 x - 1) dx \\ = \sec x \tau g x - \int \sec^3 x dx + \int \sec x dx, \text{ luego: } 2 \int \sec^3 x dx = \sec x \tau g x + \int \sec x dx$$

$$\text{Esto es: } \int \sec^3 x dx = \frac{1}{2} (\sec x \tau g x + \ln |\sec x \tau g x|) + c, \text{ ahora bien:}$$

$$\int \tau g^2 x \sec^3 x dx = \int \sec^5 x dx - \int \sec^3 x dx, \text{ con } (*) \text{ y } (**)$$

$$\int \tau g^2 x \sec^3 x dx = \sec^3 x \tau g x - 3 \int \sec^3 x \tau g^2 x dx - \frac{1}{2} (\sec x \tau g x + \ln |\sec x \tau g x|) + c$$

$$\text{De lo anterior: } 4 \int \tau g^2 x \sec^3 x dx = \sec^3 x \tau g x - \frac{1}{2} (\sec x \tau g x + \ln |\sec x \tau g x|) + c$$

$$\text{Esto es: } \int \tau g^2 x \sec^3 x dx = \frac{1}{4} \sec^3 x \tau g x - \frac{1}{8} (\sec x \tau g x + \ln |\sec x \tau g x|) + c$$

$$4.54.- \int x^3 \ell \eta^2 x dx$$

Solución.-

$$\begin{array}{ll} u = \ell \eta^2 x & dv = x^3 dx \\ \therefore du = \frac{2 \ell \eta x}{x} dx & v = \frac{x^4}{4} \end{array}$$

$$\int x^3 \ell \eta^2 x dx = \frac{x^4}{4} \ell \eta^2 x - \frac{1}{2} \int x^3 \ell \eta x dx, \text{ integral la cual se desarrolla por partes, esto es:}$$

$$\begin{array}{ll} u = \ell \eta x & dv = x^3 dx \\ du = \frac{dx}{x} & v = \frac{x^4}{4} \end{array} \\ = \frac{x^4}{4} \ell \eta^2 x - \frac{1}{2} \left( \frac{x^4}{4} \ell \eta x - \frac{1}{4} \int x^3 dx \right) = \frac{x^4}{4} \ell \eta^2 x - \frac{1}{8} x^4 \ell \eta x + \frac{1}{8} \frac{x^4}{4} + c \\ = \frac{x^4}{4} \ell \eta^2 x - \frac{1}{8} x^4 \ell \eta x + \frac{x^4}{32} + c$$

**4.55.-**  $\int x \ell \eta(9+x^2) dx$

Solución.-

$$\begin{aligned} u &= \ell \eta(9+x^2) & dv &= x dx \\ \therefore du &= \frac{2x dx}{9+x^2} & v &= \frac{x^2}{2} \\ \int x \ell \eta(9+x^2) dx &= \frac{x^2}{2} \ell \eta(9+x^2) - \int \frac{x^3}{9+x^2} dx = \frac{x^2}{2} \ell \eta(9+x^2) - \int \left( x - \frac{9x}{x^2+9} \right) dx \\ &= \frac{x^2}{2} \ell \eta(9+x^2) - \int x dx + 9 \int \frac{x dx}{9+x^2} = \frac{x^2}{2} \ell \eta(9+x^2) - \frac{x^2}{2} + \frac{9}{2} \ell \eta(x^2+9) + c \\ &= \frac{x^2}{2} [\ell \eta(9+x^2) - 1] + \frac{9}{2} \ell \eta(x^2+9) + c \end{aligned}$$

**4.56.-**  $\int \arcsen \sqrt{x} dx$

Solución.-

$$\begin{aligned} u &= \arcsen \sqrt{x} & dv &= dx \\ \therefore du &= \frac{dx}{\sqrt{1-x^2}} \cdot \frac{1}{2\sqrt{x}} & v &= x \\ \int \arcsen \sqrt{x} dx &= x \arcsen \sqrt{x} - \int \frac{x dx}{\sqrt{1-x^2}} \cdot \frac{1}{2\sqrt{x}} = x \arcsen \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x} dx}{\sqrt{1-x^2}} \end{aligned}$$

Para la integral resultante, se recomienda la siguiente sustitución:

$\sqrt{1-x} = t$ , de donde:  $x = 1-t^2$ , y  $dx = -2t dt$  ( ver capitulo 9)

$$= x \arcsen \sqrt{x} - \frac{1}{2} \frac{\sqrt{1-t^2} (-2t dt) dx}{t} = x \arcsen \sqrt{x} + \sqrt{1-t^2} dt, \quad \text{Se recomienda la}$$

sustitución:  $t = \sen \theta$ , de donde:  $\sqrt{1-t^2} = \cos \theta$ , y  $dt = \cos \theta d\theta$ . Esto es:

$$\begin{aligned} &= x \arcsen \sqrt{x} + \int \cos^2 \theta d\theta = x \arcsen \sqrt{x} + \frac{1}{2} \int (1 + \cos 2\theta) d\theta \\ &= x \arcsen \sqrt{x} + \frac{1}{2} \theta + \frac{1}{4} \sen 2\theta + c = x \arcsen \sqrt{x} + \frac{1}{2} \theta + \frac{1}{2} \sen \theta \cos \theta + c \\ &= x \arcsen \sqrt{x} + \frac{\arcsen t}{2} + \frac{t}{2} \sqrt{1-t^2} + c = x \arcsen \sqrt{x} + \frac{\arcsen \sqrt{1-x}}{2} + \frac{\sqrt{1-x}}{2} \sqrt{x} + c \end{aligned}$$

**4.57.-**  $\int x \arctg(2x+3) dx$

Solución.-

$$\begin{aligned} u &= \arctg(2x+3) & dv &= x dx \\ \therefore du &= \frac{2 dx}{1+(2x+3)^2} & v &= \frac{x^2}{2} \\ \int x \arctg(2x+3) dx &= \frac{x^2}{2} \arctg(2x+3) - \int \frac{x^2 dx}{1+4x^2+12x+9} \end{aligned}$$



$$\begin{aligned}
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \int \frac{x^2 dx}{4x^2 + 12x + 10} = \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \int \left( \frac{1}{4} - \frac{3x + \frac{5}{2}}{4x^2 + 12x + 10} \right) dx \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} \int dx + \int \frac{3x + \frac{5}{2}}{4x^2 + 12x + 10} dx \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + 3 \int \frac{x + \frac{5}{6}}{4x^2 + 12x + 10} dx \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \int \frac{8x + \frac{40}{6}}{4x^2 + 12x + 10} dx \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \int \frac{8x + 12 - \frac{32}{6}}{4x^2 + 12x + 10} dx \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \int \frac{(8x+12)dx}{4x^2 + 12x + 10} - \frac{3}{8} \frac{32}{6} \int \frac{dx}{4x^2 + 12x + 10} \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \ell \eta |4x^2 + 12x + 10| - 2 \int \frac{dx}{4x^2 + 12x + 10} \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \ell \eta |4x^2 + 12x + 10| - 2 \int \frac{dx}{(2x+3)^2 + 1} \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \ell \eta |4x^2 + 12x + 10| - \frac{2}{2} \int \frac{2dx}{(2x+3)^2 + 1} \\
&= \frac{x^2}{2} \operatorname{arc} \tau g(2x+3) - \frac{1}{4} x + \frac{3}{8} \ell \eta |4x^2 + 12x + 10| - \operatorname{arc} \tau g(2x+3) + c \\
&= \frac{1}{2} \left[ (x^2 - 2) \operatorname{arc} \tau g(2x+3) - \frac{1}{2} x + \frac{3}{4} \ell \eta |4x^2 + 12x + 10| \right] + c
\end{aligned}$$

**4.58.-**  $\int e^{\sqrt{x}} dx$

Solución.-

$$\begin{aligned}
&u = e^{\sqrt{x}} \\
\therefore \quad du &= \frac{e^{\sqrt{x}} dx}{2\sqrt{x}} & dv &= dx \\
& & v &= x
\end{aligned}$$

$$\int e^{\sqrt{x}} dx = xe^{\sqrt{x}} - \frac{1}{2} \int \frac{xe^{\sqrt{x}} dx}{2\sqrt{x}}, \text{ Se recomienda la sustitución: } z = \sqrt{x}, dz = \frac{dx}{2\sqrt{x}}$$

$$= xe^{\sqrt{x}} - \frac{1}{2} \int z^2 e^z dz, \text{ Esta integral resultante, se desarrolla por partes:}$$

$$\begin{aligned}
\therefore \quad u &= z^2 & dv &= e^z dz \\
du &= 2z dz & v &= e^z
\end{aligned}$$

$$= xe^{\sqrt{x}} - \frac{1}{2} \left( z^2 e^z - 2 \int z e^z dz \right) = xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + \int z e^z dz, \text{ integral que se desarrolla por partes:}$$

$$\begin{aligned}
\therefore \quad & u = z & dv &= e^z dz \\
& du = dz & v &= e^z \\
& = xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + ze^z - \int e^z dz = xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + ze^z - e^z + c = xe^{\sqrt{x}} - \frac{xe^{\sqrt{x}}}{2} + \sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c \\
& = e^{\sqrt{x}} \left( \frac{x}{2} + \sqrt{x} - 1 \right) + c
\end{aligned}$$

**4.59.-**  $\int \cos^2(\ell \eta x) dx$

Solución.-

$$\begin{aligned}
& u = \cos(2\ell \eta x) \\
\therefore \quad & du = -\frac{[\operatorname{sen}(2\ell \eta x)] 2dx}{x} & dv &= dx \\
& & v &= x \\
& \int \cos^2(\ell \eta x) dx = \int \frac{1 + \cos(2\ell \eta x)}{2} dx = \frac{1}{2} \int dx + \frac{1}{2} \int \cos(2\ell \eta x) dx \\
& = \frac{1}{2} x + \frac{1}{2} \left[ x \cos(2\ell \eta x) + 2 \int \operatorname{sen}(2\ell \eta x) dx \right] = \frac{x}{2} + \frac{x}{2} \cos(2\ell \eta x) + \int \operatorname{sen}(2\ell \eta x) dx *
\end{aligned}$$

Integral que se desarrolla por partes:

$$\begin{aligned}
& u = \operatorname{sen}(2\ell \eta x) \\
\therefore \quad & du = -\frac{[\cos(2\ell \eta x)] 2dx}{x} & dv &= dx \\
& & v &= x
\end{aligned}$$

$$* = \frac{x}{2} + \frac{x}{2} \cos(2\ell \eta x) + x \operatorname{sen}(2\ell \eta x) - 2 \int \cos(2\ell \eta x) dx ,$$

Dado que apareció nuevamente:  $\int \cos(2\ell \eta x) dx$ , igualamos: \*

$$\frac{x}{2} + \frac{1}{2} \int \cos(2\ell \eta x) dx = \frac{x}{2} + \frac{x}{2} \cos(2\ell \eta x) + x \operatorname{sen}(2\ell \eta x) - 2 \int \cos(2\ell \eta x) dx , \text{ de donde:}$$

$$\frac{5}{2} \int \cos(2\ell \eta x) dx = \frac{x}{2} \cos(2\ell \eta x) + x \operatorname{sen}(2\ell \eta x) + c$$

$$\frac{1}{2} \int \cos(2\ell \eta x) dx = \frac{x}{10} \cos(2\ell \eta x) + \frac{x}{5} \operatorname{sen}(2\ell \eta x) + c , \text{ Por tanto:}$$

$$\int \cos^2(\ell \eta x) dx = \frac{x}{2} + \frac{x}{10} \cos(2\ell \eta x) + \frac{x}{5} \operatorname{sen}(2\ell \eta x) + c$$

**4.60.-**  $\int \frac{\ell \eta (\ell \eta x)}{x} dx$ , Sustituyendo por:  $w = \ell \eta x, dw = \frac{dx}{x}$ , Se tiene:

Solución.-

$$\int \frac{\ell \eta (\ell \eta x)}{x} dx = \int \ell \eta w dw , \text{ Esta integral se desarrolla por partes:}$$

$$\begin{aligned}
& u = \ell \eta w \\
\therefore \quad & du = \frac{dw}{w} & dv &= dw \\
& & v &= w
\end{aligned}$$

$$= w \ell \eta w - \int dw = w \ell \eta w - w + c = w(\ell \eta w - 1) + c = \ell \eta x [\ell \eta (\ell \eta x) - 1] + c$$

$$4.61.- \int \ell \eta |x+1| dx$$

Solución.-

$$\begin{aligned} u &= \ell \eta |x+1| & dv &= dx \\ \therefore du &= \frac{dx}{x+1} & v &= x \end{aligned}$$

$$\begin{aligned} \int \ell \eta |x+1| dx &= x \ell \eta |x+1| - \int \frac{x dx}{x+1} = x \ell \eta |x+1| - \int \left(1 - \frac{1}{x+1}\right) dx \\ &= x \ell \eta |x+1| - x + \ell \eta |x+1| + c \end{aligned}$$

$$4.62.- \int x^2 e^x dx$$

Solución.-

$$\begin{aligned} u &= x^2 & dv &= e^x dx \\ \therefore du &= 2x dx & v &= e^x \end{aligned}$$

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

Integral que se desarrolla nuevamente por partes:

$$\begin{aligned} u &= x & dv &= e^x dx \\ \therefore du &= dx & v &= e^x \\ &= x^2 e^x - 2 \left[ x e^x - \int e^x dx \right] = x^2 e^x - 2 x e^x + 2 e^x + c \end{aligned}$$

$$4.63.- \int \cos^n x dx = \int \cos^{n-1} x \cos x dx$$

Solución.-

$$\begin{aligned} u &= \cos^{n-1} x & dv &= \cos x dx \\ \therefore du &= (n-1) \cos^{n-2} x (-\sin x) dx & v &= \sin x \end{aligned}$$

$$\begin{aligned} &= \cos^{n-1} x \sin x + (n-1) \int \sin^2 x \cos^{n-2} x dx \\ &= \cos^{n-1} x \sin x + (n-1) \int (1 - \cos^2 x) \cos^{n-2} x dx \\ &= \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x dx - (n-1) \int \cos^n x dx, \text{ Se tiene:} \end{aligned}$$

$$\int \cos^n x dx = \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x dx - (n-1) \int \cos^n x dx, \text{ Esto es:}$$

$$n \int \cos^n x dx = \cos^{n-1} x \sin x + (n-1) \int \cos^{n-2} x dx$$

$$\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{(n-1)}{n} \int \cos^{n-2} x dx$$

$$4.64.- \int \sin^n x dx = \int \sin^{n-1} x \sin x dx$$

Solución.-

$$\begin{aligned} u &= \sin^{n-1} x & dv &= \sin x dx \\ \therefore du &= (n-1) \sin^{n-2} x (\cos x) dx & v &= -\cos x \end{aligned}$$

$$\begin{aligned} &= -\sin^{n-1} x \cos x + (n-1) \int \cos^2 x \sin^{n-2} x dx \\ &= -\sin^{n-1} x \cos x + (n-1) \int (1 - \sin^2 x) \sin^{n-2} x dx \end{aligned}$$

$= -\operatorname{sen}^{n-1} x \cos x + (n-1) \int \operatorname{sen}^{n-2} x dx - (n-1) \int \operatorname{sen}^n x dx$ , Se tiene:

$$\int \operatorname{sen}^n x dx = -\operatorname{sen}^{n-1} x \cos x + (n-1) \int \operatorname{sen}^{n-2} x dx - (n-1) \int \operatorname{sen}^n x dx$$

$$n \int \operatorname{sen}^n x dx = -\operatorname{sen}^{n-1} x \cos x + (n-1) \int \operatorname{sen}^{n-2} x dx$$

$$\int \operatorname{sen}^n x dx = \frac{-\operatorname{sen}^{n-1} x \cos x}{n} + \frac{(n-1)}{n} \int \operatorname{sen}^{n-2} x dx$$

**4.65.-**  $\int x^m (\ell \eta x)^n dx = x^{m+1} (\ell \eta x)^n - n \int x^m (\ell \eta x)^{n-1} dx - m \int x^m (\ell \eta x)^n dx$

Solución.-

$$\begin{aligned} u &= x^m (\ell \eta x)^n & dv &= dx \\ \therefore du &= x^m n (\ell \eta x)^{n-1} \frac{dx}{x} + m x^{m-1} (\ell \eta x)^n dx & v &= x \end{aligned}$$

Se tiene:  $(m+1) \int x^m (\ell \eta x)^n dx = x^{m+1} (\ell \eta x)^n - n \int x^m (\ell \eta x)^{n-1} dx$

$$\int x^m (\ell \eta x)^n dx = \frac{x^{m+1} (\ell \eta x)^n}{(m+1)} - \frac{n}{(m+1)} \int x^m (\ell \eta x)^{n-1} dx$$

**4.66.-**  $\int x^3 (\ell \eta x)^2 dx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo:  $m=3, n=2$

$$\int x^3 (\ell \eta x)^2 dx = \frac{x^{3+1} (\ell \eta x)^2}{3+1} - \frac{2}{3+1} \int x^3 (\ell \eta x)^{2-1} dx = \frac{x^4 (\ell \eta x)^2}{4} - \frac{1}{2} \int x^3 (\ell \eta x) dx *$$

Para la integral resultante:  $\int x^3 (\ell \eta x) dx *$

$$\int x^3 (\ell \eta x) dx = \frac{x^4 (\ell \eta x)}{4} - \frac{1}{4} \int x^3 dx = \frac{x^4 (\ell \eta x)}{4} - \frac{x^4}{16} + c, \text{ introduciendo en: } *$$

$$\int x^3 (\ell \eta x)^2 dx = \frac{x^4 (\ell \eta x)^2}{4} - \frac{x^4}{8} (\ell \eta x) + \frac{x^4}{32} + c$$

**4.67.-**  $\int x^n e^x dx$

Solución.-

$$\begin{aligned} \therefore u &= x^n & dv &= e^x dx \\ du &= n x^{n-1} dx & v &= e^x \end{aligned}$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

**4.68.-**  $\int x^3 e^x dx$

Solución.-

$$\begin{aligned} \therefore u &= x^3 & dv &= e^x dx \\ du &= 3x^2 dx & v &= e^x \end{aligned}$$

Puede desarrollarse como el ejercicio anterior, haciendo:  $n=3$

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx *, \text{ Además:}$$

$$*\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx **, \text{ Además: } \int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + c$$

Reemplazando en \*\* y luego en \*:

$$\int x^3 e^x dx = x^3 e^x - 3 \left[ x^2 e^x - 2(x e^x - e^x) \right] + c$$

$$\int x^3 e^x dx = e^x (x^3 - 3x^2 + 6x - 6) + c$$

$$4.69.- \int \sec^n x dx = \int \sec^{n-2} x \sec^2 x dx$$

Solución.-

$$\begin{aligned} \therefore \quad u &= \sec^{n-2} x & dv &= \sec^2 x dx \\ du &= (n-2) \sec^{n-3} x \sec x \tau g x dx & v &= \tau g x \end{aligned}$$

$$= \sec^{n-2} x \tau g x - (n-2) \int \tau g^2 x \sec^{n-2} x dx = \sec^{n-2} x \tau g x - (n-2) \int (\sec^2 x - 1) \sec^{n-2} x dx$$

$$= \sec^{n-2} x \tau g x - (n-2) \int \sec^n x dx + (n-2) \int \sec^{n-2} x dx, \text{ Se tiene:}$$

$$\int \sec^n x dx = \sec^{n-2} x \tau g x - (n-2) \int \sec^n x dx + (n-2) \int \sec^{n-2} x dx$$

$$(n-1) \int \sec^n x dx = \sec^{n-2} x \tau g x + (n-2) \int \sec^{n-2} x dx$$

$$\int \sec^n x dx = \frac{\sec^{n-2} x \tau g x}{(n-1)} + \frac{(n-2)}{(n-1)} \int \sec^{n-2} x dx$$

$$4.70.- \int \sec^3 x dx$$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo:

$$n = 3$$

$$\int \sec^3 x dx = \frac{\sec^{3-2} x \tau g x}{3-1} + \frac{3-2}{3-1} \int \sec^{3-2} x dx = \frac{\sec x \tau g x}{2} + \frac{1}{2} \int \sec x dx$$

$$= \frac{\sec x \tau g x}{2} + \frac{1}{2} \ell \eta |\sec x \tau g x| + c$$

$$4.71.- \int x^\ell \eta x dx$$

Solución.-

$$\begin{aligned} u &= \ell \eta x & dv &= x dx \\ \therefore \quad du &= \frac{dx}{x} & v &= \frac{x^2}{2} \end{aligned}$$

$$\int x^\ell \eta x dx = \frac{x^2}{2} \ell \eta x - \int \frac{x dx}{2} = \frac{x^2}{2} \ell \eta x - \frac{1}{4} x^2 + c$$

$$4.72.- \int x^n \ell \eta |ax| dx, n \neq -1$$

Solución.-

$$\begin{aligned} u &= \ell \eta |ax| & dv &= x dx \\ \therefore \quad du &= \frac{dx}{x} & v &= \frac{x^{n+1}}{n+1} \end{aligned}$$

$$\int x^n \ell \eta |ax| dx = \frac{x^{n+1}}{n+1} \ell \eta |ax| - \frac{1}{n+1} \int x^n dx = \frac{x^{n+1}}{n+1} \ell \eta |ax| - \frac{x^{n+1}}{(n+1)^2} + c$$

**4.73.-**  $\int \arcsen ax dx$

Solución.-

$$\begin{aligned} u &= \arcsen ax \\ \therefore du &= \frac{adx}{\sqrt{1-a^2x^2}} \end{aligned} \quad \begin{aligned} dv &= dx \\ v &= x \end{aligned}$$

$$\begin{aligned} \int \arcsen ax dx &= x \arcsen ax - \int \frac{ax dx}{\sqrt{1-a^2x^2}} = x \arcsen ax + \frac{1}{2a} \int \frac{(-2a^2x)dx}{\sqrt{1-a^2x^2}} \\ &= x \arcsen ax + \frac{1}{2a} \frac{(1-a^2x^2)^{1/2}}{1/2} + c = x \arcsen ax + \frac{1}{a} \sqrt{1-a^2x^2} + c \end{aligned}$$

**4.74.-**  $\int x \sen ax dx$

Solución.-

$$\begin{aligned} u &= x \\ \therefore du &= dx \end{aligned} \quad \begin{aligned} dv &= \sen ax dx \\ v &= -\frac{1}{a} \cos ax \end{aligned}$$

$$\begin{aligned} \int x \sen ax dx &= -\frac{x}{a} \cos ax + \frac{1}{a} \int \cos ax dx = -\frac{x}{a} \cos ax + \frac{1}{a^2} \sen ax + c \\ &= \frac{1}{a^2} \sen ax - \frac{x}{a} \cos ax + c \end{aligned}$$

**4.75.-**  $\int x^2 \cos ax dx$

Solución.-

$$\begin{aligned} u &= x^2 \\ \therefore du &= 2x dx \end{aligned} \quad \begin{aligned} dv &= \cos ax dx \\ v &= \frac{1}{a} \sen ax \end{aligned}$$

$$\begin{aligned} \int x^2 \cos ax dx &= \frac{x^2}{a} \sen ax - \frac{2}{a} \int x \sen ax dx, \text{ aprovechando el ejercicio anterior:} \\ &= \frac{x^2}{a} \sen ax - \frac{2}{a} \left( \frac{1}{a^2} \sen ax - \frac{x}{a} \cos ax \right) + c = \frac{x^2}{a} \sen ax - \frac{2}{a^3} \sen ax - \frac{2x}{a^2} \cos ax + c \end{aligned}$$

**4.76.-**  $\int x \sec^2 ax dx$

Solución.-

$$\begin{aligned} u &= x \\ \therefore du &= dx \end{aligned} \quad \begin{aligned} dv &= \sec^2 ax dx \\ v &= \frac{1}{a} \tau g ax \end{aligned}$$

$$\begin{aligned} \int x \sec^2 ax dx &= \frac{x}{a} \tau g ax - \frac{1}{a} \int \tau g ax dx = \frac{x}{a} \tau g ax - \frac{1}{a} \frac{1}{a} \ell \eta |\sec ax| + c \\ &= \frac{x}{a} \tau g ax - \frac{1}{a^2} \ell \eta |\sec ax| + c \end{aligned}$$

**4.77.-**  $\int \cos(\ell \eta x) dx$

Solución.-

$$\begin{aligned}
u &= \cos(\ell \eta x) & dv &= dx \\
\therefore du &= -\frac{\operatorname{sen}(\ell \eta x)}{x} dx & v &= x \\
\int \cos(\ell \eta x) dx &= x \cos(\ell \eta x) + \int \operatorname{sen}(\ell \eta x) dx, \text{ aprovechando el ejercicio: 4.43} \\
\int \operatorname{sen}(\ell \eta x) dx &= \frac{x}{2} [\operatorname{sen}(\ell \eta x) - \cos(\ell \eta x)] + c, \text{ Luego:} \\
&= x \cos(\ell \eta x) + \frac{x}{2} [\operatorname{sen}(\ell \eta x) - \cos(\ell \eta x)] + c = x \cos(\ell \eta x) + \frac{x}{2} \operatorname{sen}(\ell \eta x) - \frac{x}{2} \cos(\ell \eta x) + c \\
&= \frac{x}{2} [\cos(\ell \eta x) + \operatorname{sen}(\ell \eta x)] + c
\end{aligned}$$

**4.78.-**  $\int \ell \eta (9 + x^2) dx$

Solución.-

$$\begin{aligned}
u &= \ell \eta (9 + x^2) & dv &= dx \\
\therefore du &= \frac{2x dx}{9 + x^2} & v &= x \\
\int \ell \eta (9 + x^2) dx &= x \ell \eta (9 + x^2) - 2 \int \frac{x^2 dx}{9 + x^2} = x \ell \eta (9 + x^2) - 2 \int \left( 1 - \frac{9}{9 + x^2} \right) dx \\
&= x \ell \eta (9 + x^2) - 2 \int dx + 18 \int \frac{dx}{9 + x^2} = x \ell \eta (9 + x^2) - 2x + 6 \operatorname{arctg} \frac{x}{3} + c
\end{aligned}$$

**4.79.-**  $\int x \cos(2x + 1) dx$

Solución.-

$$\begin{aligned}
u &= x & dv &= \cos(2x + 1) dx \\
\therefore du &= dx & v &= \frac{1}{2} \operatorname{sen}(2x + 1) \\
\int x \cos(2x + 1) dx &= \frac{x}{2} \operatorname{sen}(2x + 1) - \frac{1}{2} \int \operatorname{sen}(2x + 1) dx \\
&= \frac{x}{2} \operatorname{sen}(2x + 1) + \frac{1}{4} \cos(2x + 1) + c
\end{aligned}$$

**4.80.-**  $\int x \operatorname{arc sec} x dx$

Solución.-

$$\begin{aligned}
u &= \operatorname{arc sec} x & dv &= x dx \\
\therefore du &= \frac{dx}{x \sqrt{x^2 - 1}} & v &= \frac{x^2}{2} \\
\int x \operatorname{arc sec} x dx &= \frac{x^2}{2} \operatorname{arc sec} x - \frac{1}{2} \int \frac{x dx}{\sqrt{x^2 - 1}} = \frac{x^2}{2} \operatorname{arc sec} x - \frac{1}{2} \sqrt{x^2 - 1} + c
\end{aligned}$$

**4.81.-**  $\int \operatorname{arc sec} \sqrt{x} dx$

Solución.-

$$\begin{aligned}
 u &= \operatorname{arcsec} x \\
 \therefore du &= \frac{1}{2} \frac{dx}{x\sqrt{x-1}} & dv &= dx \\
 & & v &= x \\
 \int \operatorname{arcsec} \sqrt{x} dx &= x \operatorname{arcsec} x - \frac{1}{2} \int \frac{dx}{\sqrt{x-1}} = x \operatorname{arcsec} x - \sqrt{x-1} + c
 \end{aligned}$$

$$\begin{aligned}
 \text{4.82.-} \int \sqrt{a^2 - x^2} dx &= \int \frac{a^2 - x^2}{\sqrt{a^2 - x^2}} dx = a^2 \int \frac{dx}{\sqrt{a^2 - x^2}} - \int \frac{x^2 dx}{\sqrt{a^2 - x^2}} \\
 &= a^2 \operatorname{arcsen} \frac{x}{a} - \int x \frac{xdx}{\sqrt{a^2 - x^2}} * , \text{ integral que se desarrolla por partes:}
 \end{aligned}$$

Solución.-

$$\begin{aligned}
 u &= x & dv &= \frac{xdx}{\sqrt{a^2 - x^2}} \\
 \therefore du &= dx & v &= -\sqrt{a^2 - x^2} \\
 * &= a^2 \operatorname{arcsen} \frac{x}{a} - \left( -x\sqrt{a^2 - x^2} + \int \sqrt{a^2 - x^2} dx \right), \text{ Se tiene que:}
 \end{aligned}$$

$$\int \sqrt{a^2 - x^2} dx = a^2 \operatorname{arcsen} \frac{x}{a} + x\sqrt{a^2 - x^2} - \int \sqrt{a^2 - x^2} dx, \text{ De donde:}$$

$$2 \int \sqrt{a^2 - x^2} dx = a^2 \operatorname{arcsen} \frac{x}{a} + x\sqrt{a^2 - x^2} + c$$

$$\int \sqrt{a^2 - x^2} dx = \frac{a^2}{2} \operatorname{arcsen} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + c$$

$$\text{4.83.-} \int \ell \eta |1-x| dx$$

Solución.-

$$\begin{aligned}
 u &= \ell \eta |1-x| & dv &= dx \\
 \therefore du &= -\frac{dx}{1-x} & v &= x \\
 \int \ell \eta |1-x| dx &= x \ell \eta |1-x| - \int \frac{xdx}{x-1} = x \ell \eta |1-x| - \int \left( 1 + \frac{1}{x-1} \right) dx \\
 &= x \ell \eta |1-x| - \int dx - \int \frac{dx}{x-1} = x \ell \eta |1-x| - x - \ell \eta |x-1| + c
 \end{aligned}$$

$$\text{4.84.-} \int \ell \eta (x^2 + 1) dx$$

Solución.-

$$\begin{aligned}
 u &= \ell \eta (x^2 + 1) & dv &= dx \\
 \therefore du &= \frac{2xdx}{x^2 + 1} & v &= x \\
 \int \ell \eta (x^2 + 1) dx &= x \ell \eta (x^2 + 1) - 2 \int \frac{x^2 dx}{x^2 + 1} = x \ell \eta (x^2 + 1) - 2 \int \left( 1 - \frac{1}{x^2 + 1} \right) dx \\
 &= x \ell \eta (x^2 + 1) - 2x + 2 \operatorname{arctg} x + c
 \end{aligned}$$



$$4.85.- \int \operatorname{arc} \tau g \sqrt{x} dx$$

Solución.-

$$\begin{aligned} u &= \operatorname{arc} \tau g \sqrt{x} & dv &= dx \\ \therefore du &= \frac{dx}{1+x} \frac{1}{2\sqrt{x}} & v &= x \end{aligned}$$

$$\int \operatorname{arc} \tau g \sqrt{x} dx = x \operatorname{arc} \tau g \sqrt{x} - \frac{1}{2} \int \frac{\sqrt{x} dx}{1+x} *$$
 En la integral resultante, se recomienda la

sustitución:  $\sqrt{x} = t$ , esto es  $x = t^2$ ,  $dx = 2t dt$

$$= x \operatorname{arc} \tau g \sqrt{x} - \frac{1}{2} \int \frac{t \cancel{2} t dt}{1+t^2} = x \operatorname{arc} \tau g \sqrt{x} - \int \frac{t^2 dt}{1+t^2} = x \operatorname{arc} \tau g \sqrt{x} - \int \left( 1 - \frac{1}{1+t^2} \right) dt$$

$$= x \operatorname{arc} \tau g \sqrt{x} - \int dt + \int \frac{dt}{1+t^2} = x \operatorname{arc} \tau g \sqrt{x} - t + \operatorname{arc} \tau g t + c$$

$$= x \operatorname{arc} \tau g \sqrt{x} - \sqrt{x} + \operatorname{arc} \tau g \sqrt{x} + c$$

$$4.86.- \int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} dx$$

Solución.-

$$\begin{aligned} u &= \operatorname{arcsen} x & dv &= \frac{xdx}{\sqrt{1-x^2}} \\ \therefore du &= \frac{dx}{\sqrt{1-x^2}} & v &= -\sqrt{1-x^2} \end{aligned}$$

$$\int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \operatorname{arcsen} x + \int dx = -\sqrt{1-x^2} \operatorname{arcsen} x + x + c$$

$$4.87.- \int x \operatorname{arc} \tau g \sqrt{x^2-1} dx$$

Solución.-

$$\begin{aligned} u &= \operatorname{arc} \tau g \sqrt{x^2-1} & dv &= x dx \\ \therefore du &= \frac{dx}{x\sqrt{x^2-1}} & v &= \frac{x^2}{2} \end{aligned}$$

$$\int x \operatorname{arc} \tau g \sqrt{x^2-1} dx = \frac{x^2}{2} \operatorname{arc} \tau g \sqrt{x^2-1} - \frac{1}{2} \int \frac{xdx}{\sqrt{x^2-1}} = \frac{x^2}{2} \operatorname{arc} \tau g \sqrt{x^2-1} - \frac{1}{2} \sqrt{x^2-1} + c$$

$$4.88.- \int \frac{x \operatorname{arc} \tau g x}{(x^2+1)^2} dx$$

Solución.-

$$\begin{aligned} u &= \operatorname{arc} \tau g x & dv &= \frac{xdx}{(x^2+1)^2} \\ \therefore du &= \frac{dx}{x^2+1} & v &= \frac{-1}{2(x^2+1)} \end{aligned}$$

$$\int \frac{x \operatorname{arc} \tau g x}{(x^2+1)^2} dx = \frac{-\operatorname{arc} \tau g x}{2(x^2+1)} + \frac{1}{2} \int \frac{dx}{(x^2+1)^2} *, \text{ Se recomienda la siguiente sustitución:}$$

$x = \tau g \theta$ , de donde:  $dx = \sec^2 \theta d\theta$ ;  $x^2 + 1 = \sec^2 \theta$

$$\begin{aligned} * &= \frac{-\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{2} \int \frac{\sec^2 \theta d\theta}{\sec^4 \theta} = -\frac{\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{2} \int \cos^2 \theta d\theta = -\frac{\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{2} \int \frac{1 + \cos 2\theta d\theta}{2} \\ &= -\frac{\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{4} \theta + \frac{1}{8} \operatorname{sen} 2\theta + c = -\frac{\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{4} \operatorname{arc} \tau g x + \frac{1}{4} \operatorname{sen} \theta \cos \theta + c \\ &= -\frac{\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{4} \operatorname{arc} \tau g x + \frac{1}{4} \frac{x}{\sqrt{x^2 + 1}} \frac{1}{\sqrt{x^2 + 1}} + c \\ &= -\frac{\operatorname{arc} \tau g x}{2(x^2 + 1)} + \frac{1}{4} \operatorname{arc} \tau g x + \frac{x}{4(x^2 + 1)} + c \end{aligned}$$

**4.89.-**  $\int \operatorname{arcsen} x \frac{xdx}{\sqrt{(1-x^2)^3}}$

Solución.-

$$\begin{aligned} u &= \operatorname{arcsen} x & dv &= \frac{xdx}{(1-x^2)^{3/2}} \\ \therefore du &= \frac{dx}{\sqrt{1-x^2}} & v &= \frac{1}{\sqrt{1-x^2}} \\ \int \operatorname{arcsen} x \frac{xdx}{\sqrt{(1-x^2)^3}} &= \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} - \int \frac{dx}{1-x^2} = \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} + \frac{1}{2} \ell \eta \left| \frac{1-x}{1+x} \right| + c \end{aligned}$$

**4.90.-**  $\int x^2 \sqrt{1-x} dx$

Solución.-

$$\begin{aligned} u &= \sqrt{1-x} & dv &= x^2 dx \\ \therefore du &= -\frac{dx}{2\sqrt{1-x}} & v &= \frac{x^3}{3} \\ \int x^2 \sqrt{1-x} dx &= \frac{x^3}{3} \sqrt{1-x} + \frac{1}{6} \int \frac{x^3 dx}{\sqrt{1-x}} *, \quad \text{Se recomienda usar la siguiente} \end{aligned}$$

sustitución:  $\sqrt{1-x} = t$ , o sea:  $x = 1-t^2$ , De donde:  $dx = -2tdt$

$$\begin{aligned} &= \frac{x^3}{3} \sqrt{1-x} + \frac{1}{6} \int \frac{(1-t^2)^3 (-2t dt)}{t} = \frac{x^3}{3} \sqrt{1-x} - \frac{1}{3} \int (1-t^2)^3 dt \\ &= \frac{x^3}{3} \sqrt{1-x} - \frac{1}{3} \int (1-3t^2+3t^4-t^6) dt = \frac{x^3}{3} \sqrt{1-x} - \frac{1}{3} \left( t - t^3 + \frac{3t^5}{5} - \frac{t^7}{7} \right) + c \\ &= \frac{x^3}{3} \sqrt{1-x} - \frac{1}{3} \left[ \sqrt{1-x} - (1-x)\sqrt{1-x} + \frac{3}{5}(1-x)^2 \sqrt{1-x} - \frac{3}{7}(1-x)^3 \sqrt{1-x} \right] + c \\ &= \frac{\sqrt{1-x}}{3} \left[ x^3 - 1 - (1-x) + \frac{3}{5}(1-x)^2 - \frac{1}{7}(1-x)^3 \right] + c \end{aligned}$$

**IMPORTANTE:** En este capítulo ningún resultado, o casi ninguno, se presentaron en su forma más reducida. Esto es intencional. Una de las causas del fracaso en éstos tópicos, a veces está en el mal uso del álgebra elemental. He aquí una oportunidad para mejorar tal eficiencia. Exprese cada resultado en su forma más reducida.

## CAPITULO 5

### INTEGRACION DE FUNCIONES CUADRATICAS

Una función cuadrática, es de la forma:  $ax^2 + bx + c$  y si ésta aparece en el denominador, la integral que la contiene se hace fácil de encontrar, para la cual conviene diferenciar dos tipos esenciales en lo que se refiere al numerador.

### EJERCICIOS DESARROLLADOS

**5.1.-Encontrar:**  $\int \frac{dx}{x^2 + 2x + 5}$

Solución.- Completando cuadrados, se tiene:

$$x^2 + 2x + 5 = (x^2 + 2x + \underline{\quad}) + 5 - \underline{\quad} = (x^2 + 2x + 1) + 5 - 1 = (x^2 + 2x + 1) + 4$$

$$x^2 + 2x + 5 = (x + 1)^2 + 2^2, \text{ luego se tiene:}$$

$$\int \frac{dx}{x^2 + 2x + 5} = \int \frac{dx}{(x+1)^2 + 2^2}. \text{ Sea: } w = x + 1, dw = dx; a = 2$$

$$\int \frac{dx}{(x+1)^2 + 2^2} = \int \frac{dw}{w^2 + 2^2} = \frac{1}{2} \operatorname{arc} \tau g \frac{w}{a} + c = \frac{1}{2} \operatorname{arc} \tau g \frac{x+1}{2} + c$$

**Respuesta:**  $\int \frac{dx}{x^2 + 2x + 5} = \frac{1}{2} \operatorname{arc} \tau g \frac{x+1}{2} + c$

**5.2.-Encontrar:**  $\int \frac{dx}{4x^2 + 4x + 2}$

Solución.-  $\int \frac{dx}{4x^2 + 4x + 2} = \int \frac{dx}{4(x^2 + x + \frac{1}{2})} = \frac{1}{4} \int \frac{dx}{x^2 + x + \frac{1}{2}}$

Completando cuadrados:

$$x^2 + x + \frac{1}{2} = (x^2 + x + \underline{\quad}) + \frac{1}{2} - \underline{\quad} = (x^2 + x + \frac{1}{4}) + \frac{1}{2} - \frac{1}{4} = (x^2 + x + \frac{1}{4}) + \frac{1}{4}$$

$$(x^2 + x + \frac{1}{2}) = (x + \frac{1}{2})^2 + (\frac{1}{2})^2, \text{ luego se tiene:}$$

$$\frac{1}{4} \int \frac{dx}{x^2 + x + \frac{1}{2}} = \frac{1}{4} \int \frac{dx}{(x + \frac{1}{2})^2 + (\frac{1}{2})^2}, \text{ Sea: } w = x + \frac{1}{2}, dw = dx; a = \frac{1}{2}$$

$$= \frac{1}{4} \int \frac{dx}{(x + \frac{1}{2})^2 + (\frac{1}{2})^2} = \frac{1}{4} \int \frac{dw}{w^2 + a^2} = \frac{1}{4} \frac{1}{a} \operatorname{arc} \tau g \frac{w}{a} + c = \frac{1}{4} \frac{1}{\frac{1}{2}} \operatorname{arc} \tau g \frac{x + \frac{1}{2}}{\frac{1}{2}} + c$$

$$= \frac{1}{2} \operatorname{arc} \tau g \frac{2x+1}{1} + c = \frac{1}{2} \operatorname{arc} \tau g (2x+1) + c$$

**Respuesta:**  $\int \frac{dx}{4x^2 + 4x + 2} = \frac{1}{2} \operatorname{arctg}(2x+1) + c$

**5.3.-Encontrar:**  $\int \frac{2x dx}{x^2 - x + 1}$

Solución.-  $u = x^2 - x + 1, du = (2x - 1)dx$

$$\int \frac{2x dx}{x^2 - x + 1} = \int \frac{(2x - 1 + 1)dx}{x^2 - x + 1} = \int \frac{(2x - 1)dx}{x^2 - x + 1} + \int \frac{dx}{x^2 - x + 1} = \int \frac{du}{u} + \int \frac{dx}{x^2 - x + 1}$$

Completando cuadrados:

$$x^2 - x + 1 = (x^2 - x + \underline{\quad}) + 1 - \underline{\quad} = (x^2 - x + \frac{1}{4}) + 1 - \frac{1}{4}$$

$$x^2 - x + 1 = (x - \frac{1}{2})^2 + \frac{3}{4}, \text{ Luego se tiene:}$$

$$\int \frac{du}{u} + \int \frac{dx}{x^2 - x + 1} = \int \frac{du}{u} + \int \frac{du}{(x - \frac{1}{2})^2 + \frac{3}{4}} = \int \frac{du}{u} + \int \frac{dx}{(x - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$w = x - \frac{1}{2}, dw = dx; a = \frac{\sqrt{3}}{2}, \text{ luego:}$$

$$\int \frac{du}{u} + \int \frac{dx}{(x - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = \int \frac{du}{u} + \int \frac{dw}{w^2 + a^2} = \ell \eta |u| + \frac{1}{a} \operatorname{arctg} \frac{w}{a} + c$$

$$= \ell \eta |x^2 - x + 1| + \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \ell \eta |x^2 - x + 1| + \frac{2\sqrt{3}}{3} \operatorname{arctg} \frac{2x - 1}{\sqrt{3}} + c$$

**Respuesta:**  $\int \frac{2x dx}{x^2 - x + 1} = \ell \eta |x^2 - x + 1| + \frac{2\sqrt{3}}{3} \operatorname{arctg} \frac{2x - 1}{\sqrt{3}} + c$

**5.4.-Encontrar:**  $\int \frac{x^2 dx}{x^2 + 2x + 5}$

Solución.-

$$\int \frac{x^2 dx}{x^2 + 2x + 5} = \int \left( 1 - \frac{2x + 5}{x^2 + 2x + 5} \right) dx = \int dx - \int \frac{2x + 5}{x^2 + 2x + 5} dx,$$

Sea:  $u = x^2 + 2x + 5, du = (2x + 2)dx$

Ya se habrá dado cuenta el lector que tiene que construir en el numerador, la expresión:  $(2x + 2)dx$ . Luego se tiene:

$$= \int dx - \int \frac{(2x + 2 + 3)}{x^2 + 2x + 5} dx = \int dx - \int \frac{(2x + 2)dx}{x^2 + 2x + 5} + 3 \int \frac{dx}{x^2 + 2x + 5},$$

Completando cuadrados, se tiene:

$$x^2 + 2x + 5 = (x^2 + 2x + \underline{\quad}) + 5 - \underline{\quad} = (x^2 + 2x + 1) + 5 - 1 = (x^2 + 2x + 1) + 4 = (x + 1)^2 + 2^2$$

Luego se admite como forma equivalente a la anterior:

$$\int dx - \int \frac{du}{u} - 3 \int \frac{dx}{(x + 1)^2 + 2^2}, \text{ Sea: } w = x + 1, dw = dx; a = 2, \text{ luego:}$$

$$= \int dx - \int \frac{du}{u} - 3 \int \frac{dw}{w^2 + a^2} = x - \ell \eta |u| - 3 \frac{1}{a} \operatorname{arc} \tau g \frac{w}{a} + c$$

$$= x - \ell \eta |x^2 + 2x + 5| - \frac{3}{2} \operatorname{arc} \tau g \frac{x+1}{2} + c$$

**Respuesta:**  $\int \frac{x^2 dx}{x^2 + 2x + 5} = x - \ell \eta |x^2 + 2x + 5| - \frac{3}{2} \operatorname{arc} \tau g \frac{x+1}{2} + c$

**5.5.-Encontrar:**  $\int \frac{2x-3}{x^2+2x+2} dx$

Solución.- Sea:  $u = x^2 + 2x + 2, du = (2x+2)dx$

$$\int \frac{2x-3}{x^2+2x+2} dx = \int \frac{2x+2-5}{x^2+2x+2} dx = \int \frac{2x+2}{x^2+2x+2} dx - 5 \int \frac{dx}{x^2+2x+2}$$

$$= \int \frac{du}{u} dx - 5 \int \frac{dx}{x^2+2x+2}, \text{ Completando cuadrados:}$$

$x^2 + 2x + 2 = (x+1)^2 + 1^2$ . Luego:

$$= \int \frac{du}{u} dx - 5 \int \frac{dx}{(x+1)^2 + 1^2}, \text{ Sea: } w = x+1, du = dx; a = 1. \text{ Entonces se tiene:}$$

$$= \int \frac{du}{u} dx - 5 \int \frac{dx}{w^2 + a^2} = \ell \eta |u| - 5 \frac{1}{a} \operatorname{arc} \tau g \frac{w}{a} + c = \ell \eta |x^2 + 2x + 5| - 5 \operatorname{arc} \tau g(x+1) + c$$

**Respuesta:**  $\int \frac{2x-3}{x^2+2x+2} dx = \ell \eta |x^2 + 2x + 5| - 5 \operatorname{arc} \tau g(x+1) + c$

**5.6.-Encontrar:**  $\int \frac{dx}{\sqrt{x^2 - 2x - 8}}$

Solución.- Completando cuadrados se tiene:  $x^2 - 2x - 8 = (x-1)^2 - 3^2$

$$\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \int \frac{dx}{\sqrt{(x-1)^2 - 3^2}}, \text{ Sea: } w = x-1, dw = dx; a = 3$$

$$= \int \frac{dw}{\sqrt{w^2 - a^2}} = \ell \eta \left| w + \sqrt{w^2 - a^2} \right| + c = \ell \eta \left| x-1 + \sqrt{x^2 - 2x - 8} \right| + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \ell \eta \left| x-1 + \sqrt{x^2 - 2x - 8} \right| + c$

**5.7.-Encontrar:**  $\int \frac{xdx}{\sqrt{x^2 - 2x + 5}}$

Solución.- Sea:  $u = x^2 - 2x + 5, du = (2x-2)dx$ . Luego:

$$\int \frac{xdx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2} \int \frac{2xdx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2} \int \frac{2x-2+2}{\sqrt{x^2 - 2x + 5}} dx$$

$$= \frac{1}{2} \int \frac{(2x-2)dx}{\sqrt{x^2 - 2x + 5}} + \frac{2}{2} \int \frac{dx}{\sqrt{x^2 - 2x + 5}} = \frac{1}{2} \int \frac{du}{\sqrt{u}} + \int \frac{dx}{\sqrt{x^2 - 2x + 5}}$$

Completando cuadrados se tiene:  $x^2 + 2x + 5 = (x-1)^2 + 2^2$ . Por lo tanto:

$$\begin{aligned}
&= \frac{1}{2} \int u^{-\frac{1}{2}} du + \int \frac{dx}{\sqrt{(x-1)^2 + 2^2}}. \text{ Sea: } w = x-1, du = dx; a = 2 \\
&= \frac{1}{2} \int u^{-\frac{1}{2}} du + \int \frac{dw}{\sqrt{w^2 + a^2}} = \frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + \ell \eta \left| w + \sqrt{w^2 + a^2} \right| + c = u^{\frac{1}{2}} + \ell \eta \left| w + \sqrt{w^2 + a^2} \right| + c \\
&= \sqrt{x^2 + 2x + 5} + \ell \eta \left| x-1 + \sqrt{x^2 - 2x + 5} \right| + c
\end{aligned}$$

**Respuesta:**  $\int \frac{x dx}{\sqrt{x^2 - 2x + 5}} = \sqrt{x^2 - 2x + 5} + \ell \eta \left| x-1 + \sqrt{x^2 - 2x + 5} \right| + c$

**5.8.-Encontrar:**  $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.- Sea:  $u = 2x - x^2, du = (2 - 2x)dx$ . Luego:

$$\begin{aligned}
\int \frac{(x+1)dx}{\sqrt{2x-x^2}} &= -\frac{1}{2} \int \frac{-2(x+1)dx}{\sqrt{2x-x^2}} = -\frac{1}{2} \int \frac{(-2x-2)dx}{\sqrt{2x-x^2}} = -\frac{1}{2} \int \frac{(-2x+2-4)dx}{\sqrt{2x-x^2}} \\
&= -\frac{1}{2} \int \frac{(2-2x)dx}{\sqrt{2x-x^2}} + \frac{4}{2} \int \frac{dx}{\sqrt{2x-x^2}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}} + 2 \int \frac{dx}{\sqrt{2x-x^2}}
\end{aligned}$$

Completando cuadrados:  $2x - x^2 = -(x^2 - 2x) = -(x^2 - 2x + 1 - 1) = -(x^2 - 2x + 1) + 1$   
 $= -(x-1)^2 + 1 = 1 - (x-1)^2$ . Luego la expresión anterior es equivalente a:

$$\begin{aligned}
&= -\frac{1}{2} \int u^{-\frac{1}{2}} du + 2 \int \frac{dx}{\sqrt{1-(x-1)^2}}. \text{ Sea: } w = x-1, dw = dx; a = 1. \text{ Entonces:} \\
&= -\frac{1}{2} \int \frac{u^{\frac{1}{2}}}{\frac{1}{2}} du + 2 \int \frac{dw}{\sqrt{a^2 - w^2}} = -u^{\frac{1}{2}} + 2 \arcsen \frac{w}{a} + c = -\sqrt{2x-x^2} + 2 \arcsen(x-1) + c
\end{aligned}$$

**Respuesta:**  $\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = -\sqrt{2x-x^2} + 2 \arcsen(x-1) + c$

**5.9.-Encontrar:**  $\int \frac{x dx}{\sqrt{5x^2 - 2x + 1}}$

Solución.- Sea:  $u = 5x^2 - 2x + 1, du = (10x - 2)dx$ . Luego:

$$\begin{aligned}
\int \frac{x dx}{\sqrt{5x^2 - 2x + 1}} &= \frac{1}{10} \int \frac{10x dx}{\sqrt{5x^2 - 2x + 1}} = \frac{1}{10} \int \frac{(10x - 2 + 2)dx}{\sqrt{5x^2 - 2x + 1}} \\
&= \frac{1}{10} \int \frac{(10x - 2)dx}{\sqrt{5x^2 - 2x + 1}} + \frac{2}{10} \int \frac{dx}{\sqrt{5x^2 - 2x + 1}} = \frac{1}{10} \int \frac{du}{\sqrt{u}} + \frac{1}{5} \int \frac{dx}{\sqrt{5x^2 - 2x + 1}} \\
&= \frac{1}{10} \int \frac{du}{\sqrt{u}} + \frac{1}{5} \int \frac{dx}{\sqrt{5(x^2 - \frac{2}{5}x + \frac{1}{5})}} = \frac{1}{10} \int u^{-\frac{1}{2}} du + \frac{1}{5\sqrt{5}} \int \frac{dx}{\sqrt{(x^2 - \frac{2}{5}x + \frac{1}{5})}}
\end{aligned}$$

Completando cuadrados:  $x^2 - \frac{2}{5}x + \frac{1}{5} = (x^2 - \frac{2}{5}x + \frac{1}{25}) + \frac{1}{5} - \frac{1}{25}$

$= (x^2 - \frac{2}{5}x + \frac{1}{25}) + \frac{1}{5} - \frac{1}{25} = (x - \frac{1}{5})^2 + (\frac{2}{5})^2$ , Luego es equivalente:

$$= \frac{1}{10} \int u^{-\frac{1}{2}} du + \frac{1}{5\sqrt{5}} \int \frac{dx}{\sqrt{(x-\frac{1}{5})^2 + (\frac{2}{5})^2}}, \text{ Sea: } w = x - \frac{1}{5}, dw = dx; a = \frac{2}{5},$$

$$\begin{aligned} \text{Entonces: } &= \frac{1}{10} \int u^{-\frac{1}{2}} du + \frac{1}{5\sqrt{5}} \int \frac{dw}{\sqrt{w^2 + a^2}} = \frac{1}{10} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + \frac{1}{5\sqrt{5}} \ell \eta \left| w + \sqrt{w^2 + a^2} \right| + c \\ &= \frac{\sqrt{5x^2 - 2x + 1}}{5} + \frac{1}{5\sqrt{5}} \ell \eta \left| x - \frac{1}{5} + \frac{\sqrt{5x^2 - 2x + 1}}{\sqrt{5}} \right| + c \end{aligned}$$

$$\text{Respuesta: } \int \frac{xdx}{\sqrt{5x^2 - 2x + 1}} = \frac{\sqrt{5x^2 - 2x + 1}}{5} + \frac{\sqrt{5}}{25} \ell \eta \left| x - \frac{1}{5} + \frac{\sqrt{5x^2 - 2x + 1}}{\sqrt{5}} \right| + c$$

$$\text{5.10.-Encontrar: } \int \frac{xdx}{\sqrt{5+4x-x^2}}$$

Solución.-  $u = 5 + 4x - x^2, du = (4 - 2x)dx$ . Luego:

$$\begin{aligned} \int \frac{xdx}{\sqrt{5+4x-x^2}} &= -\frac{1}{2} \int \frac{-2xdx}{\sqrt{5+4x-x^2}} = -\frac{1}{2} \int \frac{(-2x+4-4)dx}{\sqrt{5+4x-x^2}} \\ &= -\frac{1}{2} \int \frac{(4-2x)dx}{\sqrt{5+4x-x^2}} + \frac{4}{2} \int \frac{dx}{\sqrt{5+4x-x^2}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}} + 2 \int \frac{dx}{\sqrt{5+4x-x^2}} \end{aligned}$$

Completando cuadrados:  $5 + 4x - x^2 = -(x^2 - 4x - 5) = -(x^2 - 4x + 4 - 4 - 5)$

$= -(x^2 - 4x + 4) + 9 = 9 - (x - 2)^2 = 3^2 - (x - 2)^2$ . Equivalente a:

$$= -\frac{1}{2} \int u^{-\frac{1}{2}} du + 2 \int \frac{dx}{\sqrt{3^2 - (x-2)^2}}. \text{ Sea: } w = x - 2, dw = dx; a = 3. \text{ Entonces:}$$

$$= -\frac{1}{2} \int u^{-\frac{1}{2}} du + 2 \int \frac{dw}{\sqrt{a^2 - w^2}} = -\frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + 2 \arcsen \frac{w}{a} + c$$

$$= -\sqrt{5+4x-x^2} + 2 \arcsen \frac{x-2}{3} + c$$

$$\text{Respuesta: } \int \frac{xdx}{\sqrt{5+4x-x^2}} = -\sqrt{5+4x-x^2} + 2 \arcsen \frac{x-2}{3} + c$$

$$\text{5.11.-Encontrar: } \int \frac{dx}{\sqrt{2+3x-2x^2}}$$

Solución.- Completando cuadrados se tiene:

$$2 + 3x - 2x^2 = -(2x^2 - 3x - 2) = -2(x^2 - \frac{3}{2}x - 1) = -2(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{25}{16})$$

$$= -2 \left[ (x^2 - \frac{3}{2}x + \frac{9}{16}) - \frac{25}{16} \right] = -2 \left[ (x - \frac{3}{4})^2 - (\frac{5}{4})^2 \right] = 2 \left[ (\frac{5}{4})^2 - (x - \frac{3}{4})^2 \right], \text{ luego:}$$

$$\int \frac{dx}{\sqrt{2+3x-2x^2}} = \int \frac{dx}{\sqrt{2 \left[ (\frac{5}{4})^2 - (x - \frac{3}{4})^2 \right]}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(\frac{5}{4})^2 - (x - \frac{3}{4})^2}}$$

Sea:  $w = x - \frac{3}{4}, dw = dx, a = \frac{5}{4}$ . Luego:



$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(5/4)^2 - (x - 3/4)^2}} = \frac{1}{\sqrt{2}} \int \frac{dw}{\sqrt{a^2 - w^2}} = \frac{1}{\sqrt{2}} \arcsen \frac{w}{a} + c = \frac{1}{\sqrt{2}} \arcsen \frac{x - 3/4}{5/4} + c$$

$$= \frac{\sqrt{2}}{2} \arcsen \frac{4x - 3}{5} + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt{2+3x-2x^2}} = \frac{\sqrt{2}}{2} \arcsen \frac{4x-3}{5} + c$

**5.12.-Encontrar:**  $\int \frac{dx}{3x^2+12x+42}$

Solución.-

$$\int \frac{dx}{3x^2+12x+42} = \int \frac{dx}{3(x^2+4x+14)} = \frac{1}{3} \int \frac{dx}{(x^2+4x+14)} = \frac{1}{3} \int \frac{dx}{(x^2+4x+4+10)} =$$

$$= \frac{1}{3} \int \frac{dx}{(x+2)^2+10} = \frac{1}{3} \int \frac{dx}{(x+2)^2+(\sqrt{10})^2} = \frac{1}{3} \frac{1}{\sqrt{10}} \operatorname{arctg} \frac{x+2}{\sqrt{10}} + c$$

**Respuesta:**  $\int \frac{dx}{3x^2+12x+42} = \frac{\sqrt{10}}{30} \operatorname{arctg} \frac{x+2}{\sqrt{10}} + c$

**5.13.-Encontrar:**  $\int \frac{3x-2}{x^2-4x+5} dx$

Solución.- Sea:  $u = x^2 - 4x + 5, du = (2x - 4)dx$ , Luego:

$$\int \frac{3x-2}{x^2-4x+5} dx = 3 \int \frac{x dx}{x^2-4x+5} - 2 \int \frac{dx}{x^2-4x+5} = 3 \int \frac{(x-2)+2}{x^2-4x+5} - 2 \int \frac{dx}{x^2-4x+5}$$

$$= 3 \int \frac{(x-2)}{x^2-4x+5} + 6 \int \frac{dx}{x^2-4x+5} - 2 \int \frac{dx}{x^2-4x+5} = \frac{3}{2} \int \frac{du}{u} + 4 \int \frac{dx}{x^2-4x+5}$$

$$= \frac{3}{2} \int \frac{du}{u} + 4 \int \frac{dx}{(x^2-4x+4)+1} = \frac{3}{2} \ell \eta |x^2-4x+5| + 4 \int \frac{dx}{(x-2)^2+1}$$

$$= \frac{3}{2} \ell \eta |x^2-4x+5| + 4 \operatorname{arctg}(x-2) + c$$

**Respuesta:**  $\int \frac{3x-2}{x^2-4x+5} dx = \frac{3}{2} \ell \eta |x^2-4x+5| + 4 \operatorname{arctg}(x-2) + c$

## EJERCICIOS PROPUESTOS

Usando Esencialmente la técnica tratada, encontrar las integrales siguientes:

**5.14.-**  $\int \sqrt{x^2+2x-3} dx$

**5.15.-**  $\int \sqrt{12+4x-x^2} dx$

**5.16.-**  $\int \sqrt{x^2+4x} dx$

**5.17.-**  $\int \sqrt{x^2-8x} dx$

**5.18.-**  $\int \sqrt{6x-x^2} dx$

**5.19.-**  $\int \frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}$

$$\begin{array}{lll}
5.20.- \int \frac{xdx}{\sqrt{27+6x-x^2}} & 5.21.- \int \frac{(x-1)dx}{3x^2-4x+3} & 5.22.- \int \frac{(2x-3)dx}{x^2+6x+15} \\
5.23.- \int \frac{dx}{4x^2+4x+10} & 5.24.- \int \frac{(2x+2)dx}{x^2-4x+9} & 5.25.- \int \frac{(2x+4)dx}{\sqrt{4x-x^2}} \\
5.26.- \frac{2}{3} \int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8} & 5.27.- \int \frac{(x+6)dx}{\sqrt{5-4x-x^2}} & 5.28.- \int \frac{dx}{2x^2+20x+60} \\
5.29.- \int \frac{3dx}{\sqrt{80+32x-4x^2}} & 5.30.- \int \frac{dx}{\sqrt{12x-4x^2-8}} & 5.31.- \int \frac{5dx}{\sqrt{28-12x-x^2}} \\
5.32.- \int \sqrt{12-8x-4x^2} dx & 5.33.- \int \sqrt{x^2-x+\frac{5}{4}} dx & 5.34.- \int \frac{dx}{x^2-2x+5} \\
5.35.- \int \frac{(1-x)dx}{\sqrt{8+2x-x^2}} & 5.36.- \int \frac{xdx}{x^2+4x+5} & 5.37.- \int \frac{(2x+3)dx}{4x^2+4x+5} \\
5.38.- \int \frac{(x+2)dx}{x^2+2x+2} & 5.39.- \int \frac{(2x+1)dx}{x^2+8x-2} & 5.40.- \int \frac{dx}{\sqrt{-x^2-6x}} \\
5.41.- \int \frac{(x-1)dx}{x^2+2x+2} & & 
\end{array}$$

## RESPUESTAS

$$5.14.- \int \sqrt{x^2-2x-3} dx$$

Solución.- Completando cuadrados se tiene:

$$x^2-2x-3 = (x^2-2x+1)-3-1 = (x-1)^2-4 = (x-1)^2-2^2$$

Haciendo:  $u = x-1, du = dx; a = 2$ , se tiene:

$$\begin{aligned}
\int \sqrt{x^2-2x-3} dx &= \int \sqrt{(x-1)^2-2^2} dx = \int \sqrt{u^2-a^2} du \\
&= \frac{1}{2} u \sqrt{u^2-a^2} - \frac{1}{2} a^2 \ell \eta \left| u + \sqrt{u^2-a^2} \right| + c \\
&= \frac{1}{2} (x-1) \sqrt{(x-1)^2-2^2} - \frac{1}{2} 2^2 \ell \eta \left| (x-1) + \sqrt{(x-1)^2-2^2} \right| + c \\
&= \frac{1}{2} (x-1) \sqrt{x^2-2x-3} - 2 \ell \eta \left| (x-1) + \sqrt{x^2-2x-3} \right| + c
\end{aligned}$$

$$5.15.- \int \sqrt{12+4x-x^2} dx$$

Solución.- Completando cuadrados se tiene:

$$\begin{aligned}
12+4x-x^2 &= -(x^2-4x-12) = -(x^2-4x+4-12-4) = -(x^2-4x+4)+16 \\
&= 4^2-(x-2)^2
\end{aligned}$$

Haciendo:  $u = x-2, du = dx; a = 4$ , se tiene:

$$\int \sqrt{12+4x-x^2} dx = \int \sqrt{4^2-(x-2)^2} dx = \int \sqrt{a^2-u^2} du = \frac{1}{2} u \sqrt{a^2-u^2} + \frac{1}{2} a^2 \arcsen \frac{u}{a} + c$$

$$= \frac{1}{2}(x-2)\sqrt{4^2 - (x-2)^2} + \frac{1}{2}4^2 \arcsen \frac{(x-2)}{4} + c$$

$$= \frac{1}{2}(x-2)\sqrt{12+4x-x^2} + 8\arcsen \frac{(x-2)}{4} + c$$

**5.16.-**  $\int \sqrt{x^2 + 4x} dx$

Solución.- Completando cuadrados se tiene:

$$x^2 + 4x = (x^2 + 4x + 4) - 4 = (x+2)^2 - 2^2$$

Haciendo:  $u = x + 2, du = dx; a = 2$ , se tiene:

$$\int \sqrt{x^2 + 4x} dx = \int \sqrt{(x+2)^2 - 2^2} dx = \int \sqrt{u^2 - a^2} du$$

$$= \frac{1}{2}u\sqrt{u^2 - a^2} - \frac{1}{2}a^2 \ell \eta \left| u + \sqrt{u^2 - a^2} \right| + c$$

$$= \frac{1}{2}(x+2)\sqrt{(x+2)^2 - 2^2} - \frac{1}{2}2^2 \ell \eta \left| (x+2) + \sqrt{(x+2)^2 - 2^2} \right| + c$$

$$= \frac{(x+2)}{2}\sqrt{x^2 + 4x} - 2\ell \eta \left| (x+2) + \sqrt{x^2 + 4x} \right| + c$$

**5.17.-**  $\int \sqrt{x^2 - 8x} dx$

Solución.- Completando cuadrados se tiene:

$$x^2 - 8x = (x^2 - 8x + 16) - 16 = (x-4)^2 - 4^2$$

Haciendo:  $u = x - 4, du = dx; a = 4$ , se tiene:

$$\int \sqrt{(x-4)^2 - 4^2} dx = \int \sqrt{u^2 - a^2} du = \frac{1}{2}u\sqrt{u^2 - a^2} - \frac{1}{2}a^2 \ell \eta \left| u + \sqrt{u^2 - a^2} \right| + c$$

$$= \frac{1}{2}(x-4)\sqrt{(x-4)^2 - 4^2} - \frac{1}{2}4^2 \ell \eta \left| (x-4) + \sqrt{(x-4)^2 - 4^2} \right| + c$$

$$= \frac{(x-4)}{2}\sqrt{x^2 - 8x} - 8\ell \eta \left| (x-4) + \sqrt{x^2 - 8x} \right| + c$$

**5.18.-**  $\int \sqrt{6x - x^2} dx$

Solución.- Completando cuadrados se tiene:

$$6x - x^2 = -(x^2 - 6x) = -(x^2 - 6x + 9 - 9) = -(x^2 - 6x + 9) + 9 = 3^2 - (x-3)^2$$

Haciendo:  $u = x - 3, du = dx; a = 3$ , se tiene:

$$\int \sqrt{6x - x^2} dx = \int \sqrt{3^2 - (x-3)^2} dx = \int \sqrt{a^2 - u^2} du = \frac{1}{2}u\sqrt{a^2 - u^2} + \frac{1}{2}a^2 \arcsen \frac{u}{a} + c$$

$$= \frac{1}{2}(x-3)\sqrt{3^2 - (x-3)^2} + \frac{1}{2}3^2 \arcsen \frac{x-3}{3} + c$$

$$= \frac{(x-3)}{2}\sqrt{6x - x^2} + \frac{9}{2}\arcsen \frac{x-3}{3} + c$$

**5.19.-**  $\int \frac{(5-4x)dx}{\sqrt{12x-4x^2-8}}$

Solución.- Sea:  $u = 12x - 4x^2 - 8, du = (12 - 8x)dx$

$$\begin{aligned}
\int \frac{(5-4x)dx}{\sqrt{12x-4x^2-8}} &= \int \frac{(-4x+5)dx}{\sqrt{12x-4x^2-8}} = \frac{1}{2} \int \frac{2(-4x+5)dx}{\sqrt{12x-4x^2-8}} = \frac{1}{2} \int \frac{(-8x+10)dx}{\sqrt{12x-4x^2-8}} \\
&= \frac{1}{2} \int \frac{(-8x+12-2)dx}{\sqrt{12x-4x^2-8}} = \frac{1}{2} \int \frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}} - \int \frac{dx}{\sqrt{12x-4x^2-8}} \\
&= \frac{1}{2} \int \frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}} - \int \frac{dx}{\sqrt{4(3x-x^2-2)}} = \frac{1}{2} \int \frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}} - \frac{1}{2} \int \frac{dx}{\sqrt{3x-x^2-2}}
\end{aligned}$$

Completando cuadrados se tiene:

$$\begin{aligned}
3x-x^2-2 &= -(x^2-3x+2) = -(x^2-3x+\frac{9}{4}-\frac{9}{4}+2) = -(x^2-3x+\frac{9}{4})+\frac{9}{4}-2 \\
&= -(x-\frac{3}{2})^2+\frac{1}{4} = (\frac{1}{2})^2-(x-\frac{3}{2})^2 \\
&= \frac{1}{2} \int \frac{(-8x+12)dx}{\sqrt{12x-4x^2-8}} - \frac{1}{2} \int \frac{dx}{\sqrt{(\frac{1}{2})^2-(x-\frac{3}{2})^2}}
\end{aligned}$$

Haciendo:  $u = 12x-4x^2-8$ ,  $du = (12-8x)dx$  y  $w = x-\frac{3}{2}$ ,  $dw = dx$ , entonces:

$$\begin{aligned}
&= \frac{1}{2} \int \frac{du}{\sqrt{u}} - \frac{1}{2} \int \frac{dw}{\sqrt{(\frac{1}{2})^2-w^2}} = \frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2} \arcsen \frac{w}{\frac{1}{2}} + c \\
&= u^{\frac{1}{2}} - \frac{1}{2} \arcsen 2w + c = \sqrt{12x-4x^2-8} - \frac{1}{2} \arcsen(2x-3) + c
\end{aligned}$$

**5.20.-**  $\int \frac{xdx}{\sqrt{27+6x-x^2}}$

Solución.- Sea:  $u = 27+6x-x^2$ ,  $du = (6-2x)dx$

$$\begin{aligned}
\int \frac{xdx}{\sqrt{27+6x-x^2}} &= -\frac{1}{2} \int \frac{-2xdx}{\sqrt{27+6x-x^2}} = -\frac{1}{2} \int \frac{(-2x+6-6)dx}{\sqrt{27+6x-x^2}} \\
&= -\frac{1}{2} \int \frac{(-2x+6)dx}{\sqrt{27+6x-x^2}} + 3 \int \frac{dx}{\sqrt{27+6x-x^2}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}} + 3 \int \frac{dx}{\sqrt{27+6x-x^2}}
\end{aligned}$$

Completando cuadrados se tiene:

$$\begin{aligned}
27+6x-x^2 &= -(x^2-6x-27) = -(x^2-6x+9-9-27) = -(x^2-6x+9)+36 \\
&= 6^2-(x-3)^2, \text{ Luego:}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{2} \int u^{-\frac{1}{2}} du + 3 \int \frac{dx}{\sqrt{6^2-(x-3)^2}} = -\frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + 3 \arcsen \frac{x-3}{6} + c \\
&= -u^{\frac{1}{2}} + 3 \arcsen \frac{x-3}{6} + c = -\sqrt{27+6x-x^2} + 3 \arcsen \frac{x-3}{6} + c
\end{aligned}$$

**5.21.-**  $\int \frac{(x-1)dx}{3x^2-4x+3}$

Solución.- Sea:  $u = 3x^2-4x+3$ ,  $du = (6x-4)dx$

$$\int \frac{(x-1)dx}{3x^2-4x+3} = \frac{1}{6} \int \frac{(6x-6)dx}{3x^2-4x+3} = \frac{1}{6} \int \frac{(6x-4-2)dx}{3x^2-4x+3} = \frac{1}{6} \int \frac{(6x-4)dx}{3x^2-4x+3} - \frac{1}{3} \int \frac{dx}{3x^2-4x+3}$$

$$= \frac{1}{6} \int \frac{du}{u} - \frac{1}{3} \int \frac{dx}{3x^2 - 4x + 3} = \frac{1}{6} \int \frac{du}{u} - \frac{1}{3} \int \frac{dx}{3(x^2 - \frac{4}{3}x + 1)}$$

$$= \frac{1}{6} \int \frac{du}{u} - \frac{1}{9} \int \frac{dx}{(x^2 - \frac{4}{3}x + 1)}$$

Completando cuadrados se tiene:

$$x^2 - \frac{4}{3}x + 1 = (x^2 - \frac{4}{3}x + \frac{4}{9}) + 1 - \frac{4}{9} = (x^2 - \frac{4}{3}x + \frac{4}{9}) + \frac{5}{9} = (x - \frac{2}{3})^2 + (\frac{\sqrt{5}}{3})^2$$

$$= \frac{1}{6} \int \frac{du}{u} - \frac{1}{9} \int \frac{dx}{(x - \frac{2}{3})^2 + (\frac{\sqrt{5}}{3})^2} = \frac{1}{6} \ell \eta |u| - \frac{1}{9} \frac{1}{\frac{\sqrt{5}}{3}} \operatorname{arc} \tau g \frac{x - \frac{2}{3}}{\frac{\sqrt{5}}{3}} + c$$

$$= \frac{1}{6} \ell \eta |3x^2 - 4x + 3| - \frac{\sqrt{5}}{15} \operatorname{arc} \tau g \frac{3x - 2}{\sqrt{5}} + c$$

**5.22.-**  $\int \frac{(2x-3)dx}{x^2+6x+15}$

Solución.- Sea:  $u = x^2 + 6x + 15, du = (2x + 6)dx$

$$\int \frac{(2x-3)dx}{x^2+6x+15} = \int \frac{(2x+6-9)dx}{x^2+6x+15} = \int \frac{(2x+6)dx}{x^2+6x+15} - 9 \int \frac{dx}{x^2+6x+15}$$

$$= \int \frac{du}{u} - 9 \int \frac{dx}{x^2+6x+15}, \text{ Completando cuadrados se tiene:}$$

$$x^2 + 6x + 15 = (x^2 + 6x + 9) + 15 - 9 = (x + 3)^2 + 6^2 = (x + 3)^2 + (\sqrt{6})^2$$

$$= \int \frac{du}{u} - 9 \int \frac{dx}{(x+3)^2 + (\sqrt{6})^2} = \ell \eta |x^2 + 6x + 15| - 9 \frac{1}{\sqrt{6}} \operatorname{arc} \tau g \frac{x+3}{\sqrt{6}} + c$$

$$= \ell \eta |x^2 + 6x + 15| - \frac{3\sqrt{6}}{2} \operatorname{arc} \tau g \frac{x+3}{\sqrt{6}} + c$$

**5.23.-**  $\int \frac{dx}{4x^2+4x+10}$

Solución.-

$$\int \frac{dx}{4x^2+4x+10} = \int \frac{dx}{4(x^2+x+\frac{5}{2})} = \frac{1}{4} \int \frac{dx}{(x^2+x+\frac{5}{2})}, \text{ Completando cuadrados:}$$

$$x^2 + x + \frac{5}{2} = (x^2 + x + \frac{1}{4}) + \frac{5}{2} - \frac{1}{4} = (x + \frac{1}{2})^2 + \frac{9}{4} = (x + \frac{1}{2})^2 + (\frac{3}{2})^2$$

$$= \frac{1}{4} \int \frac{dx}{(x + \frac{1}{2})^2 + (\frac{3}{2})^2} = \frac{1}{4} \frac{1}{\frac{3}{2}} \operatorname{arc} \tau g \frac{x + \frac{1}{2}}{\frac{3}{2}} + c = \frac{1}{6} \operatorname{arc} \tau g \frac{2x+1}{3} + c$$

**5.24.-**  $\int \frac{(2x+2)dx}{x^2-4x+9}$

Solución.- Sea:  $u = x^2 - 4x + 9, du = (2x - 4)dx$

$$\begin{aligned}
\int \frac{(2x+2)dx}{x^2-4x+9} &= \int \frac{(2x-4+6)dx}{x^2-4x+9} = \int \frac{(2x-4)dx}{x^2-4x+9} + 6 \int \frac{dx}{x^2-4x+9} \\
&= \int \frac{du}{u} + 6 \int \frac{dx}{x^2-4x+9}, \text{ Completando cuadrados se tiene:} \\
x^2-4x+9 &= (x^2-4x+4)+9-4 = (x-2)^2+5 = (x-2)^2+(\sqrt{5})^2, \\
&= \int \frac{du}{u} + 6 \int \frac{dx}{(x-2)^2+(\sqrt{5})^2} = \ell \eta |u| + 6 \frac{1}{\sqrt{5}} \operatorname{arc} \tau g \frac{x-2}{\sqrt{5}} + c \\
&= \ell \eta |x^2-4x+9| + \frac{6\sqrt{5}}{5} \operatorname{arc} \tau g \frac{x-2}{\sqrt{5}} + c
\end{aligned}$$

**5.25.-**  $\int \frac{(2x+4)dx}{\sqrt{4x-x^2}}$

Solución.- Sea:  $u = 4x - x^2 + 9, du = (4 - 2x)dx$

$$\begin{aligned}
\int \frac{(2x+4)dx}{\sqrt{4x-x^2}} &= -\int \frac{(-2x-4)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x+4-8)dx}{\sqrt{4x-x^2}} = -\int \frac{(-2x+4)dx}{\sqrt{4x-x^2}} + 8 \int \frac{dx}{\sqrt{4x-x^2}} \\
&= -\int u^{-1/2} du + 8 \int \frac{dx}{\sqrt{4x-x^2}}, \text{ Completando cuadrados se tiene:} \\
4x-x^2 &= -(x^2-4x) = -(x^2-4x+4-4) = -(x^2-4x+4)+4 = 2^2-(x-2)^2 \\
&= -\int u^{-1/2} du + 8 \int \frac{dx}{\sqrt{2^2-(x-2)^2}} = -2u^{1/2} + 8 \operatorname{arcsen} \frac{x-2}{2} + c \\
&= -2\sqrt{4x-x^2} + 8 \operatorname{arcsen} \frac{x-2}{2} + c
\end{aligned}$$

**5.26.-**  $\frac{2}{3} \int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8}$

Solución.- Sea:  $u = 9x^2 - 12x + 8, du = (18x - 12)dx$

$$\begin{aligned}
\frac{2}{3} \int \frac{(x+\frac{3}{2})dx}{9x^2-12x+8} &= \frac{2}{3} \frac{1}{18} \int \frac{(18x+27)dx}{9x^2-12x+8} = \frac{1}{27} \int \frac{(18x+27)dx}{9x^2-12x+8} = \frac{1}{27} \int \frac{(18x-12+39)dx}{9x^2-12x+8} \\
&= \frac{1}{27} \int \frac{(18x-12)dx}{9x^2-12x+8} + \frac{39}{27} \int \frac{dx}{9x^2-12x+8} = \frac{1}{27} \int \frac{du}{u} + \frac{39}{27} \int \frac{dx}{9(x^2-\frac{4}{3}x+\frac{8}{9})} \\
&= \frac{1}{27} \int \frac{du}{u} + \frac{39}{27 \times 9} \int \frac{dx}{(x^2-\frac{4}{3}x+\frac{8}{9})}
\end{aligned}$$

Completando cuadrados se tiene:

$$\begin{aligned}
x^2-\frac{4}{3}x+\frac{8}{9} &= (x^2-\frac{4}{3}x+\frac{4}{9})+\frac{8}{9}-\frac{4}{9} = (x-\frac{2}{3})^2+\frac{4}{9} = (x-\frac{2}{3})^2+(\frac{2}{3})^2 \\
&= \frac{1}{27} \int \frac{du}{u} + \frac{39}{27 \times 9} \int \frac{dx}{(x-\frac{2}{3})^2+(\frac{2}{3})^2} = \frac{1}{27} \ell \eta |u| + \frac{39}{27 \times 9} \frac{1}{\frac{2}{3}} \operatorname{arc} \tau g \frac{x-\frac{2}{3}}{\frac{2}{3}} + c
\end{aligned}$$

$$= -\frac{1}{27} \ell \eta |9x^2 - 12x + 8| - \frac{13}{54} \operatorname{arc} \tau g \frac{3x-2}{2} + c$$

$$\mathbf{5.27.-} \int \frac{(x+6)dx}{\sqrt{5-4x-x^2}}$$

Solución.- Sea:  $u = 5 - 4x - x^2$ ,  $du = (-4 - 2x)dx$

$$\begin{aligned} \int \frac{(x+6)dx}{\sqrt{5-4x-x^2}} &= -\frac{1}{2} \int \frac{(-2x-12)dx}{\sqrt{5-4x-x^2}} = -\frac{1}{2} \int \frac{(-2x-4-8)dx}{\sqrt{5-4x-x^2}} \\ &= -\frac{1}{2} \int \frac{(-2x-4)dx}{\sqrt{5-4x-x^2}} + 4 \int \frac{dx}{\sqrt{5-4x-x^2}} = -\frac{1}{2} \int \frac{du}{\sqrt{u}} + 4 \int \frac{dx}{\sqrt{5-4x-x^2}} \end{aligned}$$

Completando cuadrados se tiene:  $5 - 4x - x^2 = 9 - (x+2)^2 = 3^2 - (x+2)^2$

$$\begin{aligned} &= -\frac{1}{2} \int \frac{du}{\sqrt{u}} + 4 \int \frac{dx}{\sqrt{3^2 - (x+2)^2}} = -\sqrt{u} + 4 \operatorname{arcs} e n \frac{x+2}{3} + c \\ &= -\sqrt{5-4x-x^2} + 4 \operatorname{arcs} e n \frac{x+2}{3} + c \end{aligned}$$

$$\mathbf{5.28.-} \int \frac{dx}{2x^2 + 20x + 60}$$

Solución.-

$$\int \frac{dx}{2x^2 + 20x + 60} = \frac{1}{2} \int \frac{dx}{x^2 + 10x + 30}, \text{ Completando cuadrados se tiene:}$$

$$x^2 + 10x + 30 = (x^2 + 10x + 25) + 5 = (x+5)^2 + (\sqrt{5})^2$$

$$= \frac{1}{2} \int \frac{dx}{(x+5)^2 + (\sqrt{5})^2} = \frac{1}{2} \frac{1}{\sqrt{5}} \operatorname{arc} \tau g \frac{x+5}{\sqrt{5}} + c = \frac{\sqrt{5}}{10} \operatorname{arc} \tau g \frac{x+5}{\sqrt{5}} + c$$

$$\mathbf{5.29.-} \int \frac{3dx}{\sqrt{80+32x-4x^2}}$$

Solución.-

$$\int \frac{3dx}{\sqrt{80+32x-4x^2}} = \int \frac{3dx}{\sqrt{4(20+8x-x^2)}} = \frac{3}{2} \int \frac{dx}{\sqrt{(20+8x-x^2)}}$$

Completando cuadrados se tiene:

$$20+8x-x^2 = -(x^2-8x-20) = -(x^2-8x+16-20-16) = -(x^2-8x+16)+36$$

$$= -(x-4)^2 + 6^2 = 6^2 - (x-4)^2$$

$$= \frac{3}{2} \int \frac{dx}{\sqrt{6^2 - (x-4)^2}} = \frac{3}{2} \operatorname{arcs} e n \frac{x-4}{6} + c$$

$$\mathbf{5.30.-} \int \frac{dx}{\sqrt{12x-4x^2-8}}$$

Solución.-

$$\int \frac{dx}{\sqrt{12x-4x^2-8}} = \int \frac{dx}{\sqrt{4(-x^2+3x-2)}} = \frac{1}{2} \int \frac{dx}{\sqrt{(-x^2+3x-2)}}$$

Completando cuadrados se tiene:

$$\begin{aligned}
-x^2 + 3x - 2 &= -(x^2 - 3x + 2) = -(x^2 - 3x + \frac{9}{4} + 2 - \frac{9}{4}) = -(x^2 - 3x + \frac{9}{4}) + \frac{1}{4} \\
&= (\frac{1}{2})^2 - (x - \frac{3}{2})^2 \\
&= \frac{1}{2} \int \frac{dx}{\sqrt{(\frac{1}{2})^2 - (x - \frac{3}{2})^2}} = \frac{1}{2} \arcsen \frac{x - \frac{3}{2}}{\frac{1}{2}} + c = \frac{1}{2} \arcsen(2x - 3) + c
\end{aligned}$$

**5.31.-**  $\int \frac{5dx}{\sqrt{28-12x-x^2}}$

Solución.-

$$\begin{aligned}
\int \frac{5dx}{\sqrt{28-12x-x^2}} &= 5 \int \frac{dx}{\sqrt{28-12x-x^2}}, \text{ Completando cuadrados se tiene:} \\
28-12x-x^2 &= 8^2 - (x+6)^2 \\
&= 5 \int \frac{dx}{\sqrt{8^2 - (x+6)^2}} = 5 \arcsen \frac{x+6}{8} + c
\end{aligned}$$

**5.32.-**  $\int \sqrt{12-8x-4x^2} dx$

Solución.- Sea:  $u = x+1, du = dx; a = 2$

$$\int \sqrt{12-8x-4x^2} dx = \int \sqrt{4(3-2x-x^2)} dx = 2 \int \sqrt{3-2x-x^2} dx$$

Completando cuadrados se tiene:

$$\begin{aligned}
3-2x-x^2 &= -(x^2 + 2x - 3) = -(x^2 + 2x + 1) + 4 = 2^2 - (x+1)^2 \\
2 \int \sqrt{2^2 - (x+1)^2} dx &= 2 \int \sqrt{a^2 - u^2} du = 2 \left( \frac{1}{2} u \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsen \frac{u}{a} \right) + c \\
&= (x+1) \sqrt{-x^2 - 2x + 3} + 4 \arcsen \frac{x+1}{2} + c
\end{aligned}$$

**5.33.-**  $\int \sqrt{x^2 - x + \frac{5}{4}} dx$

Solución.- Sea:  $u = x - \frac{1}{2}, du = dx; a = 1$

Completando cuadrados se tiene:

$$\begin{aligned}
x^2 - x + \frac{5}{4} &= (x - \frac{1}{2})^2 + 1 \\
\sqrt{x^2 - x + \frac{5}{4}} dx &= \sqrt{(x - \frac{1}{2})^2 + 1} dx = \sqrt{u^2 + a^2} du \\
&= \frac{1}{2} u \sqrt{u^2 + a^2} + \frac{1}{2} a^2 \ell \eta \left| u + \sqrt{u^2 + a^2} \right| + c \\
&= \frac{1}{2} (x - \frac{1}{2}) \sqrt{x^2 - x + \frac{5}{4}} + \frac{1}{2} \ell \eta \left| x - \frac{1}{2} + \sqrt{x^2 - x + \frac{5}{4}} \right| + c \\
&= \frac{1}{4} (2x-1) \sqrt{x^2 - x + \frac{5}{4}} + \frac{1}{2} \ell \eta \left| x - \frac{1}{2} + \sqrt{x^2 - x + \frac{5}{4}} \right| + c
\end{aligned}$$

**5.34.-**  $\int \frac{dx}{x^2 - 2x + 5}$



Solución.- Completando cuadrados se tiene:

$$x^2 - 2x + 5 = (x^2 - 2x + 4) + 1 = (x - 2)^2 + 1$$

$$\int \frac{dx}{x^2 - 2x + 5} = \int \frac{dx}{(x - 2)^2 + 1} = \operatorname{arctg}(x - 2) + c$$

**5.35.-**  $\int \frac{(1-x)dx}{\sqrt{8+2x-x^2}}$

Solución.- Sea:  $u = 8 + 2x - x^2$ ,  $du = (2 - 2x)dx = 2(1 - x)dx$

$$\int \frac{(1-x)dx}{\sqrt{8+2x-x^2}} = \frac{1}{2} \int \frac{du}{\sqrt{u}} = \frac{1}{2} \int u^{-1/2} du = \sqrt{u} + c = \sqrt{8+2x-x^2} + c$$

**5.36.-**  $\int \frac{xdx}{x^2 + 4x + 5}$

Solución.- Sea:  $u = x^2 + 4x + 5$ ,  $du = (2x + 4)dx$

$$\begin{aligned} \int \frac{xdx}{x^2 + 4x + 5} &= \frac{1}{2} \int \frac{2xdx}{x^2 + 4x + 5} = \frac{1}{2} \int \frac{(2x + 4) - 4}{x^2 + 4x + 5} dx \\ &= \frac{1}{2} \int \frac{(2x + 4)dx}{x^2 + 4x + 5} - 2 \int \frac{dx}{x^2 + 4x + 5} = \frac{1}{2} \int \frac{du}{u} - 2 \int \frac{dx}{x^2 + 4x + 5}, \text{ Completando cuadrados se} \end{aligned}$$

tiene:  $x^2 + 4x + 5 = (x^2 + 4x + 4) + 1 = (x + 2)^2 + 1$

$$= \frac{1}{2} \int \frac{du}{u} - 2 \int \frac{dx}{(x + 2)^2 + 1} = \frac{1}{2} \ell \eta |u| - 2 \operatorname{arctg}(x + 2) + c$$

$$= \frac{1}{2} \ell \eta |x^2 + 4x + 5| - 2 \operatorname{arctg}(x + 2) + c$$

**5.37.-**  $\int \frac{(2x+3)dx}{4x^2 + 4x + 5}$

Solución.- Sea:  $u = 4x^2 + 4x + 5$ ,  $du = (8x + 4)dx$

$$\begin{aligned} \int \frac{(2x+3)dx}{4x^2 + 4x + 5} &= \frac{1}{4} \int \frac{(8x+12)dx}{4x^2 + 4x + 5} = \frac{1}{4} \int \frac{(8x+4) + 8}{4x^2 + 4x + 5} dx \\ &= \frac{1}{4} \int \frac{(8x+4)dx}{4x^2 + 4x + 5} + 2 \int \frac{dx}{4x^2 + 4x + 5} = \frac{1}{4} \int \frac{du}{u} + 2 \int \frac{dx}{4(x^2 + x + 5/4)} \end{aligned}$$

$$= \frac{1}{4} \int \frac{du}{u} + \frac{1}{2} \int \frac{dx}{(x^2 + x + 5/4)}, \text{ Completando cuadrados se tiene:}$$

$$x^2 + x + \frac{5}{4} = (x^2 + x + \frac{1}{4}) + 1 = (x + \frac{1}{2})^2 + 1$$

$$= \frac{1}{4} \int \frac{du}{u} + \frac{1}{2} \int \frac{dx}{(x + \frac{1}{2})^2 + 1} = \frac{1}{4} \ell \eta |u| + \frac{1}{2} \operatorname{arctg}(x + \frac{1}{2}) + c$$

**5.38.-**  $\int \frac{(x+2)dx}{x^2 + 2x + 2}$

Solución.- Sea:  $u = x^2 + 2x + 2$ ,  $du = (2x + 2)dx$

$$\begin{aligned}
\int \frac{(x+2)dx}{x^2+2x+2} &= \frac{1}{2} \int \frac{(2x+4)dx}{x^2+2x+2} = \frac{1}{2} \int \frac{(2x+2)+2}{x^2+2x+2} dx = \frac{1}{2} \int \frac{(2x+2)dx}{x^2+2x+2} + \int \frac{dx}{x^2+2x+2} \\
&= \frac{1}{2} \int \frac{du}{u} + \int \frac{dx}{x^2+2x+2} = \frac{1}{2} \int \frac{du}{u} + \int \frac{dx}{(x+1)^2+1} \\
&= \frac{1}{2} \ell \eta |u| + \arctan g(x+1) + c = \frac{1}{2} \ell \eta |x^2+2x+2| + \arctan g(x+1) + c
\end{aligned}$$

**5.39.-**  $\int \frac{(2x+1)dx}{x^2+8x-2}$

Solución.- Sea:  $u = x^2 + 8x - 2, du = (2x+8)dx$

$$\begin{aligned}
\int \frac{(2x+1)dx}{x^2+8x-2} &= \int \frac{(2x+8)-7dx}{x^2+8x-2} = \int \frac{(2x+8)dx}{x^2+8x-2} - 7 \int \frac{dx}{x^2+8x-2} \\
&= \int \frac{du}{u} - 7 \int \frac{dx}{(x^2+8x+16)-18} = \int \frac{du}{u} - 7 \int \frac{dx}{(x+4)^2 - (3\sqrt{2})^2} \\
&= \ell \eta |u| - 7 \frac{1}{2(3\sqrt{2})} \ell \eta \left| \frac{(x+4) - (3\sqrt{2})}{(x+4) + (3\sqrt{2})} \right| + c \\
&= \ell \eta |x^2+8x-2| - \frac{7\sqrt{2}}{12} \ell \eta \left| \frac{(x+4) - (3\sqrt{2})}{(x+4) + (3\sqrt{2})} \right| + c
\end{aligned}$$

**5.40.-**  $\int \frac{dx}{\sqrt{-x^2-6x}}$

Solución.- Completando cuadrados se tiene:

$$-x^2 - 6x = -(x^2 + 6x) = -(x^2 + 6x + 9) + 9 = 3^2 - (x+3)^2$$

$$\int \frac{dx}{\sqrt{3^2 - (x+3)^2}} = \arcsen \frac{x+3}{3} + c$$

**5.41.-**  $\int \frac{(x-1)dx}{x^2+2x+2}$

Solución.- Sea:  $u = x^2 + 2x + 2, du = (2x+2)dx$

$$\begin{aligned}
\int \frac{(x-1)dx}{x^2+2x+2} &= \frac{1}{2} \int \frac{(2x+2)-4}{x^2+2x+2} dx = \frac{1}{2} \int \frac{(2x+2)dx}{x^2+2x+2} - 2 \int \frac{dx}{x^2+2x+2} \\
&= \frac{1}{2} \int \frac{du}{u} - 2 \int \frac{dx}{x^2+2x+2} = \frac{1}{2} \int \frac{du}{u} - 2 \int \frac{dx}{(x+1)^2+1} = \frac{1}{2} \ell \eta |u| - 2 \arctan g(x+1) + c \\
&= \frac{1}{2} \ell \eta |x^2+2x+2| - 2 \arctan g(x+1) + c
\end{aligned}$$

## CAPITULO 6

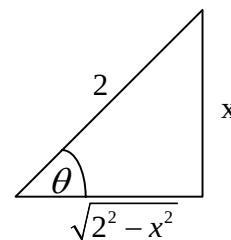
### INTEGRACION POR SUSTITUCION TRIGONOMETRICA

Existen integrales que contienen expresiones de las formas:  $a^2 - x^2$ ,  $a^2 + x^2$ ,  $x^2 - a^2$ , las que tienen fácil solución si se hace la sustitución trigonométrica adecuada. A saber, si la expresión es:  $a^2 - x^2$ , la sustitución adecuada es:  $x = a \operatorname{sen} \theta$  ó  $x = a \cos \theta$ . Si la expresión es:  $a^2 + x^2$ , entonces:  $x = a \sec \theta$

### EJERCICIOS DESARROLLADOS

**1. Encontrar:**  $\int \frac{dx}{\sqrt{(4-x^2)^3}}$

Solución.- Dada la expresión:  $4-x^2$ , la forma es:  $a^2 - x^2$ , la sustitución adecuada es:  $x = a \operatorname{sen} \theta$  o sea:  $x = 2 \operatorname{sen} \theta \therefore dx = 2 \cos \theta d\theta$ . Además:  $\operatorname{sen} \theta = \frac{x}{a}$ . Una figura auxiliar adecuada para ésta situación, es:



$$\begin{aligned} \int \frac{dx}{\sqrt{(4-x^2)^3}} &= \int \frac{dx}{\sqrt{(2^2-x^2)^3}} = \int \frac{2 \cos \theta d\theta}{\sqrt{(2^2-2^2 \operatorname{sen}^2 \theta)^3}} = \int \frac{2 \cos \theta d\theta}{\sqrt{[2^2(1-\operatorname{sen}^2 \theta)]^3}} \\ &= \int \frac{2 \cos \theta d\theta}{\sqrt{(2^2 \cos^2 \theta)^3}} = \int \frac{2 \cos \theta d\theta}{(2 \cos \theta)^3} = \int \frac{2 \cos \theta d\theta}{2^3 \cos^3 \theta} = \frac{1}{2^2} \int \frac{d\theta}{\cos^2 \theta} = \frac{1}{4} \int \sec^2 \theta d\theta \\ &= \frac{1}{4} \int \sec^2 \theta d\theta = \frac{1}{4} \tau g \theta + c. \end{aligned}$$

A partir de la figura triangular se tiene:

$$\tau g \theta = \frac{x}{\sqrt{4-x^2}}, \text{ Luego: } \frac{1}{4} \tau g \theta + c = \frac{1}{4} \frac{x}{\sqrt{4-x^2}} + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt{(4-x^2)^3}} = \frac{1}{4} \frac{x}{\sqrt{4-x^2}} + c$

**6.2.-Encontrar:**  $\int \frac{\sqrt{25-x^2}}{x} dx$

Solución.-

$$\int \frac{\sqrt{25-x^2}}{x} dx = \int \frac{\sqrt{5^2-x^2}}{x} dx, \text{ la forma es: } a^2 - x^2, \text{ luego:}$$

$$\text{Sea: } x = 5 \operatorname{sen} \theta \therefore dx = 5 \cos \theta d\theta, \sqrt{5^2 - x^2} = 5 \cos \theta$$

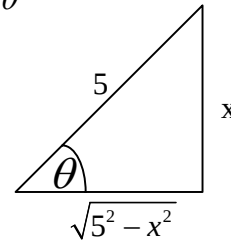
$$\text{Además: } \operatorname{sen} \theta = \frac{x}{5}$$

$$\int \frac{\sqrt{5^2-x^2}}{x} dx = \int \frac{\cancel{5} \cos \theta \cancel{5} \cos \theta d\theta}{\cancel{5} \operatorname{sen} \theta} = 5 \int \frac{\cos^2 \theta d\theta}{\operatorname{sen} \theta} = 5 \int \frac{(1-\operatorname{sen}^2 \theta) d\theta}{\operatorname{sen} \theta}$$

$$= 5 \int \frac{d\theta}{\operatorname{sen} \theta} - 5 \int \operatorname{sen} \theta d\theta = 5 \int \operatorname{cosec} \theta - 5 \int \operatorname{sen} \theta d\theta$$

$$= 5 \ell \eta |\operatorname{cosec} \theta - \operatorname{cosec} \theta| + 5 \cos \theta + c.$$

De la figura se tiene:



$$\operatorname{cosec} \theta = \frac{5}{x}, \operatorname{cosec} \theta = \frac{\sqrt{25-x^2}}{x}, \text{ luego:}$$

$$= 5 \ell \eta \left| \frac{5}{x} - \frac{\sqrt{25-x^2}}{x} \right| + \cancel{5} \frac{\sqrt{25-x^2}}{\cancel{5}} + c = 5 \ell \eta \left| \frac{5-\sqrt{25-x^2}}{x} \right| + \sqrt{25-x^2} + c$$

$$\text{Respuesta: } \int \frac{\sqrt{25-x^2}}{x} dx = 5 \ell \eta \left| \frac{5-\sqrt{25-x^2}}{x} \right| + \sqrt{25-x^2} + c$$

$$\text{6.3.-Encontrar: } \int \frac{dx}{\sqrt{(4x-x^2)^3}}$$

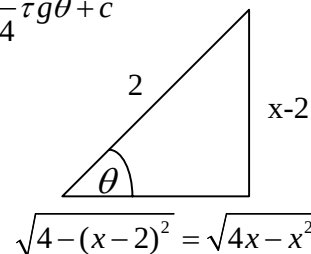
$$\text{Solución.- } 4x - x^2 = -(x^2 - 4x) = -(x^2 - 4x + 4 - 4) = 4 - (x^2 - 4x + 4) = 2^2 - (x-2)^2$$

$$\int \frac{dx}{\sqrt{(4x-x^2)^3}} = \int \frac{dx}{(\sqrt{2^2 - (x-2)^2})^3}, \text{ la forma es: } a^2 - u^2,$$

$$\text{Luego: } x-2 = 2 \operatorname{sen} \theta \therefore dx = 2 \cos \theta d\theta, \sqrt{2^2 - (x-2)^2} = 2 \cos \theta$$

$$\text{Además: } \operatorname{sen} \theta = \frac{x-2}{2}$$

$$\int \frac{dx}{(\sqrt{2^2 - (x-2)^2})^3} = \int \frac{2 \cos \theta d\theta}{2^3 \cos^3 \theta} = \frac{1}{4} \int \frac{d\theta}{\cos^2 \theta} = \frac{1}{4} \int \sec^2 \theta d\theta = \frac{1}{4} \tau g \theta + c$$



De la figura se tiene:

$$\text{Pero: } \tau g \theta = \frac{x-2}{\sqrt{4x-x^2}}, \text{ luego: } \frac{1}{4} \tau g \theta + c = \frac{x-2}{4\sqrt{4x-x^2}} + c$$

$$\text{Respuesta: } \int \frac{dx}{\sqrt{(4x-x^2)^3}} = \frac{x-2}{4\sqrt{4x-x^2}} + c$$

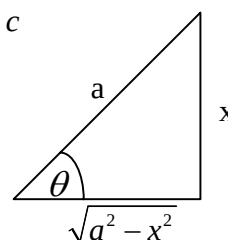
**6.4.-Encontrar:**  $\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}}$

Solución.-

$$\int \frac{x^2 dx}{(a^2 - x^2)^{3/2}} = \int \frac{x^2 dx}{(\sqrt{a^2 - x^2})^3}, \text{ la forma es: } a^2 - x^2$$

Luego:  $x = a \operatorname{sen} \theta, dx = a \cos \theta, \sqrt{a^2 - x^2} = a \cos \theta$ , además:  $\operatorname{sen} \theta = \frac{x}{a}$

$$\begin{aligned} \int \frac{x^2 dx}{(\sqrt{a^2 - x^2})^3} &= \int \frac{a^2 \operatorname{sen}^2 \theta a \cos \theta d\theta}{(a \cos \theta)^3} = \int \frac{\cancel{a^2} \operatorname{sen}^2 \theta \cancel{a} \cos \theta d\theta}{\cancel{a^2} \cancel{\cos \theta} \cos^2 \theta} = \int \frac{\operatorname{sen}^2 \theta d\theta}{\cos^2 \theta} \\ &= \int \frac{(1 - \cos^2 \theta) d\theta}{\cos^2 \theta} = \int \frac{d\theta}{\cos^2 \theta} - \int d\theta = \int \sec^2 \theta d\theta - \int d\theta = \operatorname{tg} \theta - \theta + c \end{aligned}$$



De la figura se tiene:

Pero:  $\operatorname{tg} \theta = \frac{x}{\sqrt{a^2 - x^2}}$ , además:  $\operatorname{sen} \theta = \frac{x}{a}$  y  $\theta = \arcsen \frac{x}{a}$

Luego:  $\operatorname{tg} \theta - \theta + c = \frac{x}{\sqrt{a^2 - x^2}} - \arcsen \frac{x}{a} + c$

**Respuesta:**  $\int \frac{x^2 dx}{\sqrt{(a^2 - x^2)^3}} = \frac{x}{\sqrt{a^2 - x^2}} - \arcsen \frac{x}{a} + c$

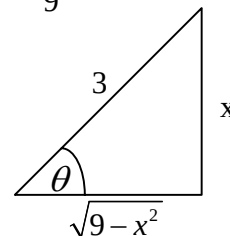
**6.5.-Encontrar:**  $\int \frac{dx}{x^2 \sqrt{9 - x^2}}$

Solución.-

$$\int \frac{dx}{x^2 \sqrt{9 - x^2}} = \int \frac{dx}{x^2 \sqrt{3^2 - x^2}}, \text{ la forma es: } a^2 - x^2$$

Luego:  $x = 3 \operatorname{sen} \theta, dx = 3 \cos \theta d\theta, \sqrt{3^2 - x^2} = 3 \cos \theta$ , además:  $\operatorname{sen} \theta = \frac{x}{3}$

$$\int \frac{dx}{x^2 \sqrt{3^2 - x^2}} = \int \frac{\cancel{3} \cos \theta d\theta}{3^2 \operatorname{sen}^2 \theta \cancel{3} \cos \theta} = \frac{1}{9} \int \frac{d\theta}{\operatorname{sen}^2 \theta} = \frac{1}{9} \int \csc^2 \theta d\theta = -\frac{1}{9} \cot \theta + c$$



De la figura se tiene:

Pero:  $\cot \theta = \frac{\sqrt{9-x^2}}{x}$ , luego:  $\frac{1}{9} \cot \theta + c = -\frac{\sqrt{9-x^2}}{9x} + c$

**Respuesta:**  $\int \frac{dx}{x^2 \sqrt{9-x^2}} = -\frac{\sqrt{9-x^2}}{9x} + c$

**6.6.-Encontrar:**  $\int \frac{x^2 dx}{\sqrt{9-x^2}}$

Solución.-

$\int \frac{x^2 dx}{\sqrt{9-x^2}} = \int \frac{x^2 dx}{\sqrt{3^2-x^2}}$ , la forma es:  $a^2 - x^2$

Luego:  $x = 3 \sin \theta$ ,  $dx = 3 \cos \theta d\theta$ ,  $\sqrt{3^2-x^2} = 3 \cos \theta$ , además:  $\sin \theta = \frac{x}{3}$

Usaremos la misma figura anterior, luego:

$\int \frac{x^2 dx}{\sqrt{3^2-x^2}} = \int \frac{3^2 \sin^2 \theta \cdot 3 \cos \theta d\theta}{3 \cos \theta} = 9 \int \sin^2 \theta d\theta = 9 \int \frac{(1-\cos 2\theta) d\theta}{2}$

$\frac{9}{2} \int \theta - \frac{9}{2} \int \cos 2\theta d\theta = \frac{9}{2} \theta - \frac{9}{4} \sin 2\theta + c = \frac{9}{2} \theta - \frac{9}{4} 2 \sin \theta \cos \theta + c$

$= \frac{9}{2} \theta - \frac{9}{2} \sin \theta \cos \theta + c$ , de la figura se tiene que:  $\sin \theta = \frac{x}{3}$ ,  $\cos \theta = \frac{\sqrt{9-x^2}}{3}$  y

$\theta = \arcsin \frac{x}{3}$ , luego es equivalente:

$= \frac{9}{2} \arcsin \frac{x}{3} - \frac{9}{4} \frac{x}{3} \frac{\sqrt{9-x^2}}{3} + c = \frac{9}{2} \left( \arcsin \frac{x}{3} - \frac{\sqrt{9-x^2}}{9} \right) + c$

**Respuesta:**  $\int \frac{x^2 dx}{\sqrt{9-x^2}} = \frac{9}{2} \left( \arcsin \frac{x}{3} - \frac{\sqrt{9-x^2}}{9} \right) + c$

**6.7.-Encontrar:**  $\int \sqrt{x^2-4} dx$

Solución.-

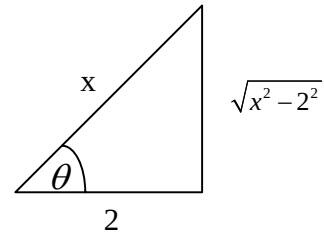
$\int \sqrt{x^2-4} dx = \int \sqrt{x^2-2^2} dx$ , la forma es:  $x^2 - a^2$

Luego:  $x = 2 \sec \theta$ ,  $dx = 2 \sec \theta \tan \theta d\theta$ ,  $\sqrt{x^2-2^2} = 2 \tan \theta$ , además:  $\sec \theta = \frac{x}{2}$

$\int \sqrt{x^2-2^2} dx = \int 2 \tan \theta \cdot 2 \sec \theta \tan \theta d\theta = 4 \int \sec \theta \tan^2 \theta d\theta = 4 \int \sec \theta (\sec^2 \theta - 1) d\theta$   
 $= 4 \int \sec^3 \theta d\theta - 4 \int \sec \theta d\theta$

Se sabe que:  $\int \sec^3 \theta d\theta = \frac{\sec \theta \tan \theta}{2} + \frac{1}{2} \ln |\sec \theta + \tan \theta| + c$ , luego lo anterior es equivalente a:

$$\begin{aligned}
&= 4 \left( \frac{1}{2} \sec \theta \tau g \theta + \frac{1}{2} \ell \eta |\sec \theta + \tau g \theta| \right) - 4 \ell \eta |\sec \theta + \tau g \theta| + c \\
&= 2 \sec \theta \tau g \theta + 2 \ell \eta |\sec \theta + \tau g \theta| - 4 \ell \eta |\sec \theta + \tau g \theta| + c \\
&= 2 \sec \theta \tau g \theta - 2 \ell \eta |\sec \theta + \tau g \theta| + c
\end{aligned}$$



De la figura se tiene:

$$\sec \theta = \frac{x}{2}, \tau g \theta = \frac{\sqrt{x^2 - 4}}{2}, \text{ luego:}$$

$$\begin{aligned}
&= \cancel{2} \frac{x}{\cancel{2}} \frac{\sqrt{x^2 - 4}}{2} - 2 \ell \eta \left| \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2} \right| + c = \frac{x \sqrt{x^2 - 4}}{2} - 2 \ell \eta \left| \frac{x + \sqrt{x^2 - 4}}{2} \right| + c \\
&= \frac{x \sqrt{x^2 - 4}}{2} - 2 \ell \eta \left| x + \sqrt{x^2 - 4} \right| - 2 \ell \eta 2 + c
\end{aligned}$$

$$\text{Respuesta: } \int \sqrt{x^2 - 4} dx = \frac{x \sqrt{x^2 - 4}}{2} - 2 \ell \eta \left| x + \sqrt{x^2 - 4} \right| + c$$

$$\text{6.8.-Encontrar: } \int \frac{x^2 dx}{\sqrt{x^2 - 16}}$$

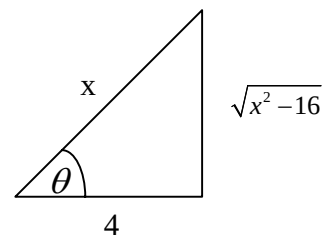
Solución.-

$$\int \frac{x^2 dx}{\sqrt{x^2 - 16}} = \int \frac{x^2 dx}{\sqrt{x^2 - 4^2}}, \text{ la forma es: } x^2 - a^2$$

$$\text{Luego: } x = 4 \sec t, dx = 4 \sec t \tau g t dt, \sqrt{x^2 - 4^2} = 4 \tau g t, \text{ además: } \sec t = \frac{x}{4}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 - 4^2}} = \int \frac{4^2 \sec^2 t (\cancel{4} \sec t \cancel{\tau g t} dt)}{\cancel{4} \tau g t} = 16 \int \sec^3 t dt$$

$$= 16 \left( \frac{1}{2} \sec t \tau g t + \frac{1}{2} \ell \eta |\sec t + \tau g t| + c \right) = 8 \sec t \tau g t + 8 \ell \eta |\sec t + \tau g t| + c$$



De la figura se tiene:

$$\sec t = \frac{x}{4}, \tau g t = \frac{\sqrt{x^2 - 16}}{4}, \text{ luego equivale a:}$$

$$\begin{aligned}
&= 8 \frac{x}{4} \frac{\sqrt{x^2 - 16}}{4} + 8 \ell \eta \left| \frac{x}{4} + \frac{\sqrt{x^2 - 16}}{4} \right| + c = \frac{x}{2} \sqrt{x^2 - 16} + 8 \ell \eta \left| \frac{x + \sqrt{x^2 - 16}}{4} \right| + c \\
&= \frac{x}{2} \sqrt{x^2 - 16} + 8 \ell \eta \left| x + \sqrt{x^2 - 16} \right| - 8 \ell \eta 4 + c = \frac{x}{2} \sqrt{x^2 - 16} + 8 \ell \eta \left| x + \sqrt{x^2 - 16} \right| + c
\end{aligned}$$

**Respuesta:**  $\int \frac{x^2 dx}{\sqrt{x^2 - 16}} = \frac{x}{2} \sqrt{x^2 - 16} + 8 \ln \eta \left| x \sqrt{x^2 - 16} \right| + c$

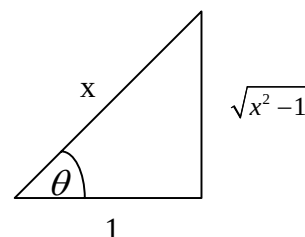
**6.9.-Encontrar:**  $\int \frac{dx}{x\sqrt{x^2 - 1}}$

Solución.-

$$\int \frac{dx}{x\sqrt{x^2 - 1}} = \int \frac{dx}{x\sqrt{x^2 - 1^2}}, \text{ la forma es: } x^2 - a^2$$

Luego:  $x = \sec t, dx = \sec t \tau g t dt, \sqrt{x^2 - 1^2} = \tau g t$ , además:

$$\int \frac{dx}{x\sqrt{x^2 - 1}} = \int \frac{\cancel{\sec t} \tau g t dt}{\cancel{\sec t} \tau g t} = \int dt = t + c,$$



De la figura se tiene:

Dado que:  $\sec t = x \Rightarrow t = \arccos x$ , luego:

$$t + c = \arccos x + c$$

**Respuesta:**  $\int \frac{dx}{x\sqrt{x^2 - 1}} = \arccos x + c$

**6.10.-Encontrar:**  $\int \frac{dx}{(\sqrt{4x^2 - 24x + 27})^3}$

Solución.-

$$\int \frac{dx}{(\sqrt{4x^2 - 24x + 27})^3} = \int \frac{dx}{\sqrt{4(x^2 - 6x + 27/4)}^3} = \int \frac{dx}{\sqrt{4^3} \left( \sqrt{x^2 - 6x + 27/4} \right)^3}$$

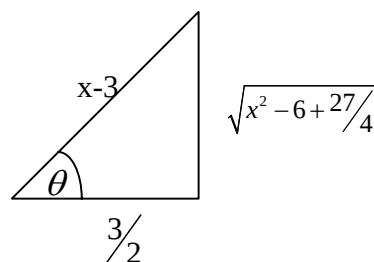
$$= \frac{1}{8} \int \frac{dx}{\sqrt{(x^2 - 6x + 27/4)}^3}, \text{ Se tiene:}$$

$$x^2 - 6x + \frac{27}{4} = (x^2 - 6x + \underline{\quad}) + \frac{27}{4} - \underline{\quad} = (x^2 - 6x + 9) + \frac{27}{4} - 9$$

$$= (x^2 - 6x + 9) - \frac{9}{4} = (x^2 - 6x + 27/4) = (x - 3)^2 - (3/2)^2, \text{ la expresión anterior equivale a:}$$

$$\frac{1}{8} \int \frac{dx}{(\sqrt{x^2 - 6x + 27/4})^3} = \frac{1}{8} \int \frac{dx}{\left[ \sqrt{(x - 3)^2 - (3/2)^2} \right]^3}, \text{ siendo la forma: } u^2 - a^2, \text{ luego:}$$

$$x - 3 = \frac{3}{2} \sec t, dx = \frac{3}{2} \sec t \tau g t dt, \text{ además: } \sec t = \frac{x - 3}{3/2}$$





De la figura se tiene:

$$\sec t = \frac{x}{4}, \tau g t = \frac{\sqrt{x^2 - 16}}{4}, \text{ luego equivale a:}$$

$$\begin{aligned} \frac{1}{8} \int \frac{dx}{\left[ \sqrt{(x-3)^2 - \left(\frac{3}{2}\right)^2} \right]^3} &= \frac{1}{8} \int \frac{\frac{3}{2} \sec t \tau g t dt}{\left(\frac{3}{2}\right)^2 \tau g^3 t} = \frac{1}{8} \frac{1}{\frac{3^2}{2^2}} \int \frac{\sec t dt}{\tau g^2 t} = \frac{1}{18} \int \frac{\frac{1}{\cos t}}{\frac{\sin^2 t}{\cos^2 t}} \\ &= \frac{1}{18} \int \frac{\cos t dt}{(\sin t)^2} = \frac{1}{18} \int (\sin t)^{-2} \cos t dt = \frac{1}{18} \frac{(\sin t)^{-1}}{-1} + c = -\frac{1}{18} \frac{1}{(\sin t)} + c \\ &= -\frac{1}{18} \cos \sec t + c, \text{ como: } \cos \sec t = \frac{x-3}{\sqrt{x^2 - 6x + 27/4}}, \text{ entonces:} \\ &= -\frac{1}{18} \frac{x-3}{\sqrt{x^2 - 6x + 27/4}} + c = -\frac{1}{18} \frac{x-3}{\sqrt{\frac{4x^2 - 24x + 27}{4}}} + c = -\frac{1}{18} \frac{x-3}{\frac{\sqrt{4x^2 - 24x + 27}}{2}} + c \\ &= -\frac{1}{9} \frac{x-3}{\sqrt{4x^2 - 24x + 27}} + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{(\sqrt{4x^2 - 24x + 27})^3} = -\frac{1}{9} \frac{x-3}{\sqrt{4x^2 - 24x + 27}} + c$

**6.11.-Encontrar:**  $\int \frac{dx}{\sqrt{(16+x^2)^4}}$

Solución.-

$$\int \frac{dx}{\sqrt{(16+x^2)^4}} = \int \frac{dx}{\sqrt{(4^2+x^2)^4}}$$

Luego:  $x = 4 \tau g t, dx = 4 \sec^2 t dt, \sqrt{4^2 + x^2} = 4 \sec t$ , además:  $\tau g t = \frac{x}{4}$

$$\begin{aligned} \int \frac{dx}{\sqrt{(4^2+x^2)^4}} &= \int \frac{4 \sec^2 t dt}{4^4 \sec^4 t} = \frac{1}{64} \int \frac{dt}{\sec^2 t} = \frac{1}{64} \int \cos^2 t dt = \frac{1}{64} \int \frac{(1+\cos 2t)}{2} dt \\ &= \frac{1}{128} \int dt + \frac{1}{128} \int \cos 2t dt = \frac{1}{128} t + \frac{1}{256} \sin 2t + c \end{aligned}$$

Como:  $\tau g t = \frac{x}{4} \Rightarrow t = \arctg \frac{x}{4}$ ,  $\sin 2t = 2 \sin t \cos t$ ; luego:

$$\frac{1}{128} t + \frac{1}{256} \sin 2t + c = 2 \frac{x}{\sqrt{16+x^2}} \frac{4}{\sqrt{16+x^2}} = \frac{8x}{16+x^2}, \text{ Se tiene:}$$

$$\frac{1}{128} \arctg \frac{x}{4} + \frac{1}{256} \frac{8x}{16+x^2} + c = \frac{1}{128} \arctg \frac{x}{4} + \frac{x}{32(16+x^2)} + c$$

**Respuesta:**  $\int \frac{dx}{\sqrt{(16+x^2)^4}} = \frac{1}{128} \arctan \frac{x}{4} + \frac{x}{32(16+x^2)} + c$

**6.12.-Encontrar:**  $\int \frac{x^2 dx}{(x^2+100)^{3/2}}$

Solución.-

$$\int \frac{x^2 dx}{(x^2+100)^{3/2}} = \int \frac{x^2 dx}{(\sqrt{x^2+10^2})^3},$$

se tiene:  $x = 10 \tau g t$ ,  $dt = 10 \sec^2 t dt$ ,  $\sqrt{x^2+10^2} = 10 \sec t$ ; además:  $\tau g t = \frac{x}{10}$ , luego:

$$\begin{aligned} \int \frac{x^2 dx}{(\sqrt{x^2+10^2})^3} &= \int \frac{\cancel{10^2} \tau g^2 t (\cancel{10} \sec^2 t) dt}{(\cancel{10^3} \sec^3 t)} = \int \frac{\tau g^2 t dt}{\sec t} = \int \frac{\cancel{\cos^2 t} \cancel{t}}{\cancel{\cos t}} dt = \int \frac{\sin^2 t}{\cos t} dt \\ &= \int \frac{(1-\cos^2 t)}{\cos t} dt = \int \frac{dt}{\cos t} - \int \cos t dt = \int \sec t dt - \int \cos t dt = \ell \eta |\sec t + \tau g t| - \sin t + c \end{aligned}$$

Como:  $\sec t = \frac{\sqrt{100+x^2}}{10}$ ,  $\tau g t = \frac{x}{10}$ , además:  $\sin t = \frac{x}{\sqrt{100+x^2}}$

$$\begin{aligned} &= \ell \eta \left| \frac{\sqrt{100+x^2}}{10} + \frac{x}{10} \right| - \frac{x}{\sqrt{x^2+100}} + c = \ell \eta \left| \frac{\sqrt{100+x^2} + x}{10} \right| - \frac{x}{\sqrt{x^2+100}} + c \\ &= \ell \eta \left| \sqrt{100+x^2} + x \right| - \frac{x}{\sqrt{x^2+100}} - \ell \eta 10 + c = \ell \eta \left| \sqrt{100+x^2} + x \right| - \frac{x}{\sqrt{x^2+100}} + c \end{aligned}$$

**Respuesta:**  $\int \frac{x^2 dx}{(x^2+100)^{3/2}} = \ell \eta \left| \sqrt{100+x^2} + x \right| - \frac{x}{\sqrt{x^2+100}} + c$

**Nota:** En los ejercicios 6.11 y 6.12 se ha omitido la figura (triángulo rectángulo). Conviene hacerla y ubicar los datos pertinentes. En adelante se entenderá que el estudiante agregará este complemento tan importante.

**6.13.-Encontrar:**  $\int \frac{x^2 dx}{(x^2+8^2)^{3/2}}$

Solución.-

$$\int \frac{x^2 dx}{(x^2+8^2)^{3/2}} = \int \frac{x^2 dx}{(\sqrt{x^2+8^2})^3},$$

se tiene:  $x = 8 \tau g t$ ,  $dt = 8 \sec^2 t dt$ ,  $\sqrt{x^2+8^2} = 8 \sec t$  además:  $\tau g t = \frac{x}{8}$ , luego:

$$\int \frac{x^2 dx}{(\sqrt{x^2+8^2})^3} = \int \frac{\cancel{8^2} \tau g^2 t (\cancel{8} \sec^2 t) dt}{\cancel{8^3} \sec^3 t} = \int \frac{\tau g^2 t dt}{\sec t} = \int \sec t dt - \int \cos t dt$$

$$= \ell \eta |\sec t + \tau g t| - \sec t + c, \text{ como: } \sec t = \frac{\sqrt{x^2 + 64}}{8}, \tau g t = \frac{x}{8}, \sec t = \frac{x}{\sqrt{x^2 + 64}}$$

Se tiene como expresión equivalente:

$$= \ell \eta \left| \frac{\sqrt{x^2 + 64}}{8} + \frac{x}{8} \right| - \frac{x}{\sqrt{x^2 + 64}} + c = \ell \eta \left| \frac{\sqrt{x^2 + 64} + x}{8} \right| - \frac{x}{\sqrt{x^2 + 64}} + c$$

$$= \ell \eta \left| \sqrt{x^2 + 64} + x \right| - \frac{x}{\sqrt{x^2 + 64}} + c$$

**Respuesta:**  $\int \frac{x^2 dx}{(x^2 + 8^2)^{3/2}} = \ell \eta \left| \sqrt{x^2 + 64} + x \right| - \frac{x}{\sqrt{x^2 + 64}} + c$

**6.14.-Encontrar:**  $\int \frac{dx}{(\sqrt{3^2 + x^2})^4}$

Solución.- se tiene:  $x = 3 \tau g t, dx = 3 \sec^2 t dt, \sqrt{3^2 + x^2} = 3 \sec t$ , además:

$$\tau g t = \frac{x}{3}$$

$$\int \frac{dx}{(\sqrt{3^2 + x^2})^4} = \int \frac{\cancel{3} \sec^2 t dt}{3^4 + \sec^4 t} = \frac{1}{3^3} \int \frac{dt}{\sec^2 t} = \frac{1}{27} \int \cos^2 t dt = \frac{1}{54} t + \frac{1}{54} \int \cos 2t dt$$

$$= \frac{1}{54} t + \frac{1}{108} \sin 2t + c_1 = \frac{1}{54} t + \frac{1}{108} 2 \sec t \cos t + c = \frac{1}{54} t + \frac{1}{54} \sec t \cos t + c$$

Como:  $\tau g t = \frac{x}{3} \Rightarrow t = \arctg \frac{x}{3}$ , además:  $\sec t = \frac{x}{\sqrt{9 + x^2}}, \cos t = \frac{3}{\sqrt{9 + x^2}}$

$$= \frac{1}{54} \arctg \frac{x}{3} + \frac{1}{54} \frac{x}{\sqrt{9 + x^2}} \frac{3}{\sqrt{9 + x^2}} + c = \frac{1}{54} \arctg \frac{x}{3} + \frac{x}{18(9 + x^2)} + c$$

**Respuesta:**  $\int \frac{dx}{(\sqrt{3^2 + x^2})^4} = \frac{1}{54} \arctg \frac{x}{3} + \frac{x}{18(9 + x^2)} + c$

**6.15.-Encontrar:**  $\int \frac{dx}{\sqrt{x^2 - 4x + 13}}$

Solución.- Completando cuadrados se tiene:

$$x^2 - 4x + 13 = (x^2 - 4x + \underline{\quad}) + 13 - \underline{\quad} = (x^2 - 4x + 4) + 13 - 4 = (x - 2)^2 + 3^2$$

Se tiene:  $x - 2 = 3 \tau g t, dx = 3 \sec^2 t dt, \sqrt{3^2 + x^2} = 3 \sec t$

$$\sqrt{(x - 2)^2 + 3^2} = \sqrt{x^2 - 4x + 13} = 3 \sec t,$$

Sea:  $x - 2 = 3 \tau g t, dx = 3 \sec^2 t dt$ ; además:  $\tau g t = \frac{x - 2}{3}$ , luego:

$$\int \frac{dx}{\sqrt{(x - 2)^2 + 3^2}} = \int \frac{\cancel{3} \sec^2 t dt}{\cancel{3} \sec t} = \int \sec t dt = \ell \eta |\sec t + \tau g t| + c$$

De la figura se tiene:

$$\sec t = \frac{\sqrt{x^2 - 4x + 13}}{3}, \tau gt = \frac{x-2}{3}, \text{ luego:}$$

$$\begin{aligned} &= \ell \eta \left| \frac{\sqrt{x^2 - 4x + 13}}{3} + \frac{x-2}{3} \right| + c = \ell \eta \left| \frac{\sqrt{x^2 - 4x + 13} + (x-2)}{3} \right| + c \\ &= \ell \eta \left| \sqrt{x^2 - 4x + 13} + (x-2) \right| + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{\sqrt{x^2 - 4x + 13}} = \ell \eta \left| \sqrt{x^2 - 4x + 13} + (x-2) \right| + c$

**6.16.-Encontrar:**  $\int \sqrt{1+4x^2} dx$

Solución.-

$$\int \sqrt{1+4x^2} dx = \int \sqrt{1^2 + (2x)^2} dx$$

Se tiene:  $2x = \tau gt, 2dx = \sec^2 t dt \Rightarrow dx = \frac{1}{2} \sec^2 t dt$ , Además:  $\tau gt = \frac{2x}{1}$

$$\int \sqrt{1^2 + (2x)^2} dx = \int \sqrt{1^2 + \tau g^2 t} \frac{1}{2} \sec^2 t dt = \frac{1}{2} \int \sec t \sec^2 t dt = \frac{1}{2} \int \sec^3 t dt$$

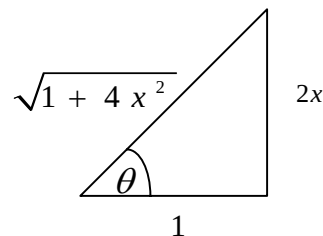
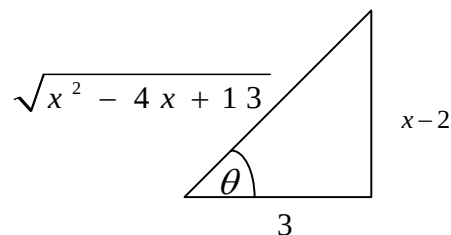
$$= \frac{1}{4} \sec t \tau gt + \frac{1}{4} \ell \eta |\sec t \tau gt| + c,$$

De la figura se tiene:

$$\sec t = \frac{\sqrt{1+4x^2}}{1}, \tau gt = 2x$$

$$= \frac{1}{4} \sqrt{1+4x^2} 2x + \frac{1}{4} \ell \eta |\sqrt{1+4x^2}| + 2x + c$$

**Respuesta:**  $\int \sqrt{1+4x^2} dx = \frac{1}{4} \sqrt{1+4x^2} 2x + \frac{1}{4} \ell \eta |\sqrt{1+4x^2}| + 2x + c$



## EJERCICIOS PROPUESTOS:

Utilizando esencialmente la técnica de sustitución por variables trigonométricas, encontrar las integrales siguientes:

**6.17.-**  $\int \sqrt{4-x^2}$

**6.18.-**  $\int \frac{dx}{\sqrt{a^2-x^2}}$

**6.19.-**  $\int \frac{dx}{x^2+a^2}$

$$6.20.- \int \frac{dx}{x^2 - a^2}$$

$$6.23.- \int \frac{dx}{x\sqrt{x^2 - 9}}$$

$$6.26.- \int \frac{x^2 dx}{\sqrt{1 - x^2}}$$

$$6.29.- \int \frac{dx}{x\sqrt{4x^2 - 16}}$$

$$6.32.- \int \sqrt{a - x^2} dx$$

$$6.35.- \int \frac{dx}{x^2 \sqrt{x^2 + 9}}$$

$$6.38.- \int x^2 \sqrt{5 - x^2} dx$$

$$6.41.- \int \frac{dx}{x^2 \sqrt{x^2 + a^2}}$$

$$6.44.- \int \frac{dx}{x^2 \sqrt{a^2 - x^2}}$$

$$6.47.- \int \frac{\sqrt{x^2 - 100}}{x} dx$$

$$6.50.- \int \frac{\sqrt{x^2 + a^2}}{x} dx$$

$$6.53.- \int \frac{dx}{\sqrt{4 + x^2}}$$

$$6.56.- \int \frac{(x+1)dx}{\sqrt{4 - x^2}}$$

$$6.59.- \int \frac{dx}{\sqrt{4 - (x-1)^2}}$$

$$6.62.- \int \frac{x^2 dx}{\sqrt{21 + 4x - x^2}}$$

$$6.65.- \int \frac{dx}{(x-1)\sqrt{x^2 - 3x + 2}}$$

$$6.68.- \int \frac{(x-1)dx}{\sqrt{x^2 - 4x + 3}}$$

$$6.21.- \int \frac{dx}{\sqrt{x^2 + a^2}}$$

$$6.24.- \int \frac{dx}{x\sqrt{x^2 - 2}}$$

$$6.27.- \int \frac{x^3 dx}{\sqrt{2 - x^2}}$$

$$6.30.- \int \frac{\sqrt{x^2 + 1}}{x} dx$$

$$6.33.- \int \sqrt{a^2 - x^2} dx$$

$$6.36.- \int \frac{dx}{\sqrt{5 - 4x^2}}$$

$$6.39.- \int \frac{dx}{x^4 \sqrt{x^2 + 3}}$$

$$6.42.- \int \frac{dx}{(x^2 + a^2)^2}$$

$$6.45.- \int \frac{\sqrt{2x^2 - 5}}{x} dx$$

$$6.48.- \int \frac{dx}{x^2 \sqrt{x^2 - 2}}$$

$$6.51.- \int \frac{xdx}{\sqrt{a^2 - x^2}}$$

$$6.54.- \int \frac{xdx}{\sqrt{4 + x^2}}$$

$$6.57.- \int \frac{dx}{\sqrt{2 - 5x^2}}$$

$$6.60.- \int \frac{x^2 dx}{\sqrt{2x - x^2}}$$

$$6.63.- \int \frac{dx}{(x^2 - 2x + 5)^{3/2}}$$

$$6.66.- \int \frac{xdx}{\sqrt{x^2 - 2x + 5}}$$

$$6.69.- \int \frac{dx}{\sqrt{x^2 - 2x - 8}}$$

$$6.22.- \int \frac{dx}{\sqrt{x^2 - a^2}}$$

$$6.25.- \int \frac{dx}{x\sqrt{1 + x^2}}$$

$$6.28.- \int \frac{\sqrt{x^2 - 9}}{x} dx$$

$$6.31.- \int \frac{dx}{x^2 \sqrt{4 - x^2}}$$

$$6.34.- \int \frac{x^2 dx}{\sqrt{x^2 + a^2}}$$

$$6.37.- \int \frac{x^2 dx}{(4 - x^2)^{3/2}}$$

$$6.40.- \int x^3 \sqrt{a^2 x^2 + b^2} dx$$

$$6.43.- \int x^3 \sqrt{a^2 x^2 - b^2} dx$$

$$6.46.- \int \frac{x^3 dx}{\sqrt{3x^2 - 5}}$$

$$6.49.- \int \frac{dx}{x\sqrt{9 - x^2}}$$

$$6.52.- \int \frac{dx}{\sqrt{1 - 4x^2}}$$

$$6.55.- \int \frac{dx}{x\sqrt{a^2 + x^2}}$$

$$6.58.- \int \frac{dx}{(a^2 - x^2)^{3/2}}$$

$$6.61.- \int \frac{x^2 dx}{\sqrt{17 - x^2}}$$

$$6.64.- \int \frac{(2x+1)dx}{\sqrt{(4x^2 - 2x+1)^3}}$$

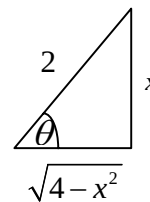
$$6.67.- \int \frac{(x+1)dx}{\sqrt{2x - x^2}}$$

$$6.70.- \int \frac{xdx}{\sqrt{x^2 + 4x + 5}}$$

## RESPUESTAS

**6.17.-**  $\int \sqrt{4-x^2}$

Solución.-



Se tiene:  $x = 2 \operatorname{sen} \theta, dx = 2 \cos \theta d\theta, \sqrt{4-x^2} = 2 \cos \theta$

$$\int \sqrt{4-x^2} = \int 2 \cos \theta 2 \cos \theta d\theta = 4 \int \cos^2 \theta d\theta = 2\theta + \operatorname{sen} 2\theta + c = 2\theta + 2 \operatorname{sen} \theta \cos \theta + c$$

$$= 2 \operatorname{arcsen} \frac{x}{2} + \frac{x\sqrt{4-x^2}}{2} + c$$

**6.18.-**  $\int \frac{dx}{\sqrt{a^2-x^2}}$

Solución.- se tiene:  $x = a \operatorname{sen} \theta, dx = a \cos \theta d\theta, \sqrt{a^2-x^2} = a \cos \theta$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{a \cancel{\cos \theta} d\theta}{a \cancel{\cos \theta}} = \int d\theta = \theta + c = \operatorname{arcsen} \frac{x}{a} + c$$

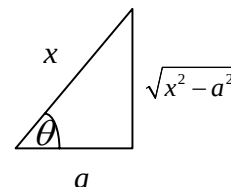
**6.19.-**  $\int \frac{dx}{x^2+a^2}$

Solución.- se tiene:  $x = a \operatorname{tg} \theta, dx = a \sec^2 \theta d\theta, \sqrt{x^2+a^2} = a \sec \theta$

$$\int \frac{dx}{x^2+a^2} = \int \frac{dx}{(\sqrt{x^2+a^2})^2} = \int \frac{a \cancel{\sec^2 \theta} d\theta}{a^2 \cancel{\sec^2 \theta}} = \frac{1}{a} \int d\theta = \frac{1}{a} \theta + c = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c$$

**6.20.-**  $\int \frac{dx}{x^2-a^2}$

Solución.-



Se tiene:  $x = a \sec \theta, dx = a \sec \theta \operatorname{tg} \theta d\theta, \sqrt{x^2-a^2} = a \operatorname{tg} \theta$

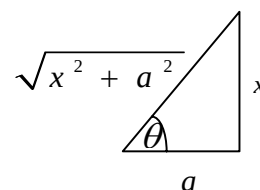
$$\int \frac{dx}{x^2-a^2} = \int \frac{dx}{(\sqrt{x^2-a^2})^2} = \int \frac{a \cancel{\sec \theta} \operatorname{tg} \theta d\theta}{a^2 \cancel{\operatorname{tg}^2 \theta}} = \frac{1}{a} \int \frac{\sec \theta d\theta}{\operatorname{tg} \theta} = \frac{1}{a} \int \operatorname{cosec} \theta d\theta$$

$$= \frac{1}{a} \ell \eta |\operatorname{cosec} \theta - \operatorname{cotg} \theta| = \frac{1}{a} \ell \eta \left| \frac{x}{\sqrt{x^2-a^2}} - \frac{a}{\sqrt{x^2-a^2}} \right| + c$$

$$= \frac{1}{a} \ell \eta \left| \frac{x-a}{\sqrt{x^2-a^2}} \right| + c = \frac{1}{a} \ell \eta \left| \sqrt{\frac{(x-a)^2}{x^2-a^2}} \right| + c = \frac{1}{2a} \ell \eta \left| \frac{x-a}{x+a} \right| + c$$

**6.21.-**  $\int \frac{dx}{\sqrt{x^2+a^2}}$

Solución.-

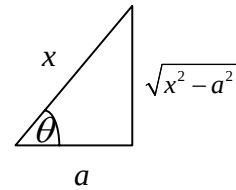


Se tiene:  $x = a \tau g \theta, dx = a \sec^2 \theta d\theta, \sqrt{x^2 + a^2} = a \sec \theta$

$$\begin{aligned} \int \frac{dx}{\sqrt{x^2 + a^2}} &= \int \frac{\cancel{a} \sec^2 \theta d\theta}{\cancel{a} \sec \theta} = \int \sec \theta d\theta = \ell \eta |\sec \theta + \tau g \theta| + c \\ &= \ell \eta \left| \frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right| + c = \ell \eta \left| \frac{\sqrt{x^2 + a^2} + x}{a} \right| + c = \ell \eta |x + \sqrt{x^2 + a^2}| - \ell \eta a + c \\ &= \ell \eta |x + \sqrt{x^2 + a^2}| + c \end{aligned}$$

**6.22.-**  $\int \frac{dx}{\sqrt{x^2 - a^2}}$

Solución.-



Se tiene:  $x = a \sec \theta, dx = a \sec \theta \tau g \theta d\theta, \sqrt{x^2 - a^2} = a \tau g \theta$

$$\begin{aligned} \int \frac{dx}{\sqrt{x^2 - a^2}} &= \int \frac{\cancel{a} \sec \theta \tau g \theta d\theta}{\cancel{a} \tau g \theta} = \int \sec \theta d\theta = \ell \eta |\sec \theta + \tau g \theta| + c \\ &= \ell \eta \left| \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right| + c = \ell \eta \left| \frac{x + \sqrt{x^2 - a^2}}{a} \right| + c = \ell \eta |x + \sqrt{x^2 - a^2}| + c \end{aligned}$$

**6.23.-**  $\int \frac{dx}{x\sqrt{x^2 - 9}}$

Solución.-

Se tiene:  $x = 3 \sec \theta, dx = 3 \sec \theta \tau g \theta d\theta, \sqrt{x^2 - 9} = 3 \tau g \theta$

$$\int \frac{dx}{x\sqrt{x^2 - 9}} = \int \frac{\cancel{3} \sec \theta \tau g \theta d\theta}{\cancel{3} \sec \theta \cancel{3} \tau g \theta} = \frac{1}{3} \int d\theta = \frac{1}{3} \theta + c = \frac{\text{arc sec } x/3}{3} + c$$

**6.24.-**  $\int \frac{dx}{x\sqrt{x^2 - 2}}$

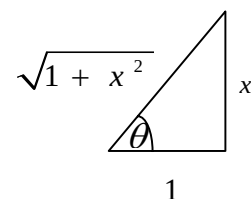
Solución.-

Se tiene:  $x = \sqrt{2} \sec \theta, dx = \sqrt{2} \sec \theta \tau g \theta d\theta, \sqrt{x^2 - 2} = \sqrt{2} \tau g \theta$

$$\int \frac{dx}{x\sqrt{x^2 - 2}} = \int \frac{\cancel{\sqrt{2}} \sec \theta \tau g \theta d\theta}{\cancel{\sqrt{2}} \sec \theta \cancel{\sqrt{2}} \tau g \theta} = \frac{\sqrt{2}}{2} \int d\theta = \frac{\sqrt{2}}{2} \theta + c = \frac{\sqrt{2}}{2} \text{arc sec } \frac{\sqrt{2}}{2} x + c$$

**6.25.-**  $\int \frac{dx}{x\sqrt{1 + x^2}}$

Solución.-

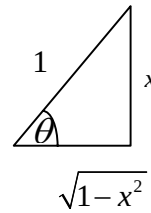


Se tiene:  $x = \tan \theta, dx = \sec^2 \theta d\theta, \sqrt{1+x^2} = \sec \theta$

$$\begin{aligned}\int \frac{dx}{x\sqrt{1+x^2}} &= \int \frac{\sec^2 \theta d\theta}{\tan \theta \sec \theta} = \int \frac{d\theta}{\sin \theta} = \int \operatorname{cosec} \theta d\theta = \ell \eta |\operatorname{cosec} \theta - \cot \theta| + c \\ &= \ell \eta \left| \frac{\sqrt{1+x^2}}{x} - \frac{1}{x} \right| + c = \ell \eta \left| \frac{\sqrt{1+x^2}-1}{x} \right| + c\end{aligned}$$

**6.26.-**  $\int \frac{x^2 dx}{\sqrt{1-x^2}}$

Solución.-

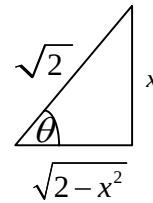


Se tiene:  $x = \sin \theta, dx = \cos \theta d\theta, \sqrt{1-x^2} = \cos \theta$

$$\begin{aligned}\int \frac{x^2 dx}{\sqrt{1-x^2}} &= \int \frac{\sin^2 \theta \cos \theta d\theta}{\cos \theta} = \int \sin^2 \theta d\theta = \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta + c \\ &= \frac{1}{2} \theta - \frac{1}{2} \sin \theta \cos \theta + c = \frac{\arcsin x}{2} - \frac{x}{2} \sqrt{1-x^2} + c\end{aligned}$$

**6.27.-**  $\int \frac{x^3 dx}{\sqrt{2-x^2}}$

Solución.-



Se tiene:  $x = \sqrt{2} \sin \theta, dx = \sqrt{2} \cos \theta d\theta, \sqrt{2-x^2} = \sqrt{2} \cos \theta$

$$\begin{aligned}\int \frac{x^3 dx}{\sqrt{2-x^2}} &= \int \frac{2\sqrt{2} \sin^3 \theta \sqrt{2} \cos \theta d\theta}{\sqrt{2} \cos \theta} = 2\sqrt{2} \int \sin^3 \theta d\theta = 2\sqrt{2} \left( -\cos \theta + \frac{\cos^3 \theta}{3} \right) + c \\ &= 2\sqrt{2} \left( -\frac{\sqrt{2-x^2}}{\sqrt{2}} + \frac{(\sqrt{2-x^2})^3}{3(\sqrt{2})^3} \right) + c = -\sqrt{2(2-x^2)} + \frac{(2-x^2)\sqrt{2-x^2}}{3} + c\end{aligned}$$

**6.28.-**  $\int \frac{\sqrt{x^2-9}}{x} dx$

Solución.-

Se tiene:  $x = 3 \sec \theta, dx = 3 \sec \theta \tan \theta d\theta, \sqrt{x^2-9} = 3 \tan \theta$

$$\begin{aligned}\int \frac{\sqrt{x^2-9}}{x} dx &= \int \frac{3 \tan \theta 3 \sec \theta \tan \theta d\theta}{3 \sec \theta} = 3 \int \tan^2 \theta d\theta = 3 \int (\sec^2 \theta - 1) d\theta \\ &= 3 \int \sec^2 \theta d\theta - 3 \int d\theta = 3 \tan \theta - 3\theta + c = \sqrt{x^2-9} - 3 \operatorname{arcsec} \frac{x}{3} + c\end{aligned}$$



**6.29.-**  $\int \frac{dx}{x\sqrt{4x^2-16}}$

Solución.-

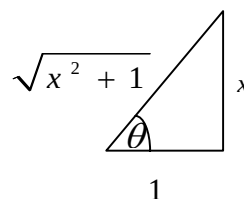
Se tiene:  $\frac{x}{2} = \sec \theta, dx = 2 \sec \theta \tan \theta d\theta, \sqrt{\frac{x^2}{4}-1} = \tan \theta$

$$\int \frac{dx}{x\sqrt{4x^2-16}} = \frac{1}{4} \int \frac{dx}{x\sqrt{(\frac{x}{2})^2-1}} = \frac{1}{4} \int \frac{\cancel{2 \sec \theta \tan \theta} d\theta}{\cancel{2 \sec \theta \tan \theta}} = \frac{1}{4} \int d\theta = \frac{1}{4} \theta + c$$

$$= \frac{1}{4} \operatorname{arcsec} \frac{x}{2} + c$$

**6.30.-**  $\int \frac{\sqrt{x^2+1}}{x} dx$

Solución.-



Se tiene:  $x = \tan \theta, dx = \sec^2 \theta d\theta, \sqrt{x^2+1} = \sec \theta$

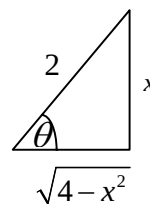
$$\int \frac{\sqrt{x^2+1}}{x} dx = \int \frac{\sec \theta \sec^2 \theta d\theta}{\tan \theta} = \int \frac{d\theta}{\cos^2 \theta \sin \theta} = \ell \eta \left| \tan \frac{\theta}{2} \right| + \frac{1}{\cos \theta} + c, \text{ o bien:}$$

$$= \ell \eta |\operatorname{cosec} \theta - \cot \theta| + \frac{1}{\cos \theta} + c = \ell \eta \left| \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} \right| + \frac{1}{\frac{1}{\sqrt{x^2+1}}} + c$$

$$= \ell \eta \left| \frac{\sqrt{x^2+1}-1}{x} \right| + \sqrt{x^2+1} + c$$

**6.31.-**  $\int \frac{dx}{x^2\sqrt{4-x^2}}$

Solución.-



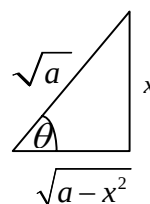
Se tiene:  $x = 2 \sin \theta, dx = 2 \cos \theta d\theta, \sqrt{4-x^2} = 2 \cos \theta$

$$\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{\cancel{2 \cos \theta} d\theta}{4 \sin^2 \theta \cancel{2 \cos \theta}} = \frac{1}{4} \int \operatorname{cosec}^2 \theta d\theta = -\frac{1}{4} \cot \theta + c$$

$$= -\frac{\sqrt{4-x^2}}{4x} + c$$

**6.32.-**  $\int \sqrt{a-x^2} dx$

Solución.-



Se tiene:  $x = \sqrt{a} \operatorname{sen} \theta, dx = \sqrt{a} \cos \theta d\theta, \sqrt{a-x^2} = \sqrt{a} \cos \theta$

$$\int \sqrt{a-x^2} dx = \int \sqrt{a} \cos \theta \sqrt{a} \cos \theta d\theta = a \int \cos^2 \theta d\theta$$

$$\frac{a}{2} \theta + \frac{a}{2} \operatorname{sen} \theta \cos \theta + c = \frac{a}{2} \arcsen \frac{x}{\sqrt{a}} + \frac{x}{2} \sqrt{a^2 - x^2} + c$$

**6.33.-**  $\int \sqrt{a^2 - x^2} dx$

Solución.-

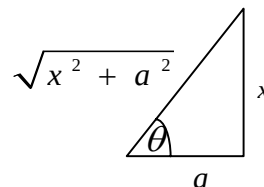
Se tiene:  $x = a \operatorname{sen} \theta, dx = a \cos \theta d\theta, \sqrt{a^2 - x^2} = a \cos \theta$

$$\int \sqrt{a^2 - x^2} dx = \int a \cos \theta a \cos \theta d\theta = a^2 \int \cos^2 \theta d\theta$$

$$\frac{a^2}{2} \theta + \frac{a^2}{2} \operatorname{sen} \theta \cos \theta + c = \frac{a^2}{2} \arcsen \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + c$$

**6.34.-**  $\int \frac{x^2 dx}{\sqrt{x^2 + a^2}}$

Solución.-



Se tiene:  $x = a \operatorname{tg} \theta, dx = a \sec^2 \theta d\theta, \sqrt{x^2 + a^2} = a \sec \theta$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = \int \frac{a^2 \operatorname{tg}^2 \theta \cancel{a} \sec^2 \theta d\theta}{a \sec \theta} = a^2 \int \operatorname{tg}^2 \theta \sec \theta d\theta = a^2 \int \frac{\operatorname{sen}^2 \theta}{\cos^3 \theta} d\theta$$

$$= a^2 \int \frac{(1 - \cos^2 \theta)}{\cos^3 \theta} d\theta = a^2 \int \sec^3 \theta d\theta - a^2 \int \sec \theta d\theta$$

$$= a^2 \left( \frac{\sec \theta \operatorname{tg} \theta}{2} + \frac{1}{2} \ell \eta |\sec \theta + \operatorname{tg} \theta| \right) - a^2 \ell \eta |\sec \theta + \operatorname{tg} \theta| + c$$

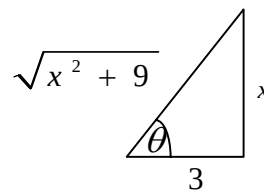
$$= \frac{a^2}{2} \sec \theta \operatorname{tg} \theta + \frac{a^2}{2} \ell \eta |\sec \theta + \operatorname{tg} \theta| - a^2 \ell \eta |\sec \theta + \operatorname{tg} \theta| + c$$

$$= \frac{a^2}{2} \sec \theta \operatorname{tg} \theta - \frac{a^2}{2} \ell \eta |\sec \theta + \operatorname{tg} \theta| + c$$

$$= \frac{a^2}{2} \frac{\sqrt{x^2 + a^2}}{a} \frac{x}{a} - \frac{a^2}{2} \ell \eta \left| \frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right| + c = \frac{x \sqrt{x^2 + a^2}}{2} - \frac{a^2}{2} \ell \eta \left| \sqrt{x^2 + a^2} + x \right| + c$$

**6.35.-**  $\int \frac{dx}{x^2 \sqrt{x^2 + 9}}$

Solución.-



Se tiene:  $x = 3\tau g\theta, dx = 3\sec^2 \theta d\theta, \sqrt{x^2 + 9} = 3\sec \theta$

$$\int \frac{dx}{x^2 \sqrt{x^2 + 9}} = \int \frac{\cancel{3} \sec^2 \theta d\theta}{9 \tau g^2 \theta \cancel{3 \sec \theta}} = \frac{1}{9} \int \frac{\sec \theta d\theta}{\tau g^2 \theta} = \frac{1}{9} \int \frac{\cos \theta}{\text{sen}^2 \theta} d\theta = -\frac{1}{9 \text{sen} \theta} + c$$

$$= -\frac{\sqrt{x^2 + 9}}{9x} + c$$

**6.36.-**  $\int \frac{dx}{\sqrt{5-4x^2}}$

Solución.-

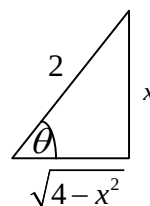
Se tiene:  $x = \sqrt{5/4} \text{sen} \theta, dx = \sqrt{5/4} \cos \theta d\theta, \sqrt{(5/4)^2 - x^2} = 5/4 \cos \theta$

$$\int \frac{dx}{\sqrt{5-4x^2}} = \frac{1}{2} \int \frac{dx}{\sqrt{5/4 - x^2}} = \frac{1}{2} \int \frac{\cancel{\sqrt{5/4}} \cos \theta d\theta}{\cancel{\sqrt{5/4}} \cos \theta} = \frac{1}{2} \int d\theta = \frac{1}{2} \theta + c$$

$$= \frac{1}{2} \arcsen \frac{x}{\sqrt{5/4}} + c = \frac{1}{2} \arcsen \frac{2x}{\sqrt{5}} + c$$

**6.37.-**  $\int \frac{x^2 dx}{(4-x^2)^{3/2}}$

Solución.-



Se tiene:  $x = 2 \text{sen} \theta, dx = 2 \cos \theta d\theta, \sqrt{4-x^2} = 2 \cos \theta$

$$\int \frac{x^2 dx}{(4-x^2)^{3/2}} = \int \frac{x^2 dx}{\sqrt{(4-x^2)^3}} = \int \frac{\cancel{4} \text{sen}^2 \theta \cancel{2} \cos \theta d\theta}{\cancel{8} \cos^3 \theta} = \int \tau g^2 \theta d\theta = \int (\sec^2 \theta - 1) d\theta$$

$$= \tau g \theta - \theta + c = \frac{x}{\sqrt{4-x^2}} - \arcsen \frac{x}{2} + c$$

**6.38.-**  $\int x^2 \sqrt{5-x^2} dx$

Solución.-

Se tiene:  $x = \sqrt{5} \text{sen} \theta, dx = \sqrt{5} \cos \theta d\theta, \sqrt{5-x^2} = \sqrt{5} \cos \theta$

$$\int x^2 \sqrt{5-x^2} dx = \int 5 \text{sen}^2 \theta \sqrt{5} \cos \theta \sqrt{5} \cos \theta d\theta = 25 \int \text{sen}^2 \theta \cos^2 \theta d\theta = \frac{25}{4} \int \text{sen}^2 2\theta d\theta$$

$$= \frac{25}{8} \int (1 - \cos 4\theta) d\theta = \frac{25}{8} \theta - \frac{25}{32} \text{sen} 4\theta + c = \frac{25}{8} \theta - \frac{25}{32} (2 \text{sen} 2\theta \cos 2\theta) + c$$

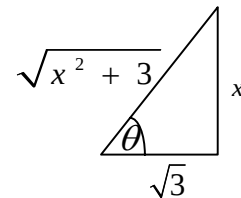
$$= \frac{25}{8} \theta - \frac{25}{32} [2 \text{sen} \theta \cos 2\theta (\cos^2 \theta - \text{sen}^2 \theta)] + c$$

$$= \frac{25}{8}\theta - \frac{25}{16}[\operatorname{sen}\theta \cos^3\theta - \operatorname{sen}^3\theta \cos\theta] + c$$

$$= \frac{25}{2} \left[ \operatorname{arcsen} \frac{x}{\sqrt{5}} - \frac{x(\sqrt{5-x^2})^3}{25} + \frac{x^3\sqrt{5-x^2}}{25} \right] + c$$

**6.39.-**  $\int \frac{dx}{x^4 \sqrt{x^2+3}}$

Solución.-



Se tiene:  $x = \sqrt{3} \operatorname{tg} \theta$ ,  $dx = \sqrt{3} \sec^2 \theta d\theta$ ,  $\sqrt{x^2+3} = \sqrt{3} \sec \theta$

$$\int \frac{dx}{x^4 \sqrt{x^2+3}} = \int \frac{\sqrt{3} \sec^2 \theta d\theta}{9 \operatorname{tg}^4 \theta \sqrt{3} \sec \theta} = \frac{1}{9} \int \frac{\sec \theta d\theta}{\operatorname{tg}^4 \theta} = \frac{1}{9} \int \frac{\cos^3 \theta d\theta}{\operatorname{sen}^4 \theta} = \frac{1}{9} \int \frac{(1-\operatorname{sen}^2 \theta) \cos \theta d\theta}{\operatorname{sen}^4 \theta}$$

$$= \frac{1}{9} \int \frac{\cos \theta d\theta}{\operatorname{sen}^4 \theta} - \frac{1}{9} \int \frac{\cos \theta d\theta}{\operatorname{sen}^2 \theta} = -\frac{1}{27} \operatorname{cosec}^3 \theta + \frac{1}{9} \operatorname{cosec} \theta + c = \frac{\sqrt{x^2+3}}{9x} - \left( \frac{\sqrt{x^2+3}}{3x} \right)^3 + c$$

**6.40.-**  $\int x^3 \sqrt{a^2 x^2 + b^2} dx$

Solución.-

Se tiene:  $ax = b \operatorname{tg} \theta$ ,  $adx = b \sec^2 \theta d\theta$ ,  $\sqrt{a^2 x^2 + b^2} = b \sec \theta$

$$\int x^3 \sqrt{a^2 x^2 + b^2} dx = \int \frac{b^3}{a^3} \operatorname{tg}^3 \theta b \sec \theta \frac{b}{a} \sec^2 \theta d\theta = \frac{b^5}{a^4} \int \operatorname{tg}^3 \theta \sec^3 \theta d\theta$$

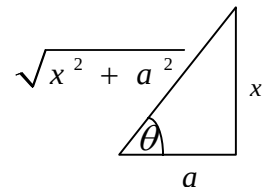
$$= \frac{b^5}{a^4} \int \operatorname{tg}^2 \theta \sec^2 \theta \operatorname{tg} \theta \sec \theta d\theta = \frac{b^5}{a^4} \int (\sec^2 \theta - 1) \sec^2 \theta \operatorname{tg} \theta \sec \theta d\theta$$

$$= \frac{b^5}{a^4} \int \sec^4 \theta \operatorname{tg} \theta \sec \theta d\theta - \frac{b^5}{a^4} \int \sec^2 \theta \operatorname{tg} \theta \sec \theta d\theta = \frac{b^5}{a^4} \frac{\sec^5 \theta}{5} + \frac{b^5}{a^4} \frac{\sec^3 \theta}{3} + c$$

$$= \frac{b^5}{a^4} \left[ \frac{(\sqrt{a^2 x^2 + b^2})^5}{5b^5} + \frac{(\sqrt{a^2 x^2 + b^2})^3}{3b^3} \right] + c = \frac{(a^2 x^2 + b^2)^{5/2}}{5a^4} - \frac{(a^2 x^2 + b^2)^{3/2} b^2}{3a^4} + c$$

**6.41.-**  $\int \frac{dx}{x^2 \sqrt{x^2+a^2}}$

Solución.-



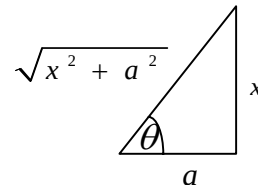
Se tiene:  $x = a \operatorname{tg} \theta$ ,  $dx = a \sec^2 \theta d\theta$ ,  $\sqrt{x^2+a^2} = a \sec \theta$

$$\int \frac{dx}{x^2 \sqrt{x^2 + a^2}} = \int \frac{\cancel{a} \sec^2 \theta d\theta}{a^2 \cancel{\tau g^2 \theta} \cancel{\sec \theta}} = \frac{1}{a^2} \int \frac{\sec \theta d\theta}{\tau g^2 \theta} = \frac{1}{a^2} \int \frac{\cos \theta d\theta}{\sec \theta} d\theta$$

$$= \frac{1}{a^2} \int \cos \theta \sec \theta d\theta = -\frac{\cos \theta}{a^2} + c = -\frac{1}{a^2 x} \sqrt{x^2 + a^2} + c$$

**6.42.-**  $\int \frac{dx}{(x^2 + a^2)^2}$

Solución.-



Se tiene:  $x = a \tau g \theta$ ,  $dx = a \sec^2 \theta d\theta$ ,  $\sqrt{x^2 + a^2} = a \sec \theta$

$$\int \frac{dx}{(x^2 + a^2)^2} = \int \frac{dx}{(\sqrt{x^2 + a^2})^4} = \int \frac{\cancel{a} \sec^2 \theta d\theta}{a^4 \sec^4 \theta} = \frac{1}{a^3} \int \cos^2 \theta d\theta = \frac{1}{2a^3} \theta + \frac{1}{2a^3} \frac{\sin 2\theta}{2} + c$$

$$= \frac{1}{2a^3} \theta + \frac{1}{2a^3} \frac{\sin \theta \cos \theta}{2} + c = \frac{1}{2a^3} \arctan g \frac{x}{a} + \frac{1}{2a^3} \left( \frac{x}{\sqrt{x^2 + a^2}} \frac{a}{\sqrt{x^2 + a^2}} \right) + c$$

$$= \frac{1}{2a^3} \arctan g \frac{x}{a} + \frac{1}{2a^3} \left( \frac{ax}{\sqrt{x^2 + a^2}} \right) + c$$

**6.43.-**  $\int x^3 \sqrt{a^2 x^2 - b^2} dx$

Solución.-

Se tiene:  $ax = b \sec \theta$ ,  $adx = b \sec \theta \tau g \theta d\theta$ ,  $\sqrt{a^2 x^2 - b^2} = b \tau g \theta$

$$\int x^3 \sqrt{a^2 x^2 - b^2} dx = \int \frac{b^3}{a^3} \sec^3 \theta b \tau g \theta \frac{b}{a} \sec \theta \tau g \theta d\theta = \frac{b^5}{a^4} \int \sec^4 \theta \tau g^2 \theta d\theta$$

$$= \frac{b^5}{a^4} \int \sec^4 \theta (\sec^2 \theta - 1) d\theta = \frac{b^5}{a^4} \int \sec^4 \theta \sec^2 \theta d\theta - \frac{b^5}{a^4} \int \sec^2 \theta \sec^2 \theta d\theta$$

$$= \frac{b^5}{a^4} \int (1 + \tau g^2 \theta)^2 \sec^2 \theta d\theta - \frac{b^5}{a^4} \int (1 + \tau g^2 \theta) \sec^2 \theta d\theta$$

$$= \frac{b^5}{a^4} \int (1 + 2\tau g^2 \theta + \tau g^4 \theta) \sec^2 \theta d\theta - \frac{b^5}{a^4} \int (1 + \tau g^2 \theta) \sec^2 \theta d\theta$$

$$= \frac{b^5}{a^4} \left[ \int \tau g^2 \theta \sec^2 \theta d\theta + \int \tau g^4 \theta \sec^2 \theta d\theta \right] = \frac{b^5}{a^4} \left[ \frac{\tau g^3 \theta}{3} + \frac{\tau g^5 \theta}{5} \right] + c$$

$$= \frac{b^5}{a^4} \left[ \frac{1}{3} \left( \frac{\sqrt{a^2 x^2 - b^2}}{b} \right)^3 + \frac{1}{5} \left( \frac{\sqrt{a^2 x^2 - b^2}}{b} \right)^5 \right] + c$$

**6.44.-**  $\int \frac{dx}{x^2 \sqrt{a^2 - x^2}}$

Solución.-

Se tiene:  $x = a \operatorname{sen} \theta, dx = a \cos \theta d\theta, \sqrt{a^2 - x^2} = a \cos \theta$

$$\begin{aligned} \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} &= \int \frac{\cancel{a \cos \theta} d\theta}{a^2 \operatorname{sen}^2 \theta \cancel{a \cos \theta}} = \frac{1}{a^2} \int \operatorname{cosec}^2 \theta d\theta = -\frac{1}{a^2} \cot \theta + c \\ &= -\frac{1}{a^2} \frac{\cos \theta}{\operatorname{sen} \theta} + c = -\frac{1}{a^2} \left( \frac{\sqrt{a^2 - x^2}}{x} \right) + c \end{aligned}$$

**6.45.-**  $\int \frac{\sqrt{2x^2 - 5}}{x} dx$

Solución.-

Se tiene:  $\sqrt{2}x = \sqrt{5} \sec \theta, \sqrt{2}dx = \sqrt{5} \sec \theta \operatorname{tg} \theta d\theta, \sqrt{2x^2 - 5} = \sqrt{5} \operatorname{tg} \theta$

$$\begin{aligned} \int \frac{\sqrt{2x^2 - 5}}{x} dx &= \int \frac{\sqrt{5} \operatorname{tg} \theta \cancel{\frac{\sqrt{5}}{\sqrt{2}}} \sec \theta \operatorname{tg} \theta d\theta}{\frac{\sqrt{5}}{\cancel{\sqrt{2}}} \sec \theta} = \sqrt{5} \int \operatorname{tg}^2 \theta d\theta = \sqrt{5} \int \sec^2 \theta d\theta - \sqrt{5} \int d\theta \\ &= \sqrt{5} \operatorname{tg} \theta - \sqrt{5} \theta + c = \sqrt{2x^2 - 5} - \sqrt{5} \operatorname{arcsec} \sqrt{\frac{2}{3}} x + c \end{aligned}$$

**6.46.-**  $\int \frac{x^3 dx}{\sqrt{3x^2 - 5}}$

Solución.-

Se tiene:  $\sqrt{3}x = \sqrt{5} \sec \theta, \sqrt{3}dx = \sqrt{5} \sec \theta \operatorname{tg} \theta d\theta, \sqrt{3x^2 - 5} = \sqrt{5} \operatorname{tg} \theta$

$$\begin{aligned} \int \frac{x^3 dx}{\sqrt{3x^2 - 5}} &= \int \frac{(\sqrt{\frac{5}{3}} \sec \theta)^3 \sqrt{\frac{5}{3}} \sec \theta \operatorname{tg} \theta d\theta}{\sqrt{\frac{5}{3}} \operatorname{tg} \theta} = \frac{5\sqrt{5}}{9} \int \sec^4 \theta d\theta \\ &= \frac{5\sqrt{5}}{9} \int \sec^2 \theta \sec^2 \theta d\theta = \frac{5\sqrt{5}}{9} \int \sec^2 \theta (1 + \operatorname{tg}^2 \theta) d\theta \\ &= \frac{5\sqrt{5}}{9} \left[ \int \sec^2 \theta d\theta + \int \sec^2 \theta \operatorname{tg}^2 \theta d\theta \right] = \frac{5\sqrt{5}}{9} \left[ \operatorname{tg} \theta + \frac{\operatorname{tg}^3 \theta}{3} \right] + c \\ &= \frac{5}{9} \left[ \sqrt{3x^2 - 5} + \frac{(\sqrt{3x^2 - 5})^3}{15} \right] + c \end{aligned}$$

**6.47.-**  $\int \frac{\sqrt{x^2 - 100}}{x} dx$

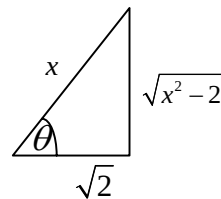
Solución.-

Se tiene:  $x = 10 \sec \theta, dx = 10 \sec \theta \operatorname{tg} \theta d\theta, \sqrt{x^2 - 100} = 10 \operatorname{tg} \theta$

$$\begin{aligned} \int \frac{\sqrt{x^2 - 100}}{x} dx &= \int \frac{10 \operatorname{tg} \theta \cancel{10 \sec \theta} \operatorname{tg} \theta d\theta}{10 \sec \theta} = 10 \int \operatorname{tg}^2 \theta d\theta = 10 \int \sec^2 \theta - 10 \int d\theta \\ &= 10(\operatorname{tg} \theta - \theta) + c = \sqrt{x^2 - 100} - 10 \operatorname{arcsen} \frac{x}{10} + c \end{aligned}$$

**6.48.-**  $\int \frac{dx}{x^2 \sqrt{x^2 - 2}}$

Solución.-

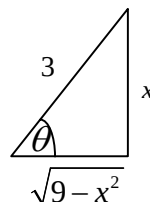


Se tiene:  $x = \sqrt{2} \sec \theta$ ,  $dx = \sqrt{2} \sec \theta \tan \theta d\theta$ ,  $\sqrt{x^2 - 2} = \sqrt{2} \tan \theta$

$$\begin{aligned} \int \frac{dx}{x^2 \sqrt{x^2 - 2}} &= \int \frac{\cancel{\sqrt{2}} \sec \theta \cancel{\tan \theta} d\theta}{2 \sec^2 \theta \cancel{\sqrt{2}} \tan \theta} = \frac{1}{2} \int \cos \theta d\theta = \frac{1}{2} \sin \theta + c = \frac{1}{2} \frac{\sqrt{x^2 - 2}}{x} + c \\ &= \frac{\sqrt{x^2 - 2}}{2x} + c \end{aligned}$$

**6.49.-**  $\int \frac{dx}{x \sqrt{9 - x^2}}$

Solución.-

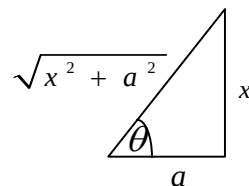


Se tiene:  $x = 3 \sin \theta$ ,  $dx = 3 \cos \theta d\theta$ ,  $\sqrt{9 - x^2} = 3 \cos \theta$

$$\begin{aligned} \int \frac{dx}{x \sqrt{9 - x^2}} &= \int \frac{\cancel{3} \cos \theta d\theta}{3 \sin \theta \cancel{3} \cos \theta} = \frac{1}{3} \int \csc \theta d\theta = \frac{1}{3} \ell \eta |\csc \theta - \cot \theta| + c \\ &= \frac{1}{3} \ell \eta \left| \frac{3 - \sqrt{9 - x^2}}{x} \right| + c \end{aligned}$$

**6.50.-**  $\int \frac{\sqrt{x^2 + a^2}}{x} dx$

Solución.-



Se tiene:  $x = a \tan \theta$ ,  $dx = a \sec^2 \theta d\theta$ ,  $\sqrt{x^2 + a^2} = a \sec \theta$

$$\begin{aligned} \int \frac{\sqrt{x^2 + a^2}}{x} dx &= \int \frac{a \sec \theta}{\cancel{a} \tan \theta} \cancel{a} \sec^2 \theta d\theta = a \int \frac{\sec^3 \theta d\theta}{\tan \theta} = a \int \frac{\sec^2 \theta \sec \theta}{\tan \theta} d\theta \\ &= a \int \frac{(1 + \tan^2 \theta) \sec \theta}{\tan \theta} d\theta = a \int \frac{\sec \theta}{\tan \theta} d\theta + a \int \sec \theta \tan \theta d\theta \end{aligned}$$

$$a \ell \eta |\csc \theta - \cot \theta| + a \sec \theta + c = a \ell \eta \left| \frac{\sqrt{x^2 + a^2} - a}{x} \right| + \sqrt{x^2 + a^2} + c$$

$$6.51.- \int \frac{x dx}{\sqrt{a^2 - x^2}}$$

Solución.-

Se tiene:  $x = a \operatorname{sen} \theta, dx = a \cos \theta d\theta, \sqrt{a^2 - x^2} = a \cos \theta$

$$\int \frac{x dx}{\sqrt{a^2 - x^2}} = \int \frac{a \operatorname{sen} \theta \cancel{a \cos \theta}}{a \cos \theta} d\theta = a \int \operatorname{sen} \theta d\theta = -a \cos \theta + c = -\sqrt{a^2 - x^2} + c$$

$$6.52.- \int \frac{dx}{\sqrt{1 - 4x^2}}$$

Solución.-

Se tiene:  $2x = \operatorname{sen} \theta, 2dx = \cos \theta d\theta, \sqrt{1 - 4x^2} = \cos \theta$

$$\int \frac{dx}{\sqrt{1 - 4x^2}} = \frac{1}{2} \int \frac{\cancel{\cos \theta}}{\cos \theta} d\theta = \frac{1}{2} \int d\theta = \frac{1}{2} \theta + c = \frac{1}{2} \operatorname{arcsen} 2x + c$$

$$6.53.- \int \frac{dx}{\sqrt{4 + x^2}}$$

Solución.-

Se tiene:  $x = 2 \operatorname{tg} \theta, dx = 2 \sec^2 \theta d\theta, \sqrt{4 + x^2} = 2 \sec \theta$

$$\int \frac{dx}{\sqrt{4 + x^2}} = \int \frac{\cancel{2} \sec^2 \theta d\theta}{2 \sec \theta} = \int \sec \theta d\theta = \ell \eta |\sec \theta + \operatorname{tg} \theta| + c = \ell \eta |\sqrt{4 + x^2} + x| + c$$

$$6.54.- \int \frac{x dx}{\sqrt{4 + x^2}}$$

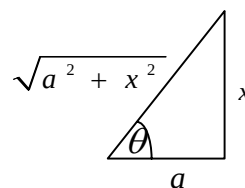
Solución.-

Se tiene:  $x = 2 \operatorname{tg} \theta, dx = 2 \sec^2 \theta d\theta, \sqrt{4 + x^2} = 2 \sec \theta$

$$\int \frac{x dx}{\sqrt{4 + x^2}} = \int \frac{2 \operatorname{tg} \theta \cancel{2} \sec^2 \theta d\theta}{2 \sec \theta} = 2 \int \operatorname{tg} \theta \sec \theta d\theta = 2 \sec \theta + c = \sqrt{4 + x^2} + c$$

$$6.55.- \int \frac{dx}{x \sqrt{a^2 + x^2}}$$

Solución.-



Se tiene:  $x = a \operatorname{tg} \theta, dx = a \sec^2 \theta d\theta, \sqrt{a^2 + x^2} = a \sec \theta$

$$\begin{aligned} \int \frac{dx}{x \sqrt{a^2 + x^2}} &= \int \frac{\cancel{a} \sec^2 \theta d\theta}{a \operatorname{tg} \theta \cancel{a \sec \theta}} = \frac{1}{a} \int \frac{\sec \theta d\theta}{\operatorname{tg} \theta} = \frac{1}{a} \int \operatorname{cosec} \theta d\theta \\ &= \frac{1}{a} \ell \eta |\operatorname{cosec} \theta - \cot \theta| + c = \frac{1}{a} \ell \eta \left| \frac{\sqrt{a^2 + x^2}}{x} - \frac{a}{x} \right| + c = \frac{1}{a} \ell \eta \left| \frac{\sqrt{a^2 + x^2} - a}{x} \right| + c \end{aligned}$$

$$6.56.- \int \frac{(x+1) dx}{\sqrt{4 - x^2}}$$

Solución.-



Se tiene:  $x = 2 \operatorname{sen} \theta, dx = 2 \cos \theta d\theta, \sqrt{4-x^2} = 2 \cos \theta$

$$\int \frac{(x+1)dx}{\sqrt{4-x^2}} = \int \frac{x dx}{\sqrt{4-x^2}} + \int \frac{dx}{\sqrt{4-x^2}} = \int \frac{2 \operatorname{sen} \theta \cancel{2 \cos \theta} d\theta}{2 \cos \theta} + \int \frac{\cancel{2 \cos \theta} d\theta}{2 \cos \theta}$$

$$2 \int \operatorname{sen} \theta d\theta + \int d\theta = -2 \cos \theta + \theta + c = -\sqrt{4-x^2} + \arcsen \frac{x}{2} + c$$

**6.57.-**  $\int \frac{dx}{\sqrt{2-5x^2}}$

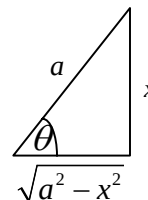
Solución.-

Se tiene:  $\sqrt{5}x = \sqrt{2} \operatorname{sen} \theta, \sqrt{5}dx = \sqrt{2} \cos \theta d\theta, \sqrt{2-5x^2} = \sqrt{2} \cos \theta$

$$\int \frac{dx}{\sqrt{2-5x^2}} = \int \frac{\cancel{\sqrt{2}} \cos \theta d\theta}{\sqrt{5} \cancel{\cos \theta}} = \frac{\sqrt{5}}{5} \int d\theta = \frac{\sqrt{5}}{5} \theta + c = \frac{\sqrt{5}}{5} \arcsen \sqrt{\frac{5}{2}} x + c$$

**6.58.-**  $\int \frac{dx}{(a^2-x^2)^{3/2}}$

Solución.-



Se tiene:  $x = a \operatorname{sen} \theta, dx = a \cos \theta d\theta, \sqrt{a^2-x^2} = a \cos \theta$

$$\int \frac{dx}{(a^2-x^2)^{3/2}} = \int \frac{dx}{(\sqrt{a^2-x^2})^3} = \int \frac{\cancel{a} \cos \theta d\theta}{a^3 \cos^3 \theta} = \frac{1}{a^2} \int \sec^2 \theta d\theta = \frac{1}{a^2} \operatorname{tg} \theta + c$$

$$= \frac{x}{a^2 \sqrt{a^2-x^2}} + c$$

**6.59.-**  $\int \frac{dx}{\sqrt{4-(x-1)^2}}$

Solución.-

Se tiene:  $x-1 = 2 \operatorname{sen} \theta, dx = 2 \cos \theta d\theta, \sqrt{4-(x-1)^2} = 2 \cos \theta$

$$\int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{\cancel{2 \cos \theta} d\theta}{2 \cos \theta} = \int d\theta = \theta + c = \arcsen \frac{x-1}{2} + c$$

**6.60.-**  $\int \frac{x^2 dx}{\sqrt{2x-x^2}}$

Solución.-

Se tiene:  $x-1 = \operatorname{sen} \theta \Rightarrow x = \operatorname{sen} \theta + 1, dx = \cos \theta d\theta, \sqrt{1-(x-1)^2} = \cos \theta$

Completando cuadrados se tiene:

$2x-x^2 = -(x^2-2x) = -(x^2-2x+1)+1 = 1-(x-1)^2$ , luego:

$$\int \frac{x^2 dx}{\sqrt{2x-x^2}} = \int \frac{x^2 dx}{\sqrt{1-(x-1)^2}} = \int \frac{(\operatorname{sen} \theta + 1)^2 \cancel{\cos \theta} d\theta}{\cos \theta} = \int (\operatorname{sen} \theta + 1)^2 d\theta$$

$$\begin{aligned}
&= \int \sin^2 \theta d\theta + 2 \int \sin \theta d\theta + \int d\theta = \frac{1}{2} \int d\theta - \frac{1}{2} \int \cos 2\theta d\theta + 2 \int \sin \theta d\theta + \int d\theta \\
&= \frac{3}{2} \int d\theta - \frac{1}{2} \int \cos 2\theta d\theta + 2 \int \sin \theta d\theta = \frac{3}{2} \theta - \frac{1}{4} \sin 2\theta - 2 \cos \theta + c \\
&= \frac{3}{2} \theta - \frac{1}{2} \sin \theta \cos \theta - 2 \cos \theta + c = \frac{3}{2} \arcsen(x-1) - \frac{1}{2} (x-1) \sqrt{2x-x^2} - 2\sqrt{2x-x^2} + c
\end{aligned}$$

**6.61.-**  $\int \frac{x^2 dx}{\sqrt{17-x^2}}$

Solución.-

Se tiene:  $x = \sqrt{17} \sin \theta, dx = \sqrt{17} \cos \theta d\theta, \sqrt{17-x^2} = \sqrt{17} \cos \theta$

$$\begin{aligned}
\int \frac{x^2 dx}{\sqrt{17-x^2}} &= \int \frac{17 \sin^2 \theta \cancel{\sqrt{17} \cos \theta} d\theta}{\cancel{\sqrt{17} \cos \theta}} = 17 \int \sin^2 \theta d\theta = \frac{17}{2} \int d\theta - \frac{17}{2} \int \cos 2\theta d\theta \\
&= \frac{17}{2} \theta - \frac{17}{4} \sin 2\theta + c = \frac{17}{2} \theta - \frac{17}{2} \sin \theta \cos \theta + c \\
&= \frac{17}{2} \arcsen \frac{x}{\sqrt{17}} - \frac{17}{2} \frac{x}{\sqrt{17}} \frac{\sqrt{17-x^2}}{\sqrt{17}} + c = \frac{17}{2} \arcsen \frac{x}{\sqrt{17}} - \frac{1}{2} x \sqrt{17-x^2} + c
\end{aligned}$$

**6.62.-**  $\int \frac{x^2 dx}{\sqrt{21+4x-x^2}}$

Solución.-

Se tiene:  $x-2 = 5 \sin \theta \Rightarrow x = 5 \sin \theta + 2, dx = 5 \cos \theta d\theta, \sqrt{5^2-(x-2)^2} = 5 \cos \theta$

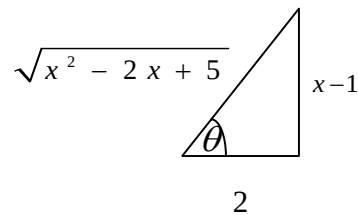
Completando cuadrados se tiene:

$21+4x-x^2 = -(x^2-4x+4-4)+21 = -(x^2-4x+4)+25 = 5^2-(x-2)^2$ , luego:

$$\begin{aligned}
\int \frac{x^2 dx}{\sqrt{21+4x-x^2}} &= \int \frac{x^2 dx}{\sqrt{5^2-(x-2)^2}} = \int \frac{(5 \sin \theta + 2)^2 \cancel{5 \cos \theta} d\theta}{\cancel{5 \cos \theta}} = \int (5 \sin \theta + 2)^2 d\theta \\
&= \int (25 \sin^2 \theta + 20 \sin \theta + 4) d\theta = 25 \int \frac{1-\cos 2\theta}{2} d\theta + 20 \int \sin \theta d\theta + 4 \int d\theta \\
&= \frac{25}{2} \int d\theta - \frac{25}{2} \int \cos 2\theta d\theta + 20 \int \sin \theta d\theta = \frac{25}{2} \theta - \frac{25}{4} \sin 2\theta - 20 \cos \theta + 4\theta + c \\
&= \frac{33}{2} \theta - \frac{25}{2} \sin \theta \cos \theta - 20 \cos \theta + c \\
&= \frac{33}{2} \arcsen \frac{x-2}{5} - \frac{25}{2} \frac{x-2}{5} \left( \frac{\sqrt{21+4x-x^2}}{5} \right) - 20 \left( \frac{\sqrt{21+4x-x^2}}{5} \right) + c \\
&= \frac{33}{2} \arcsen \frac{x-2}{5} - \sqrt{21+4x-x^2} \left( \frac{x-2}{2} + 4 \right) + c \\
&= \frac{33}{2} \arcsen \frac{x-2}{5} - \sqrt{21+4x-x^2} \left( \frac{x+6}{2} \right) + c
\end{aligned}$$

**6.63.-**  $\int \frac{dx}{(x^2 - 2x + 5)^{3/2}}$

Solución.-



Se tiene:  $x-1 = 2 \tan \theta$ ,  $dx = 2 \sec^2 \theta d\theta$ ,  $\sqrt{(x-1)^2 + 2^2} = 2 \sec \theta$

Completando cuadrados se tiene:

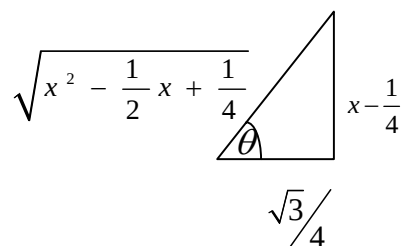
$x^2 - 2x + 5 = (x^2 - 2x + 1) + 5 - 1 = (x^2 - 2x + 1) + 4 = (x-1)^2 + 2^2$ , luego:

$$\begin{aligned} \int \frac{dx}{(x^2 - 2x + 5)^{3/2}} &= \int \frac{dx}{\left[\sqrt{(x-1)^2 + 2^2}\right]^3} = \int \frac{2 \sec^2 \theta d\theta}{2^3 \sec^3 \theta} = \frac{1}{4} \int \cos \theta d\theta = \frac{1}{4} \sin \theta + c \\ &= \frac{1}{4} \frac{x-1}{\sqrt{x^2 - 2x + 5}} + c \end{aligned}$$

**6.64.-**  $\int \frac{(2x+1)dx}{\sqrt{(4x^2 - 2x + 1)^3}}$

Solución.-

Sea:  $u = 4x^2 - 2x + 1$ ,  $du = (8x - 2)dx$



Se tiene:  $x - \frac{1}{4} = \frac{\sqrt{3}}{4} \tan \theta$ ,  $dx = \frac{\sqrt{3}}{4} \sec^2 \theta d\theta$ ,  $\sqrt{(x - \frac{1}{4})^2 + (\frac{\sqrt{3}}{4})^2} = \frac{\sqrt{3}}{4} \sec \theta$

Completando cuadrados se tiene:

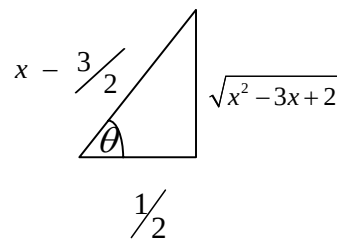
$x^2 - \frac{1}{2}x + \frac{1}{4} = (x^2 - \frac{1}{2}x + \frac{1}{16}) + \frac{1}{4} - \frac{1}{16} = (x - \frac{1}{4})^2 + \frac{3}{16} = (x - \frac{1}{4})^2 + (\frac{\sqrt{3}}{4})^2$ , luego:

$$\begin{aligned} \int \frac{(2x+1)dx}{\sqrt{(4x^2 - 2x + 1)^3}} &= \frac{1}{4} \int \frac{(8x+4)dx}{\sqrt{(4x^2 - 2x + 1)^3}} = \frac{1}{4} \int \frac{(8x-2+6)dx}{\sqrt{(4x^2 - 2x + 1)^3}} \\ &= \frac{1}{4} \int \frac{(8x-2)dx}{\sqrt{(4x^2 - 2x + 1)^3}} + \frac{3}{2} \int \frac{dx}{\sqrt{(4x^2 - 2x + 1)^3}} \\ &= \frac{1}{4} \int \frac{du}{(u)^{3/2}} + \frac{3}{2} \int \frac{dx}{\sqrt{4(x^2 - \frac{1}{2}x + \frac{1}{4})^3}} = \frac{1}{4} \int (u)^{-3/2} du + \frac{3}{2} \frac{1}{8} \int \frac{dx}{\sqrt{(x^2 - \frac{1}{2}x + \frac{1}{4})^3}} \\ &= \frac{1}{4} \int (u)^{-3/2} du + \frac{3}{16} \int \frac{dx}{\left[\sqrt{(x - \frac{1}{4})^2 + (\frac{\sqrt{3}}{4})^2}\right]^3} = \frac{1}{4} \int (u)^{-3/2} du + \frac{3}{16} \int \frac{\frac{\sqrt{3}}{4} \sec^2 \theta d\theta}{(\frac{\sqrt{3}}{4} \sec \theta)^3} \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} \int (u)^{-\frac{3}{2}} du + \int \frac{d\theta}{\sec \theta} = \frac{1}{4} \frac{u^{-\frac{1}{2}}}{(-\frac{1}{2})} + \text{sen } \theta + c = -\frac{1}{2u^{\frac{1}{2}}} + \text{sen } \theta + c \\
&= \frac{-1}{2\sqrt{4x^2 - 2x + 1}} + \frac{x - \frac{1}{4}}{\sqrt{x^2 - \frac{1}{2}x + \frac{1}{4}}} + c = \frac{4x - 2}{4\sqrt{x^2 - \frac{1}{2}x + \frac{1}{4}}} + c
\end{aligned}$$

**6.65.-**  $\int \frac{dx}{(x-1)\sqrt{x^2 - 3x + 2}}$

Solución.-



Se tiene:  $x - \frac{3}{2} = \frac{1}{2} \sec \theta \Rightarrow x - 1 = \frac{1}{2} (\sec \theta + 1), dx = \frac{1}{2} \sec \theta \tan \theta d\theta$ ,

$$\sqrt{(x - \frac{3}{2})^2 + (\frac{1}{2})^2} = \frac{1}{2} \sec \theta$$

Completando cuadrados se tiene:

$$x^2 - 3x + 2 = (x^2 - 3x + \frac{9}{4}) - \frac{1}{4} = (x - \frac{3}{2})^2 - (\frac{1}{2})^2, \text{ luego:}$$

$$\begin{aligned}
\int \frac{dx}{(x-1)\sqrt{x^2 - 3x + 2}} &= \int \frac{dx}{(x-1)\sqrt{(x - \frac{3}{2})^2 - (\frac{1}{2})^2}} = \int \frac{\frac{1}{2} \sec \theta \tan \theta d\theta}{\frac{1}{2} (\sec \theta + 1) \frac{1}{2} \sec \theta} \\
&= \int \frac{\sec \theta d\theta}{\frac{1}{2} (\sec \theta + 1)} = 2 \int \frac{\sec \theta d\theta}{(\sec \theta + 1)} = 2 \int \frac{\sec \theta (\sec \theta - 1) d\theta}{\sec^2 \theta - 1} = 2 \int \frac{\sec^2 \theta d\theta}{\tan^2 \theta} - 2 \int \frac{\sec \theta d\theta}{\tan^2 \theta} \\
&= 2 \int \sec^2 \theta d\theta - 2 \int \frac{\csc \theta d\theta}{\text{sen}^2 \theta} = -2 \cot \theta + 2 \csc \theta + c \\
&= -2 \frac{1/2}{\sqrt{x^2 - 3x + 2}} + 2 \frac{x - 3/2}{\sqrt{x^2 - 3x + 2}} + c = \frac{2x - 4}{\sqrt{x^2 - 3x + 2}} + c
\end{aligned}$$

**6.66.-**  $\int \frac{x dx}{\sqrt{x^2 - 2x + 5}}$

Solución.-

Se tiene:  $x - 1 = 2 \tan \theta, dx = 2 \sec^2 \theta d\theta, \sqrt{(x-1)^2 + (2)^2} = 2 \sec \theta$

Completando cuadrados se tiene:

$$x^2 - 2x + 5 = (x^2 - 2x + 1) + 4 = (x - 1)^2 + 2^2, \text{ luego:}$$

$$\begin{aligned}
\int \frac{x dx}{\sqrt{x^2 - 2x + 5}} &= \int \frac{x dx}{\sqrt{(x-1)^2 - 2^2}} = \int \frac{(2\tau g\theta + 1) \cancel{\sec^2 \theta} d\theta}{2\sec \theta} \\
&= 2 \int \tau g\theta \sec \theta d\theta + \int \sec \theta d\theta = 2 \sec \theta + \ell \eta |\sec \theta + \tau g\theta| + c \\
&= \sqrt{x^2 - 2x + 5} + \ell \eta \left| \frac{\sqrt{x^2 - 2x + 5} + x - 1}{2} \right| + c
\end{aligned}$$

**6.67.-**  $\int \frac{(x+1)dx}{\sqrt{2x-x^2}}$

Solución.-

Se tiene:  $x-1 = \sec \theta \Rightarrow x+1 = \sec \theta + 2, dx = \cos \theta d\theta, \sqrt{1-(x-1)^2} = \cos \theta$

Completando cuadrados se tiene:

$2x - x^2 = -(x^2 - 2x) = -(x^2 - 2x + 1 - 1) = -(x^2 - 2x + 1) + 1 = 1 - (x-1)^2$ , luego:

$$\begin{aligned}
\int \frac{(x+1)dx}{\sqrt{2x-x^2}} &= \int \frac{(x+1)dx}{\sqrt{1-(x-1)^2}} = \int \frac{(\sec \theta + 2) \cos \theta d\theta}{\cos \theta} = \int \sec \theta d\theta + 2 \int d\theta \\
&= -\cos \theta + 2\theta + c = -\sqrt{2x-x^2} + 2 \arcsen(x-1) + c
\end{aligned}$$

**6.68.-**  $\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}}$

Solución.-

Se tiene:  $x-2 = \sec \theta \Rightarrow x-1 = \sec \theta + 1, dx = \sec \theta \tau g\theta d\theta, \sqrt{(x-2)^2 - 1} = \tau g\theta$

Completando cuadrados se tiene:

$x^2 - 4x + 3 = x^2 - 4x + 4 - 1 = (x-2)^2 - 1$ , luego:

$$\begin{aligned}
\int \frac{(x-1)dx}{\sqrt{x^2-4x+3}} &= \int \frac{(x-1)dx}{\sqrt{(x-2)^2 - 1}} = \int \frac{(\sec \theta + 1) \sec \theta \cancel{\tau g\theta} d\theta}{\tau g\theta} \\
&= \int \sec^2 \theta d\theta + \int \sec \theta d\theta = \tau g\theta + \ell \eta |\sec \theta + \tau g\theta| + c \\
&= \sqrt{x^2 - 4x + 3} + \ell \eta \left| x - 2 + \sqrt{x^2 - 4x + 3} \right| + c
\end{aligned}$$

**6.69.-**  $\int \frac{dx}{\sqrt{x^2-2x-8}}$

Solución.-

Se tiene:  $x-1 = 3\sec \theta, dx = 3\sec \theta \tau g\theta d\theta, \sqrt{(x-1)^2 - 3^2} = 3\tau g\theta$

Completando cuadrados se tiene:

$x^2 - 2x - 8 = x^2 - 2x + 1 - 9 = (x-1)^2 - 3^2$ , luego:

$$\begin{aligned}
\int \frac{dx}{\sqrt{x^2-2x-8}} &= \int \frac{dx}{\sqrt{(x-1)^2 - 3^2}} = \int \frac{\cancel{3} \sec \theta \cancel{\tau g\theta} d\theta}{3\tau g\theta} = \int \sec \theta d\theta = \ell \eta |\sec \theta + \tau g\theta| + c \\
&= \ell \eta \left| \frac{x-1}{3} + \frac{\sqrt{x^2-2x-8}}{3} \right| + c = \ell \eta \left| x-1 + \sqrt{x^2-2x-8} \right| + c
\end{aligned}$$

**6.70.-**  $\int \frac{xdx}{\sqrt{x^2 + 4x + 5}}$

Solución.-

Se tiene:  $x + 2 = \tau g\theta$ ,  $dx = \sec^2 \theta d\theta$ ,  $\sqrt{(x + 2)^2 + 1^2} = \sec \theta$

Completando cuadrados se tiene:

$x^2 + 4x + 5 = (x^2 + 4x + 4) + 1 = (x + 2)^2 + 1^2$ , luego:

$$\int \frac{xdx}{\sqrt{x^2 + 4x + 5}} = \int \frac{xdx}{\sqrt{(x + 2)^2 + 1^2}} = \int \frac{(\tau g\theta - 2)\sec^2 \theta d\theta}{\sec \theta} = \int \tau g\theta \sec \theta d\theta - 2 \int \sec \theta d\theta$$

$$= \sec \theta - 2\ell\eta |\sec \theta + \tau g\theta| + c = \sqrt{x^2 + 4x + 5} - 2\ell\eta |\sqrt{x^2 + 4x + 5} + x + 2| + c$$

## CAPITULO 7

### INTEGRACIÓN DE FUNCIONES RACIONALES

Mediante el recurso de la descomposición en fracciones simples, el proceso de integración de funciones racionales se puede simplificar notablemente.

### EJERCICIOS DESARROLLADOS

**7.1.-Encontrar:**  $\int \frac{dx}{x^2-9}$

Solución.- Descomponiendo el denominador en factores:  $x^2-9=(x+3)(x-3)$ ,  
Como los factores son ambos lineales y diferentes se tiene:

$$\frac{1}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}, \text{ de donde:}$$

$$\frac{1}{\cancel{x^2}-9} = \frac{A}{\cancel{x}+3} + \frac{B}{\cancel{x}-3} \Rightarrow 1 = A(x-3) + B(x+3)(*) \Rightarrow 1 = (A+B)x + (-3A+3B)$$

Para calcular las constantes A y B, se pueden identificar los coeficientes de igual potencia x en la última expresión, y se resuelve el sistema de ecuaciones dado; obteniendo así los valores de las constantes en referencia (método general) luego:

$$\begin{pmatrix} A+B=0 \\ -3A+3B=1 \end{pmatrix} \Rightarrow \begin{pmatrix} 3A+3B=0 \\ -3A+3B=1 \end{pmatrix} \Rightarrow 6B=1 \Rightarrow B = \frac{1}{6}, \text{ además:}$$

$$A+B=0 \Rightarrow A=-B \Rightarrow A = -\frac{1}{6}$$

También es frecuente usar otro mecanismo, que consiste en la expresión (\*)

Sustituyendo a x por los valores que anulen los denominadores de las fracciones:

$$x=3 \Rightarrow 1=6B \Rightarrow B = \frac{1}{6}$$

$$x=-3 \Rightarrow 1=-6A \Rightarrow A = -\frac{1}{6}$$

Usando cualquier método de los señalados anteriormente, se establece que:

$$\frac{1}{x^2-9} = \frac{-\frac{1}{6}}{x+3} + \frac{\frac{1}{6}}{x-3}, \text{ Luego se tiene:}$$

$$\begin{aligned} \int \frac{dx}{x^2-9} &= -\frac{1}{6} \int \frac{dx}{x+3} + \frac{1}{6} \int \frac{dx}{x-3} = -\frac{1}{6} \ell \eta |x+3| + \frac{1}{6} \ell \eta |x-3| + c \\ &= \frac{1}{6} (\ell \eta |x-3| - \ell \eta |x+3|) + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{x^2-9} = \frac{1}{6} \ell \eta \left| \frac{x-3}{x+3} \right| + c$

**7.2.-Encontrar:**  $\int \frac{dx}{x^2+7x-6}$

Solución.- Sea:  $x^2+7x+6=(x+6)(x+1)$ , factores lineales y diferentes; luego:

$$\frac{1}{x^2+7x+6} = \frac{A}{x+6} + \frac{B}{x+1},$$

De donde:

$1 = A(x+1) + B(x+6)(*) \Rightarrow 1 = (A+B)x + (A+6B)$ , calculando las constantes  $A$  y  $B$  por el método general, se tiene:  $1 = (A+B)x + (A+6B)$

$$\begin{pmatrix} A+B=0 \\ A+6B=1 \end{pmatrix} \Rightarrow -\begin{pmatrix} -A-B=0 \\ A+6B=1 \end{pmatrix} \Rightarrow 5B=1 \Rightarrow B = \frac{1}{5}, \text{ además:}$$

$$A+B=0 \Rightarrow A=-B \Rightarrow A = -\frac{1}{5}$$

Ahora utilizando el método abreviado se tiene:

$$x=-1 \Rightarrow 1=5B \Rightarrow B = \frac{1}{5}$$

$$x=-6 \Rightarrow 1=-5A \Rightarrow A = -\frac{1}{5}$$

Usando cualquier método se puede establecer:

$$\frac{1}{x^2+7x+6} = \frac{-\frac{1}{5}}{x+6} + \frac{\frac{1}{5}}{x+1}, \text{ Luego se tiene:}$$

$$\begin{aligned} \int \frac{dx}{x^2+7x+6} &= -\frac{1}{5} \int \frac{dx}{x+6} + \frac{1}{5} \int \frac{dx}{x+1} = -\frac{1}{5} \ell \eta |x+6| + \frac{1}{5} \ell \eta |x+1| + c \\ &= \frac{1}{5} (\ell \eta |x+1| - \ell \eta |x+6|) + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{x^2+7x+6} = \frac{1}{5} \ell \eta \left| \frac{x+1}{x+6} \right| + c$

**7.3.-Encontrar:**  $\int \frac{x dx}{x^2-4x+4}$

Solución.- Sea:  $x^2-4x+4=(x-2)^2$ , factores lineales con repetición; luego:

$$\frac{x}{x^2-x+4} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \Rightarrow \frac{x}{\cancel{x^2-x+4}} = \frac{A(x-2)+B}{(\cancel{x-2})^2},$$

De donde:

$x = A(x-2) + B(*)$ , calculando las constantes  $A$  y  $B$  por el método general, se tiene:  $x = Ax + (-2A+B)$ , luego:

$$\begin{pmatrix} A=1 \\ -2A+B=0 \end{pmatrix} \Rightarrow B=2A \Rightarrow B=2(1) \Rightarrow B=2$$



Usando el método abreviado, se sustituye en  $x$ , el valor que anula el denominador (o los denominadores), y si este no es suficiente se usan para sustituir cualquier valor conveniente de  $x$ , esto es:  $x = 0, x = -1$ ; luego en (\*)

$$x = 2 \Rightarrow 2 = B \Rightarrow B = 2$$

$$x = 0 \Rightarrow 0 = -2A + B \Rightarrow 2A + B \Rightarrow A = \frac{B}{2} \Rightarrow A = 1$$

Usando cualquier método se establece:

$$\int \frac{x dx}{x^2 - 4x + 4} = \int \frac{dx}{x-2} + 2 \int \frac{dx}{(x-2)^2} = \ell \eta |x-2| - \frac{2}{x-2} + c$$

**Respuesta:**  $\int \frac{x dx}{x^2 - 4x + 4} = \ell \eta |x-2| - \frac{2}{x-2} + c$

**7.4.-Encontrar:**  $\int \frac{(2x^2 + 3)dx}{x^3 - 2x^2 + x}$

Solución.- Sea:  $x^3 - 2x^2 + x = x(x^2 - 2x + 1) = x(x-1)^2$ , factores lineales:

$x, x-1$ ; donde este último es con repetición; luego:

$$\frac{2x^2 + 3}{x^3 - 2x^2 + x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \Rightarrow \frac{2x^2 + 3}{\cancel{x^3} - 2x^2 + x} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{x\cancel{(x-1)^2}}$$

De donde:

$2x^2 + 3 = A(x-1)^2 + Bx(x-1) + Cx(*)$ , calculando las constantes  $A$  y  $B$  por el método general, se tiene:  $2x^2 + 3 = (A+B)x^2 + (-2A-B+C)x + A$ , de donde identificando los coeficientes de igual potencia de  $x$  se puede obtener el siguiente sistema de ecuaciones:

$$\begin{pmatrix} A+B & = 2 \\ -2A-B+C & = 0 \\ A & = 3 \end{pmatrix} \Rightarrow B = 2 - A \Rightarrow B = 2 - 3 \Rightarrow B = -1, \text{ tomando la segunda ecuación}$$

del sistema:  $C = 2A + B \Rightarrow C = 2(3) - 1 \Rightarrow C = 5$ , también es posible usar el método abreviado, utilizando para ello la expresión (\*) en la cual:

$$x = 1 \Rightarrow 2(1) + 3 = C \Rightarrow C = 5$$

$$x = 0 \Rightarrow 3 = A \Rightarrow A = 3$$

Usando un valor arbitrario para  $x$ , sea este  $x = -1$ :

$$x = -1 \Rightarrow 2(-1)^2 + 3 = A(-2)^2 + B(-1)(-2) + C(-1) \Rightarrow 5 = 4A + 2B - C, \text{ luego:}$$

$$2B = 5 - 4A + C \Rightarrow 2B = 5 - 4(3) + 5 \Rightarrow 2B = -2 \Rightarrow B = -1, \text{ S, e establece que:}$$

$$\frac{2x^2 + 3}{x^3 - 2x^2 + x} = \frac{3}{x} - \frac{1}{x-1} + \frac{5}{(x-1)^2}, \text{ entonces:}$$

$$\frac{2x^2 + 3}{x^3 - 2x^2 + x} = 3 \int \frac{dx}{x} - \int \frac{dx}{x-1} + 5 \int \frac{dx}{(x-1)^2} = 3\ell \eta |x| - \ell \eta |x-1| - \frac{5}{x-1} + c$$

**Respuesta:**  $\int \frac{(2x^2 + 3)dx}{x^3 - 2x^2 + x} = \ell \eta \left| \frac{x^3}{x-1} \right| - \frac{5}{x-1} + c$

**7.5.-Encontrar:**  $\int \frac{dx}{x^3 - 2x^2 + x}$

**Solución.-**  $x^3 - 2x^2 + x = x(x-1)^2$  ,factores lineales:

$x, x-1$ ; donde este último es con repetición; luego:

$$\frac{1}{x^3 - 2x^2 + x} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \Rightarrow \frac{1}{\cancel{x^3 - 2x^2 + x}} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{\cancel{x(x-1)^2}}$$

De donde:

$1 = A(x-1)^2 + Bx(x-1) + Cx(*)$ , calculando las constantes  $A$  y  $B$  por el método general, se tiene:  $1 = (A+B)x^2 + (-2A-B+C)x + A$ , de donde identificando los coeficientes de igual potencia de  $x$  se puede obtener el siguiente sistema de ecuaciones:

$$\begin{pmatrix} A+B & =0 \\ -2A-B+C & =0 \\ A & =1 \end{pmatrix} \Rightarrow B=-A \Rightarrow B=-1, \text{ tomando la segunda ecuación del}$$

sistema:  $C = 2A + B \Rightarrow C = 2(1) - 1 \Rightarrow C = 1$ , a partir de lo cual se tiene:

$$\frac{1}{x^3 - 2x^2 + x} = \frac{1}{x} - \frac{1}{x-1} + \frac{1}{(x-1)^2}$$

$$\int \frac{dx}{x^3 - 2x^2 + x} = \int \frac{dx}{x} - \int \frac{dx}{x-1} + \int \frac{dx}{(x-1)^2} = \ell \eta |x| - \ell \eta |x-1| - \frac{1}{x-1} + c$$

**Respuesta:**  $\int \frac{dx}{x^3 - 2x^2 + x} = \ell \eta \left| \frac{x}{x-1} \right| - \frac{1}{x-1} + c$

**7.6.-Encontrar:**  $\int \frac{x^4 - 6x^3 + 12x^2 + 6}{x^3 - 6x^2 + 12x - 8} dx$

Solución.- Se sabe que si el grado del polinomio dividendo, es igual o superior al grado del polinomio divisor, previamente conviene efectuar la división de tales polinomios.

$$\begin{array}{r} x^4 - 6x^3 + 12x^2 + 0x + 6 \quad | \quad x^3 - 6x^2 + 12x - 8 \\ -x^4 + 6x^3 - 12x^2 + 8x \quad \phantom{|} \quad x \\ \hline \phantom{x^4 - 6x^3 + 12x^2 + 0x + 6 - x^4 + 6x^3 - 12x^2 + 8x} \boxed{8x + 6} \end{array}$$

Luego se tiene:  $\int \frac{x^4 - 6x^3 + 12x^2 + 6}{x^3 - 6x^2 + 12x - 8} dx = \int x dx + \int \frac{(8x + 6)dx}{x^3 - 6x^2 + 12x - 8}$

La descomposición de:  $x^3 - 6x^2 + 12x - 8$ :

$$\begin{array}{c|cccc} & 1 & -6 & 12 & -8 \\ 2 & & 2 & -8 & 8 \\ \hline & 1 & -4 & 4 & \boxed{0} \end{array} \quad x = 2 \Rightarrow (x-2)$$

$$x^2 - 4x + 4 = (x - 2)^2$$

$$x^3 - 6x^2 + 12x - 8 = (x - 2)^3$$

Esto es factores lineales:  $[(x-2)]$  con repetición por tanto:

$$\frac{8x+6}{x^3-6x^2+12x-8} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$

$$\frac{8x+6}{\cancel{x^3-6x^2+12x-8}} = \frac{A(x-2)^2 + B((x-2) + C}{\cancel{(x-2)^3}}$$

Luego:

$$8x+6 = A(x-2)^2 + B(x-2) + C \Rightarrow 8x+6 = A(x^2-4x+4) + B(x-2) + C$$

$$8x+6 = Ax^2 + (-4A+B)x + (4A-2B+C)$$

Calculando las constantes  $A$  y  $B$  por el método general, se tiene:

$$\begin{pmatrix} A & = 0 \\ -4A + B & = 8 \\ +4A - 2B + C & = 6 \end{pmatrix} \Rightarrow B = 8 + 4A \Rightarrow B = 8 + 4(0) \Rightarrow B = 8,$$

Resolviendo el sistema:  $C = 6 - 4A + 2B \Rightarrow C = 6 - 4(0) + 2(8) \Rightarrow C = 22$ , luego:

$$\frac{8x+6}{x^3-6x^2+12x-8} = \frac{0}{\cancel{x-2}} + \frac{8}{(x-2)^2} + \frac{22}{(x-2)^3}, \text{ de donde:}$$

$$\int \frac{(8x+6)dx}{x^3-6x^2+12x-8} = 8 \int \frac{dx}{(x-2)^2} + 22 \int \frac{dx}{(x-2)^3}, \text{ o sea:}$$

$$= \int x dx + 8 \int \frac{dx}{(x-2)^2} + 22 \int \frac{dx}{(x-2)^3} = \int x dx + 8 \int (x-2)^{-2} dx + 22 \int (x-2)^{-3} dx$$

$$\frac{x^2}{2} - \frac{8}{x-2} - \frac{11}{(x-2)^2} + c$$

**Respuesta:**  $\int \frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8} dx = \frac{x^2}{2} - \frac{8}{x-2} - \frac{11}{(x-2)^2} + c$

**7.7.-Encontrar:**  $\int \frac{x^3+x^2+x+3}{x^4+4x^2+3} dx$

Solución.-  $x^4+4x^2+3 = (x^2+3)(x^2+1)$ , la descomposición es en factores cuadráticos sin repetición, por lo tanto:

$$\frac{x^3+x^2+x+3}{x^4+4x^2+3} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+1}$$

$$\frac{x^3+x^2+x+3}{\cancel{x^4+4x^2+3}} = \frac{(Ax+B)(x^2+1) + (Cx+D)(x^2+3)}{\cancel{(x^2+3)(x^2+1)}}$$

$$x^3+x^2+x+3 = A(x^3+x) + B(x^2+1) + C(x^3+3x) + D(x^2+3)$$

$$x^3+x^2+x+3 = (A+C)x^3 + (B+D)x^2 + (A+3C)x + (B+3D), \text{ luego:}$$

$$\begin{aligned} (1) & \begin{pmatrix} A + C = 1 \\ B + D = 1 \\ A + 3C = 1 \\ B + 3D = 3 \end{pmatrix} \\ (2) & \\ (3) & \\ (4) & \end{aligned}$$

Con (1) y (3), se tiene:  $\begin{pmatrix} A + C = 1 \\ A + 3C = 1 \end{pmatrix} \Rightarrow A = 1, C = 0$

Con (2) y (4), se tiene:  $\begin{pmatrix} B + D = 1 \\ B + 3D = 3 \end{pmatrix} \Rightarrow B = 0, D = 1$

Por lo tanto:  $\frac{x^3 + x^2 + x + 3}{x^4 + 4x^2 + 3} = \frac{x}{x+3} + \frac{1}{x^2+1}$ , o sea:

$$\int \frac{x^3 + x^2 + x + 3}{x^4 + 4x^2 + 3} dx = \int \frac{x dx}{x+3} + \int \frac{dx}{x^2+1}, \text{ sea: } u = x^2 + 3, du = 2x dx, \text{ luego:}$$

$$\begin{aligned} \int \frac{x^3 + x^2 + x + 3}{x^4 + 4x^2 + 3} dx &= \frac{1}{2} \int \frac{2x dx}{x+3} + \int \frac{dx}{x^2+1} = \frac{1}{2} \int \frac{du}{u} + \int \frac{dx}{x^2+1} \\ &= \frac{1}{2} \ell \eta |u| + \arctan x + c = \frac{1}{2} \ell \eta |x^2 + 3| + \arctan x + c \end{aligned}$$

**Respuesta:**  $\int \frac{x^3 + x^2 + x + 3}{x^4 + 4x^2 + 3} dx = \frac{1}{2} \ell \eta |x^2 + 3| + \arctan x + c$

**7.8.-Encontrar:**  $\int \frac{x^4 dx}{x^4 + 2x^2 + 1}$

Solución.-

$$\begin{array}{r} x^4 \quad \quad \quad |x^4 + 2x^2 + 1 \\ -x^4 - 2x^2 - 1 \quad \quad 1 \\ \hline \quad \quad \quad -2x^2 - 1 \end{array}$$

Luego  $\int \frac{x^4 dx}{x^4 + 2x^2 + 1} = \int \left( 1 - \frac{2x^2 + 1}{x^4 + 2x^2 + 1} \right) dx = \int dx - \int \frac{2x^2 + 1}{x^4 + 2x^2 + 1} dx$

La descomposición del denominador es:  $x^4 + 2x^2 + 1 = (x^2 + 1)^2$ , entonces:

$$\frac{2x^2 + 1}{x^4 + 2x^2 + 1} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{(x^2 + 1)^2} \Rightarrow \frac{2x^2 + 1}{x^4 + 2x^2 + 1} = \frac{(Ax + B)(x^2 + 1)(Cx + D)}{(x^2 + 1)^2}$$

$$2x^2 + 1 = (Ax + B)(x^2 + 1) + (Cx + D) \Rightarrow 2x^2 + 1 = A(x^3 + x) + B(x^2 + 1) + Cx + D$$

$$2x^2 + 1 = Ax^3 + Bx^2 + (A + C)x + (B + D)$$

Calculando las constantes por el método general, se tiene:

$$\begin{pmatrix} A & & & = 0 \\ & B & & = 2 \\ A & + C & & = 0 \\ & B & + D & = 1 \end{pmatrix}$$

Resolviendo el sistema:  $C = -A \Rightarrow A = 0 \therefore C = 0$ ,  $B + D = 1 \Rightarrow D = 1 - B \Rightarrow D = -1$   
 luego:

$$\frac{2x^2+1}{x^4+2x^2+1} = \frac{2}{x^2+1} - \frac{1}{(x^2+1)^2}, \text{ o sea:}$$

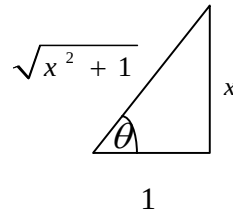
$$\int \frac{2x^2+1}{x^4+2x^2+1} = 2 \int \frac{dx}{x^2+1^2} - \int \frac{dx}{(x^2+1)^2} = 2 \int \frac{dx}{x^2+1^2} - \int \frac{dx}{(\sqrt{x^2+1})^4}$$

Sea:  $x = \tau g \theta$ ,  $dx = \sec^2 \theta d\theta$ ;  $\sqrt{x^2+1} = \sec \theta$ , luego:

$$= 2 \operatorname{arc} \tau g x - \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = 2 \operatorname{arc} \tau g x - \int \frac{d\theta}{\sec^2 \theta} = 2 \operatorname{arc} \tau g x - \int \cos^2 \theta$$

$$= 2 \operatorname{arc} \tau g x - \int \frac{1+\cos 2\theta}{2} d\theta = 2 \operatorname{arc} \tau g x - \frac{1}{2} \int d\theta - \frac{1}{2} \int \cos 2\theta d\theta$$

$$\operatorname{arc} \tau g x - \frac{1}{2} \theta - \frac{1}{2} \operatorname{sen} 2\theta + c = 2 \operatorname{arc} \tau g x - \frac{1}{2} \theta - \frac{1}{2} \operatorname{sen} \theta \cos \theta + c$$



De la figura se tiene que:

$$\tau g \theta = x, \theta \operatorname{arc} \tau g \theta, \operatorname{sen} \theta = \frac{x}{\sqrt{x^2+1}}, \cos \theta = \frac{1}{\sqrt{x^2+1}}$$

$$\text{Luego: } = 2 \operatorname{arc} \tau g x - \frac{1}{2} \operatorname{arc} \tau g x - \frac{1}{2} \frac{x}{\sqrt{x^2+1}} \frac{1}{\sqrt{x^2+1}} + c = 2 \operatorname{arc} \tau g x - \frac{1}{2} \operatorname{arc} \tau g x - \frac{x}{2(x^2+1)} + c$$

Recordando que:

$$\frac{x^4 dx}{x^4+2x^2+1} = \int dx - \int \frac{(2x^2+1)dx}{x^4+2x^2+1} = x - 2 \operatorname{arc} \tau g x + \frac{1}{2} \operatorname{arc} \tau g x + \frac{1}{2} \frac{x}{(x^2+1)} + c$$

$$\text{Respuesta: } \int \frac{x^4 dx}{x^4+2x^2+1} = x - \frac{3}{2} \operatorname{arc} \tau g x + \frac{x}{2(x^2+1)} + c$$

$$\text{7.9.-Encontrar: } \int \frac{x^4 dx}{x^4-1}$$

Solución.-

$$\frac{x^4}{-x^4+1} \quad \frac{x^4-1}{1}$$

Luego:

$$\int \frac{x^4 dx}{x^4-1} = \int \left( 1 + \frac{1}{x^4-1} \right) dx = \int dx + \int \frac{dx}{x^4-1}$$

Descomponiendo en factores el denominador:

$x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x^2 + 1)(x + 1)(x - 1)$ , es decir factores lineales y cuadráticos sin repetición por tanto:

$$\frac{1}{\cancel{x^4-1}} = \frac{(Ax+B)(x^2-1)+C(x^2+1)(x-1)+D(x+1)(x^2+1)}{(\cancel{x^2+1})(\cancel{x+1})(x+1)}$$

$$1 = (A + C + D)x^3 + (B - C + D)x^2 + (-A + C + D)x + (-B - C + D)$$

$$\begin{array}{l} (1) \quad A + C + D = 0 \\ (2) \quad B - C + D = 0 \\ (3) \quad -A + C + D = 0 \\ (4) \quad -B - C + D = 1 \end{array}$$

Con (2) y (4), se tiene:  $\begin{pmatrix} B-C+D=0 \\ -B-C+D=1 \end{pmatrix} \Rightarrow -2C+2D=1$  **(6)**

Además:  $A = 0, B = -\frac{1}{2}$ , luego:

$$= -\frac{1}{2} \arccos \tau g x - \frac{1}{4} \ell \eta |x+1| + \frac{1}{4} \ell \eta |x-1| + c$$

**Respuesta:**  $\int \frac{1}{x^4 - 1} = x - \frac{1}{2} \operatorname{arctg} x + \frac{1}{4} \ell \eta \left| \frac{x-1}{x+1} \right| + c$

Solución.-

$$-x^4 + 2x^3 - 3x^2 \quad x$$

$$\boxed{-x+3}$$

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$$\int \frac{x^4 - 2x^3 + 3x^2 - x + 3}{x^3 - 2x^2 + 3x} dx = \int \left( x - \frac{x-3}{x^3 - 2x^2 + 3x} \right) dx = \int x dx - \int \frac{x-3}{x^3 - 2x^2 + 3x} dx$$

Descomponiendo en factores el denominador:

$x^3 - 2x^2 + 3x = x(x^2 - 2x + 3)$ , es decir un factor lineal y uno cuadrático; por lo cual:

$$\frac{x-3}{x^3 - 2x^2 + 3x} = \frac{A}{x} + \frac{Bx+C}{x^2 - 2x + 3} \Rightarrow \frac{x-3}{\cancel{x^3 - 2x^2 + 3x}} = \frac{A(\cancel{x^2 - 2x + 3}) + (Bx+C)x}{x(\cancel{x^2 - 2x + 3})}$$

$$x-3 = A(x^2 - 2x + 3) + (Bx+C)x \Rightarrow x-3 = (A+B)x^2 + (-2A+C)x + 3A$$

De donde:

$$\begin{cases} A+B = 0 \\ -2A+C = 1 \\ 3A = -3 \end{cases} \Rightarrow \begin{cases} A = -1 \\ B = -A \Rightarrow B = 1 \\ C = 1+2A \Rightarrow C = -1 \end{cases}$$

Luego:

$$\frac{x-3}{x^3 - 2x^2 + 3x} = -\frac{1}{x} + \frac{x-1}{x^2 - 2x + 3}, \text{ de donde:}$$

$$\int \frac{x-3}{x^3 - 2x^2 + 3x} dx = -\int \frac{dx}{x} + \int \frac{x-1}{x^2 - 2x + 3} dx = -\ell \eta |x| + \int \frac{x-1}{x^2 - 2x + 3} dx$$

$$\begin{aligned} \int \frac{x^4 - 2x^3 + 3x^2 - x + 3}{x^3 - 2x^2 + 3x} dx &= \int x dx + \ell \eta |x| - \int \frac{x-1}{x^2 - 2x + 3} dx \\ &= \frac{x^2}{2} + \ell \eta |x| - \int \frac{x-1}{x^2 - 2x + 3} dx = \frac{x^2}{2} + \ell \eta |x| - \frac{1}{2} \int \frac{2(x-1)dx}{x^2 - 2x + 3} \end{aligned}$$

$$\text{Sea: } u = x^2 - 2x + 3, du = (2x-2)dx \Rightarrow du = 2(x-1)dx$$

$$= \frac{x^2}{2} + \ell \eta |x| - \frac{1}{2} \int \frac{du}{u} = \frac{x^2}{2} + \ell \eta |x| - \frac{1}{2} \ell \eta |x^2 - 2x + 3| + c$$

$$\text{Respuesta: } \int \frac{x^4 - 2x^3 + 3x^2 - x + 3}{x^3 - 2x^2 + 3x} dx = \frac{x^2}{2} + \ell \eta \left| \frac{x}{\sqrt{x^2 - 2x + 3}} \right| + c$$

## EJERCICIOS PROPUESTOS

Usando La técnica de descomposición en fracciones simples parciales, calcular las siguientes integrales:

$$7.11.- \int \frac{(x^5 + 2)dx}{x^2 - 1}$$

$$7.12.- \int \frac{xdx}{(x+1)^2}$$

$$7.13.- \int \frac{x^3 dx}{x^2 - 2x - 3}$$

$$7.14.- \int \frac{(3x+7)dx}{(x-1)(x-2)(x-3)}$$

$$7.15.- \int \frac{dx}{x^3 + 1} dx$$

$$7.16.- \int \frac{(x+5)dx}{x^2 - x + 6}$$

$$7.17.- \int \frac{(x^2 + 1)dx}{x^3 + 1}$$

$$7.18.- \int \frac{(x^2 + 6)dx}{(x-1)^2(x-2)}$$

$$7.19.- \int \frac{(x^2 - 1)dx}{(x^2 + 1)(x-2)}$$

$$\begin{array}{lll}
7.20.- \int \frac{x dx}{x^2 - 4x - 5} & 7.21.- \int \frac{x dx}{x^2 - 2x - 3} & 7.22.- \int \frac{(x+1) dx}{x^2 + 4x - 5} \\
7.23.- \int \frac{x^2 dx}{x^2 + 2x + 1} & 7.24.- \int \frac{dx}{x(x+1)^2} & 7.25.- \int \frac{dx}{(x+1)(x^2+1)} \\
7.26.- \int \frac{dx}{x(x^2+x+1)} & 7.27.- \int \frac{2x^2+5x-1}{x^3+x^2-2x} dx & 7.28.- \int \frac{(x^2+2x+3) dx}{(x-1)(x+1)^2} \\
7.29.- \int \frac{3x^2+2x-2}{x^3-1} dx & 7.30.- \int \frac{x^4-x^3+2x^2-x+2}{(x-1)(x^2+2)^2} dx & 7.31.- \int \frac{(2x^2-7x-1) dx}{x^3+x^2-x-1} \\
7.32.- \int \frac{3x^2+3x+1}{x^3+2x^2+2x+1} dx & 7.33.- \int \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^2} dx & 7.34.- \int \frac{2x dx}{(x^2+x+1)^2} \\
7.35.- \int \frac{x^2+2x+3}{x^3-x} dx & 7.36.- \int \frac{(2x^2-3x+5) dx}{(x+2)(x-1)(x-3)} & 7.37.- \int \frac{(3x^2+x-2) dx}{(x-1)(x^2+1)} \\
7.38.- \int \frac{(x+5) dx}{x^3-3x+2} & 7.39.- \int \frac{2x^3+3x^2+x-1}{(x+1)(x^2+2x+2)^2} dx & 7.40.- \int \frac{(2x+1) dx}{3x^3+2x-1} \\
7.41.- \int \frac{(2x^2+3x-1) dx}{x^3+2x^2+4x+2} & 7.42.- \int \frac{x^4-2x^2+3x+4}{(x-1)^3(x^2+2x+2)} dx & 7.43.- \int \frac{e^t dt}{e^{2t}+3e^t+2} \\
7.44.- \int \frac{\sec \theta d\theta}{\cos^2 \theta + \cos \theta - 2} & 7.45.- \int \frac{4x^4-2x^3-x^2+3x+1}{(x^3+x^2-x-1)} dx & 7.46.- \int \frac{3x^4 dx}{(x^2+1)^2} \\
7.47.- \int \frac{(2x^2+41x-91) dx}{x^3-2x^2-11x+12} & 7.48.- \int \frac{(2x^4+3x^3-x-1) dx}{(x-1)(x^2+2x+2)^2} & 7.49.- \int \frac{dx}{e^{2x}+e^x-2} \\
7.50.- \int \frac{\sec x dx}{\cos x(1+\cos^2 x)} & 7.51.- \int \frac{(2+\tau g^2 \theta) \sec^2 \theta d\theta}{1+\tau g^3 \theta} & 7.52.- \int \frac{(5x^3+2) dx}{x^3-5x^2+4x} \\
7.53.- \int \frac{x^5 dx}{(x^3+1)(x^3+8)} & & 
\end{array}$$

## RESPUESTAS

$$7.11.- \int \frac{(x^5+2) dx}{x^2-1}$$

Solución.-

$$\int \frac{(x^5+2) dx}{x^2-1} = \int \left( x^3 + x + \frac{x+2}{x^2-1} \right) dx = \int x^3 dx + \int x dx + \int \frac{x+2}{x^2-1} dx$$

$$= \frac{x^4}{4} + \frac{x^2}{2} + \int \frac{(x+2) dx}{(x+1)(x-1)} (*) , \text{ luego:}$$

$$\frac{x+2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1} \Rightarrow x+2 = A(x-1) + B(x+1)$$



$$\begin{aligned} \therefore \begin{cases} x=1 \Rightarrow 3=2B \Rightarrow B=3/2 \\ x=-1 \Rightarrow 1=-2A \Rightarrow A=-1/2 \end{cases} \\ (*) = \frac{x^4}{4} + \frac{x^2}{2} - \frac{1}{2} \int \frac{dx}{x+1} + \frac{3}{2} \int \frac{dx}{x-1} = \frac{x^4}{4} + \frac{x^2}{2} - \frac{1}{2} \ell \eta |x+1| + \frac{3}{2} \ell \eta |x-1| + c \\ = \frac{x^4}{4} + \frac{x^2}{2} + \eta \left| \frac{(x-1)^{3/2}}{\sqrt{x+1}} \right| + c \end{aligned}$$

**7.12.-**  $\int \frac{xdx}{(x+1)^2}$

Solución.-

$$\int \frac{xdx}{(x+1)^2} = \int \frac{Adx}{x+1} + \int \frac{Bdx}{(x+1)^2} \quad (*) \text{ , luego:}$$

$$\frac{x}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \Rightarrow x = A(x+1) + B$$

$$\therefore \begin{cases} x=-1 \Rightarrow -1=B \\ x=0 \Rightarrow 0=A+B \Rightarrow A=-B \Rightarrow A=-1 \end{cases}$$

$$(*) \int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} = \ell \eta |x+1| + (x+1)^{-1} + c = \ell \eta |x+1| + \frac{1}{x+1} + c$$

**7.13.-**  $\int \frac{x^3 dx}{x^2 - 2x - 3}$

Solución.-

$$\int \frac{x^3 dx}{x^2 - 2x - 3} = \int \left( x + 2 + \frac{7x+6}{x^2 - 2x - 3} \right) dx = \int x dx + 2 \int dx + \int \frac{(7x+6)dx}{x^2 - 2x - 3}$$

$$= \frac{x^2}{2} + 2x + \int \frac{(7x+6)dx}{(x-3)(x+1)} \quad (*), \text{ luego:}$$

$$\frac{(7x+6)}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1} \Rightarrow 7x+6 = A(x+1) + B(x-3)$$

$$\therefore \begin{cases} x=3 \Rightarrow 27=4A \Rightarrow A=27/4 \\ x=-1 \Rightarrow -1=-4B \Rightarrow B=1/4 \end{cases}$$

$$(*) = \frac{x^2}{2} + 2x + \frac{27}{4} \int \frac{dx}{x-3} + \frac{1}{4} \int \frac{dx}{x+1} = \frac{x^2}{2} + 2x + \frac{27}{4} \ell \eta |x-3| + \frac{1}{4} \ell \eta |x+1| + c$$

$$= \frac{x^2}{2} + 2x + \frac{1}{4} \ell \eta |(x-3)^{27}(x+1)| + c$$

**7.14.-**  $\int \frac{(3x+7)dx}{(x-1)(x-2)(x-3)}$

Solución.-

$$\int \frac{(3x+7)dx}{(x-1)(x-2)(x-3)} = \int \frac{Adx}{x-1} + \int \frac{Bdx}{x-2} + \int \frac{Cdx}{x-3} \quad (*)$$

$$\frac{(3x+7)}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$3x-7 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ , luego:

$$\therefore \begin{cases} x=1 \Rightarrow -4 = 2A \Rightarrow A = -2 \\ x=2 \Rightarrow -1 = -B \Rightarrow B = 1 \\ x=3 \Rightarrow 2 = 2C \Rightarrow C = 1 \end{cases}$$

$$\begin{aligned} (*) &= -2 \int \frac{dx}{x-1} + \int \frac{dx}{x-2} + \int \frac{dx}{x-3} = -2\ell\eta|x-1| + \ell\eta|x-2| + \ell\eta|x-3| + c \\ &= \ell\eta \left| \frac{(x-2)(x-3)}{(x-1)^2} \right| + c \end{aligned}$$

**7.15.-**  $\int \frac{dx}{x^3+1} dx$

Solución.-

$$\int \frac{dx}{x^3+1} = \int \frac{dx}{(x+1)(x^2-x+1)} = \int \frac{Adx}{x+1} + \int \frac{(Bx+C)dx}{(x^2-x+1)} \quad (*), \text{ luego:}$$

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{(Bx+C)}{(x^2-x+1)} \Rightarrow 1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$\begin{aligned} \therefore \begin{cases} x=-1 \Rightarrow 1 = 3A \Rightarrow A = \frac{1}{3} \\ x=0 \Rightarrow 1 = A+C \Rightarrow C = 1-A \Rightarrow C = \frac{2}{3} \\ x=1 \Rightarrow 1 = A+(B+C)2 \Rightarrow 1 = \frac{1}{3} + 2B + 2C \Rightarrow \frac{1}{3} = B+C \Rightarrow B = \frac{1}{3} - C \end{cases} \\ \Rightarrow B = -\frac{1}{3} \end{aligned}$$

$$\begin{aligned} (*) &= \frac{1}{3} \int \frac{dx}{x+1} + \int \frac{(-\frac{1}{3}x + \frac{2}{3})dx}{(x^2-x+1)} = \frac{1}{3} \ell\eta|x+1| - \frac{1}{3} \int \frac{(x-2)dx}{x^2-x+1} \\ &= \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \int \frac{(2x-4)dx}{x^2-x+1} = \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \int \frac{(2x-1-3)dx}{x^2-x+1} \\ &= \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \int \frac{(2x-1)dx}{x^2-x+1} + \frac{1}{2} \int \frac{dx}{x^2-x+1} \\ &= \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \ell\eta|x^2-x+1| + \frac{1}{2} \int \frac{dx}{(x^2-x+\frac{1}{4})+\frac{3}{4}} \\ &= \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \ell\eta|x^2-x+1| + \frac{1}{2} \int \frac{dx}{(x-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} \\ &= \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \ell\eta|x^2-x+1| + \frac{1}{2} \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arc} \tau g \frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c \\ &= \frac{1}{3} \ell\eta|x+1| - \frac{1}{6} \ell\eta|x^2-x+1| + \frac{\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2x-1}{\sqrt{3}} + c \end{aligned}$$

$$= \ell \eta \left| \frac{\sqrt[3]{x+1}}{\sqrt{x^2-x+1}} \right| + \frac{\sqrt{3}}{3} \operatorname{arctg} \frac{2x-1}{\sqrt{3}} + c$$

**7.16.-**  $\int \frac{(x+5)dx}{x^2-x+6}$

Solución.-

$$\int \frac{(x+5)dx}{x^2-x+6} = \int \frac{(x+5)dx}{(x+3)(x-2)} = \int \frac{A dx}{(x+3)} + \int \frac{B dx}{(x-2)} \quad (*), \text{ luego:}$$

$$\frac{(x+5)}{(x^2+x-6)} = \frac{A}{(x+3)} + \frac{B}{(x-2)} \Rightarrow x+5 = A(x-2) + B(x+3)$$

$$\therefore \begin{cases} x=2 \Rightarrow 7=5B \Rightarrow B=7/5 \\ x=-3 \Rightarrow 2=-5A \Rightarrow A=-2/5 \end{cases}$$

$$(*) = -\frac{2}{5} \int \frac{dx}{x+3} + \frac{7}{5} \int \frac{dx}{x-2} = -\frac{2}{5} \ell \eta |x+3| + \frac{7}{5} \ell \eta |x-2| + c = \frac{1}{5} \ell \eta \left| \frac{(x-2)^7}{(x+3)^2} \right| + c$$

**7.17.-**  $\int \frac{(x^2+1)dx}{x^3+1}$

Solución.-

$$\int \frac{(x^2+1)dx}{x^3+1} = \int \frac{(x^2+1)dx}{(x+1)(x^2-x+1)} = \int \frac{A dx}{(x+1)} + \int \frac{(Bx+C)dx}{(x^2-x+1)} \quad (*), \text{ luego:}$$

$$\frac{(x^2+1)}{x^3+1} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)} \Rightarrow x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$\therefore \begin{cases} x=-1 \Rightarrow 2=3A \Rightarrow A=2/3 \\ x=0 \Rightarrow 1=A+C \Rightarrow C=1/3 \\ x=1 \Rightarrow 2=A+(B+C)2 \Rightarrow B=1/3 \end{cases}$$

$$\begin{aligned} (*) \int \frac{(x^2+1)dx}{x^3+1} &= \int \frac{(x^2+1)dx}{(x+1)(x^2-x+1)} = \frac{2}{3} \int \frac{dx}{(x+1)} + \frac{1}{3} \int \frac{(x+1)dx}{(x^2-x+1)} \\ &= \frac{2}{3} \ell \eta |x+1| + \frac{1}{3} \int \frac{\left[ \frac{1}{2}(2x-1) + \frac{2}{3} \right] dx}{(x^2-x+1)} = \frac{2}{3} \ell \eta |x+1| + \frac{1}{6} \int \frac{(2x-1)dx}{(x^2-x+1)} + \frac{1}{2} \int \frac{dx}{(x^2-x+1)} \\ &= \frac{2}{3} \ell \eta |x+1| + \frac{1}{6} \ell \eta |x^2-x+1| + \frac{1}{2} \int \frac{dx}{(x^2-x+1)} \\ &= \frac{2}{3} \ell \eta |x+1| + \frac{1}{6} \ell \eta |x^2-x+1| + \frac{1}{2} \int \frac{dx}{(x^2-x+\frac{1}{4})+\frac{3}{4}} \\ &= \frac{4}{6} \ell \eta |x+1| + \frac{1}{6} \ell \eta |x^2-x+1| + \frac{1}{2} \int \frac{dx}{(x-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} \end{aligned}$$

$$= \frac{1}{6} \ell \eta \left| (x+1)^4 (x^2 - x + 1) \right| + \frac{1}{2} \frac{1}{\sqrt{3}} \operatorname{arc} \tau g \frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} + c$$

$$= \frac{1}{6} \ell \eta \left| (x+1)^4 (x^2 - x + 1) \right| + \frac{\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2x-1}{\sqrt{3}} + c$$

**7.18.-**  $\int \frac{(x^2+6)dx}{(x-1)^2(x-2)}$

Solución.-

$$\int \frac{(x^2+6)dx}{(x-1)^2(x-2)} = \int \frac{Adx}{(x+1)} + \int \frac{Bdx}{(x-1)^2} + \int \frac{Cdx}{(x+2)} \quad (*), \text{ luego:}$$

$$\frac{(x^2+6)}{(x-1)^2(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x^2+6 = A(x+1) + (x+2) + B(x+2) + C(x-1)^2$$

$$\begin{cases} x=1 \Rightarrow 7=3B \Rightarrow B=7/3 \\ x=-2 \Rightarrow 10=9C \Rightarrow C=10/9 \\ x=0 \Rightarrow 6=-2A+B+C \Rightarrow A=-1/9 \end{cases}$$

$$(*) = -\frac{1}{9} \int \frac{dx}{(x+1)} + \frac{7}{3} \int \frac{dx}{(x-1)^2} + \frac{10}{9} \int \frac{dx}{(x+2)} = -\frac{1}{9} \ell \eta |x-1| - \frac{7}{3} \frac{1}{x-1} + \frac{10}{9} \ell \eta |x+2| + c$$

$$= \frac{1}{9} \ell \eta \left| \frac{(x+2)^{10}}{x-1} \right| - \frac{7}{3(x-1)} + c$$

**7.19.-**  $\int \frac{(x^2-1)dx}{(x^2+1)(x-2)}$

Solución.-

$$\int \frac{(x^2-1)dx}{(x^2+1)(x-2)} = \int \frac{Ax+B}{(x^2+1)} dx + \int \frac{Cdx}{(x-2)} \quad (*), \text{ luego:}$$

$$\frac{(x^2-1)}{(x^2+1)(x-2)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-2)} \Rightarrow x^2-1 = (Ax+B)(x-2) + C(x^2+1)$$

$$\begin{cases} x=2 \Rightarrow 3=5C \Rightarrow C=3/5 \\ x=0 \Rightarrow -1=-2B+C \Rightarrow B=4/5 \\ x=1 \Rightarrow 0=-(A+B)+2C \Rightarrow A=2/5 \end{cases}$$

$$(*) = \int \frac{(2/5 x + 4/5)dx}{(x^2+1)} + \int \frac{3/5 dx}{(x-2)} = \frac{1}{5} \int \frac{2xdx}{(x^2+1)} + \frac{4}{5} \int \frac{dx}{(x^2+1)} + \frac{3}{5} \int \frac{dx}{x-2}$$

$$= \frac{1}{5} \ell \eta |x^2+1| + \frac{4}{5} \operatorname{arc} x + \frac{3}{5} \ell \eta |x-2| + c = \frac{1}{5} \ell \eta |(x^2+1)(x-2)^3| + \frac{4}{5} \operatorname{arc} x + c$$

$$7.20.- \int \frac{x dx}{x^2 - 4x - 5}$$

Solución.-

$$\int \frac{x dx}{x^2 - 4x - 5} = \int \frac{x dx}{(x+5)(x-1)} = \int \frac{A dx}{(x+5)} + \int \frac{B dx}{(x-1)} \quad (*), \text{ luego:}$$

$$\frac{x}{(x+5)(x-1)} = \frac{A}{(x+5)} + \frac{B}{(x-1)} \Rightarrow x = A(x-1) + B(x+5)$$

$$\therefore \begin{cases} x=1 \Rightarrow 1=6B \Rightarrow B=1/6 \\ x=-5 \Rightarrow -5=-6A \Rightarrow A=5/6 \end{cases}$$

$$(*) = \frac{5}{6} \int \frac{dx}{(x+5)} + \frac{1}{6} \int \frac{dx}{(x-1)} = \frac{5}{6} \ell \eta |x+5| + \frac{1}{6} \ell \eta |x-1| + c = \frac{5}{6} \ell \eta |(x+5)^5(x-1)| + c$$

$$7.21.- \int \frac{x dx}{x^2 - 2x - 3}$$

Solución.-

$$\int \frac{x dx}{x^2 - 2x - 3} = \int \frac{x dx}{(x-3)(x+1)} = \int \frac{A dx}{(x-3)} + \int \frac{B dx}{(x+1)} \quad (*), \text{ luego:}$$

$$\frac{x}{(x-3)(x+1)} = \frac{A}{(x-3)} + \frac{B}{(x+1)} \Rightarrow x = A(x+1) + B(x-3)$$

$$\therefore \begin{cases} x=-1 \Rightarrow -1=-4B \Rightarrow B=1/4 \\ x=3 \Rightarrow 3=4A \Rightarrow A=3/4 \end{cases}$$

$$(*) = \frac{3}{4} \int \frac{dx}{(x-3)} + \frac{1}{4} \int \frac{B}{(x+1)} = \frac{3}{4} \ell \eta |x-3| + \frac{1}{4} \ell \eta |x+1| + c = \frac{1}{4} \ell \eta |(x-3)^3(x+1)| + c$$

$$7.22.- \int \frac{(x+1) dx}{x^2 + 4x - 5}$$

Solución.-

$$\int \frac{(x+1) dx}{x^2 + 4x - 5} = \int \frac{(x+1) dx}{(x+5)(x-1)} = \int \frac{A dx}{(x+5)} + \int \frac{B dx}{(x-1)} \quad (*), \text{ luego:}$$

$$\frac{x+1}{(x^2 + 4x - 5)} = \frac{A}{(x+5)} + \frac{B}{(x-1)} \Rightarrow x+1 = A(x-1) + B(x+5)$$

$$\therefore \begin{cases} x=1 \Rightarrow 2=6B \Rightarrow B=1/3 \\ x=-5 \Rightarrow 3=-4A \Rightarrow -6A=2/3 \end{cases}$$

$$(*) = \frac{2}{3} \int \frac{dx}{(x+5)} + \frac{1}{3} \int \frac{B}{(x-1)} = \frac{2}{3} \ell \eta |x+5| + \frac{1}{3} \ell \eta |x-1| + c = \frac{1}{3} \ell \eta |(x+5)^2(x-1)| + c$$

$$7.23.- \int \frac{x^2 dx}{x^2 + 2x + 1}$$

Solución.-

$$\begin{aligned}
\int \frac{x^2 dx}{x^2 + 2x + 1} &= \int \left( 1 - \frac{2x+1}{x^2 + 2x + 1} \right) dx = \int dx - \int \frac{(2x+1)dx}{x^2 + 2x + 1} = \int dx - \int \frac{(2x+1)dx}{(x+1)^2} \\
&= x - \left[ \int \frac{Adx}{(x+1)} + \int \frac{Bdx}{(x+1)^2} \right] (*), \text{ luego:} \\
\frac{2x+1}{(x+1)^2} &= \frac{A}{(x+1)} + \frac{B}{(x+1)^2} \Rightarrow 2x+1 = A(x+1) + B \\
\therefore \begin{cases} x = -1 \Rightarrow -1 = B \Rightarrow B = -1 \\ x = 0 \Rightarrow 1 = A + B \Rightarrow A = 2 \end{cases} \\
(*) &= x - \left[ 2 \int \frac{dx}{(x+1)} - \int \frac{dx}{(x+1)^2} \right] = x - \left[ 2\ell\eta|x+1| + \frac{1}{x+5} \right] + c = x - 2\ell\eta|x+1| - \frac{1}{x+5} + c
\end{aligned}$$

**7.24.-**  $\int \frac{dx}{x(x+1)^2}$

Solución.-

$$\begin{aligned}
\int \frac{dx}{x(x+1)^2} &= \int \frac{Adx}{x} + \int \frac{Bdx}{(x+1)} + \int \frac{Cdx}{(x+1)^2} (*), \text{ luego:} \\
\frac{1}{x(x+1)^2} &= \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \Rightarrow 1 = A(x+1)^2 + Bx(x+1) + Cx \\
\therefore \begin{cases} x = -1 \Rightarrow 1 = -C \Rightarrow C = -1 \\ x = 0 \Rightarrow 1 = A \Rightarrow A = 1 \\ x = 1 \Rightarrow 1 = 4A + 2B + C \Rightarrow B = -1 \end{cases} \\
(*) &= \int \frac{dx}{x} - \int \frac{dx}{(x+1)} - \int \frac{dx}{(x+1)^2} = \ell\eta|x| - \ell\eta|x+1| + \frac{1}{x+1} + c = \ell\eta \left| \frac{x}{x+1} \right| + \frac{1}{x+1} + c
\end{aligned}$$

**7.25.-**  $\int \frac{dx}{(x+1)(x^2+1)}$

Solución.-

$$\begin{aligned}
\int \frac{dx}{(x+1)(x^2+1)} &= \int \frac{Adx}{x+1} + \int \frac{Bx+C}{(x^2+1)} dx (*), \text{ luego:} \\
\frac{1}{(x+1)(x^2+1)} &= \frac{A}{x+1} + \frac{Bx+C}{(x^2+1)} \Rightarrow 1 = A(x^2+1) + (Bx+C)(x+1) \\
\therefore \begin{cases} x = -1 \Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2} \\ x = 0 \Rightarrow 1 = A + C \Rightarrow C = \frac{1}{2} \\ x = 1 \Rightarrow 1 = 2A + (B+C)2 \Rightarrow B = -\frac{1}{2} \end{cases} \\
(*) &= \frac{1}{2} \int \frac{dx}{(x+1)} + \int \frac{(-\frac{1}{2}x + \frac{1}{2})dx}{(x^2+1)} = \frac{1}{2} \ell\eta|x+1| - \frac{1}{2} \int \frac{x-1}{(x^2+1)} dx \\
&= \frac{1}{2} \ell\eta|x+1| - \frac{1}{4} \int \frac{2xdx}{(x^2+1)} + \frac{1}{2} \int \frac{dx}{(x^2+1)} = \frac{1}{2} \ell\eta|x+1| - \frac{1}{4} \ell\eta|x^2+1| + \frac{1}{2} \operatorname{arctg} x + c
\end{aligned}$$

$$= \frac{1}{4} \ell \eta \left| \frac{(x+1)^2}{x^2+1} \right| + \frac{1}{2} \operatorname{arc} \tau g x + c$$

**7.26.-**  $\int \frac{dx}{x(x^2+x+1)}$

Solución.-

$$\int \frac{dx}{x(x^2+x+1)} = \int \frac{A dx}{x} + \int \frac{Bx+C}{(x^2+x+1)} dx \quad (*), \text{ luego:}$$

$$\frac{1}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+x+1)} \Rightarrow 1 = A(x^2+x+1) + (Bx+C)x$$

$$\therefore \begin{cases} x=0 \Rightarrow 1=A \Rightarrow A=1 \\ x=1 \Rightarrow 1=3A+B+C \Rightarrow B+C=-2 \\ x=-1 \Rightarrow 1=A+B-C \Rightarrow B-C=0 \end{cases}$$

$$\begin{aligned} (*) &= \int \frac{dx}{x} - \int \frac{(x+1)dx}{(x^2+x+1)} = \ell \eta |x+1| - \frac{1}{2} \int \frac{(2x+2)dx}{(x^2+x+1)} \\ &= \ell \eta |x| - \frac{1}{2} \int \frac{(2x+1)+1}{(x^2+x+1)} dx = \ell \eta |x| - \frac{1}{2} \int \frac{(2x+1)dx}{(x^2+x+1)} - \frac{1}{2} \int \frac{dx}{(x^2+x+1)} \\ &= \ell \eta |x| - \frac{1}{2} \ell \eta |x^2+x+1| - \frac{1}{2} \int \frac{dx}{(x^2+x+\frac{1}{4})+\frac{3}{4}} \\ &= \ell \eta |x| - \frac{1}{2} \ell \eta |x^2+x+1| - \frac{1}{2} \int \frac{dx}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} \\ &= \ell \eta |x| - \frac{1}{2} \ell \eta |x^2+x+1| - \frac{1}{2} \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arc} \tau g \frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c \\ &= \ell \eta |x| - \frac{1}{2} \ell \eta |x^2+x+1| - \frac{\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2x+1}{\sqrt{3}} + c \end{aligned}$$

**7.27.-**  $\int \frac{2x^2+5x-1}{x^3+x^2-2x} dx$

Solución.-

$$\int \frac{(2x^2+5x-1)dx}{(x^3+x^2-2x)} = \int \frac{A dx}{x} + \int \frac{B dx}{(x-1)} + \int \frac{C dx}{(x+2)} \quad (*), \text{ luego:}$$

$$\frac{2x^2+5x-1}{(x^3+x^2-2x)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+2)}$$

$$2x^2+5x-1 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$$\therefore \begin{cases} x=0 \Rightarrow -1=-2A \Rightarrow A=\frac{1}{2} \\ x=1 \Rightarrow 6=3B \Rightarrow B=2 \\ x=-2 \Rightarrow -3=6C \Rightarrow C=-\frac{1}{2} \end{cases}$$

$$(*) = \frac{1}{2} \int \frac{dx}{x} + 2 \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{dx}{(x+2)} = \frac{1}{2} \ell \eta |x| + 2 \ell \eta |x-1| - \frac{1}{2} \ell \eta |x+2| + c$$

$$\mathbf{7.28.-} \int \frac{x^2 + 2x + 3}{(x-1)(x+1)^2} dx$$

Solución.-

$$\int \frac{x^2 + 2x + 3}{(x-1)(x+1)^2} dx = \int \frac{A dx}{(x-1)} + \int \frac{B dx}{(x+1)} + \int \frac{C dx}{(x+1)^2} \quad (*), \text{ luego:}$$

$$\frac{x^2 + 2x + 3}{(x-1)(x+1)^2} = \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$x^2 + 2x + 3 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 6 = 4A \Rightarrow A = \frac{3}{2} \\ x=-1 \Rightarrow 2 = -2C \Rightarrow C = -1 \\ x=0 \Rightarrow 3 = A - B - C \Rightarrow B = -\frac{1}{2} \end{cases}$$

$$(*) = \frac{3}{2} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} = \frac{3}{2} \ell \eta |x-1| - \frac{1}{2} \ell \eta |x+1| + \frac{1}{x+1} + c$$

$$= \frac{1}{2} \ell \eta \left| \frac{(x-1)^3}{x+1} \right| + \frac{1}{x+1} + c$$

$$\mathbf{7.29.-} \int \frac{3x^2 + 2x - 2}{x^3 - 1} dx$$

Solución.-

$$\int \frac{3x^2 + 2x - 2}{x^3 - 1} dx = \int \frac{3x^2 + 2x - 2}{(x-1)(x^2 + x + 1)} dx = \int \frac{A dx}{x-1} + \int \frac{(Bx + C) dx}{(x^2 + x + 1)} \quad (*), \text{ luego:}$$

$$\frac{3x^2 + 2x - 2}{(x-1)(x^2 + x + 1)} = \frac{A}{x-1} + \frac{Bx + C}{(x^2 + x + 1)}$$

$$3x^2 + 2x - 2 = A(x^2 + x + 1) + (Bx + C)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 3 = 3A \Rightarrow A = 1 \\ x=0 \Rightarrow -2 = A - C \Rightarrow C = 3 \\ x=-1 \Rightarrow -1 = A + (-B + C)(-2) \Rightarrow B = 2 \end{cases}$$

$$(*) = \int \frac{dx}{x-1} + \int \frac{(2x+3)dx}{(x^2 + x + 1)} = \ell \eta |x-1| + \int \frac{(2x+1) + 2}{(x^2 + x + 1)} dx$$

$$= \ell \eta |x-1| + \int \frac{(2x+1)dx}{(x^2 + x + 1)} + 2 \int \frac{dx}{(x^2 + x + 1)}$$

$$= \ell \eta |x-1| + \ell \eta |x^2 + x + 1| + 2 \int \frac{dx}{(x + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$



$$= \ell \eta \left| (x-1)(x^2+x+1) \right| + 2 \frac{1}{\sqrt{3}} \operatorname{arc} \tau g \frac{x+1/2}{\sqrt{3}} + c$$

$$= \ell \eta \left| (x-1)(x^2+x+1) \right| + \frac{4\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2x+1}{\sqrt{3}} + c$$

**7.30.-**  $\int \frac{x^4 - x^3 + 2x^2 - x + 2}{(x-1)(x^2+2)^2} dx$

Solución.-

$$\int \frac{x^4 - x^3 + 2x^2 - x + 2}{(x-1)(x^2+2)^2} dx = \int \frac{A dx}{x-1} + \int \frac{(Bx+C) dx}{(x^2+2)} + \int \frac{(Dx+E) dx}{(x^2+2)^2} (*) , \text{ luego:}$$

$$\frac{x^4 - x^3 + 2x^2 - x + 2}{(x-1)(x^2+2)^2} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+2)} + \frac{Dx+E}{(x^2+2)^2}$$

$$x^4 - x^3 + 2x^2 - x + 2 = A(x^2+2)^2 + (Bx+C)(x-1)(x^2+2) + (Dx+E)(x-1)$$

$$= A(x^4 + 4x^2 + 4) + (Bx+C)(x^3 + 2x - x^2 - 2) + Dx^2 - Dx + Ex - E$$

$$= Ax^4 + 4Ax^2 + 4A + Bx^4 + 2Bx^2 - Bx^3 - 2Bx + Cx^3 + 2Cx - Cx^2 - 2C$$

$$\Rightarrow +Dx^2 - Dx + Ex - E$$

$$= (A+B)x^4 + (C-B)x^3 + (4A-C+2B+D)x^2 + (-2B+2C-D+E)x + (4A-2C-E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A+B & & & & =1 \\ -B+C & & & & =-1 \\ 4A+2B-C+D & & & & =2 \\ -2B+2C-D+E & & & & =-1 \\ 4A & -2C & -E & & =2 \end{pmatrix} \therefore A = \frac{1}{3}, B = \frac{2}{3}, C = -\frac{1}{3}, D = -1, E = 0$$

$$\begin{aligned} (*) &= \frac{1}{3} \int \frac{dx}{x-1} + \int \frac{(\frac{2}{3}x - \frac{1}{3})dx}{(x^2+2)} - \int \frac{xdx}{(x^2+2)^2} \\ &= \frac{1}{3} \int \frac{dx}{x-1} + \frac{1}{3} \int \frac{2xdx}{(x^2+2)} - \frac{1}{3} \int \frac{dx}{(x^2+2)} - \frac{1}{2} \int \frac{2xdx}{(x^2+2)^2} \\ &= \frac{1}{3} \ell \eta |x-1| + \frac{1}{3} \ell \eta |x^2+2| - \frac{\sqrt{2}}{6} \operatorname{arc} \tau g \frac{x}{\sqrt{2}} + \frac{1}{2} \frac{1}{x^2+2} + c \\ &= \frac{1}{3} \ell \eta |(x-1)(x^2+2)| - \frac{\sqrt{2}}{6} \operatorname{arc} \tau g \frac{x}{\sqrt{2}} + \frac{1}{2(x^2+2)} + c \end{aligned}$$

**7.31.-**  $\int \frac{2x^2 - 7x - 1}{x^3 + x^2 - x - 1} dx$

Solución.-

$$\int \frac{2x^2 - 7x - 1}{x^3 + x^2 - x - 1} dx = \int \frac{2x^2 - 7x - 1}{(x-1)(x+1)^2} dx = \int \frac{A dx}{x-1} + \int \frac{B dx}{(x+1)} + \int \frac{C dx}{(x+1)^2} (*), \text{ luego:}$$

$$\frac{2x^2 - 7x - 1}{(x^3 + x^2 - x - 1)} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$2x^2 - 7x - 1 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$\therefore \begin{cases} x = -1 \Rightarrow 8 = -2C \Rightarrow C = -4 \\ x = 1 \Rightarrow -6 = 4A \Rightarrow A = -\frac{3}{2} \\ x = 0 \Rightarrow -1 = A - B - C \Rightarrow B = \frac{7}{2} \end{cases}$$

$$(*) = -\frac{3}{2} \int \frac{dx}{x-1} + \frac{7}{2} \int \frac{dx}{x+1} - 4 \int \frac{dx}{(x+1)^2} = -\frac{3}{2} \ell \eta |x-1| + \frac{7}{2} \ell \eta |x+1| + \frac{4}{x+1} + c$$

$$= -\frac{1}{2} \ell \eta \left| \frac{(x+1)^7}{(x-1)^3} \right| + \frac{4}{x+1} + c$$

$$\mathbf{7.32.-} \int \frac{3x^2 + 3x + 1}{x^3 + 2x^2 + 2x + 1} dx$$

Solución.-

$$\int \frac{3x^2 + 3x + 1}{x^3 + 2x^2 + 2x + 1} dx = \int \frac{(3x^2 + 3x + 1)dx}{(x+1)(x^2 + x + 1)} = \int \frac{A dx}{x+1} + \int \frac{(Bx + C)dx}{(x^2 + x + 1)} \quad (*), \text{ luego:}$$

$$\frac{3x^2 + 3x + 1}{(x+1)(x^2 + x + 1)} = \frac{A}{x+1} + \frac{Bx + C}{(x^2 + x + 1)}$$

$$3x^2 + 3x + 1 = A(x^2 + x + 1) + (Bx + C)(x+1)$$

$$\therefore \begin{cases} x = -1 \Rightarrow A = 1 \\ x = 0 \Rightarrow 1 = A + C \Rightarrow C = 0 \\ x = 1 \Rightarrow 7 = 3A + (B + C)(2) \Rightarrow B = 2 \end{cases}$$

$$(*) = \int \frac{dx}{x+1} + \int \frac{2x dx}{(x^2 + x + 1)} = \ell \eta |x+1| + \int \frac{(2x+1) - 1}{(x^2 + x + 1)} dx$$

$$= \ell \eta |x+1| + \int \frac{(2x+1)dx}{(x^2 + x + 1)} - \int \frac{dx}{(x^2 + x + 1)}$$

$$= \ell \eta |x+1| + \ell \eta |x^2 + x + 1| - \int \frac{dx}{(x^2 + x + \frac{1}{4}) + (\frac{\sqrt{3}}{2})^2}$$

$$= \ell \eta |x+1| + \ell \eta |x^2 + x + 1| - \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arc} \tau g \frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}} + c$$

$$= \ell \eta |(x+1)(x^2 + x + 1)| - \frac{2\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2x+1}{\sqrt{3}} + c$$

$$\mathbf{7.33.-} \int \frac{x^3 + 7x^2 - 5x + 5}{(x-1)^2(x+1)^2} dx$$

Solución.-

$$\int \frac{x^3 + 7x^2 - 5x + 5}{(x-1)^2(x+1)^3} dx = \int \frac{A dx}{x-1} + \int \frac{B dx}{(x-1)^2} + \int \frac{C dx}{(x+1)} + \int \frac{D dx}{(x+1)^2} + \int \frac{E dx}{(x+1)^3} \quad (*), \text{ luego:}$$

$$\frac{x^3 + 7x^2 - 5x + 5}{(x-1)^2(x+1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{(x+1)^3}$$

$$x^3 + 7x^2 - 5x + 5 = A(x-1)(x+1)^3 + B(x+1)^3 + C(x-1)^2(x+1)^2$$

$$\Rightarrow +D(x-1)^2(x+1) + E(x-1)^2$$

$$= Ax^4 + 2Ax^3 - 2Ax - A + Bx^3 + 3Bx^2 + 3Bx + B + Cx^4 - 2Cx^2 + C$$

$$\Rightarrow +Dx^3 - Dx^2 - Dx + D + Ex^2 - 2Ex + E$$

$$= (A+C)x^4 + (2A+B+D)x^3 + (3B-2C-D+E)x^2$$

$$\Rightarrow +(-2A+3B-D-2E)x + (-A+B+C+D+E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A & +C & & & = 0 \\ 2A & + B & & +D & = 1 \\ & +3B-2C- & D+ & E = 7 \\ -2A & +3B & & -D-2E = -5 \\ - & A & + B & + C + D + E = 2 \end{pmatrix} \therefore A=0, B=1, C=0, D=0, E=4$$

$$(*) = \int \frac{dx}{(x-1)^2} + 4 \int \frac{dx}{(x+1)^3} = -\frac{1}{x-1} - \frac{2}{(x+1)^2} + c = -\frac{x^2 - 4x - 1}{(x-1)(x+1)^2} + c$$

$$\mathbf{7.34.-} \int \frac{2x dx}{(x^2 + x + 1)^2}$$

Solución.-

$$\int \frac{2x dx}{(x^2 + x + 1)^2} = \int \frac{(Ax+B)dx}{x^2 + x + 1} + \int \frac{(Cx+D)dx}{(x^2 + x + 1)^2} \quad (*), \text{ luego:}$$

$$\frac{2x}{(x^2 + x + 1)^2} = \frac{Ax+B}{x^2 + x + 1} + \frac{Cx+D}{(x^2 + x + 1)^2}$$

$$2x = (Ax+B)(x^2 + x + 1) + Cx + D \Rightarrow 2x = Ax^3 + Ax^2 + Ax + Bx^2 + Bx + B + Cx + D$$

$$= Ax^3 + (A+B)x^2 + (A+B+C)x + B+D, \text{ igualando coeficientes se tiene:}$$

$$\begin{pmatrix} A & & & = 0 \\ A+B & & & = 0 \\ A+B+C & & & = 2 \\ & +D & = 0 \end{pmatrix}$$

$$\therefore A=0, B=0, C=2, D=0$$

$$(*) = \int \frac{2x dx}{(x^2 + x + 1)}, \text{ de donde el método sugerido pierde aplicabilidad; tal como se}$$

había planteado la técnica trabajada debe ser sustituida por otra:

$$\int \frac{2x dx}{(x^2 + x + 1)} = \int \frac{(2x+1)dx}{(x^2 + x + 1)} - \int \frac{dx}{(x^2 + x + 1)^2}$$

$$= \int \frac{(2x+1)dx}{(x^2+x+1)} - \frac{16}{9} \int \frac{dx}{\left\{ \left[ \frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right) \right]^2 + 1 \right\}} \quad (**)$$

sea:  $u = \frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right), dx = \frac{\sqrt{3}}{2} du$ , entonces:

$$(**) - \frac{1}{x^2+x+1} - \frac{16}{9} \frac{\sqrt{3}}{2} \int \frac{du}{(u^2+1)^2}, \text{ trabajando la integral sustituyendo}$$

trigonométricamente:

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9} \int \frac{\sec^2 \theta d\theta}{\sec^4 \theta}, \text{ ya que: } u = \tau g \theta, du = \sec^2 \theta d\theta$$

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9} \left[ \frac{1}{2} \operatorname{arc} \tau g u + \frac{1}{2} \frac{u}{(u^2+1)} \right]$$

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9} \left\{ \frac{1}{2} \operatorname{arc} \tau g \frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right) + \frac{\frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right)}{2 \left[ \frac{4}{3} \left( x + \frac{1}{2} \right)^2 + 1 \right]} \right\} + c$$

$$= -\frac{1}{x^2+x+1} - \frac{8\sqrt{3}}{9} \left\{ \frac{1}{2} \operatorname{arc} \tau g \frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right) + \frac{x + \frac{1}{2}}{\sqrt{3} \left[ \frac{4}{3} \left( x + \frac{1}{2} \right)^2 + 1 \right]} \right\} + c$$

$$= -\frac{1}{x^2+x+1} - \frac{4\sqrt{3}}{9} \operatorname{arc} \tau g \frac{2}{\sqrt{3}} \left( x + \frac{1}{2} \right) - \frac{8}{9} \frac{\left( x + \frac{1}{2} \right)}{\left[ \frac{4}{3} \left( x + \frac{1}{2} \right)^2 + 1 \right]} + c$$

**7.35.-**  $\int \frac{x^2+2x+3}{x^3-x} dx$

Solución.-

$$\int \frac{x^2+2x+3}{x^3-x} dx = \int \frac{x^2+2x+3}{x(x-1)(x+1)} dx = \int \frac{A dx}{x} + \int \frac{B dx}{(x-1)} + \int \frac{C dx}{(x+1)} \quad (*), \text{ luego:}$$

$$\frac{x^2+2x+3}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x+1)}$$

$$x^2+2x+3 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

$$\therefore \begin{cases} x=0 \Rightarrow 3 = -A \Rightarrow A = -3 \\ x=-1 \Rightarrow 2 = 2C \Rightarrow C = 1 \\ x=1 \Rightarrow 6 = 2B \Rightarrow B = 3 \end{cases}$$

$$(*) = -3 \int \frac{dx}{x} + 3 \int \frac{dx}{(x-1)} + \int \frac{dx}{(x+1)} = -3 \ell \eta |x| + 3 \ell \eta |x-1| + \ell \eta |x+1| + c$$

$$= \ell \eta \left| \frac{(x-1)^3(x+1)}{x^3} \right| + c$$

$$7.36.- \int \frac{(2x^2 - 3x + 5)dx}{(x+2)(x-1)(x-3)}$$

Solución.-

$$\int \frac{2x^2 - 3x + 5}{(x+2)(x-1)(x-3)} dx = \int \frac{Adx}{(x+2)} + \int \frac{Bdx}{(x-1)} + \int \frac{Cdx}{(x-3)} \quad (*), \text{ luego:}$$

$$\frac{2x^2 - 3x + 5}{(x+2)(x-1)(x-3)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x-3}$$

$$2x^2 - 3x + 5 = A(x-1)(x-3) + B(x+2)(x-3) + C(x+2)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 4 = -6B \Rightarrow B = -\frac{2}{3} \\ x=3 \Rightarrow 14 = 10C \Rightarrow C = \frac{7}{5} \\ x=-2 \Rightarrow 19 = 15A \Rightarrow A = \frac{19}{15} \end{cases}$$

$$(*) = \frac{19}{15} \int \frac{dx}{x+2} - \frac{2}{3} \int \frac{dx}{x-1} + \frac{7}{5} \int \frac{dx}{x-3} = \frac{19}{15} \ell \eta |x+2| - \frac{2}{3} \ell \eta |x-1| + \frac{7}{5} \ell \eta |x-3| + c$$

$$7.37.- \int \frac{3x^2 + x - 2}{(x-1)(x^2 + 1)} dx$$

Solución.-

$$\int \frac{3x^2 + x - 2}{(x-1)(x^2 + 1)} dx = \int \frac{Adx}{(x-1)} + \int \frac{(Bx+C)dx}{(x^2 + 1)} \quad (*), \text{ luego:}$$

$$\frac{3x^2 + x - 2}{(x-1)(x^2 + 1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2 + 1}$$

$$3x^2 + x - 2 = A(x^2 + 1) + (Bx+C)(x-1)$$

$$\therefore \begin{cases} x=1 \Rightarrow 2 = 2A \Rightarrow A=1 \\ x=0 \Rightarrow -2 = A-C \Rightarrow C=3 \\ x=2 \Rightarrow 12 = 5A+2B+C \Rightarrow B=2 \end{cases}$$

$$\begin{aligned} (*) &= \int \frac{dx}{x-1} + \int \frac{(2x+3)dx}{x^2 + 1} = \int \frac{dx}{x-1} + \int \frac{2xdx}{x^2 + 1} + 3 \int \frac{dx}{x^2 + 1} \\ &= \ell \eta |x-1| + \ell \eta |x^2 + 1| + 3 \operatorname{arc} \tau g x + c = \ell \eta |(x-1)(x^2 + 1)| + 3 \operatorname{arc} \tau g x + c \end{aligned}$$

$$7.38.- \int \frac{(x+5)dx}{x^3 - 3x + 2}$$

Solución.-

$$\int \frac{(x+5)dx}{x^3 - 3x + 2} = \int \frac{(x+5)dx}{(x-1)^2(x+2)} = \int \frac{Adx}{(x-1)} + \int \frac{Bdx}{(x-1)^2} + \int \frac{Cdx}{(x+2)} \quad (*), \text{ luego:}$$

$$\frac{x+5}{x^3 - 3x + 2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x+5 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

$$\therefore \begin{cases} x=1 \Rightarrow 6=3B \Rightarrow B=2 \\ x=-2 \Rightarrow 3=9C \Rightarrow C=\frac{1}{3} \\ x=0 \Rightarrow 5=-2A+B+C \Rightarrow A=-\frac{1}{3} \end{cases}$$

$$(*) = -\frac{1}{3} \int \frac{dx}{(x-1)} + 2 \int \frac{dx}{(x-1)^2} + \frac{1}{3} \int \frac{dx}{(x+2)} = -\frac{1}{3} \ell \eta |x-1| - \frac{2}{x-1} + \frac{1}{3} \ell \eta |x+2| + c$$

$$= \frac{1}{3} \ell \eta \left| \frac{x+2}{x-1} \right| - \frac{2}{x-1} + c$$

**7.39.-**  $\int \frac{2x^3 + 3x^2 + x - 1}{(x+1)(x^2 + 2x + 2)^2} dx$

Solución.-

$$\int \frac{(2x^3 + 3x^2 + x - 1)dx}{(x+1)(x^2 + 2x + 2)^2} = \int \frac{Adx}{x+1} + \int \frac{(Bx+C)dx}{(x^2 + 2x + 2)} + \int \frac{(Dx+E)dx}{(x^2 + 2x + 2)^2} \quad (*), \text{ luego:}$$

$$\frac{2x^3 + 3x^2 + x - 1}{(x+1)(x^2 + 2x + 2)^2} = \frac{A}{x+1} + \frac{Bx+C}{(x^2 + 2x + 2)} + \frac{Dx+E}{(x^2 + 2x + 2)^2}$$

$$2x^3 + 3x^2 + x - 1 = A(x^2 + 2x + 2)^2 + (Bx+C)(x^2 + 2x + 2)(x+1) + (Dx+E)(x+1)$$

$$= Ax^4 + 4Ax^3 + 8Ax^2 + 8Ax + 4A + Bx^4 + 3Bx^3 + 4Bx^2 + 2Bx + Cx^3 + 3Cx^2 + 4Cx$$

$$\Rightarrow +2C + Dx^2 + Dx + Ex + E$$

$$= (A+B)x^4 + (4A+3B+C)x^3 + (+8A+4B+3C+D)x^2$$

$$\Rightarrow +(8A+2B+4C+D+E)x + (4A+2C+E)$$

Igualando coeficientes, se tiene:

$$\begin{pmatrix} A + B & & & & = 0 \\ 4A + 3B + C & & & & = 2 \\ 8A + 4B + 3C + D & & & & = 3 \\ 8A + 2B + 4C + D + E & & & & = 1 \\ 4A & + 2C & + E & = -1 \end{pmatrix} \therefore A = -1, B = 1, C = 3, D = -2, E = -3$$

$$(*) = -\int \frac{dx}{x+1} + \int \frac{(x+3)dx}{(x^2 + 2x + 2)} - \int \frac{(2x+3)dx}{(x^2 + 2x + 2)^2}$$

$$= -\ell \eta |x-1| + \frac{1}{2} \int \frac{(2x+6)dx}{(x^2 + 2x + 2)} - \int \frac{(2x+2)+1dx}{(x^2 + 2x + 2)^2}$$

$$= -\ell \eta |x-1| + \frac{1}{2} \int \frac{(2x+2)+4}{(x^2 + 2x + 2)} dx - \int \frac{(2x+2)dx}{(x^2 + 2x + 2)^2} - \int \frac{dx}{(x^2 + 2x + 2)^2}$$

$$= -\ell \eta |x-1| + \frac{1}{2} \int \frac{(2x+2)dx}{(x^2 + 2x + 2)} + 2 \int \frac{dx}{(x^2 + 2x + 2)} - \int \frac{(2x+2)dx}{(x^2 + 2x + 2)^2} - \int \frac{dx}{(x^2 + 2x + 2)^2}$$

$$= -\ell \eta |x-1| + \frac{1}{2} \ell \eta |x^2 + 2x + 2| + 2 \int \frac{dx}{(x+1)^2 + 1} + \frac{1}{2} \frac{1}{x^2 + 2x + 2} - \int \frac{dx}{[(x+1)^2 + 1]^2}$$

$$\begin{aligned}
&= -\ell \eta |x-1| + \frac{1}{2} \ell \eta |x^2 + 2x + 2| + 2 \operatorname{arc} \tau g(x+1) \\
&\Rightarrow + \frac{1}{2} \frac{1}{x^2 + 2x + 2} - \frac{1}{2} \frac{x+1}{x^2 + 2x + 2} - \frac{1}{2} \operatorname{arc} \tau g(x+1) + c \\
&= \ell \eta \left| \frac{\sqrt{x^2 + 2x + 2}}{x+1} \right| + \frac{3}{2} \operatorname{arc} \tau g(x+1) - \frac{1}{2} \frac{x}{x^2 + 2x + 2} + c
\end{aligned}$$

**7.40.-**  $\int \frac{(2x^2 + 3x - 1)dx}{x^3 + 2x^2 + 4x + 2}$

Solución.-

$$\int \frac{(2x^2 + 3x - 1)dx}{x^3 + 2x^2 + 4x + 2} = \int \frac{(2x^2 + 3x - 1)dx}{(x+1)(x^2 + 2x + 2)} = \int \frac{Adx}{(x+1)} + \int \frac{(Bx+C)dx}{(x^2 + 2x + 2)} \quad (*), \text{ luego:}$$

$$\frac{(2x^2 + 3x - 1)}{(x+1)(x^2 + 2x + 2)} = \frac{A}{(x+1)} + \frac{(Bx+C)}{(x^2 + 2x + 2)}$$

$$2x^2 + 3x - 1 = A(x^2 + 2x + 2) + (Bx + C)(x+1)$$

$$\therefore \begin{cases} x = -1 \Rightarrow -2 = A \Rightarrow A = -2 \\ x = 0 \Rightarrow -1 = 2A + C \Rightarrow C = 3 \\ x = 1 \Rightarrow 4 = 5A + (B+C)(2) \Rightarrow B = 4 \end{cases}$$

$$(*) = -2 \int \frac{dx}{(x+1)} + \int \frac{(4x+3)dx}{x^2 + 2x + 2} = -2\ell \eta |x+1| + 2 \int \frac{(2x+2)-1}{x^2 + 2x + 2} dx$$

$$= -2\ell \eta |x+1| + 2 \int \frac{(2x+2)dx}{x^2 + 2x + 2} - 2 \int \frac{dx}{x^2 + 2x + 2}$$

$$= -2\ell \eta |x+1| + 2\ell \eta |x^2 + 2x + 2| - 2 \operatorname{arc} \tau g(x+1) + c$$

**7.41.-**  $\int \frac{(2x+1)dx}{3x^3 + 2x - 1}$

Solución.-

$$\int \frac{(2x+1)dx}{3x^3 - 2x - 1} = \int \frac{(2x+1)dx}{(x-1)(3x^2 + 3x + 1)} = \int \frac{Adx}{(x-1)} + \int \frac{(Bx+C)dx}{(3x^2 + 3x + 1)} \quad (*), \text{ luego:}$$

$$\frac{(2x+1)}{(3x^3 - 2x - 1)} = \frac{A}{(x-1)} + \frac{(Bx+C)}{(3x^2 + 3x + 1)}$$

$$2x+1 = A(3x^2 + 3x + 1) + (Bx+C)(x-1)$$

$$\therefore \begin{cases} x = 1 \Rightarrow 3 = 7A \Rightarrow A = \frac{3}{7} \\ x = 0 \Rightarrow 1 = A - C \Rightarrow C = -\frac{4}{7} \\ x = -1 \Rightarrow -1 = A + (-B+C)(-2) \Rightarrow B = -\frac{9}{7} \end{cases}$$

$$(*) = \frac{3}{7} \int \frac{dx}{(x-1)} - \frac{1}{7} \int \frac{(9x+4)dx}{3x^2 + 3x + 1} = \frac{3}{7} \ell \eta |x-1| - \frac{1}{7} \frac{9}{6} \int \frac{(6x+3 - \frac{1}{3})dx}{3x^2 + 3x + 1}$$

$$\begin{aligned}
&= \frac{3}{7} \ell \eta |x-1| - \frac{3}{14} \int \frac{(6x+3)dx}{3x^2+3x+1} + \frac{1}{14} \int \frac{dx}{3x^2+3x+1} \\
&= \frac{3}{7} \ell \eta |x-1| - \frac{3}{14} \ell \eta |3x^2+3x+1| + \frac{1}{14} \int \frac{dx}{3(x+\frac{1}{2})^2 + \frac{1}{4}} \\
&= \frac{3}{7} \ell \eta |x-1| - \frac{3}{14} \ell \eta |3x^2+3x+1| + \frac{2}{7} \int \frac{dx}{12(x+\frac{1}{2})^2 + 1} \\
&= \frac{3}{7} \ell \eta |x-1| - \frac{3}{14} \ell \eta |3x^2+3x+1| + \frac{\sqrt{3}}{21} \operatorname{arctg} 2\sqrt{3}(x+\frac{1}{2}) + c
\end{aligned}$$

**7.42.-**  $\int \frac{x^4 - 2x^2 + 3x + 4}{(x-1)^3(x^2 + 2x + 2)} dx$

Solución.-

$$\int \frac{x^4 - 2x^2 + 3x + 4}{(x-1)^3(x^2 + 2x + 2)} dx = \int \frac{A dx}{(x-1)} + \int \frac{B dx}{(x-1)^2} + \int \frac{C dx}{(x-1)^3} + \int \frac{(Dx + E) dx}{(x^2 + 2x + 2)} \quad (*), \text{ luego:}$$

$$\frac{x^4 - 2x^2 + 3x + 4}{(x-1)^3(x^2 + 2x + 2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{Dx + E}{(x^2 + 2x + 2)}$$

$$x^4 - 2x^2 + 3x + 4 = A(x-1)^2(x^2 + 2x + 2) + B(x-1)(x^2 + 2x + 2)$$

$$\Rightarrow +C(x^2 + 2x + 2) + (Dx + E)(x-1)^3$$

$$x^4 - 2x^2 + 3x + 4 = A(x^2 - 2x + 1)(x^2 + 2x + 2) + B(x^3 + 2x^2 + 2x - x^2 - 2x - 2)$$

$$\Rightarrow +C(x^2 + 2x + 2) + (Dx + E)(x^3 - 3x^2 + 3x - 1)$$

$$x^4 - 2x^2 + 3x + 4 = Ax^4 - Ax^2 - 2Ax + 2A + Bx^3 + Bx^2 - 2B + Cx^2 + 2Cx + 2C$$

$$\Rightarrow +Dx^4 - 3Dx^3 + 3Dx^2 - Dx + Ex^3 - 3Ex^2 + 3Ex - E$$

$$x^4 - 2x^2 + 3x + 4 = (A + D)x^4 + (B - 3D + E)x^3 + (-A + B + C + 3D - 3E)x^2$$

$$\Rightarrow +(-2A + 2C - D + 3E)x + (-2A - 2B + 2C - E)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix}
A & & +D & & = & 1 \\
& B & & -3D & + & E & = & 0 \\
- & A & + & B & + & C & +3D & -3E & = & -2 \\
-2A & & +2C & -D & +3E & = & 3 \\
2A & -2B & +2C & & -E & = & 4
\end{pmatrix}$$

$$\therefore A = 106/125, B = 9/25, C = 6/5, D = 19/125, E = 102/125$$

$$(*) = \frac{106}{125} \int \frac{dx}{x+1} - \frac{9}{25} \int \frac{dx}{(x-1)^2} + \frac{6}{5} \int \frac{dx}{(x-1)^3} + \frac{1}{125} \int \frac{(19x+102)dx}{(x^2+2x+2)}$$

$$= \frac{106}{125} \ell \eta |x-1| + \frac{9}{25} \frac{1}{x-1} + \frac{6}{5} \frac{1}{(-2)(x-1)^2} + \frac{19}{125} \int \frac{(x+102/19)dx}{(x^2+2x+2)}$$



$$\begin{aligned}
&= \frac{106}{125} \ell \eta |x-1| + \frac{9}{25(x-1)} - \frac{3}{5(x-1)^2} + \frac{19}{250} \int \frac{(2x+2)+8\frac{14}{19}}{(x^2+2x+2)} dx \\
&= \frac{106}{125} \ell \eta |x-1| + \frac{9}{25(x-1)} - \frac{3}{5(x-1)^2} + \frac{19}{250} \ell \eta |x^2+2x+2| + \frac{19}{250} \frac{166}{19} \int \frac{dx}{(x^2+2x+1)+1} \\
&= \frac{106}{125} \ell \eta |x-1| + \frac{9}{25(x-1)} - \frac{3}{5(x-1)^2} + \frac{19}{250} \ell \eta |x^2+2x+2| + \frac{166}{250} \int \frac{dx}{(x+1)^2+1} \\
&= \frac{106}{125} \ell \eta |x-1| + \frac{9}{25(x-1)} - \frac{3}{5(x-1)^2} + \frac{19}{250} \ell \eta |x^2+2x+2| + \frac{166}{250} \arctan(x+1) + c
\end{aligned}$$

**7.43.-**  $\int \frac{e^t dt}{e^{2t} + 3e^t + 2}$

Solución.-

$$\int \frac{e^t dt}{e^{2t} + 3e^t + 2} = \int \frac{e^t dt}{(e^t + 2)(e^t + 1)} \quad (*), \text{ Sea: } u = e^t + 1, du = e^t dt; e^t + 2 = u + 1$$

Luego:

$$(*) \int \frac{du}{(u+1)u} = \int \frac{Adu}{(u+1)} + \int \frac{Bdu}{u} \quad (**)$$

$$\frac{1}{(u+1)u} = \frac{A}{(u+1)} + \frac{B}{u} \Rightarrow 1 = Au + B(u+1)$$

$$\therefore \begin{cases} u=0 \Rightarrow 1=B \Rightarrow B=1 \\ u=-1 \Rightarrow 1=-A \Rightarrow A=-1 \end{cases}$$

$$(**) = -\int \frac{du}{(u+1)} + \int \frac{du}{u} = -\ell \eta |u+1| + \ell \eta |u| + c = -\ell \eta |e^t + 2| + \ell \eta |e^t + 1| + c$$

$$= \ell \eta \left| \frac{e^t + 1}{e^t + 2} \right| + c$$

**7.44.-**  $\int \frac{\operatorname{sen} \theta d\theta}{\cos^2 \theta + \cos \theta - 2}$

Solución.-

$$\int \frac{\operatorname{sen} \theta d\theta}{\cos^2 \theta + \cos \theta - 2} = \int \frac{\operatorname{sen} \theta d\theta}{(\cos \theta + 2)(\cos \theta - 1)} \quad (*),$$

$$\text{Sea: } u = \cos \theta - 1, du = -\operatorname{sen} \theta d\theta, \cos \theta + 2 = u + 3$$

Luego:

$$(*) \int \frac{-du}{(u+3)u} = -\int \frac{du}{u(u+3)} = -\int \frac{Adu}{u} - \int \frac{Bdu}{u+3} \quad (**)$$

$$\frac{1}{u(u+3)} = \frac{A}{u} + \frac{B}{u+3} \Rightarrow 1 = A(u+3) + Bu$$

$$\therefore \begin{cases} u=0 \Rightarrow 1=3A \Rightarrow A=\frac{1}{3} \\ u=-3 \Rightarrow 1=-3B \Rightarrow B=-\frac{1}{3} \end{cases}$$

$$\begin{aligned}
 (**) &= -\frac{1}{3} \int \frac{du}{u} + \frac{1}{3} \int \frac{du}{(u+3)} = -\frac{1}{3} \ell \eta |u| + \frac{1}{3} \ell \eta |u+3| + c \\
 &= -\frac{1}{3} \ell \eta |\cos \theta - 1| + \frac{1}{3} \ell \eta |\cos \theta + 2| + c, \text{ Como: } |\cos \theta| < 1, \text{ se tiene:} \\
 &= -\frac{1}{3} \ell \eta |1 - \cos \theta| + \frac{1}{3} \ell \eta |2 + \cos \theta| + c = \frac{1}{3} \ell \eta \left| \frac{2 + \cos \theta}{1 - \cos \theta} \right| + c
 \end{aligned}$$

**7.45.-**  $\int \frac{4x^4 - 2x^3 - x^2 + 3x + 1}{(x^3 + x^2 - x - 1)} dx$

Solución.-

$$\begin{aligned}
 \int \frac{4x^4 - 2x^3 - x^2 + 3x + 1}{(x^3 + x^2 - x - 1)} dx &= \int \left( 4x - 6 + \frac{9x^2 + x - 5}{x^3 + x^2 - x - 1} \right) dx \\
 &= \int 4x dx - \int 6 dx + \int \frac{(9x^2 + x - 5) dx}{x^3 + x^2 - x - 1} = 2x^2 - 6x + \int \frac{(9x^2 + x - 5) dx}{x^3 + x^2 - x - 1} (*)
 \end{aligned}$$

Trabajando sólo la integral resultante:

$$\int \frac{(9x^2 + x - 5) dx}{x^3 + x^2 - x - 1} = \int \frac{(9x^2 + x - 5) dx}{(x+1)^2(x-1)} = \int \frac{A dx}{(x+1)} + \int \frac{B dx}{(x+1)^2} + \int \frac{C dx}{(x-1)} (**), \text{ luego:}$$

$$\frac{(9x^2 + x - 5)}{(x^3 + x^2 - x - 1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$$

$$= 9x^2 + x - 5 = A(x+1)(x-1) + B(x-1) + C(x+1)^2$$

$$\therefore \begin{cases} x=1 \Rightarrow 5 = 4C \Rightarrow C = \frac{5}{4} \\ x=-1 \Rightarrow 3 = -2B \Rightarrow B = -\frac{3}{2} \\ x=0 \Rightarrow -5 = -A - B + C \Rightarrow A = \frac{31}{4} \end{cases}$$

$$(**) = \frac{31}{4} \int \frac{dx}{(x+1)} - \frac{3}{2} \int \frac{dx}{(x+1)^2} + \frac{5}{4} \int \frac{dx}{(x-1)} = \frac{31}{4} \ell \eta |x+1| + \frac{3}{2(x+1)} + \frac{5}{4} \ell \eta |x-1| + c$$

$$(*) = 2x^2 - 6x + \frac{31}{4} \ell \eta |x+1| + \frac{3}{2(x+1)} + \frac{5}{4} \ell \eta |x-1| + c$$

**7.46.-**  $\int \frac{3x^4 dx}{(x^2 + 1)^2}$

Solución.-

$$\int \frac{3x^4 dx}{(x^2 + 1)^2} = \int \frac{3x^4 dx}{(x^4 + 2x^2 + 1)} = 3 \int \left[ 1 - \frac{2x^2 + 1}{(x^2 + 1)^2} \right] dx = 3 \int dx - 3 \int \frac{2x^2 + 1}{(x^2 + 1)^2} dx$$

$$= 3x - 3 \int \frac{2x^2 + 1}{(x^2 + 1)^2} dx (*)$$

Trabajando sólo la integral resultante:

$$\int \frac{(2x^2 + 1) dx}{(x^2 + 1)^2} = \int \frac{(Ax + B) dx}{(x^2 + 1)} + \int \frac{(Cx + D) dx}{(x^2 + 1)^2} (**), \text{ luego:}$$

$$\frac{(2x^2+1)}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} \Rightarrow 2x^2+1 = (Ax+B)(x^2+1) + Cx+D$$

$$\Rightarrow 2x^2+1 = Ax^3 + Ax + Bx^2 + B + Cx + D \Rightarrow 2x^2+1 = Ax^3 + Bx^2 + (A+C)x + (B+D)$$

Igualando coeficientes:  $A=0, B=2, A+C=0 \Rightarrow C=0, B+D=1 \Rightarrow D=-1$

$$(**) = 2 \int \frac{dx}{(x^2+1)} - \int \frac{dx}{(x^2+1)^2} = 2 \operatorname{arctg} x - \frac{1}{2} \left( \operatorname{arctg} x + \frac{x}{1+x^2} \right) + c$$

$$= \frac{3}{2} \operatorname{arctg} x - \frac{x}{2(1+x^2)} + c$$

$$(*) = 3x - \frac{9}{2} \operatorname{arctg} x - \frac{x}{2(1+x^2)} + c$$

**7.47.-**  $\int \frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12}$

Solución.-

$$\int \frac{(2x^2+41x-91)dx}{x^3-2x^2-11x+12} = \int \frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)}$$

$$= \int \frac{(2x^2+41x-91)dx}{(x-1)(x+3)(x-4)} = \int \frac{Adx}{x-1} + \int \frac{Bdx}{x+3} + \int \frac{Cdx}{x-4} \quad (*)$$

$$\frac{(2x^2+41x-91)}{(x-1)(x+3)(x-4)} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{x-4}$$

$$(2x^2+41x-91) = A(x+3)(x-4) + B(x-1)(x-4) + C(x-1)(x+3)$$

$$\begin{cases} x=-3 \Rightarrow 18-123-91 = B(-4)(-7) \Rightarrow B=-7 \\ x=4 \Rightarrow 32+164-91 = C(3)(7) \Rightarrow C=5 \\ x=1 \Rightarrow 2+41-91 = A(4)(-3) \Rightarrow A=4 \end{cases}$$

$$(*) = 4 \int \frac{dx}{(x-1)} - 7 \int \frac{dx}{(x+3)} + 5 \int \frac{dx}{(x-4)} = 4\ell \eta |x-1| - 7\ell \eta |x+3| + 5\ell \eta |x-4| + c$$

$$= \ell \eta \left| \frac{(x-1)^4(x-4)^5}{(x+3)^7} \right| + c$$

**7.48.-**  $\int \frac{(2x^4+3x^3-x-1)dx}{(x-1)(x^2+2x+2)^2}$

Solución.-

$$\int \frac{2x^4+3x^3-x-1}{(x-1)(x^2+2x+2)^2} dx = \int \frac{Adx}{(x-1)} + \int \frac{(Bx+C)dx}{(x^2+2x+2)} + \int \frac{(Dx+E)dx}{(x^2+2x+2)^2} \quad (*), \text{ luego:}$$

$$\frac{2x^4+3x^3-x-1}{(x-1)(x^2+2x+2)^2} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+2x+2)} + \frac{Dx+E}{(x^2+2x+2)^2}$$

$$2x^4+3x^3-x-1 = A(x^2+2x+2)^2 + (Bx+C)(x-1)(x^2+2x+2) + (Dx+E)(x-1)$$

$$2x^4+3x^3-x-1 = A(x^4+4x^2+4+4x^3+4x^2+8x) + B(x^4+2x^3+2x^2-x^3-2x^2-2x)$$

$$\Rightarrow +C(x^3+2x^2+2x-x^2-2x-2) + D(x^2-x) + E(x-1)$$

$$2x^4 + 3x^3 - x - 1 = (A+B)x^4 + (4A+B+C)x^3 + (8A+C+D)x^2$$

$$\Rightarrow +(8A-2B-D+E)x + (4A-2C-E)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A + B & & & & = 2 \\ 4A + B + C & & & & = 3 \\ 8A & & + C + D & & = 0 \\ 8A & -2B & & -D + E & = -1 \\ 4A & & -2C & & -E = -1 \end{pmatrix}$$

$$\therefore A = \frac{3}{25}, B = \frac{47}{25}, C = \frac{16}{25}, D = -\frac{8}{5}, E = \frac{1}{5}$$

$$(*) = \frac{3}{25} \int \frac{dx}{x-1} + \frac{1}{25} \int \frac{(47x+16)dx}{(x^2+2x+2)} - \frac{1}{5} \int \frac{(8x-1)dx}{(x^2+2x+2)^2}$$

$$= \frac{3}{25} \ell \eta |x-1| + \frac{47}{25} \int \frac{(x+\frac{16}{47})dx}{(x^2+2x+2)} - \frac{8}{5} \int \frac{(x-\frac{1}{8})dx}{(x^2+2x+2)^2}$$

$$= \frac{3}{25} \ell \eta |x-1| + \frac{47}{50} \int \frac{(2x+2)-\frac{62}{47}}{(x^2+2x+2)} dx - \frac{4}{5} \int \frac{(2x+2)-\frac{9}{4}}{(x^2+2x+2)^2} dx$$

$$= \frac{3}{25} \ell \eta |x-1| + \frac{47}{50} \int \frac{(2x+2)dx}{(x^2+2x+2)} - \frac{62}{50} \int \frac{dx}{(x^2+2x+2)} - \frac{4}{5} \int \frac{(2x+2)dx}{(x^2+2x+2)^2}$$

$$\Rightarrow + \frac{9}{5} \int \frac{dx}{(x^2+2x+2)^2}$$

$$= \frac{3}{25} \ell \eta |x-1| + \frac{47}{50} \ell \eta |x^2+2x+2| - \frac{62}{50} \int \frac{dx}{(x+1)^2+1} + \frac{4}{5} \int \frac{1}{(x^2+2x+2)}$$

$$\Rightarrow + \frac{9}{5} \int \frac{dx}{[(x+1)^2+1]^2}$$

$$= \frac{3}{25} \ell \eta |x-1| + \frac{47}{50} \ell \eta |x^2+2x+2| - \frac{62}{50} \operatorname{arc} \tau g(x+1) + \frac{4}{5(x^2+2x+2)}$$

$$\Rightarrow + \frac{9}{5} \left[ \frac{1}{2} \operatorname{arc} \tau g(x+1) + \frac{1}{2} \frac{x+1}{x^2+2x+2} \right] + c$$

$$= \frac{3}{25} \ell \eta |x-1| + \frac{47}{50} \ell \eta |x^2+2x+2| - \frac{17}{50} \operatorname{arc} \tau g(x+1) + \frac{9x+17}{10(x^2+2x+2)} + c$$

**7.49.-**  $\int \frac{dx}{e^{2x} + e^x - 2}$

Solución.-

$$\int \frac{dx}{e^{2x} + e^x - 2} = \int \frac{dx}{(e^x)^2 + e^x - 2} = \int \frac{dx}{[(e^x)^2 + e^x + \frac{1}{4}] - 2 - \frac{1}{4}}$$

$$= \int \frac{dx}{\left[e^x + \frac{1}{2}\right]^2 - \left(\frac{3}{2}\right)^2} (*) , \text{ Sea: } u = e^x + \frac{1}{2}, du = e^x dx \Rightarrow dx = \frac{du}{u - \frac{1}{2}}$$

Luego:

$$(*) \int \frac{\frac{du}{u - \frac{1}{2}}}{u^2 - \left(\frac{3}{2}\right)^2} = \int \frac{du}{(u - \frac{1}{2})(u + \frac{3}{2})(u - \frac{3}{2})} = \int \frac{Adu}{u - \frac{1}{2}} - \int \frac{Bdu}{(u + \frac{3}{2})} + \int \frac{Cdu}{(u - \frac{3}{2})} (**)$$

$$\frac{1}{(u - \frac{1}{2})(u + \frac{3}{2})(u - \frac{3}{2})} = \frac{A}{(u - \frac{1}{2})} - \frac{B}{(u + \frac{3}{2})} + \frac{C}{(u - \frac{3}{2})}$$

$$1 = A(u + \frac{3}{2})(u - \frac{3}{2}) - B(u - \frac{1}{2})(u - \frac{3}{2}) + C(u - \frac{1}{2})(u + \frac{3}{2})$$

$$\therefore \begin{cases} u = \frac{1}{2} \Rightarrow 1 = A(2)(-1) \Rightarrow A = -\frac{1}{2} \\ u = -\frac{3}{2} \Rightarrow 1 = B(-2)(-3) \Rightarrow B = \frac{1}{6} \\ u = \frac{3}{2} \Rightarrow 1 = C(1)(3) \Rightarrow C = \frac{1}{3} \end{cases}$$

$$(**) = -\frac{1}{2} \int \frac{du}{(u - \frac{1}{2})} + \frac{1}{6} \int \frac{du}{(u + \frac{3}{2})} + \frac{1}{3} \int \frac{du}{(u - \frac{3}{2})}$$

$$= -\frac{1}{2} \ell \eta \left| (u - \frac{1}{2}) \right| + \frac{1}{6} \ell \eta \left| (u + \frac{3}{2}) \right| + \frac{1}{3} \ell \eta \left| (u - \frac{3}{2}) \right| + c$$

$$= \frac{1}{6} \ell \eta \left| \frac{(u + \frac{3}{2})(u - \frac{3}{2})^2}{(u - \frac{1}{2})^3} \right| + c = \frac{1}{6} \ell \eta \left| \frac{(e^x + 2)(e^x - 1)^2}{(e^x)^3} \right| + c = \frac{1}{6} \ell \eta \left| \frac{(e^x + 2)(e^x - 1)^2}{e^{3x}} \right| + c$$

**7.50.-**  $\int \frac{\operatorname{sen} x dx}{\cos x (1 + \cos^2 x)}$

Solución.-

$$\int \frac{\operatorname{sen} x dx}{\cos x (1 + \cos^2 x)} = \int \frac{-\operatorname{sen} x dx}{\cos x (1 + \cos^2 x)} = - \int \frac{du}{u(1 + u^2)} = - \int \frac{Adu}{u} - \int \frac{(Bu + C)du}{(1 + u^2)} (*)$$

Sea:  $u = \cos x, du = -\operatorname{sen} x dx$

$$\frac{1}{u(1 + u^2)} = \frac{A}{u} + \frac{(Bu + C)}{(1 + u^2)} \Rightarrow 1 = A(1 + u^2) + (Bu + C)u$$

$$1 = A + Au^2 + Bu^2 + Cu \Rightarrow 1 = (A + B)u^2 + Cu + A$$

Igualando Coeficientes se tiene:

$$\therefore \begin{cases} A + B = 0 \Rightarrow B = -A \Rightarrow B = -(1) \Rightarrow B = -1 \\ C = 0, \\ A = 1 \end{cases}$$

$$(*) = - \int \frac{du}{u} + \int \frac{udu}{1 + u^2} = -\ell \eta |u| + \ell \eta \left| \sqrt{1 + u^2} \right| + c = -\ell \eta |\cos x| + \ell \eta \left| \sqrt{1 + (\cos x)^2} \right| + c$$

$$= \ell \eta \left| \frac{\sqrt{1 + (\cos x)^2}}{\cos x} \right| + c$$

**7.51.-**  $\int \frac{(2 + \tau g^2 \theta) \sec^2 \theta d\theta}{1 + \tau g^3 \theta}$

Solución.-

$$\int \frac{(2 + \tau g^2 \theta) \sec^2 \theta d\theta}{1 + \tau g^3 \theta} = \int \frac{(2 + u^2) du}{(1 + u^3)} = \int \frac{(2 + u^2) du}{(1 + u)(u^2 - u + 1)} \quad (*)$$

Sea:  $u = \tau g \theta, du = -\sec^2 \theta d\theta$

$$\int \frac{(2 + u^2) du}{(1 + u^3)} = \int \frac{A du}{(1 + u)} + \int \frac{Bu + C}{(u^2 - u + 1)}, \text{ luego:}$$

$$\frac{(2 + u^2)}{(1 + u^3)} = \frac{A}{(1 + u)} + \frac{Bu + C}{(u^2 - u + 1)} \Rightarrow (2 + u^2) = A(u^2 - u + 1) + (Bu + C)(1 + u)$$

$$(2 + u^2) = Au^2 - Au + A + Bu^2 + Bu + C + Cu$$

$$(2 + u^2) = (A + B)u^2 + (-A + B + C)u + A + C$$

Igualando Coeficientes se tiene:

$$\begin{pmatrix} A + B & = 1 \\ -A + B + C & = 0 \\ A & + C = 2 \end{pmatrix} \therefore A = 1, B = 0, C = 1$$

$$(*) = \int \frac{du}{1 + u} + \int \frac{du}{u^2 - u + 1} = \int \frac{du}{1 + u} + \int \frac{du}{(u - 1/2)^2 + (\sqrt{3}/2)^2}$$

$$= \ell \eta |1 + u| + \frac{1}{\sqrt{3}/2} \arctan \tau g \frac{u - 1/2}{\sqrt{3}/2} + c = \ell \eta |1 + u| + \frac{2}{\sqrt{3}} \arctan \tau g \frac{2u - 1}{\sqrt{3}} + c$$

$$= \ell \eta |1 + \tau g \theta| + \frac{2}{\sqrt{3}} \arctan \tau g \frac{(2\tau g \theta - 1)}{\sqrt{3}} + c$$

**7.52.-**  $\int \frac{(5x^3 + 2) dx}{x^3 - 5x^2 + 4x}$

Solución.-

$$\int \frac{(5x^3 + 2) dx}{x^3 - 5x^2 + 4x} = \int \frac{(5x^3 + 2) dx}{x(x-1)(x-4)} = \int \frac{A dx}{x} + \int \frac{B dx}{(x-1)} + \int \frac{C dx}{(x-4)} \quad (*)$$

$$\frac{(5x^3 + 2)}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-4)}, \text{ Luego:}$$

$$(5x^3 + 2) = A(x-1)(x-4) + Bx(x-4) + Cx(x-1)$$

Igualando Coeficientes se tiene:

$$\therefore \begin{cases} x=0 \Rightarrow 2=4A \Rightarrow A=1/2 \\ x=1 \Rightarrow 7=-3B \Rightarrow B=-7/3 \\ x=4 \Rightarrow 322=12C \Rightarrow C=161/6 \end{cases}$$

$$(*) = \frac{1}{2} \int \frac{dx}{x} - \frac{7}{3} \int \frac{dx}{x-1} + \frac{161}{6} \int \frac{dx}{x-4} = \frac{1}{2} \ell \eta |x| - \frac{7}{3} \ell \eta |x-1| + \frac{161}{6} \ell \eta |x-4| + c$$

$$= \frac{3}{6} \ell \eta |x| - \frac{14}{3} \ell \eta |x-1| + \frac{161}{6} \ell \eta |x-4| + c = \frac{1}{6} \ell \eta \left| \frac{x^3(x-4)^{161}}{(x-1)^{14}} \right| + c$$

**7.53.-**  $\int \frac{x^5 dx}{(x^3+1)(x^3+8)}$

Solución.-

$$\int \frac{x^5 dx}{(x^3+1)(x^3+8)} = \int \frac{x^5 dx}{(x+1)(x^2-x+1)(x+2)(x^2-2x+4)}$$

$$= \int \frac{A dx}{(x+1)} + \int \frac{B dx}{(x+2)} + \int \frac{(Cx+D) dx}{(x^2-x+1)} + \int \frac{(Ex+F) dx}{(x^2-2x+4)} \quad (*), \text{ luego:}$$

$$\frac{x^5}{(x^3+1)(x^3+8)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{Cx+D}{(x^2-x+1)} + \frac{Ex+F}{(x^2-2x+4)}, \text{ luego:}$$

$$x^5 = A(x+2)(x^2-x+1)(x^2-2x+4) + B(x+1)(x^2-x+1)(x^2-2x+4)$$

$$\Rightarrow + (Cx+D)(x+1)(x+2)(x^2-2x+4) + (Ex+F)(x+1)(x+1)(x^2-x+1)$$

$$x^5 = A(x^5+8x^2-x^4-8x+x^3+8) + B(x^5-2x^4+4x^3+x^2-2x+4)$$

$$\Rightarrow + (Cx+D)(x^4+8x+x^3+8) + (Ex+F)(x^4+2x^3+x+2)$$

$$x^5 = (A+B+C+E)x^5 + (-A-2B+C+D+2E+F)x^4 + (A+4B+D+2F)x^3$$

$$\Rightarrow + (8A+B+8C+E)x^2 + (-8A-2B+8C+8D+2E+F)x + (8A+4B+8D+2F)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A + B + C + E = 1 \\ -A - 2B + C + D + 2E + F = 0 \\ A + 4B + D + 2F = 0 \\ 8A + B + 8C + E = 0 \\ 8A - 2B + 8C + 8D + 2E + F = 0 \\ 8A + 4B + 8D + 2F = 0 \end{pmatrix}$$

$$\therefore A = -1/21, B = 8/21, C = -2/21, D = 1/21, E = 16/21, F = -16/21$$

$$(*) = -\frac{1}{21} \int \frac{dx}{x+1} + \frac{8}{21} \int \frac{dx}{(x+2)} - \frac{1}{21} \int \frac{(2x-1)dx}{(x^2-x+1)} + \frac{16}{21} \int \frac{(x-1)dx}{(x^2-2x+4)}$$

$$= -\frac{1}{21} \ell \eta |x+1| + \frac{8}{21} \ell \eta |x+2| - \frac{1}{21} \ell \eta |x^2-x+1| + \frac{8}{21} \int \frac{(2x-2)dx}{x^2-2x+4}$$

$$\begin{aligned}
&= -\frac{1}{21} \ell \eta |x+1| + \frac{8}{21} \ell \eta |x+2| - \frac{1}{21} \ell \eta |x^2 - x + 1| - \frac{8}{21} \ell \eta |x^2 - 2x + 4| + c \\
&= \frac{1}{21} \ell \eta \left| \frac{[(x+2)(x^2 - 2x + 4)]^8}{(x+1)(x^2 - x + 1)} \right| + c
\end{aligned}$$



## CAPITULO 8

### INTEGRACION DE FUNCIONES RACIONALES D SENO Y COSENO

Existen funciones racionales que conllevan formas trigonométricas, reducibles por si a: seno y coseno. Lo conveniente en tales casos es usar las siguientes sustituciones:  $z = \tau g \frac{x}{2}$ , de donde:  $x = 2 \arctan z$  y  $dx = \frac{2dz}{1+z^2}$ . Es fácil llegar a verificar

que de lo anterior se consigue:  $\sin x = \frac{2z}{1+z^2}$  y  $\cos x = \frac{1-z^2}{1+z^2}$

### EJERCICIOS DESARROLLADOS

**8.1.-Encontrar:**  $\int \frac{dx}{2 - \cos x}$

Solución.- La función racional con expresión trigonométrica es:  $\frac{1}{2 - \cos x}$ , y su solución se hace sencilla, usando sustituciones recomendadas, este es:

$$z = \tau g \frac{x}{2}, x = 2 \arctan z, dx = \frac{2dz}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2} \therefore$$

$$\begin{aligned} \int \frac{dx}{2 - \cos x} &= \int \frac{\frac{2dz}{1+z^2}}{2 - \frac{1-z^2}{1+z^2}} = \int \frac{\cancel{2}dz}{\cancel{1+z^2} \frac{2+2z-1+z^2}{\cancel{1+z^2}}} = \int \frac{2dz}{3z^2+1} = \int \frac{2dz}{3(z^2 + \frac{1}{3})} \\ &= \frac{2}{3} \int \frac{dz}{z^2 + (\sqrt{\frac{1}{3}})^2} = \frac{2}{3} \sqrt{3} \arctan \sqrt{3}z + c, \text{ recordando que: } z = \tau g \frac{x}{2}, \text{ se tiene:} \\ &= \frac{2}{3} \sqrt{3} \arctan \sqrt{3} \tau g \frac{x}{2} + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{2 - \cos x} = \frac{2}{3} \arctan \sqrt{3} \tau g \frac{x}{2} + c$

**8.2.-Encontrar:**  $\int \frac{dx}{2 - \sin x}$

Solución.- Forma racional:  $\frac{1}{2 - \sin x}$ ,

$$\text{sustituciones: } z = \tau g \frac{x}{2}, x = 2 \arctan z, dx = \frac{2dz}{1+z^2}, \sin x = \frac{2z}{1+z^2} \therefore$$

$$\int \frac{dx}{2 - \sin x} = \int \frac{\frac{2dz}{1+z^2}}{2 - \frac{2z}{1+z^2}} = \int \frac{\cancel{2}dz}{\cancel{1+z^2} \frac{2+2z^2-2z}{\cancel{1+z^2}}} = \int \frac{\cancel{2}dz}{\cancel{2}(1+z^2-z)} = \int \frac{dz}{(z^2 - z + 1)}$$

Ahora bien:  $z^2 - z + 1 = (z^2 - z + \frac{1}{4}) + 1 - \frac{1}{4} = (z - \frac{1}{2})^2 + \frac{3}{4} = (z - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2$

$$\begin{aligned} \therefore \int \frac{dx}{(z - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} &= \frac{1}{\frac{\sqrt{3}}{2}} \operatorname{arc} \tau g \frac{z - \frac{1}{2}}{\frac{\sqrt{3}}{2}} + c = \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2z-1}{\sqrt{3}} + c \\ &= \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2z-1}{\sqrt{3}} + c, \text{ recordando que: } z = \tau g \frac{x}{2}, \text{ se tiene:} \\ &= \frac{2\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2\tau g \frac{x}{2} - 1}{\sqrt{3}} + c \end{aligned}$$

**Respuesta:**  $\int \frac{dx}{2 - \operatorname{sen} x} = \frac{2\sqrt{3}}{3} \operatorname{arc} \tau g \frac{2\tau g \frac{x}{2} - 1}{\sqrt{3}} + c$

**8.3.-Encontrar:**  $\int \frac{d\theta}{4 - 5\cos \theta}$

Solución.- Forma racional:  $\frac{1}{4 - 5\cos \theta}$ ,

sustituciones:  $z = \tau g \frac{\theta}{2}$ ,  $x = 2 \operatorname{arc} \tau g z$ ,  $dx = \frac{2dz}{1+z^2}$ ,  $\cos x = \frac{1-z^2}{1+z^2}$

$$\begin{aligned} \therefore \int \frac{dx}{4 - 5\cos \theta} &= \int \frac{\frac{2dz}{1+z^2}}{4 - 5\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{\frac{2dz}{1+z^2}}{\frac{4+4z^2-5+5z^2}{1+z^2}} = \int \frac{2dz}{9z^2-1} = \int \frac{2dz}{9(z^2 - \frac{1}{9})} \\ &= \frac{2}{9} \int \frac{dz}{z^2 - (\frac{1}{3})^2} = \frac{2}{9} \frac{1}{\frac{1}{3}} \ell \eta \left| \frac{z - \frac{1}{3}}{z + \frac{1}{3}} \right| + c = \frac{1}{3} \ell \eta \left| \frac{3z-1}{3z+1} \right| + c \end{aligned}$$

Recordando que:  $z = \tau g \frac{\theta}{2}$ , se tiene:  $= \frac{1}{3} \ell \eta \left| \frac{3\tau g \frac{\theta}{2} - 1}{3\tau g \frac{\theta}{2} + 1} \right| + c$

**Respuesta:**  $\int \frac{d\theta}{4 - 5\cos \theta} = \frac{1}{3} \ell \eta \left| \frac{3\tau g \frac{\theta}{2} - 1}{3\tau g \frac{\theta}{2} + 1} \right| + c$

**8.4.-Encontrar:**  $\int \frac{d\theta}{3\cos \theta + 4\operatorname{sen} \theta}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{d\theta}{3\cos \theta + 4\operatorname{sen} \theta} = \int \frac{\frac{2dz}{1+z^2}}{3\left(\frac{1-z^2}{1+z^2}\right) + 4\left(\frac{2z}{1+z^2}\right)} = \int \frac{\frac{2dz}{1+z^2}}{\frac{3-3z^2+8z}{1+z^2}}$$

$$\begin{aligned}
&= \int \frac{2dz}{-3(z^2 - \frac{8}{3}z - 1)} = -\frac{2}{3} \int \frac{dz}{z^2 - \frac{8}{3}z - 1}, \text{ pero:} \\
&z^2 - \frac{8}{3}z - 1 = (z^2 - \frac{8}{3}z + \frac{16}{9}) - 1 - \frac{16}{9} = (z - \frac{4}{3})^2 - (\frac{5}{3})^2, \text{ luego:} \\
&= -\frac{2}{3} \int \frac{dz}{(z - \frac{4}{3})^2 - (\frac{5}{3})^2}, \text{ sea: } w = z - \frac{4}{3}, dw = dz; \text{ de donde:} \\
&= -\frac{2}{3} \frac{1}{2(\frac{5}{3})} \ell \eta \left| \frac{z - \frac{4}{3} - \frac{5}{3}}{z - \frac{4}{3} + \frac{5}{3}} \right| + c = -\frac{1}{5} \ell \eta \left| \frac{3z - 9}{3z + 1} \right| + c, \text{ como: } z = \tau g \frac{\theta}{2}, \text{ se tiene:} \\
&= -\frac{1}{5} \ell \eta \left| \frac{3\tau g \frac{\theta}{2} - 9}{3\tau g \frac{\theta}{2} + 1} \right| + c
\end{aligned}$$

**Respuesta:**  $\int \frac{d\theta}{3\cos\theta + 4\sin\theta} = -\frac{1}{5} \ell \eta \left| \frac{3\tau g \frac{\theta}{2} - 9}{3\tau g \frac{\theta}{2} + 1} \right| + c$

**8.5.-Encontrar:**  $\int \frac{d\theta}{3 + 2\cos\theta + 2\sin\theta}$

Solución.- usando las sustituciones recomendadas:

$$\begin{aligned}
\int \frac{d\theta}{3 + 2\cos\theta + 2\sin\theta} &= \int \frac{\frac{2dz}{1+z^2}}{3 + 2\left(\frac{1-z^2}{1+z^2}\right) + 2\left(\frac{2z}{1+z^2}\right)} = \int \frac{\frac{2dz}{1+z^2}}{3 + \frac{2-2z^2}{1+z^2} + \frac{4z}{1+z^2}} \\
&= \int \frac{\frac{2dz}{1+z^2}}{\frac{3+3z^2+2-2z^2+4z}{1+z^2}} = \int \frac{2dz}{z^2 + 4z + 5} = \int \frac{2dz}{(z+2)^2 + 1} = 2 \operatorname{arc} \tau g(z+2) + c
\end{aligned}$$

Como:  $z = \tau g \frac{\theta}{2}$ , se tiene:  $= 2 \operatorname{arc} \tau g(\tau g \frac{\theta}{2} + 2) + c$

**Respuesta:**  $\int \frac{d\theta}{3 + 2\cos\theta + 2\sin\theta} = 2 \operatorname{arc} \tau g(\tau g \frac{\theta}{2} + 2) + c$

**8.6.-Encontrar:**  $\int \frac{dx}{\tau g\theta - \sin\theta}$

Solución.- Antes de hacer las sustituciones recomendadas, se buscará la equivalencia correspondiente a  $\tau g\theta$

$$\tau g\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2z}{1+z^2}}{\frac{1-z^2}{1+z^2}} = \frac{2z}{1-z^2}, \text{ procédase ahora como antes:}$$

$$\int \frac{dx}{\tau g \theta - \operatorname{sen} \theta} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z}{1-z^2} + \frac{2z}{1+z^2}} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2z(1+z^2) - 2z(1-z^2)}{(1-z^2)(1+z^2)}} = \int \frac{2(1-z^2)dz}{2z + 2z^3 - 2z + 2z^3}$$

$$= \int \frac{(2-2z^2)dz}{4z^3} = \frac{1}{2} \int z^{-3} dz - \frac{1}{2} \int \frac{dz}{z} = -\frac{1}{4z^2} - \frac{1}{2} \ell \eta |z| + c$$

Como:  $z = \tau g \theta/2$ , se tiene:  $= -\frac{1}{4}(\operatorname{co} \tau g^2 \theta/2) - \frac{1}{2} \ell \eta |\tau g \theta/2| + c$

**Respuesta:**  $\int \frac{dx}{\tau g \theta - \operatorname{sen} \theta} = -\frac{1}{4}(\operatorname{co} \tau g^2 \theta/2) - \frac{1}{2} \ell \eta |\tau g \theta/2| + c$

**8.7.-Encontrar:**  $\int \frac{dx}{2 + \operatorname{sen} x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{2 + \operatorname{sen} x} = \int \frac{\frac{2dz}{1+z^2}}{2 + \frac{2z}{1+z^2}} = \int \frac{\frac{2dz}{1+z^2}}{\frac{2+2z^2+2z}{1+z^2}} = \int \frac{dz}{z^2 + z + 1} = \int \frac{dz}{(z^2 + z + 1/4) + 3/4}$$

$$= \int \frac{2dz}{(z + 1/2)^2 + (\sqrt{3}/2)^2} = \frac{1}{\sqrt{3}/2} \operatorname{arc} \tau g \frac{(z + 1/2)}{\sqrt{3}/2} + c = \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2z + 1}{\sqrt{3}} + c$$

Como:  $z = \tau g x/2$ , se tiene:  $= \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2\tau g x/2 + 1}{\sqrt{3}} + c$

**Respuesta:**  $\int \frac{dx}{2 + \operatorname{sen} x} = \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2\tau g x/2 + 1}{\sqrt{3}} + c$

**8.8.-Encontrar:**  $\int \frac{\cos x dx}{1 + \cos x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{\cos x dx}{1 + \cos x} = \int \frac{\left(\frac{1-z^2}{1+z^2}\right) \left(\frac{2dz}{1+z^2}\right)}{1 + \frac{1-z^2}{1+z^2}} = \int \frac{\left(\frac{1-z^2}{1+z^2}\right) \left(\frac{2dz}{1+z^2}\right)}{\frac{1+z^2+1-z^2}{1+z^2}} = \int \frac{\cancel{2}(1-z^2)dz}{(1+z^2)\cancel{2}} = \int \frac{(1-z^2)dz}{(1+z^2)}$$

$$= \int \frac{(-z^2+1)dz}{(z^2+1)} = \int \left(-1 + \frac{2}{z^2+1}\right) dz = \int dz + 2 \int \frac{dz}{z^2+1} = -z + 2 \operatorname{arc} \tau g z + c$$

Como:  $z = \tau g x/2$ , se tiene:  $= -\tau g \frac{x}{2} + 2 \operatorname{arc} \tau g (\tau g \frac{x}{2}) + c$

**Respuesta:**  $\int \frac{\cos x dx}{1 + \cos x} = -\tau g \frac{x}{2} + x + c$

**8.9.-Encontrar:**  $\int \frac{dx}{1+\operatorname{sen} x+\cos x}$

Solución.- usando las sustituciones recomendadas:

$$\begin{aligned}\int \frac{dx}{1+\operatorname{sen} x+\cos x} &= \int \frac{\frac{2dz}{1+z^2}}{1+\left(\frac{2z}{1+z^2}\right)+\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{2dz}{1-\cancel{z^2}+2z+1-\cancel{z^2}} \\ &= \int \frac{2dz}{2z+2} = \int \frac{dz}{z+1} = \ell \eta |z+1| + c, \text{ como: } z = \tau g \frac{x}{2}, \text{ se tiene: } = \ell \eta \left| \tau g \frac{x}{2} + 1 \right| + c\end{aligned}$$

**Respuesta:**  $\int \frac{dx}{1+\operatorname{sen} x+\cos x} = \ell \eta \left| \tau g \frac{x}{2} + 1 \right| + c$

**8.10.-Encontrar:**  $\int \frac{dx}{\cos x+2\operatorname{sen} x+3}$

Solución.- usando las sustituciones recomendadas:

$$\begin{aligned}\int \frac{dx}{\cos x+2\operatorname{sen} x+3} &= \int \frac{\frac{2dz}{1+z^2}}{\left(\frac{1-z^2}{1+z^2}\right)+\left(\frac{4z}{1+z^2}\right)+3} = \int \frac{2dz}{1-z^2+4z+3+3z^2} = \int \frac{2dz}{2z^2+2z+2} \\ &= \int \frac{dz}{z^2+2z+2} = \int \frac{dz}{(z+1)^2+1} = \operatorname{arc} \tau g(z+1) + c, \text{ como: } z = \tau g \frac{\theta}{2},\end{aligned}$$

Se tiene:  $= \operatorname{arc} \tau g(\tau g \frac{x}{2} + 1) + c$

**Respuesta:**  $\int \frac{dx}{\cos x+2\operatorname{sen} x+3} = \operatorname{arc} \tau g(\tau g \frac{x}{2} + 1) + c$

**8.11.-Encontrar:**  $\int \frac{\operatorname{sen} x dx}{1+\operatorname{sen}^2 x}$

Solución.- usando las sustituciones recomendadas:

$$\begin{aligned}\int \frac{\operatorname{sen} x dx}{1+\operatorname{sen}^2 x} &= \int \frac{\left(\frac{2z}{1+z^2}\right)\left(\frac{2dz}{1+z^2}\right)}{1+\left(\frac{2z}{1+z^2}\right)^2} = \int \frac{\frac{4zdz}{(1+z^2)^2}}{1+\frac{4z^2}{(1+z^2)^2}} = \int \frac{4zdz}{(1+z^2)^2+4z^2} = \int \frac{4zdz}{1+2z^2+z^4+4z^2} \\ &= \int \frac{4zdz}{z^4+6z^2+1} = \int \frac{4zdz}{(z^4+6z^2+9)-8} = \int \frac{4zdz}{(z^2+3)^2-(\sqrt{8})^2}\end{aligned}$$

Sea:  $w = z^2 + 3, dw = 2zdz$

$$= 2 \int \frac{dw}{w^2 - (\sqrt{8})^2} = \frac{\cancel{2}}{\cancel{2}\sqrt{8}} \ell \eta \left| \frac{w - \sqrt{8}}{w + \sqrt{8}} \right| + c = \frac{\sqrt{8}}{8} \ell \eta \left| \frac{w - \sqrt{8}}{w + \sqrt{8}} \right| + c = \frac{\sqrt{8}}{8} \ell \eta \left| \frac{z^2 + 3 - \sqrt{8}}{z^2 + 3 + \sqrt{8}} \right| + c$$

Como:  $z = \tau g \frac{\theta}{2}$ , se tiene:  $= \frac{\sqrt{2}}{4} \ell \eta \left| \frac{z^2 + 3 - \sqrt{8}}{z^2 + 3 + \sqrt{8}} \right| + c = \frac{\sqrt{2}}{4} \ell \eta \left| \frac{\tau g^2 \frac{x}{2} + 3 - 2\sqrt{2}}{\tau g^2 \frac{x}{2} + 3 + 2\sqrt{2}} \right| + c$

**Respuesta:**  $\int \frac{\operatorname{sen} x dx}{1 + \operatorname{sen}^2 x} = \frac{\sqrt{2}}{4} \ell \eta \left| \frac{\tau g^{\frac{x}{2}} + 3 - 2\sqrt{2}}{\tau g^{\frac{x}{2}} + 3 + 2\sqrt{2}} \right| + c$

**8.12.-Encontrar:**  $\int \frac{d\theta}{5 + 4\cos \theta}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{5 + 4\cos \theta} = \int \frac{\frac{2dz}{1+z^2}}{5 + 4\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{2dz}{5 + 5z^2 + 4 - 4z^2} = \int \frac{2dz}{z^2 + 9} = 2 \int \frac{dz}{z^2 + 3^2}$$

$$= \frac{2}{3} \operatorname{arc} \tau g \frac{z}{3} + c, \text{ como: } z = \tau g \frac{\theta}{2}, \text{ se tiene: } = \frac{2}{3} \operatorname{arc} \tau g \frac{\tau g \frac{\theta}{2}}{3} + c$$

**Respuesta:**  $\int \frac{d\theta}{5 + 4\cos \theta} = \frac{2}{3} \operatorname{arc} \tau g \frac{\tau g \frac{\theta}{2}}{3} + c$

**8.14.-Encontrar:**  $\int \frac{dx}{\operatorname{sen} x + \cos x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{\operatorname{sen} x + \cos x} = \int \frac{\frac{2dz}{1+z^2}}{\left(\frac{2z}{1+z^2}\right) + \left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{2dz}{2z + 1 - z^2} = 2 \int \frac{dz}{(-z^2 + 2z + 1)}$$

$$= -2 \int \frac{dz}{(z^2 - 2z + 1) - 2} = -2 \int \frac{dz}{(z-1)^2 - (\sqrt{2})^2} = -\cancel{2} \frac{1}{\cancel{2}\sqrt{2}} \ell \eta \left| \frac{z-1-\sqrt{2}}{z-1+\sqrt{2}} \right| + c$$

$$= -\frac{\sqrt{2}}{2} \ell \eta \left| \frac{z-1-\sqrt{2}}{z-1+\sqrt{2}} \right| + c, \text{ como: } z = \tau g \frac{x}{2}, \text{ se tiene: } = -\frac{\sqrt{2}}{2} \ell \eta \left| \frac{\tau g \frac{x}{2} - 1 - \sqrt{2}}{\tau g \frac{x}{2} - 1 + \sqrt{2}} \right| + c$$

**Respuesta:**  $\int \frac{dx}{\operatorname{sen} x + \cos x} = -\frac{\sqrt{2}}{2} \ell \eta \left| \frac{\tau g \frac{x}{2} - 1 - \sqrt{2}}{\tau g \frac{x}{2} - 1 + \sqrt{2}} \right| + c$

**8.14.-Encontrar:**  $\int \frac{\sec x dx}{\sec x + 2\tau g x - 1}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{\sec x dx}{\sec x + 2\tau g x - 1} = \int \frac{\frac{1}{\cos x} dx}{\frac{1}{\cos x} + \frac{2\operatorname{sen} x}{\cos x} - 1} = \int \frac{dx}{1 + 2\operatorname{sen} x - \cos x} = \int \frac{\frac{2dz}{1+z^2}}{1 + \left(\frac{4z}{1+z^2}\right) - \left(\frac{1-z^2}{1+z^2}\right)}$$

$$= \int \frac{\frac{2dz}{1+z^2}}{\frac{1+z^2}{1+z^2} + \frac{4z}{1+z^2}} = \int \frac{2dz}{2z^2+4z} = \int \frac{\cancel{2}dz}{\cancel{2}(z^2+2z)} = \int \frac{dz}{z(z+2)} (*)$$

Ahora bien:  $\frac{1}{z(z+2)} = \frac{A}{z} + \frac{B}{z+2}$ , de donde:

$$\frac{1}{\cancel{z(z+2)}} = \frac{A(\cancel{z+2}) + B(\cancel{z})}{\cancel{z(z+2)}} \Rightarrow 1 = A(z+2) + B(z), \text{ de donde: } A = \frac{1}{2}, B = -\frac{1}{2}$$

$$(*) \int \frac{dz}{z(z+2)} = \int \frac{\frac{1}{2}dz}{z} - \int \frac{\frac{1}{2}dz}{z+2} = \frac{1}{2} \int \frac{dz}{z} - \frac{1}{2} \int \frac{dz}{z+2} = \frac{1}{2} \ell \eta |z| - \frac{1}{2} \ell \eta |z+2| + c$$

$$= \frac{1}{2} \ell \eta \left| \frac{z}{z+2} \right| + c, \text{ como: } z = \tau g^{x/2}, \text{ se tiene: } = \frac{1}{2} \ell \eta \left| \frac{\tau g^{x/2}}{\tau g^{x/2} + 2} \right| + c$$

**Respuesta:**  $\int \frac{\sec x dx}{\sec x + 2 \tau g x - 1} = \frac{1}{2} \ell \eta \left| \frac{\tau g^{x/2}}{\tau g^{x/2} + 2} \right| + c$

**8.15.-Encontrar:**  $\int \frac{dx}{1 - \cos x + \sec x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{1 - \cos x + \sec x} = \int \frac{\frac{2dz}{1+z^2}}{1 - \left( \frac{1-z^2}{1+z^2} \right) + \left( \frac{2z}{1+z^2} \right)} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{\cancel{1+z^2} - \cancel{1+z^2} + 2z}{\cancel{1+z^2}}} = \int \frac{2dz}{2z^2+2z}$$

$$= \int \frac{\cancel{2}dz}{\cancel{2}(z^2+z)} = \int \frac{dz}{z(z+1)} (*)$$

Ahora bien:  $\frac{1}{z(z+1)} = \frac{A}{z} + \frac{B}{z+1}$ , de donde se tiene:

$$\frac{1}{\cancel{z(z+1)}} = \frac{A(\cancel{z+1}) + B(\cancel{z})}{\cancel{z(z+1)}} \Rightarrow 1 = A(z+1) + B(z), \text{ de donde: } A = 1, B = -1, \text{ luego:}$$

$$\int \frac{dz}{z(z+1)} = \int \frac{dz}{z} - \int \frac{dz}{z+1} = \ell \eta |z| - \ell \eta |z+1| + c = \ell \eta \left| \frac{z}{z+1} \right| + c, \text{ como: } z = \tau g^{x/2},$$

$$\text{Se tiene: } = \ell \eta \left| \frac{\tau g^{x/2}}{\tau g^{x/2} + 1} \right| + c$$

**Respuesta:**  $\int \frac{dx}{1 - \cos x + \sec x} = \ell \eta \left| \frac{\tau g^{x/2}}{\tau g^{x/2} + 1} \right| + c$

**8.16.-Encontrar:**  $\int \frac{dx}{8 - 4 \sec x + 7 \cos x}$

Solución.- usando las sustituciones recomendadas:

$$\int \frac{dx}{8-4\operatorname{sen} x+7\cos x} = \int \frac{\frac{2dz}{1+z^2}}{8-\left(\frac{8z}{1+z^2}\right)+7\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{8+8z^2-8z+7-7z^2}{\cancel{1+z^2}}}$$

$$= \int \frac{2dz}{z^2-8z+15} = \int \frac{2dz}{(z-3)(z-5)} \quad (*)$$

Ahora bien:  $\frac{2}{(z-3)(z-5)} = \frac{A}{(z-3)} + \frac{B}{(z-5)}$ , de donde se tiene:

$\Rightarrow 2 = A(z-5) + B(z-3)$ , de donde:  $A = -1, B = 1$ , luego:

$$\int \frac{2dz}{(z-3)(z-5)} = -\int \frac{dz}{z-3} + \int \frac{dz}{z-5} = -\ell\eta|z-3| + \ell\eta|z-5| + c = \ell\eta\left|\frac{z-5}{z-3}\right| + c,$$

como:  $z = \tau g \frac{x}{2}$ , se tiene:  $= \ell\eta\left|\frac{\tau g \frac{x}{2}-5}{\tau g \frac{x}{2}-3}\right| + c$

**Respuesta:**  $\int \frac{dx}{8-4\operatorname{sen} x+7\cos x} = \ell\eta\left|\frac{\tau g \frac{x}{2}-5}{\tau g \frac{x}{2}-3}\right| + c$

## EJERCICIOS PROPUESTOS

8.17.-  $\int \frac{dx}{1+\cos x}$

8.18.-  $\int \frac{dx}{1-\cos x}$

8.19.-  $\int \frac{\operatorname{sen} x dx}{1+\cos x}$

8.20.-  $\int \frac{\cos x dx}{2-\cos x}$

8.21.-  $\int \frac{d\theta}{5-4\cos \theta}$

8.22.-  $\int \frac{\operatorname{sen} \theta d\theta}{\cos^2 \theta - \cos \theta - 2}$

8.23.-  $\int \sec x dx$

8.24.-  $\int \frac{\cos \theta d\theta}{5+4\cos \theta}$

8.25.-  $\int \frac{d\theta}{\cos \theta + \operatorname{co} \tau g \theta}$

## RESPUESTAS

8.17.-  $\int \frac{dx}{1+\cos x}$

Solución.-

$$\int \frac{dx}{1+\cos x} = \int \frac{\frac{2dz}{1+z^2}}{1+\left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{1+z^2+1-z^2}{\cancel{1+z^2}}} = \int dz = z + c = \tau g \frac{x}{2} + c$$

8.18.-  $\int \frac{dx}{1-\cos x}$

Solución.-



$$\int \frac{dx}{1 - \cos x} = \int \frac{\frac{2dz}{1+z^2}}{1 - \left( \frac{1-z^2}{1+z^2} \right)} = \int \frac{\frac{2dz}{\cancel{1+z^2}}}{\frac{\cancel{1+z^2} - 1 - z^2}{\cancel{1+z^2}}} = \int \frac{\cancel{2}dz}{\cancel{2}z^2} = -\frac{1}{z} + c = -\operatorname{cog} \frac{x}{2} + c$$

**8.19.-**  $\int \frac{\operatorname{sen} x dx}{1 + \cos x}$

Solución.-

$$\begin{aligned} \int \frac{\operatorname{sen} x dx}{1 + \cos x} &= \int \frac{\left( \frac{2z}{1+z^2} \right) \left( \frac{2dz}{1+z^2} \right)}{1 + \left( \frac{1-z^2}{1+z^2} \right)} = \int \frac{\frac{4zdz}{\cancel{(1+z^2)^2}}}{\frac{1 + \cancel{1+z^2} + 1 - \cancel{z^2}}{\cancel{1+z^2}}} = \int \frac{4zdz}{2(1+z^2)} = \int \frac{2zdz}{(1+z^2)} \\ &= \ell \eta |1+z^2| + c = \ell \eta |1 + \tau g^2 \frac{x}{2}| + c \end{aligned}$$

**8.20.-**  $\int \frac{\cos x dx}{2 - \cos x}$

Solución.-

$$\begin{aligned} \int \frac{\cos x dx}{2 - \cos x} &= \int \left( -1 + \frac{2}{2 - \cos x} \right) dx = -\int dx + 2 \int \frac{dx}{2 - \cos x} = -\int dx + 2 \int \frac{\left( \frac{2dz}{1+z^2} \right)}{2 - \left( \frac{1-z^2}{1+z^2} \right)} \\ &= -\int dx + 2 \int \frac{\frac{2dz}{\cancel{(1+z^2)}}}{\frac{2 + 2z^2 - 1 + z^2}{\cancel{1+z^2}}} = -\int dx + 2 \int \frac{2dz}{3z^2 + 1} = -\int dx + \frac{4}{3} \int \frac{dz}{(z^2 + 1/3)} \\ &= -\int dx + \frac{4}{3} \int \frac{dz}{z^2 + (1/\sqrt{3})^2} = -x + \frac{4}{3} \frac{1}{1/\sqrt{3}} \operatorname{arc} \tau g \frac{z}{1/\sqrt{3}} + c = -x + \frac{4\sqrt{3}}{3} \operatorname{arc} \tau g \sqrt{3}z + c \\ &= -x + \frac{4\sqrt{3}}{3} \operatorname{arc} \tau g (\sqrt{3} \tau g \frac{x}{2}) + c \end{aligned}$$

**8.21.-**  $\int \frac{d\theta}{5 - 4 \cos \theta}$

Solución.-

$$\begin{aligned} \int \frac{d\theta}{5 - 4 \cos \theta} &= \int \frac{\left( \frac{2dz}{1+z^2} \right)}{5 - 4 \left( \frac{1-z^2}{1+z^2} \right)} = \int \frac{\frac{2dz}{\cancel{(1+z^2)}}}{\frac{5 + 5z^2 - 4 + 4z^2}{\cancel{1+z^2}}} = \int \frac{2dz}{9z^2 + 1} = \frac{2}{9} \int \frac{dz}{(z^2 + 1)} \\ &= \frac{2}{9} \int \frac{dz}{z^2 + (1/3)^2} = \frac{2}{9} \frac{1}{1/3} \operatorname{arc} \tau g \frac{z}{1/3} + c = \frac{2}{3} \operatorname{arc} \tau g 3z + c = \frac{2}{3} \operatorname{arc} \tau g (3 \tau g \frac{x}{2}) + c \end{aligned}$$

$$8.22.- \int \frac{\operatorname{sen} \theta d\theta}{\cos^2 \theta - \cos \theta - 2}$$

Solución.-

$$\begin{aligned} \int \frac{\operatorname{sen} \theta d\theta}{\cos^2 \theta - \cos \theta - 2} &= \int \frac{\left(\frac{2z}{1+z^2}\right) \left(\frac{2dz}{1+z^2}\right)}{\left(\frac{1-z^2}{1+z^2}\right)^2 - \left(\frac{1-z^2}{1+z^2}\right) - 2} = \int \frac{\frac{4zdz}{(1+z^2)^2}}{\frac{(1-z^2)^2 - (1-z^2)(1+z^2) - 2(1+z^2)^2}{(1+z^2)^2}} \\ &= \int \frac{4zdz}{-6z^2 - 2} = -\frac{1}{3} \int \frac{2zdz}{(z^2 - 1/3)} = -\frac{1}{3} \ell \eta \left| z^2 - 1/3 \right| + c = -\frac{1}{3} \ell \eta \left| \tau g^2 \frac{x}{2} - 1/3 \right| + c \end{aligned}$$

$$8.23.- \int \sec x dx$$

Solución.-

$$\int \sec x dx = \int \frac{dx}{\cos x} = \int \frac{\frac{2dz}{1+z^2}}{\frac{1-z^2}{1+z^2}} = \int \frac{2dz}{(1-z^2)} = \int \frac{2dz}{(1+z)(1-z)} (*)$$

Ahora bien:  $\frac{2}{(1+z)(1-z)} = \frac{A}{1+z} + \frac{B}{1-z}$ , de donde:  $A=1, B=1$ , luego:

$$(*) \int \frac{2dz}{(1+z)(1-z)} = \int \frac{dz}{1+z} - \int \frac{dz}{1-z} = \ell \eta |1+z| - \ell \eta |1-z| + c = \ell \eta \left| \frac{1+z}{1-z} \right| + c$$

$$\text{Como: } z = \tau g \frac{x}{2}, \text{ Se tiene: } = \ell \eta \left| \frac{1 + \tau g \frac{x}{2}}{1 - \tau g \frac{x}{2}} \right| + c$$

$$8.24.- \int \frac{\cos \theta d\theta}{5 + 4 \cos \theta}$$

Solución.-

$$\int \frac{d\theta}{5 + 4 \cos \theta} = \int \frac{\left(\frac{1-z^2}{1+z^2}\right) \left(\frac{2dz}{1+z^2}\right)}{5 + 4 \left(\frac{1-z^2}{1+z^2}\right)} = \int \frac{\frac{2(1-z^2)dz}{(1+z^2)^2}}{\frac{(5+5z^2+4-4z^2)}{(1+z^2)}} = \int \frac{(2-2z^2)dz}{(1+z^2)(9+z^2)}$$

Ahora bien:  $\frac{2-2z^2}{(z^2+1)(z^2+9)} = \frac{Az+B}{z^2+1} + \frac{Cz+D}{z^2+9}$ , de donde:  $A=0, B=1/2, C=0, D=-5/2$ ,

luego:

$$\begin{aligned} \int \frac{(2-2z^2)}{(z^2+1)(z^2+9)} &= \frac{1}{2} \int \frac{dz}{z^2+1} - \frac{5}{2} \int \frac{dz}{z^2+9} = \frac{1}{2} \operatorname{arc} \tau g z + \frac{5}{2} \operatorname{arc} \tau g \frac{z}{3} + c \\ &= \frac{1}{2} \operatorname{arc} \tau g \frac{\theta}{2} - \frac{5}{6} \operatorname{arc} \tau g \left( \frac{\tau g \frac{\theta}{2}}{3} \right) + c = \frac{\theta}{4} - \frac{5}{6} \operatorname{arc} \tau g \left( \frac{\tau g \frac{\theta}{2}}{3} \right) + c \end{aligned}$$

$$8.25.- \int \frac{d\theta}{\cos \theta + \operatorname{co} \tau g \theta}$$

Solución.-

$$\int \frac{d\theta}{\cos \theta + \coth \tau g \theta} = \int \frac{\left( \frac{2dz}{1+z^2} \right)}{\left( \frac{1-z^2}{1+z^2} \right) + \left( \frac{1-z^2}{2z} \right)} = \int \frac{\frac{2dz}{\cancel{(1+z^2)}}}{\frac{2z(1-z^2) + (1-z^2)(1+z^2)}{\cancel{(1+z^2)}2z}}$$

$$= \int \frac{4zdz}{2z(1-z^2) + (1-z^2)(1+z^2)} = \int \frac{4zdz}{(1-z^2)(z^2 + 2z + 1)} = \int \frac{4zdz}{(1+z^3)(1-z)} \quad (*)$$

Ahora bien:  $\frac{4z}{(1+z^3)(1-z)} = \frac{A}{1+z} + \frac{B}{(1+z)^2} + \frac{C}{(1+z)^3} + \frac{D}{(1-z)}$

De donde:  $A = \frac{1}{2}, B = 1, C = -2, D = \frac{1}{2}$ , luego:

$$(*) \int \frac{4z}{(1+z^3)(1-z)} = \frac{1}{2} \int \frac{dz}{1+z} + \int \frac{dz}{(1+z)^2} - 2 \int \frac{dz}{(1+z)^3} + \frac{1}{2} \int \frac{dz}{1-z}$$

$$= \frac{1}{2} \ell \eta \left| \frac{1+z}{1-z} \right| - \frac{1}{1+z} + \frac{1}{(1+z)^2} - \frac{1}{2} \ell \eta \left| 1-z \right| + c = \frac{1}{2} \ell \eta \left| \frac{1+z}{1-z} \right| - \frac{1}{1+z} + \frac{1}{(1+z)^2} + c$$

$$= \frac{1}{2} \ell \eta \left| \frac{1+z}{1-z} \right| + \frac{-(1+z)+1}{(1+z)^2} + c = \frac{1}{2} \ell \eta \left| \frac{1+z}{1-z} \right| - \frac{z}{(1+z)^2} + c = \frac{1}{2} \ell \eta \left| \frac{1+\tau g \frac{\theta}{2}}{1-\tau g \frac{\theta}{2}} \right| - \frac{\tau g \frac{\theta}{2}}{(1+\tau g \frac{\theta}{2})^2} + c$$

## CAPITULO 9

### INTEGRACION DE FUNCIONES IRRACIONALES

En el caso de que el integrando contiene potencias fraccionarias de la variable de integración, estas se simplifican usando una sustitución del tipo:

$x = t^n$ ,  $\sqrt[n]{x} = t$ , siendo "n" el m.c.m. de los denominadores de los exponentes.

### EJERCICIOS DESARROLLADOS

**9.1.-Encontrar:**  $\int \frac{\sqrt{x}dx}{1+x}$

Solución.- La única expresión "irracional" es  $\sqrt{x}$ , por lo tanto:

$\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$ , luego:

$$\int \frac{\sqrt{x}dx}{1+x} = \int \frac{t(2tdt)}{1+t^2} = 2 \int \frac{t^2 dt}{1+t^2} = 2 \int \left(1 - \frac{1}{1+t^2}\right) dt = 2 \int dt - 2 \int \frac{dt}{t^2+1} = 2t - 2 \operatorname{arctg} t + c$$

Dado que:  $t = \sqrt{x}$ , se tiene:  $= 2\sqrt{x} - 2 \operatorname{arctg} \sqrt{x} + c$

**Respuesta:**  $\int \frac{\sqrt{x}dx}{1+x} = 2\sqrt{x} - 2 \operatorname{arctg} \sqrt{x} + c$

**9.2.-Encontrar:**  $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$

Solución.- Análogamente al caso anterior:  $\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$ , luego:

$$\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = \int \frac{2t dt}{t(1+t)} = \int \frac{2dt}{1+t} = 2\ell\eta|t+1| + c$$

Dado que:  $t = \sqrt{x}$ , se tiene:  $= 2\ell\eta|\sqrt{x}+1| + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})} = 2\ell\eta|\sqrt{x}+1| + c$

**9.3.-Encontrar:**  $\int \frac{dx}{3+\sqrt{x+2}}$

Solución.- La expresión "irracional" es ahora  $\sqrt{x+2}$ , por lo tanto:

$\sqrt{x+2} = t \Rightarrow x = t^2 - 2, dx = 2tdt$ , luego:

$$\int \frac{dx}{3+\sqrt{x+2}} = \int \frac{2tdt}{3+t} = 2 \int \left(1 - \frac{3}{t+3}\right) dt = 2 \int dt - 6 \int \frac{dt}{t+3} = 2t - 6\ell\eta|t+3| + c$$

Dado que:  $t = \sqrt{x+2}$ , se tiene:  $= 2\sqrt{x+2} - 6\ell\eta|\sqrt{x+2}+3| + c$

**Respuesta:**  $\int \frac{dx}{3+\sqrt{x+2}} = 2\sqrt{x+2} - 6\ell\eta|\sqrt{x+2}+3| + c$

**9.4.-Encontrar:**  $\int \frac{1-\sqrt{3x+2}}{1+\sqrt{3x+2}} dx$

Solución.- La expresión "irracional" es ahora  $\sqrt{3x+2}$ , por lo tanto:

$\sqrt{3x+2} = t \Rightarrow 3x = t^2 - 2, dx = \frac{2}{3} t dt$ , luego:

$$\begin{aligned} \int \frac{1-\sqrt{3x+2}}{1+\sqrt{3x+2}} dx &= \int \frac{1-t}{1+t} \frac{2}{3} t dt = \frac{2}{3} \int \frac{t-t^2}{1+t} dt = \frac{2}{3} \int \left( -t + 2 - \frac{2}{t+1} \right) dt \\ &= -\frac{2}{3} \int t dt + \frac{4}{3} \int dt - \frac{4}{3} \int \frac{dt}{t+1} = -\frac{1}{3} t^2 + \frac{4}{3} t - \frac{4}{3} \ell \eta |t+1| + c \end{aligned}$$

Dado que:  $t = \sqrt{3x+2}$ , se tiene:

$$\begin{aligned} &= -\frac{1}{3} (3x+2) + \frac{4}{3} \sqrt{3x+2} - \frac{4}{3} \ell \eta |\sqrt{3x+2} + 1| + c \\ &= -x - \frac{2}{3} + \frac{4}{3} \sqrt{3x+2} - \frac{4}{3} \ell \eta |\sqrt{3x+2} + 1| + c = -x - \frac{2}{3} + \frac{4}{3} \left( \sqrt{3x+2} - \ell \eta |\sqrt{3x+2} + 1| \right) + c \end{aligned}$$

**Respuesta:**  $\int \frac{1-\sqrt{3x+2}}{1+\sqrt{3x+2}} dx = -x - \frac{2}{3} + \frac{4}{3} \left( \sqrt{3x+2} - \ell \eta |\sqrt{3x+2} + 1| \right) + c$

**9.5.- Encontrar:**  $\int \sqrt{1+\sqrt{x}} dx$

Solución.- La expresión "irracional" es ahora  $\sqrt{x}$ , por lo tanto:

$\sqrt{x} = t \Rightarrow x = t^2, dx = 2t dt$ , luego:  $\int (\sqrt{1+\sqrt{x}}) dx = \int \sqrt{1+t} 2t dt$ , como apareció la expresión:  $\sqrt{1+t}$ ; se procede análogamente:  $w = \sqrt{1+t} \Rightarrow t = w^2 - 1, dt = 2w dw$ , esto es:  $\sqrt{1+t} 2t dt = \int w 2(w^2 - 1) 2w dw = 4 \int (w^4 - w^2) dw = \frac{4w^5}{5} - \frac{4w^3}{3} + c$

Dado que:  $w = \sqrt{1+t}$ , se tiene:  $= \frac{4(1+t)^{5/2}}{5} - \frac{4(1+t)^{3/2}}{3} + c$

**Respuesta:**  $\int \sqrt{1+\sqrt{x}} dx = \frac{4(1+\sqrt{x})^{5/2}}{5} - \frac{4(1+\sqrt{x})^{3/2}}{3} + c$

**9.6.-Encontrar:**  $\int \frac{dx}{\sqrt{x+1} + \sqrt[4]{x+1}}$

Solución.- Previamente se tiene que el m.c.m. de los índices de Las raíces es: 4, por lo cual:  $x+1 = t^4, dx = 4t^3 dt$ , de donde:

$$\begin{aligned} \int \frac{dx}{\sqrt{x+1} + \sqrt[4]{x+1}} &= \int \frac{4t^3 dt}{t^2 + t} = 4 \int \left( t - 1 + \frac{t}{t^2 + t} \right) dt = 4 \int t dt - 4 \int dt + 4 \int \frac{dt}{t+1} \\ &= 2t^2 - 4t + 4 \ell \eta |t+1| + c, \text{ dado que: } t = \sqrt[4]{x+1} \end{aligned}$$

Se tiene:  $= 2(x+1)^{1/2} - 4(x+1)^{1/4} + 4 \ell \eta |(x+1)^{1/4} + 1| + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{x+1} + \sqrt[4]{x+1}} = 2(x+1)^{1/2} - 4(x+1)^{1/4} + 4 \ell \eta |(x+1)^{1/4} + 1| + c$

**9.7.-Encontrar:**  $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$

Solución.- Previamente se tiene que el m.c.m. de los índices de Las raíces es: 6 , por lo cual:  $x = t^6 \Rightarrow t = \sqrt[6]{x}, dx = 6t^5 dt$ , de donde:

$$\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = \int \frac{6t^5 dt}{t^3 + t^2} = 6 \int \frac{t^3 dt}{t+1} = 6 \int \left( t^2 - t + 1 - \frac{1}{t+1} \right) dt = 6 \int t^2 dt - 6 \int t dt + 6 \int dt - 6 \int \frac{dt}{t+1}$$

$$= 2t^3 - 3t^2 + 6t - 6\ell\eta|t+1| + c$$

Dado que:  $t = \sqrt[6]{x}$

Se tiene:  $= 2(\sqrt[6]{x})^3 - 3(\sqrt[6]{x})^2 + 6\sqrt[6]{x} - 6\ell\eta|\sqrt[6]{x} + 1| + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}} = 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ell\eta|\sqrt[6]{x} + 1| + c$

**9.8.-Encontrar:**  $\int \frac{dx}{\sqrt{x+1} + \sqrt{(x+1)^3}}$

Solución.- Previamente se tiene igual índice por lo cual:  $\sqrt{x+1} = t \Rightarrow x = t^2 - 1, dx = 2t dt$ , de donde:

$$\int \frac{dx}{\sqrt{x+1} + \sqrt{(x+1)^3}} = \int \frac{2t dt}{t + t^3} = 2 \int \frac{dt}{1+t^2} = 2 \arctan t + c$$

Dado que:  $t = \sqrt{x+1}$ , Se tiene:  $= 2 \arctan \sqrt{x+1} + c$

**Respuesta:**  $\int \frac{dx}{\sqrt{x+1} + \sqrt{(x+1)^3}} = 2 \arctan \sqrt{x+1} + c$

**9.9.-Encontrar:**  $\int \frac{\sqrt{x}-1}{\sqrt[3]{x+1}} dx$

Solución.- Previamente se tiene que el m.c.m. de los índices de Las raíces es: 6 , por lo cual:  $x = t^6 \Rightarrow t = \sqrt[6]{x}, dx = 6t^5 dt$ , de donde:

$$\int \frac{\sqrt{x}-1}{\sqrt[3]{x+1}} dx = \int \frac{t^3-1}{t^2+1} 6t^5 dt = 6 \int \frac{t^8-t^5}{t^2+1} dt = 6 \int \left( t^6 - t^4 - t^3 + t^2 + t - 1 - \frac{t-1}{t^2+1} \right) dt$$

$$= \frac{6}{7}t^7 - \frac{6}{5}t^5 - \frac{3}{2}t^4 + 2t^3 + 3t^2 - 6t + c_1 - 3 \int \frac{2t-2}{t^2+1} dt$$

$$= \frac{6}{7}t^7 - \frac{6}{5}t^5 - \frac{3}{2}t^4 + 2t^3 + 3t^2 - 6t + c_1 - 3 \int \frac{2t-2}{t^2+1} dt + 6 \int \frac{dt}{t^2+1}$$

$$= \frac{6}{7}t^7 - \frac{6}{5}t^5 - \frac{3}{2}t^4 + 2t^3 + 3t^2 - 6t - 3\ell\eta|t^2+1| + 6 \arctan t + c$$

Dado que:  $t = \sqrt[6]{x}$ , se tiene:

$$= \frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} - \frac{3}{2}\sqrt[3]{x^2} + 2\sqrt{x} + 3\sqrt[3]{x} - 6\sqrt[6]{x} - 3\ell\eta|1 + \sqrt[3]{x}| + 6 \arctan \sqrt[6]{x} + c$$

**Respuesta:**

$$\int \frac{\sqrt{x}-1}{\sqrt[3]{x}+1} dx = \frac{6}{7} x \sqrt[6]{x} - \frac{6}{5} \sqrt[6]{x^5} - \frac{3}{2} \sqrt[3]{x^2} + 2\sqrt{x} + 3\sqrt[3]{x} - 6\sqrt[6]{x} - 3\ell\eta \left| 1 + \sqrt[3]{x} \right| + 6 \operatorname{arc} \tau g \sqrt[6]{x} + c$$

**9.10.-Encontrar:**  $\int \frac{\sqrt{x} dx}{x+2}$

Solución.- La expresión “irracional” es  $\sqrt{x}$ , por lo tanto:

$$\sqrt{x} = t \Rightarrow x = t^2, dx = 2t dt,$$

$$\begin{aligned} \text{luego: } \int \frac{\sqrt{x} dx}{x+2} &= \int \frac{t(2t dt)}{t^2+2} = 2 \int \frac{t^2 dt}{t^2+2} = 2 \int \left( 1 - \frac{2}{t^2+2} \right) dt = 2 \int dt - 4 \int \frac{dt}{t^2+2} \\ &= 2t - \frac{4}{\sqrt{2}} \operatorname{arc} \tau g \frac{t}{\sqrt{2}} + c, \text{ dado que: } t = \sqrt{x}, \text{ se tiene: } = 2\sqrt{x} - 2\sqrt{2} \operatorname{arc} \tau g \sqrt{\frac{x}{2}} + c \end{aligned}$$

**Respuesta:**  $\int \frac{\sqrt{x} dx}{x+2} = 2\sqrt{x} - 2\sqrt{2} \operatorname{arc} \tau g \sqrt{\frac{x}{2}} + c$

**9.11.-Encontrar:**  $\int \frac{(\sqrt{x+1}+2)dx}{(x+1)^2 - \sqrt{x+1}}$

Solución.- Previamente se tiene igual índice por lo cual:  $\sqrt{x+1} = t \Rightarrow x = t^2 - 1, dx = 2t dt$ , de donde:

$$\begin{aligned} \int \frac{(\sqrt{x+1}+2)dx}{(x+1)^2 - \sqrt{x+1}} &= \int \frac{[(x+1)^{1/2} + 2]dx}{(x+1)^2 - (x+1)^{1/2}} = \int \frac{t+2}{t^4 - t} 2t dt = 2 \int \frac{(t+2) \cancel{t} dt}{\cancel{t}(t^3 - 1)} \\ &= 2 \int \frac{(t+2)dt}{(t-1)(t^2+t+1)} (*), \text{ considerando que:} \end{aligned}$$

$$\frac{t+2}{(t-1)(t^2+t+1)} = \frac{A}{(t-1)} + \frac{Bt+C}{(t^2+t+1)} \Rightarrow A=1, B=-1, C=-1$$

Dado que:  $t = \sqrt{x+1}$ , Se tiene:  $= 2 \operatorname{arc} \tau g \sqrt{x+1} + c$

$$\begin{aligned} (*) \quad 2 \int \frac{(t+2)dt}{(t-1)(t^2+t+1)} &= 2 \int \frac{dt}{(t-1)} + 2 \int \frac{-t-1}{(t^2+t+1)} dt = 2 \int \frac{dt}{(t-1)} - 2 \int \frac{t+1}{(t^2+t+1)} dt \\ &= 2 \int \frac{dt}{(t-1)} - 2 \int \frac{\frac{1}{2}(2t+1) + \frac{1}{2}}{(t^2+t+1)} dt = 2 \int \frac{dt}{(t-1)} - \int \frac{(2t+1)dt}{(t^2+t+1)} - \int \frac{dt}{(t^2+t+1)} \\ &= 2 \int \frac{dt}{(t-1)} - \int \frac{(2t+1)dt}{(t^2+t+1)} - \int \frac{dt}{(t^2+t+\frac{1}{4}) + \frac{3}{4}} \\ &= 2\ell\eta |t-1| - \ell\eta |t^2+t+1| - \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2t+1}{\sqrt{3}} + c \\ &= \ell\eta \left| \frac{(t-1)^2}{(t^2+t+1)} \right| - \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2t+1}{\sqrt{3}} + c \end{aligned}$$

Dado que:  $t = \sqrt{x+1}$ , se tiene

**Respuesta:**  $\int \frac{(\sqrt{x+1}+2)dx}{(x+1)^2 - \sqrt{x+1}} = \ell \eta \left| \frac{(\sqrt{x+1}-1)^2}{(\sqrt{x+1}+x+2)} \right| - \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \frac{2\sqrt{x+1}+1}{\sqrt{3}} + c$

## EJERCICIOS PROPUESTOS

**9.12.-**  $\int \frac{1+x}{1+\sqrt{x}} dx$

**9.13.-**  $\int \frac{1-x}{1+\sqrt{x}} dx$

**9.14.-**  $\int \frac{dx}{a+b\sqrt{x}}$

**9.15.-**  $\int \frac{\sqrt{x+a}}{x+a} dx$

**9.16.-**  $\int \frac{\sqrt{x} dx}{1+\sqrt[4]{x}}$

**9.17.-**  $\int \frac{\sqrt{x}-\sqrt[6]{x}}{\sqrt[3]{x}+1} dx$

**9.18.-**  $\int \frac{dx}{x-2-\sqrt{x}} dx$

**9.19.-**  $\int \sqrt{\frac{1+x}{1-x}} dx$

**9.20.-**  $\int \frac{\sqrt{x+a}}{x+b} dx$

**9.21.-**  $\int \frac{\sqrt[3]{x+1}}{x} dx$

**9.22.-**  $\int \frac{\sqrt{a^2-x^2}}{x^3} dx$

**9.23.-**  $\int x^2 \sqrt{x+ad} dx$

**9.24.-**  $\int \frac{dx}{\sqrt{x} + \sqrt[4]{x} + 2\sqrt[8]{x}}$

**9.25.-**  $\int x^3 \sqrt{x^2+a^2} dx$

## RESPUESTAS

**9.12.-**  $\int \frac{1+x}{1+\sqrt{x}} dx$

Solución.- Sea:  $\sqrt{x} = t \Rightarrow x = t^2, dx = 2t dt$

$$\begin{aligned} \int \frac{1+x}{1+\sqrt{x}} dx &= \int \frac{1+t^2}{1+t} 2t dt = 2 \int \frac{t+t^3}{1+t} dt = 2 \int \left( t^2 - t + 2 - \frac{2}{t+1} \right) dt \\ &= 2 \int t^2 dt - 2 \int t dt + 4 \int dt - 4 \int \frac{dt}{t+1} = \frac{2t^3}{3} - \frac{2t^2}{2} + 4t - 4\ell \eta |t+1| + c \\ &= \frac{2\sqrt{x^3}}{3} - x + 4\sqrt{x} - 4\ell \eta |\sqrt{x}+1| + c \end{aligned}$$

**9.13.-**  $\int \frac{1-x}{1+\sqrt{x}} dx$

Solución.- Sea:  $\sqrt{x} = t \Rightarrow x = t^2, dx = 2t dt$

$$\begin{aligned} \int \frac{1-x}{1+\sqrt{x}} dx &= \int \frac{1-t^2}{1+t} 2t dt = 2 \int \frac{t-t^3}{1+t} dt = -2 \int t dt + 4 \int dt - 4 \int \frac{dt}{t+1} = -t^2 + 4t - 4\ell \eta |t+1| + c \\ &= -x + 4\sqrt{x} - 4\ell \eta |\sqrt{x}+1| + c \end{aligned}$$

**9.14.-**  $\int \frac{dx}{a+b\sqrt{x}}$

Solución.- Sea:  $\sqrt{x} = t \Rightarrow x = t^2, dx = 2t dt$



$$\int \frac{dx}{a+b\sqrt{x}} = \int \frac{2tdt}{a+bt} = 2 \int \frac{tdt}{a+bt} = 2 \int \left( \frac{1}{b} - \frac{a}{b} \frac{1}{a+bt} \right) dt = \frac{2}{b} \int dt - \frac{2a}{b^2} \int \frac{bdt}{a+bt}$$

$$= \frac{2}{b} t - \frac{2a}{b^2} \ell \eta |a+bt| + c = \frac{2}{b} \sqrt{x} - \frac{2a}{b^2} \ell \eta |a+b\sqrt{x}| + c$$

**9.15.-**  $\int \frac{\sqrt{x+a}}{x+a} dx$

Solución.- Sea:  $\sqrt{x+a} = t \Rightarrow x = t^2 - a, dx = 2tdt$

$$\int \frac{\sqrt{x+a}}{x+a} dx = \int \frac{t \cancel{2} dt}{\cancel{t}} = 2 \int dt = 2t + c = 2\sqrt{x+a} + c$$

**9.16.-**  $\int \frac{\sqrt{x} dx}{1+\sqrt[4]{x}}$

Solución.- m.c.m: 4 ; Sea:  $\sqrt[4]{x} = t \Rightarrow x = t^4, dx = 4t^3 dt$

$$\int \frac{\sqrt{x} dx}{1+\sqrt[4]{x}} = \int \frac{t^2 4t^3 dt}{1+t} = 4 \int \frac{t^5 dt}{1+t} = 4 \int \left( t^4 - t^3 + t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$= 4 \left( \frac{t^5}{5} - \frac{t^4}{4} + \frac{t^3}{3} - \frac{t^2}{2} + t - \ell \eta |t+1| \right) + c = \frac{4t^5}{5} - t^4 + \frac{4t^3}{3} - 2t^2 + 4t - 4\ell \eta |t+1|$$

$$= \frac{4x^{5/4}}{5} - x + \frac{4x^{3/4}}{3} - 2x^{1/2} + 4x^{1/4} - 4\ell \eta |x^{1/4} + 1|$$

**9.17.-**  $\int \frac{\sqrt{x} - \sqrt[6]{x}}{\sqrt[3]{x} + 1} dx$

Solución.- m.c.m: 6 ; Sea:  $\sqrt[6]{x} = t \Rightarrow x = t^6, dx = 6t^5 dt$

$$\int \frac{\sqrt{x} - \sqrt[6]{x}}{\sqrt[3]{x} + 1} dx = \int \frac{t^3 - t}{t^2 + 1} 6t^5 dt = 6 \int \frac{(t^8 - t^6) dt}{t^2 + 1} = 6 \int t^6 dt - 2 \int t^4 dt + 2 \int t^2 dt - 2 \int dt + 2 \int \frac{dt}{1+t^2}$$

$$= 6 \left( \frac{t^7}{7} - \frac{2t^5}{5} + \frac{2t^3}{3} - 2t + 2 \operatorname{arc} \tau gt \right) + c = \frac{6t^7}{7} - \frac{12t^5}{5} + 4t^3 - 12t + 12 \operatorname{arc} \tau gt + c$$

$$= \frac{6x^{7/6}}{7} - \frac{12x^{5/6}}{5} + 4x^{1/2} - 12x^{1/6} + 12 \operatorname{arc} \tau gx^{1/6} + c$$

**9.18.-**  $\int \frac{dx}{x-2-\sqrt{x}}$

Solución.- Sea:  $\sqrt{x} = t \Rightarrow x = t^2, dx = 2tdt$

$$\int \frac{dx}{x-2-\sqrt{x}} = \int \frac{2tdt}{t^2-2-t} = \int \frac{(2t-1)+1}{t^2-t-2} dt = \int \frac{2t-1}{t^2-t-2} dt + \int \frac{dt}{t^2-t-2}$$

$$= \int \frac{2t-1}{t^2-t-2} dt + \int \frac{dt}{(t-1/2)^2 - 9/4} = \ell \eta |t^2-t-2| + \frac{1}{\cancel{2} \cancel{3} \cancel{2}} \ell \eta \left| \frac{t-\cancel{3}/\cancel{2}}{t+\cancel{3}/\cancel{2}} \right| + c$$

$$= \ell \eta |t^2 - t - 2| + \frac{1}{3} \ell \eta \left| \frac{2t-3}{2t+3} \right| + c = \ell \eta |x - \sqrt{x} - 2| + \frac{1}{3} \ell \eta \left| \frac{2\sqrt{x}-3}{2\sqrt{x}+3} \right| + c$$

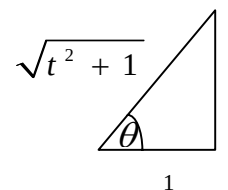
**9.19.-**  $\int \sqrt{\frac{1+x}{1-x}} dx$

Solución.- Notará el lector, que este caso se diferencia de los anteriores, sin embargo la técnica que se seguirá, tiene la misma fundamentación y la información que se consiga es valiosa. (\*)

Sea:  $\sqrt{\frac{1+x}{1-x}} = t \Rightarrow \frac{1+x}{1-x} = t^2 \Rightarrow 1+x = t^2 - t^2 x \Rightarrow x(1+t^2) = t^2 - 1$

$x = \frac{t^2-1}{t^2+1} \Rightarrow dx = \frac{4t dt}{(t^2+1)^2}$ , luego:

(\*)  $\int \sqrt{\frac{1+x}{1-x}} dx = \int \frac{t 4t dt}{(t^2+1)^2} = \int \frac{4t^2 dt}{(t^2+1)^2} = 4 \int \frac{t^2 dt}{(\sqrt{t^2+1})^4}$  (\*\*), haciendo uso de sustituciones trigonométricas convenientes en (\*\*), y de la figura se tiene:



Se tiene:  $t = \tau g \theta, dt = \sec^2 \theta d\theta; \sqrt{t^2+1} = \sec \theta$

(\*\*)  $4 \int \frac{t^2 dt}{(\sqrt{t^2+1})^4} = \int \frac{4 \tau g^2 \theta \sec^2 \theta d\theta}{\sec^4 \theta} = 4 \int \frac{\tau g^2 \theta}{\sec^2 \theta} d\theta$

$= 4 \int \sin^2 \theta d\theta = 2 \int d\theta - 2 \int \cos 2\theta d\theta = 2\theta - \sin 2\theta + c = 2\theta - 2 \sin \theta \cos \theta + c$

$= 2 \arctan t - 2 \frac{t}{\sqrt{t^2+1}} \frac{1}{\sqrt{t^2+1}} + c = 2 \arctan t - \frac{2t}{t^2+1} + c = 2 \arctan t g \sqrt{\frac{1+x}{1-x}} - \frac{2\sqrt{\frac{1+x}{1-x}}}{\frac{1+x}{1-x} + 1} + c$

$= 2 \arctan t g \sqrt{\frac{1+x}{1-x}} - (1-x) \sqrt{\frac{1+x}{1-x}} + c$

**9.20.-**  $\int \frac{\sqrt{x+a}}{x+b} dx$

Solución.- Sea:  $\sqrt{x+a} = t \Rightarrow x = t^2 - a, dx = 2t dt$

$\int \frac{\sqrt{x+a}}{x+b} dx = \int \frac{t 2t dt}{t^2 - a + b} = 2 \int \frac{t^2 dt}{t^2 + (b-a)} = 2 \int \left( 1 - \frac{b-a}{t^2 + (b-a)} \right) dt$

$= 2 \int dt - 2(b-a) \int \frac{dt}{t^2 + (b-a)} = 2t - 2(b-a) \frac{1}{\sqrt{b-a}} \arctan t g \frac{t}{\sqrt{b-a}} + c$

$$= 2\sqrt{x+a} - 2\sqrt{b-a} \operatorname{arc} \tau g \sqrt{\frac{x+a}{b-a}} + c$$

**9.21.-**  $\int \frac{\sqrt[3]{x+1}}{x} dx$

Solución.- Sea:  $\sqrt[3]{x+1} = t \Rightarrow x = t^3 - 1, dx = 3t^2 dt$

$$\int \frac{\sqrt[3]{x+1}}{x} dx = \int \frac{t 3t^2 dt}{t^3 - 1} = 3 \int \frac{t^3 dt}{t^3 - 1} = 3 \int \left( 1 + \frac{1}{t^3 - 1} \right) dt = 3 \int dt + 3 \int \frac{dt}{t^3 - 1}$$

$$= 3 \int dt + 3 \int \frac{dt}{(t-1)(t^2+t+1)} (*), \text{ por fracciones parciales:}$$

$$\frac{3}{(t-1)(t^2+t+1)} = \frac{A}{(t-1)} + \frac{Bt+C}{(t^2+t+1)} \Rightarrow 3 = A(t^2+t+1) + (Bt+C)(t-1), \text{ de donde:}$$

$A=1, B=-1, C=-2$ , luego:

$$(*) = 3 \int dt + \int \frac{dt}{t-1} - \int \frac{t+2}{t^2+t+1} dt = 3t + \ell \eta |t-1| - \frac{1}{2} \ell \eta |t^2+t+1| - \sqrt{3} \operatorname{arc} \tau g \left( \frac{2t+1}{\sqrt{3}} \right) + c$$

**9.22.-**  $\int \frac{\sqrt{a^2-x^2}}{x^3} dx$

Solución.- Sea:  $\sqrt{a^2-x^2} = t \Rightarrow x^2 = a^2 - t^2, x dx = -t dt$

$$\int \frac{\sqrt{a^2-x^2}}{x^3} dx = \int \frac{\sqrt{a^2-x^2} x dx}{x^4} = - \int \frac{t dt}{(a^2-t^2)^2} = \int \frac{-t^2 dt}{(a^2-t^2)^2} = \int \frac{-t^2 dt}{(a+t)^2 (a-t)^2} (*)$$

Por fracciones parciales:

$$\frac{-t^2}{(t+a)^2 (t-a)^2} = \frac{A}{(t+a)} + \frac{B}{(t+a)^2} + \frac{C}{(t-a)} + \frac{D}{(t-a)^2}, \text{ de donde:}$$

$A = \frac{1}{4}a, B = -\frac{1}{4}, C = -\frac{1}{4}a, D = -\frac{1}{4}$ , luego:

$$(*) \int \frac{-t^2 dt}{(a+t)^2 (a-t)^2} = \frac{1}{4a} \int \frac{dt}{(t+a)} - \frac{1}{4a} \int \frac{dt}{(t+a)^2} - \frac{1}{4a} \int \frac{dt}{(t-a)} - \frac{1}{4a} \int \frac{dt}{(t-a)^2}$$

$$= \frac{1}{4a} \ell \eta |(t+a)| + \frac{1}{4(t+a)} - \frac{1}{4a} \ell \eta |(t-a)| + \frac{1}{4(t-a)} + c$$

$$= \frac{1}{4a} \ell \eta \left| \frac{(t+a)}{(t-a)} \right| + \frac{1}{4(t+a)} + \frac{1}{4(t-a)} + c$$

$$= \frac{1}{4a} \ell \eta \left| \frac{\sqrt{a^2-x^2}+a}{\sqrt{a^2-x^2}-a} \right| + \frac{\sqrt{a^2-x^2}}{2(a^2-x^2-a)} + c = \frac{1}{4a} \ell \eta \left| \frac{\sqrt{a^2-x^2}+a}{\sqrt{a^2-x^2}-a} \right| - \frac{\sqrt{a^2-x^2}}{2x^2} + c$$

$$= \frac{1}{4a} \ell \eta \left| \frac{(\sqrt{a^2-x^2}+a)^2}{a^2-x^2-a^2} \right| - \frac{\sqrt{a^2-x^2}}{2x^2} + c = \frac{1}{2a} \ell \eta |\sqrt{a^2-x^2}+a| - \frac{1}{2a} \ell \eta |x| - \frac{\sqrt{a^2-x^2}}{2x^2} + c$$

**9.23.-**  $\int x^2 \sqrt{x+a} dx$

Solución.- Sea:  $\sqrt{x+a} = t \Rightarrow x = t^2 - a, dx = 2t dt$

$$\begin{aligned}
\int x^2 \sqrt{x+a} dx &= \int (t^2 - a)^2 t 2t dt = 2 \int t^2 (t^2 - a)^2 dt = 2 \int (t^6 - 2at^4 + a^2 t^2) dt \\
&= 2 \int t^6 dt - 4a \int t^4 dt + 2a^2 \int t^2 dt = \frac{2t^7}{7} - \frac{4at^5}{5} + \frac{2a^2 t^3}{3} + c \\
&= \frac{2(x+a)^{7/2}}{7} - \frac{4a(x+a)^{5/2}}{5} + \frac{2a^2(x+a)^{3/2}}{3} + c
\end{aligned}$$

**9.24.-**  $\int \frac{dx}{\sqrt{x} + \sqrt[4]{x} + 2\sqrt[8]{x}}$

Solución.- Sea:  $\sqrt[8]{x} = t \Rightarrow x = t^8, dx = 8t^7 dt$

$$\int \frac{dx}{\sqrt{x} + \sqrt[4]{x} + 2\sqrt[8]{x}} = \int \frac{8t^7 dt}{t^4 + t^2 + 2t} = 8 \int \frac{t^6 dt}{t^3 + t + 2} = 8 \int \left( t^3 - t - 2 + \frac{t^2 + 4t + 4}{t^3 + t + 2} \right) dt$$

$$= 8 \int t^3 dt - 8 \int t dt - 16 \int dt + 8 \int \frac{t^2 + 4t + 4}{t^3 + t + 2} dt = 8 \frac{t^4}{4} - \frac{8t^2}{2} - 16t + 8 \int \frac{t^2 + 4t + 4}{t^3 + t + 2} dt$$

$$= 2t^4 - 4t^2 - 16t + 8 \int \frac{t^2 + 4t + 4}{t^3 + t + 2} dt (*) , \text{ por fracciones parciales:}$$

$$\frac{t^2 + 4t + 4}{(t^3 + t + 2)} = \frac{t^2 + 4t + 4}{(t+1)(t^2 - t + 2)} = \frac{A}{(t+1)} + \frac{Bt + C}{(t^2 - t + 2)} \Rightarrow A = 1/4, B = 3/4, C = 14/4, \text{ luego:}$$

$$(*) = 2t^4 - 4t^2 - 16t + 8 \left( \int \frac{1/4 dt}{t+1} + \int \frac{3/4 t + 14/4}{t^2 - t + 2} dt \right)$$

$$= 2t^4 - 4t^2 - 16t + 8 \left( \frac{1}{4} \int \frac{dt}{t+1} + \frac{1}{4} \int \frac{3t+14}{t^2 - t + 2} dt \right) = 2t^4 - 4t^2 - 16t + 2 \int \frac{dt}{t+1} + 2 \int \frac{3t+14}{t^2 - t + 2} dt$$

$$= 2t^4 - 4t^2 - 16t + 2 \ell \eta |t+1| + 2 \int \frac{3t+14}{t^2 - t + 2} dt$$

$$= 2t^4 - 4t^2 - 16t + 2 \ell \eta |t+1| + 3 \int \frac{(2t-1)}{t^2 - t + 2} dt + 31 \int \frac{dt}{t^2 - t + 2}$$

$$= 2t^4 - 4t^2 - 16t + 2 \ell \eta |t+1| + 3 \ell \eta |t^2 - t + 2| + 31 \int \frac{dt}{(t - 1/2)^2 + 7/4}$$

$$= 2t^4 - 4t^2 - 16t + 2 \ell \eta |t+1| + 3 \ell \eta |t^2 - t + 2| + 31 \frac{2}{\sqrt{7}} \operatorname{arc} \tau g \frac{t - 1/2}{\sqrt{7}/2} + c$$

$$= 2t^4 - 4t^2 - 16t + 2 \ell \eta |t+1| + 3 \ell \eta |t^2 - t + 2| + \frac{62}{\sqrt{7}} \operatorname{arc} \tau g \frac{2t-1}{\sqrt{7}} + c$$

$$= 2x^{1/2} - 4x^{1/4} - 16x^{1/8} + 2 \ell \eta |x^{1/8} + 1| + 3 \ell \eta |x^{1/4} - x^{1/8} + 2| + \frac{62}{\sqrt{7}} \operatorname{arc} \tau g \frac{2x^{1/8} - 1}{\sqrt{7}} + c$$

**9.25.-**  $\int x^3 \sqrt{x^2 + a^2} dx$

Solución.- Sea:  $\sqrt{x^2 + a^2} = t \Rightarrow x^2 = t^2 - a^2, x dx = t dt$

$$\begin{aligned}
\int x^3 \sqrt{x^2 + a^2} dx &= \int x^2 \sqrt{x^2 + a^2} x dx = \int (t^2 - a^2) t dt = \int (t^2 - a^2) t^2 dt = \int (t^4 - a^2 t^2) dt \\
&= \frac{t^5}{5} - \frac{a^2 t^3}{3} + c = \frac{(x^2 + a^2)^{5/2}}{5} - \frac{a^2 (x^2 + a^2)^{3/2}}{3} + c = (x^2 + a^2)^{3/2} \left( \frac{x^2 + a^2}{5} - \frac{a^2}{3} \right) + c \\
&= (x^2 + a^2)^{3/2} \left( \frac{3x^2 - 2a^2}{15} \right) + c
\end{aligned}$$

## EJERCICIOS COMPLEMENTARIOS

A continuación, se adjunta un listado de ejercicios que se proponen al lector. Observará que no se indica técnica alguna solicitada para el desarrollo de los mismos, y que además no se han respetado normas relativas a niveles de dificultad, ni a las técnicas mismas. Como siempre, se adjuntarán las soluciones cuyos desarrollos pueden diferir de los aquí presentados. No importa, eso es posible; además una consulta con su profesor aclarará cualquier discrepancia.

**Encontrar:**

1.-  $\int t^3 e^{\sin t^4} \cos t^4 dt$

4.-  $\int e^{\tau g 3\theta} \sec^2 3\theta d\theta$

7.-  $\int \frac{dx}{(2-x)\sqrt{1-x}}$

10.-  $\int \frac{(t+1)dt}{t^2 + 2t - 5}$

13.-  $\int \frac{\eta^2}{a} \operatorname{sen} \frac{\eta}{b} d\eta$

16.-  $\int \sec^2(1-x) dx$

19.-  $\int \frac{dx}{\sqrt{x+4} - \sqrt{x+3}}$

22.-  $\int t(1-t^2)^{1/2} \operatorname{arcsen} t dt$

25.-  $\int \frac{e^x dx}{\sqrt{9-e^{2x}}}$

28.-  $\int \frac{ds}{\sqrt{4-s^2}}$

2.-  $\int \frac{\theta d\theta}{(1+\theta)^2}$

5.-  $\int \frac{x dx}{\sqrt[3]{ax+b}}$

8.-  $\int e^{2-x} dx$

11.-  $\int \sec \frac{\varphi}{2} d\varphi$

14.-  $\int \varphi \sec^2 \varphi d\varphi$

17.-  $\int \frac{x dx}{\sqrt{16-x^4}}$

20.-  $\int \operatorname{cosec} \theta d\theta$

23.-  $\int \frac{1+\cos 2x}{\operatorname{sen}^2 2x} dx$

26.-  $\int \frac{dx}{(x-1)^3}$

29.-  $\int \frac{dx}{x^2 \sqrt{x^2 + e}}$

3.-  $\int \frac{\theta e^\theta d\theta}{(1+\theta)^2}$

6.-  $\int \sqrt{\frac{x^2-1}{x+1}}$

9.-  $\int \frac{e^x dx}{ae^x - b}$

12.-  $\int \tau g \theta d\theta$

15.-  $\int \frac{dx}{5^x}$

18.-  $\int \frac{dy}{\sqrt{1+\sqrt{1+y}}}$

21.-  $\int t(1-t^2)^{1/2} dt$

24.-  $\int \frac{x^2+1}{x^3-x} dx$

27.-  $\int \frac{(3x+4)dx}{\sqrt{2x+x^2}}$

30.-  $\int \frac{x dx}{\sqrt{1+x}}$

$$\begin{array}{lll}
31.- \int \frac{y^2 dy}{\sqrt{y+1}} & 32.- \int \frac{y^3 dy}{\sqrt{y^2-1}} & 33.- \int \frac{d\theta}{1+2\cos\theta} \\
34.- \int \frac{t^4-t^3+4t^2-2t+1}{t^3+1} dt & 35.- \int \frac{d\phi}{\ell \eta e} & 36.- \int x(10+8x^2)^9 dx \\
37.- \int \frac{dx}{\sqrt{(16+x^2)^3}} & 38.- \int \frac{x^3 dx}{\sqrt{x^2+4}} & 39.- \int \frac{x^3 dx}{\sqrt{16-x^2}} \\
40.- \int a(x^2+1)^{1/2} dy & 41.- \int \frac{dx}{(\sqrt{6-x^2})^3} & 42.- \int \frac{dx}{x(3+\ell \eta x)} \\
43.- \int \frac{e^x}{16+e^{2x}} dx & 44.- \int \cos \sqrt{1-x} dx & 45.- \int \frac{x^3 dx}{\sqrt{x-1}} \\
46.- \int \frac{2y^5-7y^4+7y^3-19y^2+7y-6}{(y-1)^2(y^2+1)^2} dy & 47.- \int \sec \sqrt{x+1} dx & 48.- \int \frac{9x^2+7x-6}{x^3-x} dx \\
49.- \int \frac{5w^3-5w^2+2w-1}{w^4+w^2} dw & 50.- \int \frac{3dx}{1+2x} & 51.- \int \frac{(1-x)^2 dx}{x} \\
52.- \int \frac{x e^{-2x^2}}{2} dx & 53.- \int e^{2t} \cos(e^t) dt & 54.- \int \sqrt{x}(x^{3/2}-4)^3 dx \\
55.- \int \frac{\sec x e^{\sec x}}{\cos^2 x} dx & 56.- \int \frac{ds}{s^{1/3}(1+s^{1/3})} & 57.- \int \frac{1}{z^3} \left( \frac{1-z^2}{z^2} \right)^{10} dz \\
58.- \int \frac{x \ell \eta (1+x^2)}{1+x^2} dx & 59.- \int \frac{\cot g x dx}{\ell \eta |\sec x|} & 60.- \int \frac{ax^2-bx+c}{ax^2+bx-c} dx \\
61.- \int \frac{dx}{\cos^2 5x} & 62.- \int \frac{dx}{12-7x} & 63.- \int \tau g 16x dx \\
64.- \int \tau g 4\theta \sec^2 4\theta d\theta & 65.- \int \frac{x dx}{\sqrt{x-5}} & 66.- \int \frac{7t-2}{\sqrt{7-2t^2}} dt \\
67.- \int (1+x) \cos \sqrt{x} dx & 68.- \int \frac{dx}{x(\sqrt{1+x}-1)} & 69.- \int \frac{dx}{\cot g 6x} \\
70.- \int \cot g (2x-4) dx & 71.- \int (e^t - e^{-2t})^2 dt & 72.- \int \frac{(x+1)dx}{(x+2)^2(x+3)} \\
73.- \int (\cot g e^x) e^x dx & 74.- \int \frac{\sec \theta + \theta}{\cos \theta + 1} d\theta & 75.- \int \frac{\arctan x dx}{(1+x^2)^{3/2}} \\
76.- \int x \cot g (x^2/5) dx & 77.- \int x \sqrt{4x^2-2} dx & 78.- \int \frac{(x^2+9)^{1/2} dx}{x^4} \\
79.- \int x^2 \sec^5 x^3 \cos x^3 dx & 80.- \int \frac{x dx}{\sqrt{5x^2+7}} & 81.- \int \frac{x^3 dx}{x^2-x-6} \\
82.- \int \sec 2\theta e^{\sec^2 \theta} d\theta & 83.- \int \frac{dx}{e^x - 9e^{-x}} & 84.- \int \frac{dw}{1+\cos w}
\end{array}$$

$$85.- \int e^{\left(\frac{1-\operatorname{sen}^2 \frac{x}{2}}{3}\right)^2} (\cos^3 \frac{x}{2} \operatorname{sen} \frac{x}{2}) dx$$

$$88.- \int (\sec \varphi + \tau g \varphi)^2 d\varphi$$

$$91.- \int \operatorname{sen}^{\frac{1}{2}} \varphi \cos^3 \varphi d\varphi$$

$$94.- \int (e^{2s} - 1)(e^{2s} + 1) ds$$

$$97.- \int (\arcsen \sqrt{1-x^2})^0 dx$$

$$100.- \int \frac{d\varphi}{a^2 \operatorname{sen}^2 \varphi + b^2 \cos^2 \varphi}$$

$$103.- \int (2 \cos \alpha \operatorname{sen} \alpha - \operatorname{sen} 2\alpha) d\alpha$$

$$106.- \int \frac{(\varphi + \operatorname{sen} 3\varphi) d\varphi}{3\varphi^2 - 2 \cos 3\varphi}$$

$$109.- \int \sqrt{u} (1+u^2)^2 du$$

$$112.- \int \frac{dx}{\sqrt{x^2 - 2x - 8}}$$

$$115.- \int \frac{x^3 + 7x^2 - 5x + 5}{x^2 + 2x - 3} dx$$

$$118.- \int \frac{xdx}{\sqrt{x^2 + 4x + 5}}$$

$$121.- \int \ell \eta \exp \sqrt{x-1} dx$$

$$124.- \int \frac{\operatorname{sen} x dx}{1 + \operatorname{sen} x + \cos x}$$

$$127.- \int \frac{(1 + \operatorname{sen} x) dx}{\operatorname{sen} x (2 + \cos x)}$$

$$86.- \int \frac{x^3 dx}{\sqrt{19-x^2}}$$

$$89.- \int \frac{dt}{t(4 + \ell \eta^2 t)^{\frac{1}{2}}}$$

$$92.- \int \frac{\sec^2 \theta d\theta}{9 + \tau g^2 \theta}$$

$$95.- \int \frac{dx}{5x^2 + 8x + 5}$$

$$98.- \int \frac{3dy}{1 + \sqrt{y}}$$

$$101.- \int \frac{tdt}{(2t+1)^{\frac{1}{2}}}$$

$$104.- \int t^4 \ell \eta^2 t dt$$

$$107.- \int \frac{(y^{\frac{1}{2}} + 1) dy}{y^{\frac{1}{2}} (y+1)}$$

$$110.- \int \frac{(x^3 + x^2) dx}{x^2 + x - 2}$$

$$113.- \int \frac{(x+1) dx}{\sqrt{2x-x^2}}$$

$$116.- \int e^{\ell \eta |1+x+x^2|} dx$$

$$119.- \int \frac{4dx}{x^3 + 4x}$$

$$122.- \int \frac{\sqrt{1+x^3}}{x} dx$$

$$125.- \int \frac{dx}{3 + 2 \cos x}$$

$$128.- \int \frac{dx}{x^4 + 4}$$

$$87.- \int \frac{\operatorname{sen} \varphi d\varphi}{\cos^{\frac{1}{2}} \varphi}$$

$$90.- \int a^\theta b^{2\theta} c^{3\theta} d\theta$$

$$93.- \int \frac{dx}{\sqrt{e^{2x} - 16}}$$

$$96.- \int \frac{x^3 + 1}{x^3 - x} dx$$

$$99.- \int x(1+x)^{\frac{1}{5}} dx$$

$$102.- \int \frac{s \ell \eta |s| ds}{(1-s^2)^{\frac{1}{2}}}$$

$$105.- \int u^2 (1+v)^{11} dx$$

$$108.- \int \frac{ds}{s^3 (s^2 - 4)^{\frac{1}{2}}}$$

$$111.- \int adb$$

$$114.- \int f(x) f'(x) dx$$

$$117.- \int \frac{(x-1) dx}{\sqrt{x^2 - 4x + 3}}$$

$$120.- \int \frac{\cos \tau g x dx}{\ell \eta |\operatorname{sen} x|}$$

$$123.- \int \sqrt{\frac{x-1}{x+1}} \frac{1}{x} dx$$

$$126.- \int \frac{xdx}{\sqrt{x^2 - 2x + 5}}$$

## RESPUESTAS

$$1.- \int t^3 e^{\operatorname{sen} t^4} \cos t^4 dt$$

Solución.- Sea:  $u = \operatorname{sen} t^4$ ,  $du = (\cos t^4) 4t^3 dt$ ; luego:

$$\int t^3 e^{\operatorname{sen} t^4} \cos t^4 dt = \frac{1}{4} \int 4t^3 e^{\operatorname{sen} t^4} \cos t^4 dt = \frac{1}{4} \int e^u du = \frac{1}{4} e^u + c = \frac{1}{4} e^{\operatorname{sen} t^4} + c$$

$$2.- \int \frac{\theta d\theta}{(1+\theta)^2}$$

Solución.-

$$\int \frac{\theta d\theta}{(1+\theta)^2} = \int \frac{A d\theta}{1+\theta} + \int \frac{B d\theta}{(1+\theta)^2} (*)$$

$$\frac{\theta}{(1+\theta)^2} = \frac{A}{1+\theta} + \frac{B}{(1+\theta)^2} \Rightarrow \theta = A(1+\theta) + B \Rightarrow \theta = A\theta + (A+B), \text{ de donde:}$$

$$A=1, B=-1, \text{ entonces: } (*) \int \frac{\theta d\theta}{(1+\theta)^2} = \int \frac{d\theta}{1+\theta} - \int \frac{d\theta}{(1+\theta)^2} = \ell \eta |1+\theta| + \frac{1}{1+\theta} + c$$

$$3.- \int \frac{\theta e^\theta d\theta}{(1+\theta)^2}$$

Solución.-

$$\text{Sea: } \begin{aligned} u &= e^\theta & dv &= \frac{\theta d\theta}{(1+\theta)^2} \\ du &= e^\theta d\theta & v &= \ell \eta |1+\theta| + \frac{1}{1+\theta} \end{aligned}$$

$$\int \frac{\theta e^\theta d\theta}{(1+\theta)^2} = e^\theta \ell \eta |1+\theta| + \frac{e^\theta}{1+\theta} - \int (\ell \eta |1+\theta| + \frac{1}{1+\theta}) e^\theta d\theta$$

$$= e^\theta \ell \eta |1+\theta| + \frac{e^\theta}{1+\theta} - \int e^\theta \ell \eta |1+\theta| d\theta - \int \frac{e^\theta d\theta}{1+\theta} (*), \text{ resolviendo por partes la segunda}$$

$$\text{integral se tiene: } \begin{aligned} u &= e^\theta & dv &= \frac{\theta d\theta}{1+\theta} \\ du &= e^\theta d\theta & v &= \ell \eta |1+\theta| \end{aligned}$$

$$\text{Luego: } \int \frac{e^\theta d\theta}{1+\theta} = e^\theta \ell \eta |1+\theta| - \int e^\theta \ell \eta |1+\theta| d\theta, \text{ esto es:}$$

$$(*) = \cancel{e^\theta \ell \eta |1+\theta|} + \frac{e^\theta}{1+\theta} - \cancel{\int e^\theta \ell \eta |1+\theta| d\theta} - \cancel{e^\theta \ell \eta |1+\theta|} + \cancel{\int e^\theta \ell \eta |1+\theta| d\theta} \\ = \frac{e^\theta}{1+\theta}$$

$$4.- \int e^{\tau g 3\theta} \sec^2 3\theta d\theta$$

$$\text{Solución.- Sea: } u = \tau g 3\theta, du = 3 \sec^2 3\theta d\theta$$

$$\int e^{\tau g 3\theta} \sec^2 3\theta d\theta = \frac{1}{3} \int e^u du = \frac{1}{3} e^u + c = \frac{e^{\tau g 3\theta}}{3} + c$$

$$5.- \int \frac{x dx}{\sqrt[3]{ax+b}}$$

$$\text{Solución.- Sea: } ax+b = t^3 \Rightarrow x = \frac{t^3-b}{a}, dx = \frac{3t^2}{a} dt$$



$$\begin{aligned}\int \frac{x dx}{\sqrt[3]{ax+b}} &= \int \frac{\left(\frac{t^3-b}{a}\right) \frac{3t^2}{a} dt}{t} = \int \frac{3t(t^3-b)}{a^2} dt = \frac{3}{a^2} \int (t^4 - bt) dt = \frac{3}{a^2} \left( \frac{t^5}{5} - \frac{bt^2}{2} \right) + c \\&= \frac{3t^5}{5a^2} - \frac{3bt^2}{2a^2} + c = \frac{3(ax+b)^{5/3}}{5a^2} - \frac{3b(ax+b)^{2/3}}{2a^2} + c \\&= \frac{3(ax+b)\sqrt[3]{(ax+b)^2}}{5a^2} - \frac{3b\sqrt[3]{(ax+b)^2}}{2a^2} + c\end{aligned}$$

6.-  $\int \sqrt{\frac{x^2-1}{x+1}} dx$

Solución.-

$$\begin{aligned}\int \sqrt{\frac{x^2-1}{x+1}} dx &= \int \sqrt{\frac{(x+1)(x-1)}{x+1}} = \int (x-1)^{1/2} dx = \frac{(x-1)^{3/2}}{3/2} + c = \frac{2(x-1)^{3/2}}{3} + c \\&= \frac{2(x-1)\sqrt{x-1}}{3} + c\end{aligned}$$

7.-  $\int \frac{dx}{(2-x)\sqrt{1-x}}$

Solución.- Sea:  $1-x=t^2 \Rightarrow x=1-t^2, dx=-2tdt$

$$\int \frac{dx}{(2-x)\sqrt{1-x}} = \int \frac{-2tdt}{[2-(1-t^2)]t} = -2 \int \frac{dt}{1+t^2} = -2 \operatorname{arc} \tau g t + c = -2 \operatorname{arc} \tau g \sqrt{1-x} + c$$

8.-  $\int e^{2-x} dx$

Solución.- Sea:  $u=2-x, du=-dx$

$$\int e^{2-x} dx = -\int e^u du = -e^u + c = -e^{2-x} + c$$

9.-  $\int \frac{e^x dx}{ae^x - b}$

Solución.- Sea:  $u = ae^x - b, du = ae^x dx$

$$\int \frac{e^x dx}{ae^x - b} = \frac{1}{a} \int \frac{du}{u} = \frac{1}{a} \ell \eta |u| + c = \frac{1}{a} \ell \eta |ae^x - b| + c$$

10.-  $\int \frac{(t+1)dt}{t^2+2t-5}$

Solución.- Sea:  $u = t^2 + 2t - 5, du = 2(t+1)dt$

$$\int \frac{(t+1)dt}{t^2+2t-5} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ell \eta |u| + c = \frac{1}{2} \ell \eta |t^2+2t-5| + c$$

11.-  $\int \sec \frac{\varphi}{2} d\varphi$

Solución.- Sea:  $u = \sec \frac{\varphi}{2} + \tau g \frac{\varphi}{2}, du = \frac{1}{2} (\sec \frac{\varphi}{2} \tau g \frac{\varphi}{2} + \sec^2 \frac{\varphi}{2}) d\varphi$

$$\int \sec \frac{\varphi}{2} d\varphi = \int \frac{\sec \frac{\varphi}{2} (\sec \frac{\varphi}{2} + \tau g \frac{\varphi}{2})}{\sec \frac{\varphi}{2} + \tau g \frac{\varphi}{2}} d\varphi = \int \frac{\sec^2 \frac{\varphi}{2} + \sec \frac{\varphi}{2} \tau g \frac{\varphi}{2}}{\sec \frac{\varphi}{2} + \tau g \frac{\varphi}{2}} d\varphi$$

$$= 2 \int \frac{du}{u} = 2 \ell \eta |u| + c = 2 \ell \eta \left| \sec \frac{\varphi}{2} + \tau g \frac{\varphi}{2} \right| + c$$

**12.-**  $\int \tau g \theta d\theta$

Solución.- Sea:  $u = \cos \theta, du = -\operatorname{sen} \theta d\theta$

$$\int \tau g \theta d\theta = \int \frac{\operatorname{sen} \theta}{\cos \theta} d\theta = - \int \frac{du}{u} = -\ell \eta |u| + c = -\ell \eta |\cos \theta| + c = -\ell \eta \left| \frac{1}{\sec \theta} \right| + c$$

$$= -\ell \eta \left| \frac{1}{\sec \theta} \right| + \ell \eta |\sec \theta| + c = \ell \eta |\sec \theta| + c$$

**13.-**  $\int \frac{\eta^2}{a} \operatorname{sen} \frac{\eta}{b} d\eta$

Solución.-

Sea:  $u = \frac{\eta^2}{a} \quad dv = \operatorname{sen} \frac{\eta}{b} d\eta$

$$du = \frac{2\eta d\eta}{a} \quad v = -b \cos \frac{\eta}{b}$$

$$\int \frac{\eta^2}{a} \operatorname{sen} \frac{\eta}{b} d\eta = -\frac{a}{b} \eta^2 \cos \frac{\eta}{b} + \frac{2b}{a} \int \eta \cos \frac{\eta}{b} d\eta (*), \text{ resolviendo por partes la segunda}$$

integral se tiene:  $u = \eta \quad dv = \cos \frac{\eta}{b} d\eta$

$$du = d\eta \quad v = b \operatorname{sen} \frac{\eta}{b}$$

$$(*) = -\frac{a}{b} \eta^2 \cos \frac{\eta}{b} + \frac{2b}{a} \left( b \eta \operatorname{sen} \frac{\eta}{b} - b \int \operatorname{sen} \frac{\eta}{b} d\eta \right)$$

$$= -\frac{a}{b} \eta^2 \cos \frac{\eta}{b} + \frac{2b^2}{a} \eta \operatorname{sen} \frac{\eta}{b} + \frac{2b^3}{a} \cos \frac{\eta}{b} + c$$

**14.-**  $\int \varphi \sec^2 \varphi d\varphi$

Solución.-

Sea:  $u = \varphi \quad dv = \sec^2 \varphi d\varphi$

$$du = d\varphi \quad v = \tau g \varphi$$

$$\int \varphi \sec^2 \varphi d\varphi = \varphi \tau g \varphi - \int \tau g \varphi d\varphi = \varphi \tau g \varphi - \ell \eta |\sec \varphi| + c$$

**15.-**  $\int \frac{dx}{5^x}$

Solución.- Sea:  $u = -x, du = -dx$

$$\int \frac{dx}{5^x} = \int 5^{-x} dx = - \int 5^u du = -\frac{5^u}{\ell \eta 5} + c = -\frac{5^{-x}}{\ell \eta 5} + c = -\frac{1}{5^x \ell \eta 5} + c$$

$$16.- \int \sec^2(1-x) dx$$

Solución.- Sea:  $u = 1-x, du = -dx$

$$\int \sec^2(1-x) dx = -\int \sec^2 u du = -\tau g u + c = -\tau g(1-x) + c$$

$$17.- \int \frac{xdx}{\sqrt{16-x^4}}$$

Solución.- Sea:  $u = x^2, du = 2xdx$

$$\begin{aligned} \int \frac{xdx}{\sqrt{16-x^4}} &= \int \frac{xdx}{\sqrt{4^2-(x^2)^2}} = \frac{1}{2} \int \frac{2xdx}{\sqrt{4^2-(x^2)^2}} = \frac{1}{2} \int \frac{du}{\sqrt{4^2-u^2}} = \frac{1}{2} \arcsen \frac{u}{4} + c \\ &= \frac{1}{2} \arcsen \frac{x^2}{4} + c \end{aligned}$$

$$18.- \int \frac{dy}{\sqrt{1+\sqrt{1+y}}}$$

Solución.- Sea:  $t = [1+(1+y)^{\frac{1}{2}}]^{\frac{1}{2}} \Rightarrow t^2 = 1+(1+y)^{\frac{1}{2}} \Rightarrow t^2 - 1 = (1+y)^{\frac{1}{2}}$

$$\Rightarrow (t^2 - 1)^2 = 1+y \Rightarrow y = (t^2 - 1)^2 - 1, dy = 4t(t^2 - 1)dt$$

$$\int \frac{dy}{\sqrt{1+\sqrt{1+y}}} = \int \frac{4t(t^2-1)dt}{t} = 4 \int (t^2 - 1)dt = 4\left(\frac{t^3}{3} - t\right) + c = 4t\left(\frac{t^2}{3} - 1\right) + c$$

$$= 4\sqrt{1+\sqrt{1+y}}\left(\frac{1+\sqrt{1+y}}{3} - 1\right) + c = \frac{4}{3}\sqrt{1+\sqrt{1+y}}(\sqrt{1+y} - 2) + c$$

$$19.- \int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}}$$

Solución.-

$$\int \frac{dx}{\sqrt{x+4}-\sqrt{x+3}} = \int \frac{(x+4)^{\frac{1}{2}} + (x+3)^{\frac{1}{2}}}{(x+4)-(x+3)} dx = \int [(x+4)^{\frac{1}{2}} + (x+3)^{\frac{1}{2}}] dx$$

$$\int (x+4)^{\frac{1}{2}} + \int (x+3)^{\frac{1}{2}} = \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2\sqrt{(x+4)^3}}{3} + \frac{2\sqrt{(x+3)^3}}{3} + c$$

$$= \frac{2}{3} \left( \sqrt{(x+4)^3} + \sqrt{(x+3)^3} \right) + c$$

$$20.- \int \operatorname{cosec} \theta d\theta$$

Solución.- Sea:  $u = \operatorname{cosec} \theta + \cot g \theta, du = -(\operatorname{cosec} \theta \cot g \theta + \operatorname{cosec}^2 \theta) d\theta$

$$\int \operatorname{cosec} \theta d\theta = \int \frac{\operatorname{cosec} \theta (\operatorname{cosec} \theta + \cot g \theta) d\theta}{\operatorname{cosec} \theta + \cot g \theta} = \int \frac{\operatorname{cosec}^2 \theta + \operatorname{cosec} \theta \cot g \theta d\theta}{\operatorname{cosec} \theta + \cot g \theta}$$

$$= -\int \frac{du}{u} = -\ell \eta |u| + c = -\ell \eta |(\operatorname{cosec} \theta + \cot g \theta)| + c$$

$$21.- \int t(1-t^2)^{\frac{1}{2}} dt$$

Solución.- Sea:  $u = 1-t^2, du = -2tdt$

$$\int t(1-t^2)^{\frac{1}{2}} dt = -\frac{1}{2} \int u^{\frac{1}{2}} du = -\frac{1}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{1}{3} u^{\frac{3}{2}} + c = -\frac{1}{3} (1-t^2)^{\frac{3}{2}} + c$$

**22.-**  $\int t(1-t^2)^{\frac{1}{2}} \arcsen t dt$

Solución.-

$$u = \arcsen t \quad dv = t(1-t^2)^{\frac{1}{2}} dt$$

Sea:  $du = \frac{dt}{\sqrt{1-t^2}} \quad v = -\frac{1}{3} (1-t^2)^{\frac{3}{2}}$

$$\begin{aligned} \int t(1-t^2)^{\frac{1}{2}} \arcsen t dt &= -\frac{1}{3} (1-t^2)^{\frac{3}{2}} \arcsen t + \frac{1}{3} \int (1-t^2) \frac{dt}{\sqrt{1-t^2}} \\ &= -\frac{(1-t^2)^{\frac{3}{2}}}{3} \arcsen t + \frac{1}{3} \int (1-t^2) dt = -\frac{(1-t^2)^{\frac{3}{2}}}{3} \arcsen t + \frac{1}{3} \left( t - \frac{t^3}{3} \right) + c \\ &= -\frac{1}{3} \left[ (1-t^2)^{\frac{3}{2}} \arcsen t - t + \frac{t^3}{3} \right] + c \end{aligned}$$

**23.-**  $\int \frac{1+\cos 2x}{\sen^2 2x} dx$

Solución.-

$$\begin{aligned} \int \frac{1+\cos 2x}{\sen^2 2x} dx &= \int \frac{1+\cos 2x}{1-\cos^2 x} dx = \int \frac{dx}{1-\cos 2x} = \int \frac{dx}{2 \left( \frac{1-\cos 2x}{2} \right)} = \frac{1}{2} \int \frac{dx}{\sen^2 x} \\ &= \frac{1}{2} \int \sec^2 x dx = -\frac{1}{2} \cot x + c \end{aligned}$$

**24.-**  $\int \frac{x^2+1}{x^3-x} dx$

Solución.-

$$\int \frac{x^2+1}{x^3-x} dx = \int \frac{(x^2+1)dx}{x(x^2-1)} = \int \frac{(x^2+1)dx}{x(x+1)(x-1)} = \int \frac{A dx}{x} + \int \frac{B dx}{(x+1)} + \int \frac{C dx}{(x-1)} (*)$$

$$\frac{(x^2+1)}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \Rightarrow (x^2+1) = A(x^2-1) + Bx(x-1) + Cx(x+1)$$

$$x=0 \Rightarrow 1 = -A \Rightarrow A = -1$$

De donde:  $x = -1 \Rightarrow 2 = B(-1)(-2) \Rightarrow B = 1$

$$x = 1 \Rightarrow 2 = C(1)(2) \Rightarrow C = 1$$

Entonces:

$$\begin{aligned} (*) \int \frac{(x^2+1)dx}{x(x+1)(x-1)} &= -\int \frac{dx}{x} + \int \frac{dx}{(x+1)} + \int \frac{dx}{(x-1)} = -\ell \eta |x| + \ell \eta |x+1| + \ell \eta |x-1| + c \\ &= \ell \eta \left| \frac{x^2-1}{x} \right| + c \end{aligned}$$

$$25.- \int \frac{e^x dx}{\sqrt{9-e^{2x}}}$$

Solución.- Sea:  $u = e^x$ ,  $du = e^x dx$

$$\int \frac{e^x dx}{\sqrt{9-e^{2x}}} = \int \frac{e^x dx}{\sqrt{3^2-(e^x)^2}} = \int \frac{du}{\sqrt{3^2-u^2}} = \arcsen \frac{u}{3} + c = \arcsen \frac{e^x}{3} + c$$

$$26.- \int \frac{dx}{(x-1)^3}$$

Solución.-

$$\int \frac{dx}{(x-1)^3} = \int (x-1)^{-3} dx = -\frac{(x-1)^{-2}}{2} + c = -\frac{1}{(x-1)^2} + c$$

$$27.- \int \frac{(3x+4)dx}{\sqrt{2x+x^2}}$$

Solución.- Sea:  $u = 2x+x^2$ ,  $du = 2(1+x)dx$

$$\begin{aligned} \int \frac{(3x+4)dx}{\sqrt{2x+x^2}} &= \int \frac{(3x+3)+1}{\sqrt{2x+x^2}} dx = 3 \int \frac{(x+1)dx}{\sqrt{2x+x^2}} + \int \frac{dx}{\sqrt{2x+x^2}} = \frac{3}{2} \int \frac{du}{u^{1/2}} + \int \frac{dx}{\sqrt{2x+x^2}} \\ &= \frac{3}{2} \int \frac{du}{u^{1/2}} + \int \frac{dx}{\sqrt{(x^2+2x+1)-1}} = \frac{3}{2} \frac{u^{1/2}}{1/2} + \int \frac{dx}{\sqrt{(x+1)^2-1}} = 3\sqrt{2x+x^2} + \int \frac{dx}{\sqrt{(x+1)^2-1}} \end{aligned}$$

Sustituyendo por:  $x+1 = \sec \theta$ ,  $dx = \sec \theta \tan \theta d\theta$ ,  $\sqrt{(x+1)^2-1} = \tan \theta$

$$\begin{aligned} &= 3\sqrt{2x+x^2} + \int \frac{\sec \theta \tan \theta}{\tan \theta} d\theta = 3\sqrt{2x+x^2} + \int \sec \theta d\theta = 3\sqrt{2x+x^2} + \ell \eta |\sec \theta + \tan \theta| + c \\ &= 3\sqrt{2x+x^2} + \ell \eta |x+1 + \sqrt{2x+x^2}| + c \end{aligned}$$

$$28.- \int \frac{ds}{\sqrt{4-s^2}}$$

Solución.- Sea:  $s = 2 \sen \theta$ ,  $ds = 2 \cos \theta d\theta$ ,  $\sqrt{4-s^2} = 2 \cos \theta$

$$\int \frac{ds}{\sqrt{4-s^2}} = \int \frac{2 \cos \theta d\theta}{2 \cos \theta} = \int d\theta = \theta = \arcsen \frac{s}{2} + c$$

$$29.- \int \frac{dx}{x^2 \sqrt{x^2+e}}$$

Solución.- Sea:  $x = \sqrt{e} \tan \theta$ ,  $dx = \sqrt{e} \sec^2 \theta d\theta$ ,  $\sqrt{x^2+e} = \sqrt{e} \sec \theta$

$$\int \frac{dx}{x^2 \sqrt{x^2+e}} = \int \frac{\sqrt{e} \sec^2 \theta d\theta}{e \tan^2 \theta \sqrt{e} \sec \theta} = \frac{1}{e} \int \frac{\sec \theta d\theta}{\tan^2 \theta} = \frac{1}{e} \int \frac{\frac{1}{\cos \theta} d\theta}{\frac{\sen^2 \theta}{\cos^2 \theta}} = \frac{1}{e} \int \frac{\cos \theta}{\sen^2 \theta} d\theta (*)$$

Sea:  $u = \sen \theta$ ,  $du = \cos \theta d\theta$ , luego:

$$\begin{aligned}
 (*) &= \frac{1}{e} \int \frac{du}{u^2} = \frac{1}{e} \int u^{-2} du = \frac{1}{e} \frac{u^{-1}}{-1} + c = -\frac{1}{eu} + c = -\frac{1}{e \operatorname{sen} \theta} + c = -\frac{1}{e \frac{x}{\sqrt{x^2+e}}} + c \\
 &= -\frac{\sqrt{x^2+e}}{ex} + c
 \end{aligned}$$

**30.-**  $\int \frac{x dx}{\sqrt{1+x}}$

Solución.- Sea:  $x+1=t^2 \Rightarrow x=t^2-1, dx=2t dt$

$$\begin{aligned}
 \int \frac{x dx}{\sqrt{1+x}} &= \int \frac{(t^2-1)2t dt}{2t} = 2 \int (t^2-1) dt = 2 \left( \frac{t^3}{3} - t \right) + c = 2t \left( \frac{t^2}{3} - 1 \right) + c \\
 &= 2\sqrt{x+1} \left( \frac{x+1}{3} - 1 \right) + c = 2\sqrt{x+1} \left( \frac{x-2}{3} \right) + c
 \end{aligned}$$

**31.-**  $\int \frac{y^2 dy}{\sqrt{y+1}}$

Solución.- Sea:  $y+1=t^2 \Rightarrow y=t^2-1, dy=2t dt$

$$\begin{aligned}
 \int \frac{y^2 dy}{\sqrt{y+1}} &= \int \frac{(t^2-1)^2 2t dt}{2t} = 2 \int (t^2-1)^2 dt = 2 \int (t^4 - 2t^2 + 1) dt = 2 \left( \frac{t^5}{5} - \frac{2t^3}{3} + t \right) + c \\
 &= 2t \left( \frac{t^4}{5} - \frac{2t^2}{3} + 1 \right) + c = 2\sqrt{y+1} \left( \frac{(\sqrt{y+1})^4}{5} - \frac{2(\sqrt{y+1})^2}{3} + 1 \right) + c \\
 &= 2\sqrt{y+1} \left( \frac{(y+1)^2}{5} - \frac{2y+2}{3} + 1 \right) + c = 2\sqrt{y+1} \left( \frac{y^2+2y+1}{5} - \frac{2y+2}{3} + 1 \right) + c \\
 &= 2\sqrt{y+1} \left( \frac{3y^2-4y+8}{15} \right) + c
 \end{aligned}$$

**32.-**  $\int \frac{y^3 dy}{\sqrt{y^2-1}}$

Solución.- Sea:  $u = y^2 - 1 \Rightarrow y^2 = u + 1, dy = 2y dy$

$$\begin{aligned}
 \int \frac{y^3 dy}{\sqrt{y^2-1}} &= \int \frac{y^2 y dy}{\sqrt{y^2-1}} = \frac{1}{2} \int \frac{(u+1) du}{u^{1/2}} = \frac{1}{2} \int (u^{1/2} + u^{-1/2}) du = \frac{1}{2} \left( \frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} \right) + c \\
 &= \frac{u^{3/2}}{3} + u^{1/2} + c = u^{1/2} \left( \frac{1}{3} u + 1 \right) + c = \sqrt{y^2-1} \left( \frac{y^2-1}{3} + 1 \right) + c = \sqrt{y^2-1} \left( \frac{y^2+2}{3} \right) + c
 \end{aligned}$$

**33.-**  $\int \frac{d\theta}{1+2\cos\theta}$

Solución.- Sea:  $d\theta = \frac{2dz}{1+z^2}, \cos\theta = \frac{1-z^2}{1+z^2}, \theta = 2 \arctan z$

$$\begin{aligned}
\int \frac{d\theta}{1+2\cos\theta} &= \int \frac{\frac{2dz}{1+z^2}}{1+\frac{2(1-z^2)}{1+z^2}} = \int \frac{2dz}{1+z^2+2(1-z^2)} = \int \frac{2dz}{1+z^2+2-2z^2} = \int \frac{2dz}{3-z^2} \\
&= \int \frac{2dz}{3-z^2} = -2 \int \frac{dz}{z^2-3} = -2 \int \frac{dz}{z^2-(\sqrt{3})^2} = -\cancel{2} \frac{1}{\cancel{2}\sqrt{3}} \ell \eta \left| \frac{z-\sqrt{3}}{z+\sqrt{3}} \right| + c \\
&= -\frac{1}{\sqrt{3}} \ell \eta \left| \frac{\tau g \theta/2 - \sqrt{3}}{\tau g \theta/2 + \sqrt{3}} \right| + c
\end{aligned}$$

**34.-**  $\int \frac{t^4 - t^3 + 4t^2 - 2t + 1}{t^3 + 1} dt$

Solución.-

$$\begin{aligned}
\int \frac{t^4 - t^3 + 4t^2 - 2t + 1}{t^3 + 1} dt &= \int \left( t - 1 + \frac{3t^2 - t + 1}{t^3 + t} \right) dt = \int t dt - \int dt + \int \frac{3t^2 - t + 1}{t^3 + t} dt \\
&= \frac{t^2}{2} - t + \int \frac{3t^2 - t + 1}{t^3 + t} dt (*) \\
\frac{3t^2 - t + 1}{t(t^2 + 1)} &= \frac{A}{t} + \frac{Bt + C}{(t^2 + 1)} \Rightarrow 3t^2 - t + 1 = A(t^2 + 1) + (Bt + C)t
\end{aligned}$$

$$\begin{aligned}
t = 0 &\Rightarrow 1 = A \Rightarrow A = 1 \\
\text{De donde: } \left. \begin{aligned} t = 1 &\Rightarrow 3 = 2A + B + C \Rightarrow B + C = 1 \\ t = -1 &\Rightarrow 5 = 2A - (C - B) \Rightarrow B - C = 3 \end{aligned} \right\} B = 2, C = -1
\end{aligned}$$

$$\begin{aligned}
(*) &= \frac{t^2}{2} - t + \int \frac{Adt}{t} + \int \frac{Bt + C}{t^2 + 1} dt = \frac{t^2}{2} - t + \int \frac{dt}{t} + \int \frac{2t - 1}{t^2 + 1} dt \\
&= \frac{t^2}{2} - t + \ell \eta |t| + \int \frac{2tdt}{t^2 + 1} - \int \frac{dt}{t^2 + 1} = \frac{t^2}{2} - t + \ell \eta |t| + \ell \eta |t^2 + 1| - \arctan t + c \\
&= \frac{t^2}{2} - t + \ell \eta |t(t^2 + 1)| - \arctan t + c
\end{aligned}$$

**35.-**  $\int \frac{d\varphi}{\ell \eta e}$

Solución.-

$$\int \frac{d\varphi}{\ell \eta e} = \int d\varphi = \varphi + c$$

**36.-**  $\int x(10 + 8x^2)^9 dx$

Solución.- Sea:  $u = 10 + 8x^2, du = 16x dx$

$$\begin{aligned}
\int x(10 + 8x^2)^9 dx &= \frac{1}{16} \int 16x(10 + 8x^2)^9 dx = \frac{1}{16} \int u^9 du = \frac{1}{16} \frac{u^{10}}{10} + c = \frac{u^{10}}{160} + c \\
&= \frac{(10 + 8x^2)^{10}}{160} + c
\end{aligned}$$

$$37.- \int \frac{dx}{\sqrt{(16+x^2)^3}}$$

Solución.- Sea:  $x = 4 \tan \theta, dx = 4 \sec^2 \theta d\theta$

$$\int \frac{dx}{\sqrt{(16+x^2)^3}} = \int \frac{4 \sec^2 \theta d\theta}{4^3 \sec^3 \theta} = \frac{1}{16} \int \frac{d\theta}{\sec \theta} = \frac{1}{16} \int \cos \theta d\theta = \frac{1}{16} \sin \theta + c = \frac{x}{16\sqrt{16+x^2}} + c$$

$$38.- \int \frac{x^3 dx}{\sqrt{x^2+4}}$$

Solución.- Sea:  $u = x^2 + 4 \Rightarrow x^2 = u - 4, du = 2x dx$

$$\begin{aligned} \int \frac{x^3 dx}{\sqrt{x^2+4}} &= \int \frac{x^2 x dx}{\sqrt{x^2+4}} = \frac{1}{2} \int \frac{(u-4) du}{u^{1/2}} = \frac{1}{2} \int (u^{1/2} - 4u^{-1/2}) du = \frac{1}{2} \int u^{1/2} du - 2 \int u^{-1/2} du \\ &= \frac{1}{2} \frac{u^{3/2}}{3/2} - \frac{2u^{1/2}}{1/2} + c = \frac{u^{3/2}}{3} - 4u^{1/2} + c = u^{1/2} \left( \frac{u}{3} - 4 \right) + c = \sqrt{x^2+4} \left( \frac{x^2+4}{3} - 4 \right) + c \\ &= \sqrt{x^2+4} \left( \frac{x^2-8}{3} \right) + c \end{aligned}$$

$$39.- \int \frac{x^3 dx}{\sqrt{16-x^2}}$$

Solución.- Sea:  $u = 16 - x^2 \Rightarrow x^2 = 16 - u, du = -2x dx$

$$\begin{aligned} \int \frac{x^3 dx}{\sqrt{16-x^2}} &= \int \frac{x^2 x dx}{\sqrt{16-x^2}} = -\frac{1}{2} \int \frac{(16-u) du}{u^{1/2}} = -\frac{1}{2} \int (16u^{-1/2} - u^{1/2}) du \\ &= -\frac{1}{2} \frac{16u^{1/2}}{1/2} + \frac{1}{2} \frac{u^{3/2}}{3/2} = -16u^{1/2} + \frac{u^{3/2}}{3} + c = -16u^{1/2} + \frac{\sqrt{u} u}{3} + c = \sqrt{u} \left( -16 + \frac{u}{3} \right) + c \\ &= \sqrt{16-x^2} \left( -16 + \frac{16-x^2}{3} \right) + c = -\sqrt{16-x^2} \left( \frac{32+x^2}{3} \right) + c \end{aligned}$$

$$40.- \int a(x^2+1)^{1/2} dy$$

Solución.-

$$\int a(x^2+1)^{1/2} dy = a(x^2+1)^{1/2} \int dy = a(x^2+1)^{1/2} y + c$$

$$41.- \int \frac{dx}{(\sqrt{6-x^2})^3}$$

Solución.- Sea:  $x = \sqrt{6} \sin \theta, dx = \sqrt{6} \cos \theta d\theta, \sqrt{6-x^2} = \sqrt{6} \cos \theta$

$$\int \frac{dx}{(\sqrt{6-x^2})^3} = \int \frac{\sqrt{6} \cos \theta d\theta}{(\sqrt{6})^3 \cos^3 \theta} = \frac{1}{6} \int \frac{d\theta}{\cos^2 \theta} = \frac{1}{6} \sec^2 \theta d\theta = \frac{1}{6} \tan \theta + c = \frac{1}{6} \frac{x}{\sqrt{6-x^2}} + c$$

$$42.- \int \frac{dx}{x(3+\ln x)}$$



Solución.- Sea:  $u = 3 + \ell \eta x, du = \frac{dx}{x}$

$$\int \frac{dx}{x(3 + \ell \eta x)} = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |3 + \ell \eta x| + c$$

**43.-**  $\int \frac{e^x}{16 + e^{2x}} dx$

Solución.- Sea:  $u = e^x, du = e^x dx$

$$\int \frac{e^x}{16 + e^{2x}} dx = \int \frac{du}{4^2 + u^2} = \frac{1}{4} \operatorname{arc} \tau g \frac{u}{4} + c = \frac{1}{4} \operatorname{arc} \tau g \frac{e^x}{4} + c$$

**44.-**  $\int \cos \sqrt{1-x} dx$

Solución.- Sea:  $1-x = t^2 \Rightarrow x = 1-t^2, dx = -2t dt$

$\int \cos \sqrt{1-x} dx = -2 \int \cos t dt (*)$ , integrando por partes se tiene:

Sea:  $u = t \quad dv = \cos t dt$   
 $du = dt \quad v = \operatorname{sen} t$

$$(*) = -2 \left( t \operatorname{sen} t - \int \operatorname{sen} t dt \right) = -2t \operatorname{sen} t + 2 \int \operatorname{sen} t dt = -2t \operatorname{sen} t - 2 \cos t + c$$

$$= -2\sqrt{1-x} \operatorname{sen} \sqrt{1-x} - 2 \cos \sqrt{1-x} + c$$

**45.-**  $\int \frac{x^3 dx}{\sqrt{x-1}}$

Solución.- Sea:  $x-1 = t^2 \Rightarrow x = t^2+1, dx = 2t dt$

$$\int \frac{x^3 dx}{\sqrt{x-1}} = \int \frac{(t^2+1)^3 2t dt}{t} = 2 \int (t^6 + 3t^4 + 3t^2 + 1) dt = \frac{2t^7}{7} + \frac{6t^5}{5} + 2t^3 + 2t + c$$

$$= t \left( \frac{2t^6}{7} + \frac{6t^4}{5} + 2t^2 + 2 \right) + c = \sqrt{x-1} \left[ \frac{2(x-1)^3}{7} + \frac{6(x-1)^2}{5} + 2(x-1) + 2 \right] + c$$

$$= 2\sqrt{x-1} \left[ \frac{(x-1)^3}{7} + \frac{3(x-1)^2}{5} + x \right] + c$$

**46.-**  $\int \frac{2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6}{(y-1)^2(y^2+1)^2} dy$

Solución.-

$$\int \frac{2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6}{(y-1)^2(y^2+1)^2} dy (*)$$

$$\frac{2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6}{(y-1)^2(y^2+1)^2} = \frac{A}{y-1} + \frac{B}{(y-1)^2} + \frac{Cy+D}{(y^2+1)} + \frac{Ey+F}{(y^2+1)^2}$$

$$2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6 = A(y-1)(y^2+1)^2 + B(y^2+1)^2$$

$$\Rightarrow +(Cy+D)(y-1)^2(y^2+1) + (Ey+F)(y-1)^2, \text{ luego:}$$

$$2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6 = (A+C)y^5 + (-A+B-2C+D)y^4$$

$$\Rightarrow +(2A+2C-2D+E)y^3 + (-2A+2B-2C+2D-2E+F)y^2$$

$\Rightarrow +(A+C-2D+E-2F)y+(-A+B+D+F)$ , Igualando coeficientes se tiene:

$$\left( \begin{array}{cccccc} A & & +C & & & = 2 \\ -A & + B & -2C & +D & & = -7 \\ 2A & & +2C & -2D & +E & = 7 \\ -2A & +2B & -2C & +2D & -2E & +F = -19 \\ A & & +C & -2D & +E & -2F = 7 \\ -A & + B & & +D & & +F = -6 \end{array} \right) \Rightarrow A=1, B=-4, C=1$$

$$D=0, E=3, F=-1$$

$$\begin{aligned} (*) \int \frac{2y^5 - 7y^4 + 7y^3 - 19y^2 + 7y - 6}{(y-1)^2(y^2+1)^2} dy &= \int \frac{dy}{y-1} - 4 \int \frac{dy}{(y-1)^2} + \int \frac{y dy}{(y^2+1)} + \int \frac{(3y-1) dy}{(y^2+1)^2} \\ &= \ell \eta |y-1| + \frac{4}{y-1} + \frac{1}{2} \ell \eta |y^2+1| + 3 \int \frac{y dy}{(y^2+1)} - \int \frac{dy}{(y^2+1)^2} \\ &= \ell \eta |y-1| + \frac{4}{y-1} + \ell \eta |\sqrt{y^2+1}| - \frac{3}{2} \ell \eta |y^2+1| - \left[ \frac{1}{2} \frac{y}{y^2+1} + \frac{1}{2} \arctan y \right] + c \\ &= \ell \eta |(y-1)\sqrt{y^2+1}| + \frac{4}{y-1} - \frac{3}{2} \ell \eta |y^2+1| - \frac{y}{2(y^2+1)} - \frac{1}{2} \arctan y + c \\ &= \ell \eta \left| \frac{(y-1)}{\sqrt{y^2+1}} \right| + \frac{4}{y-1} - \frac{y}{2(y^2+1)} - \frac{1}{2} \arctan y + c \end{aligned}$$

**47.-**  $\int \operatorname{sen} \sqrt{x+1} dx$

Solución.- Sea:  $x+1=t^2 \Rightarrow x=t^2-1, dx=2t dt$

$$\int \operatorname{sen} \sqrt{x+1} dx = 2 \int (\operatorname{sen} t) t dt (*), \text{ trabajando por partes}$$

Sea:  $u=t \quad dv = \operatorname{sen} t dt$   
 $du = dt \quad v = -\cos t$

$$\begin{aligned} (*) 2 \int (\operatorname{sen} t) t dt &= 2 \left( -t \cos t + \int \cos t dt \right) = -2t \cos t + 2 \operatorname{sen} t + c \\ &= -2\sqrt{x+1} \cos \sqrt{x+1} + 2 \operatorname{sen} \sqrt{x+1} + c \end{aligned}$$

**48.-**  $\int \frac{9x^2+7x-6}{x^3-x} dx$

Solución.-

$$\int \frac{9x^2+7x-6}{x^3-x} dx = \int \frac{9x^2+7x-6}{x(x+1)(x-1)} dx = \int \frac{A dx}{x} + \int \frac{B dx}{x+1} + \int \frac{C dx}{x-1} (*)$$

$$\frac{9x^2+7x-6}{x^3-x} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \Rightarrow 9x^2+7x-6 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$$

De donde:  $\begin{cases} x=0 \Rightarrow -6 = -A \Rightarrow A=6 \\ x=1 \Rightarrow 10 = 2C \Rightarrow C=5 \\ x=-1 \Rightarrow -4 = 2B \Rightarrow B=-2 \end{cases}$

$$(*) = 6 \int \frac{dx}{x} - 2 \int \frac{dx}{x+1} + 5 \int \frac{dx}{x-1} = 6 \ell \eta |x| - 2 \ell \eta |x+1| + 5 \ell \eta |x-1| + c$$

$$= \ell \eta |x^6| - \ell \eta |(x+1)^2| + \ell \eta |(x-1)^5| + c = \ell \eta \left| \frac{x^6(x-1)^5}{(x+1)^2} \right| + c$$

$$49.- \int \frac{5w^3 - 5w^2 + 2w - 1}{w^4 + w^2} dw$$

Solución.-

$$\int \frac{5w^3 - 5w^2 + 2w - 1}{w^4 + w^2} dw = \int \frac{5w^3 - 5w^2 + 2w - 1}{w^2(w^2 + 1)} dw (*)$$

$$\frac{5w^3 - 5w^2 + 2w - 1}{w^2(w^2 + 1)} = \frac{Aw + B}{w^2} + \frac{Cw + D}{w^2 + 1}$$

$$5w^3 - 5w^2 + 2w - 1 = (Aw + B)(w^2 + 1) + (Cw + D)w^2$$

$$\Rightarrow Aw^3 + Aw + Bw^2 + B + Cw^3 + Dw^2 \Rightarrow (A + C)w^3 + (B + D)w^2 + Aw + B$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A & +C & = & 5 \\ & B & +D & = -5 \\ A & & = & 2 \\ & B & = & -1 \end{pmatrix} \Rightarrow A = 2, B = -1, C = 3, D = -4$$

$$(*) \int \frac{Aw + B}{w^2} dw + \int \frac{Cw + D}{w^2 + 1} dw = \int \frac{2w - 1}{w^2} dw + \int \frac{3w - 4}{w^2 + 1} dw$$

$$= \int \frac{2w dw}{w^2} - \int w^{-2} dw + \frac{3}{2} \int \frac{2w dw}{w^2 + 1} - 4 \int \frac{dw}{w^2 + 1}$$

$$= \ell \eta |w^2| + \frac{1}{w} + \ell \eta |\sqrt{(w^2 + 1)^3}| - 4 \operatorname{arc} \tau g w + c = \ell \eta |w^2 \sqrt{(w^2 + 1)^3}| + \frac{1}{w} - 4 \operatorname{arc} \tau g w + c$$

$$50.- \int \frac{3dx}{1 + 2x}$$

Solución.- Sea:  $u = 1 + 2x, du = 2dx$

$$\int \frac{3dx}{1 + 2x} = 3 \int \frac{dx}{1 + 2x} = \frac{3}{2} \int \frac{du}{u} = \frac{3}{2} \ell \eta |u| + c = \frac{3}{2} \ell \eta |1 + 2x| + c = \ell \eta |\sqrt{(1 + 2x)^3}| + c$$

$$51.- \int \frac{(1-x)^2 dx}{x}$$

Solución.-

$$\int \frac{(1-x)^2 dx}{x} = \int \frac{1 - 2x + x^2 dx}{x} = \int \frac{dx}{x} - 2 \int dx + \int x dx = \ell \eta |x| - 2x + \frac{x^2}{2} + c$$

$$52.- \int \frac{x e^{-2x^2}}{2} dx$$

Solución.- Sea:  $u = -2x^2, du = -4x dx$

$$\int \frac{x e^{-2x^2}}{2} dx = \frac{1}{2} \int x e^{-2x^2} dx = -\frac{1}{8} \int e^u du = -\frac{1}{8} e^u + c = -\frac{1}{8} e^{-2x^2} + c$$

$$53.- \int e^{2t} \cos(e^t) dt$$

Solución.- Sea:  $w = e^t, dw = e^t dt$

$$\int e^t \cos(e^t) e^t dt = \int w \cos w dw (*) , \text{trabajando por partes}$$

$$\text{Sea: } \begin{array}{ll} u = w & dv = \cos w dw \\ du = dw & v = \operatorname{sen} w \end{array}$$

$$(*) \int w \cos w dw = w \operatorname{sen} w - \int \operatorname{sen} w dw = w \operatorname{sen} w + \cos w + c = e^t \operatorname{sen}(e^t) + \cos(e^t) + c$$

$$54.- \int \sqrt{x}(x^{3/2} - 4)^3 dx$$

$$\text{Solución.- Sea: } u = x^{3/2} - 4, du = \frac{3}{2} \sqrt{x} dx$$

$$\int \sqrt{x}(x^{3/2} - 4)^3 dx = \frac{2}{3} \int u^3 du = \frac{2}{3} \frac{u^4}{4} + c = \frac{1}{6} u^4 + c = \frac{(x^{3/2} - 4)^4}{6} + c$$

$$55.- \int \frac{\operatorname{sen} x e^{\sec x}}{\cos^2 x} dx = \int \frac{\operatorname{sen} x}{\cos x} \frac{1}{\cos x} e^{\sec x} dx = \int \tau g x \sec x e^{\sec x} dx (*)$$

$$\text{Solución.- Sea: } u = \sec x, du = \sec x \tau g x dx$$

$$(*) = \int e^u du = e^u + c = e^{\sec x} + c$$

$$56.- \int \frac{ds}{s^{1/3}(1+s^{2/3})}$$

$$\text{Solución.- Sea: } t = s^{1/3} \Rightarrow s = t^3, ds = 3t^2 dt$$

$$\int \frac{ds}{s^{1/3}(1+s^{2/3})} = \int \frac{3t^2 dt}{t(1+t^2)} = \int \frac{3t dt}{(1+t^2)} = 3 \int \frac{t dt}{(1+t^2)} = \frac{3}{2} \ell \eta |1+t^2| + c$$

$$57.- \int \frac{1}{z^3} \left( \frac{1-z^2}{z^2} \right)^{10} dz$$

$$\text{Solución.- Sea: } u = \frac{1-z^2}{z^2}, du = \frac{-2dz}{z^3}$$

$$\int \frac{1}{z^3} \left( \frac{1-z^2}{z^2} \right)^{10} dz = -\frac{1}{2} \int u^{10} du = -\frac{1}{2} \frac{u^{11}}{11} + c = -\frac{u^{11}}{22} + c = -\frac{1}{22} \left( \frac{1-z^2}{z^2} \right)^{11} + c$$

$$58.- \int \frac{x \ell \eta (1+x^2)}{1+x^2} dx$$

$$\text{Solución.- Sea: } u = \ell \eta (1+x^2), du = \frac{2x dx}{1+x^2}$$

$$\int \frac{x \ell \eta (1+x^2)}{1+x^2} dx = \frac{1}{2} \int u du = \frac{1}{2} \frac{u^2}{2} + c = \frac{u^2}{4} + c = \frac{[\ell \eta (1+x^2)]^2}{4} + c$$

$$59.- \int \frac{\operatorname{co} \tau g x dx}{\ell \eta |\operatorname{sen} x|}$$

$$\text{Solución.- Sea: } u = \ell \eta |\operatorname{sen} x|, du = \operatorname{co} \tau g x dx$$

$$\int \frac{\operatorname{co} \tau g x dx}{\ell \eta |\operatorname{sen} x|} = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |\ell \eta |\operatorname{sen} x|| + c$$

$$60.- \int \frac{ax^2 - bx + c}{ax^2 + bx - c} dx$$

Solución.-

$$\int \frac{ax^2 - bx + c}{ax^2 + bx - c} dx = \frac{ax^2 - bx + c}{ax^2 + bx - c} \int dt = \frac{ax^2 - bx + c}{ax^2 + bx - c} t + c$$

$$61.- \int \frac{dx}{\cos^2 5x}$$

Solución.- Sea:  $u = 5x, du = 5dx$

$$\int \frac{dx}{\cos^2 5x} = \int \sec^2 5x dx = \frac{1}{5} \int \sec^2 u du = \frac{1}{5} \tau g u + c = \frac{1}{5} \tau g 5x + c$$

$$62.- \int \frac{dx}{12 - 7x}$$

Solución.- Sea:  $u = 12 - 7x, du = -7dx$

$$\int \frac{dx}{12 - 7x} = -\frac{1}{7} \int \frac{du}{u} = -\frac{1}{7} \ell \eta |u| + c = -\frac{1}{7} \ell \eta |12 - 7x| + c$$

$$63.- \int \tau g 16x dx$$

Solución.- Sea:  $u = \cos(16x), du = -16 \operatorname{sen}(16x) dx$

$$\int \tau g 16x dx = \int \frac{\operatorname{sen}(16x)}{\cos(16x)} dx = -\frac{1}{16} \int \frac{du}{u} = -\frac{1}{16} \ell \eta |u| + c = -\frac{1}{16} \ell \eta |\cos(16x)| + c$$

$$64.- \int \tau g 4\theta \sec^2 4\theta d\theta$$

Solución.- Sea:  $u = \tau g 4\theta, du = 4 \sec^2 4\theta d\theta$

$$\int \tau g 4\theta \sec^2 4\theta d\theta = \frac{1}{4} \int u du = \frac{1}{4} \frac{u^2}{2} + c = \frac{u^2}{8} + c = \frac{\tau g^2 4\theta}{8} + c$$

$$65.- \int \frac{x dx}{\sqrt{x-5}}$$

Solución.- Sea:  $u = x - 5 \Rightarrow x = u + 5, du = dx$

$$\int \frac{x dx}{\sqrt{x-5}} = \int \frac{u+5}{u^{1/2}} du = \int u^{1/2} du + 5 \int u^{-1/2} du = \frac{u^{3/2}}{3/2} + 5 \frac{u^{1/2}}{1/2} + c = \frac{2u^{3/2}}{3} + 10u^{1/2} + c$$

$$= \frac{2}{3} u \sqrt{u} + 10 \sqrt{u} + c = \frac{2}{3} (x-5) \sqrt{x-5} + 10 \sqrt{x-5} + c = 2 \sqrt{x-5} \left( \frac{x+10}{3} \right) + c$$

$$66.- \int \frac{7t-2}{\sqrt{7-2t^2}} dt$$

Solución.-

$$\int \frac{7t-2}{\sqrt{7-2t^2}} dt = \int \frac{7t dt}{\sqrt{7-2t^2}} - \int \frac{2 dt}{\sqrt{7-2t^2}} = -\frac{7}{4} \int \frac{-4t dt}{\sqrt{7-2t^2}} - \sqrt{2} \int \frac{dt}{\sqrt{7/2 - t^2}}$$

$$= -\frac{7}{2} \sqrt{7-2t^2} - \sqrt{2} \operatorname{arcsen} \sqrt{\frac{2}{7}} t + c$$

$$67.- \int (1+x) \cos \sqrt{x} dx$$

Solución.- Sea:  $\sqrt{x} = t \Rightarrow x = t^2, dx = 2t dt$

$$\int (1+x) \cos \sqrt{x} dx = \int (1+t^2)(\cos t) 2t dt = 2 \int (t+t^3)(\cos t) dt = 2 \int t \cos t dt + 2 \int t^3 \cos t dt (*)$$

Trabajando por partes:  $\int t^3 \cos t dt$

$$\begin{array}{ll} \text{Sea: } u = t^3 & dv = \cos t dt \\ & du = 3t^2 dt \quad v = \sin t \end{array}$$

$$\int t^3 \cos t dt = t^3 \sin t - 3 \int t^2 \sin t dt$$

Trabajando por partes:  $\int t^2 \sin t dt$

$$\begin{array}{ll} \text{Sea: } u = t^2 & dv = \sin t dt \\ & du = 2t dt \quad v = -\cos t \end{array}$$

$$\int t^2 \sin t dt = -t^2 \cos t + 2 \int t \cos t dt$$

Trabajando por partes:  $\int t \cos t dt$

$$\begin{array}{ll} \text{Sea: } u = t & dv = \cos t dt \\ & du = dt \quad v = \sin t \end{array}$$

$$\int t \cos t dt = t \sin t - \int \sin t dt = t \sin t + \cos t + c_1$$

$$\begin{aligned} (*) \quad 2 \int t \cos t dt + 2 \int t^3 \cos t dt &= 2 \int t \cos t dt + 2(t^3 \sin t - 3 \int t^2 \sin t dt) \\ &= 2 \int t \cos t dt + 2t^3 \sin t - 6 \int t^2 \sin t dt = 2 \int t \cos t dt + 2t^3 \sin t - 6(-t^2 \cos t + 2 \int t \cos t dt) \\ &= 2 \int t \cos t dt + 2t^3 \sin t + 6t^2 \cos t - 12 \int t \cos t dt = 2t^3 \sin t + 6t^2 \cos t - 10 \int t \cos t dt \\ &= 2t^3 \sin t + 6t^2 \cos t - 10(t \sin t + \cos t) + c \\ &= 2t^3 \sin t + 6t^2 \cos t - 10t \sin t - 10 \cos t + c \\ &= 2\sqrt{x^3} \sin \sqrt{x} + 6x \cos \sqrt{x} - 10\sqrt{x} \sin \sqrt{x} - 10 \cos \sqrt{x} + c \end{aligned}$$

**68.-**  $\int \frac{dx}{x(\sqrt{1+x}-1)}$

Solución.- Sea:  $(1+x)^{1/2} = t \Rightarrow 1+x = t^2 \Rightarrow x = t^2 - 1, dx = 2t dt$

$$\int \frac{dx}{x(\sqrt{1+x}-1)} = \int \frac{2t dt}{(t^2-1)(t-1)} (*)$$

$$\frac{t}{(t+1)(t^2-1)} = \frac{A}{t+1} + \frac{B}{t-1} + \frac{C}{(t-1)^2} \Rightarrow t = A(t-1)^2 + B(t^2-1) + C(t+1)$$

$$\text{De donde: } \begin{cases} t=1 \Rightarrow 1=2C \Rightarrow C=1/2 \\ t=-1 \Rightarrow -1=4A \Rightarrow A=-1/4 \\ t=0 \Rightarrow 0=A-B+C \Rightarrow B=1/4 \end{cases}$$

$$(*) = 2 \left[ \int \frac{Adt}{t+1} + \int \frac{Bdt}{t-1} + \int \frac{Cdt}{(t-1)^2} \right] = 2 \left[ -\frac{1}{4} \int \frac{dt}{t+1} + \frac{1}{4} \int \frac{dt}{t-1} + \frac{1}{2} \int \frac{dt}{(t-1)^2} \right]$$

$$= -\frac{1}{2} \int \frac{dt}{t+1} + \frac{1}{2} \int \frac{dt}{t-1} + \int \frac{dt}{(t-1)^2} = -\frac{1}{2} \ell \eta |t+1| + \frac{1}{2} \ell \eta |t-1| - \frac{1}{t-1} + c$$

$$= \frac{1}{2} \ell \eta \left| \frac{t-1}{t+1} \right| - \frac{1}{t-1} + c = \frac{1}{2} \ell \eta \left| \frac{\sqrt{1+x}-1}{\sqrt{1+x}+1} \right| - \frac{1}{\sqrt{1+x}-1} + c$$

**69.-**  $\int \frac{dx}{\cos \tau g 6x}$

Solución.- Sea:  $u = \cos 6x$ ,  $du = -6 \operatorname{sen} 6x dx$

$$\int \frac{dx}{\cos \tau g 6x} = \int \tau g 6x dx = \int \frac{\operatorname{sen} 6x}{\cos 6x} dx = -\frac{1}{6} \int \frac{du}{u} = -\frac{1}{6} \ell \eta |u| + c = -\frac{1}{6} \ell \eta |\cos 6x| + c$$

**70.-**  $\int \cos \tau g(2x-4) dx$

Solución.- Sea:  $u = \operatorname{sen}(2x-4)$ ,  $du = 2 \cos(2x-4) dx$

$$\int \cos \tau g(2x-4) dx = \int \frac{\cos(2x-4)}{\operatorname{sen}(2x-4)} dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ell \eta |u| + c = \frac{1}{2} \ell \eta |(2x-4)| + c$$

**71.-**  $\int (e^t - e^{-2t})^2 dt$

Solución.-

$$\int (e^t - e^{-2t})^2 dt = \int (e^{2t} - 2e^{t-2t} + e^{-4t}) dt = \int e^{2t} dt - 2 \int e^{-t} dt + \int e^{-4t} dt$$

$$= \frac{1}{2} e^{2t} + 2e^{-t} - \frac{1}{2} e^{-4t} + c$$

**72.-**  $\int \frac{(x+1)dx}{(x+2)^2(x+3)}$

Solución.-

$$\int \frac{(x+1)dx}{(x+2)^2(x+3)} \Rightarrow \frac{(x+1)}{(x+2)^2(x+3)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x+3} (*)$$

$$\Rightarrow x+1 = A(x+2)(x+3) + B(x+3) + C(x+2)^2$$

De donde: 
$$\begin{cases} x = -2 \Rightarrow -1 = B \Rightarrow B = -1 \\ x = -3 \Rightarrow -2 = C \Rightarrow C = -2 \\ x = 0 \Rightarrow 1 = 6A + 3B + 4C \Rightarrow A = 2 \end{cases}$$

$$(*) \int \frac{A dx}{x+2} + \int \frac{B dx}{(x+2)^2} + \int \frac{C dx}{x+3} = 2 \int \frac{dx}{x+2} - \int \frac{dx}{(x+2)^2} - 2 \int \frac{dx}{x+3}$$

$$= 2 \ell \eta |x+2| + \frac{1}{x+2} - 2 \ell \eta |x+3| + c = \ell \eta \left| \frac{x+2}{x+3} \right| + \frac{1}{x+2} + c$$

**73.-**  $\int (\cos \tau g e^x) e^x dx$

Solución.- Sea:  $u = \operatorname{sen} e^x$ ,  $du = (\cos e^x) e^x dx$

$$\int (\cos \tau g e^x) e^x dx = \int \frac{(\cos e^x) e^x dx}{\operatorname{sen} e^x} = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |\operatorname{sen} e^x| + c$$

**74.-**  $\int \frac{\operatorname{sen} \theta + \theta}{\cos \theta + 1} d\theta$

Solución.-

$$\begin{aligned}\int \frac{\operatorname{sen} \theta + \theta}{\cos \theta + 1} d\theta &= \int \frac{\operatorname{sen} \theta d\theta}{\cos \theta + 1} + \int \frac{\theta d\theta}{\cos \theta + 1} = -\int \frac{-\operatorname{sen} \theta d\theta}{\cos \theta + 1} + \int \frac{\theta(\cos \theta - 1)d\theta}{\cos^2 \theta + 1} \\ &= -\ell \eta |\cos \theta + 1| - \int \frac{\theta \cos \theta d\theta}{\operatorname{sen}^2 \theta} + \int \frac{\theta d\theta}{\operatorname{sen}^2 \theta} \\ &= -\ell \eta |\cos \theta + 1| - \int \theta \operatorname{co} \tau g \theta \operatorname{cosec} \theta d\theta + \int \theta \operatorname{cosec}^2 \theta d\theta (*)\end{aligned}$$

Trabajando por partes:  $\int \theta \operatorname{co} \tau g \theta \operatorname{cosec} \theta d\theta$

Sea:  $u = \theta \quad dv = \operatorname{co} \tau g \theta \operatorname{cosec} \theta d\theta$   
 $du = d\theta \quad v = -\operatorname{cosec} \theta$

$$\int \theta \operatorname{co} \tau g \theta \operatorname{cosec} \theta d\theta = -\theta \operatorname{cosec} \theta + \int \operatorname{cosec} \theta d\theta = -\theta \operatorname{cosec} \theta - \ell \eta |\operatorname{cosec} \theta - \operatorname{co} \tau g \theta| + c_1$$

Trabajando por partes:  $\int \theta \operatorname{cosec}^2 \theta d\theta$

Sea:  $u = \theta \quad dv = \operatorname{cosec}^2 \theta d\theta$   
 $du = d\theta \quad v = -\operatorname{co} \tau g \theta$

$$\int \theta \operatorname{cosec}^2 \theta d\theta = -\theta \operatorname{co} \tau g \theta + \int \operatorname{co} \tau g \theta d\theta = -\theta \operatorname{co} \tau g \theta + \ell \eta |\operatorname{sen} \theta| + c_2$$

$$(*) = -\ell \eta |\cos \theta + 1| + \theta \operatorname{cosec} \theta + \ell \eta |\operatorname{cosec} \theta - \operatorname{co} \tau g \theta| - \theta \operatorname{co} \tau g \theta + \ell \eta |\operatorname{sen} \theta| + c$$

$$= \ell \eta \left| \frac{(\operatorname{cosec} \theta - \operatorname{co} \tau g \theta) \operatorname{sen} \theta}{\cos \theta + 1} \right| + \theta (\operatorname{cosec} \theta - \operatorname{co} \tau g \theta) + c$$

$$= \ell \eta \left| \frac{1 - \cos \theta}{1 + \cos \theta} \right| + \theta \left( \frac{1 - \cos \theta}{\operatorname{sen} \theta} \right) + c$$

**75.-**  $\int \frac{\operatorname{arc} \tau g x dx}{(1+x^2)^{3/2}}$

Solución.- Sea:  $x = \tau g \theta \Rightarrow \theta = \operatorname{arc} \tau g x, dx = \sec^2 \theta d\theta, \sqrt{1+x^2} = \sec \theta$

$$\int \frac{\operatorname{arc} \tau g x dx}{(1+x^2)^{3/2}} = \int \frac{\theta \sec^2 \theta d\theta}{\sec^3 \theta} = \int \frac{\theta d\theta}{\sec \theta} = \int \theta \cos \theta d\theta (*), \text{trabajando por partes}$$

Sea:  $u = \theta \quad dv = \cos \theta d\theta$   
 $du = d\theta \quad v = \operatorname{sen} \theta$

$$= \theta \operatorname{sen} \theta - \int \operatorname{sen} \theta d\theta = \theta \operatorname{sen} \theta + \cos \theta + c = (\operatorname{arc} \tau g x) \frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}} + c$$

$$= \frac{1}{\sqrt{1+x^2}} (x \operatorname{arc} \tau g x + 1) + c$$

**76.-**  $\int x \operatorname{co} \tau g(x^2/5) dx$

Solución.- Sea:  $u = \operatorname{sen} \frac{x^2}{5}, du = \frac{2}{5} x \cos \frac{x^2}{5} dx$



$$\int x \cot g \left( \frac{x^2}{5} \right) dx = \int \frac{x \cos \frac{x^2}{5}}{\operatorname{sen} \frac{x^2}{5}} dx = \frac{5}{2} \int \frac{du}{u} = \frac{5}{2} \ell \eta |u| + c = \frac{5}{2} \ell \eta \left| \operatorname{sen} \frac{x^2}{5} \right| + c$$

**77.-**  $\int x \sqrt{4x^2 - 2} dx$

Solución.- Sea:  $u = 4x^2 - 2, dx = 8x dx$

$$\int x \sqrt{4x^2 - 2} dx = \frac{1}{8} \int u^{\frac{1}{2}} du = \frac{1}{8} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{u^{\frac{3}{2}}}{12} + c = \frac{\sqrt{(4x^2 - 2)^3}}{12} + c$$

**78.-**  $\int \frac{(x^2 + 9)^{\frac{1}{2}} dx}{x^4}$

Solución.- Sea:  $x = 3 \operatorname{tg} \theta, dx = 3 \sec^2 \theta, \sqrt{x^2 + 9} = 3 \sec \theta$

$$\int \frac{(x^2 + 9)^{\frac{1}{2}} dx}{x^4} = \int \frac{3 \sec \theta 3 \sec^2 \theta d\theta}{3^4 \operatorname{tg}^4 \theta} = \frac{1}{9} \int \frac{\sec^3 \theta d\theta}{\operatorname{tg}^4 \theta} = \frac{1}{9} \int \frac{\frac{1}{\cos^3 \theta} d\theta}{\frac{\operatorname{sen}^4 \theta}{\cos^4 \theta}} = \frac{1}{9} \int \frac{\cos \theta d\theta}{\operatorname{sen}^4 \theta}$$

$$= \frac{1}{9} \left( -\frac{1}{3 \operatorname{sen}^3 \theta} \right) + c = -\frac{1}{27 \operatorname{sen}^3 \theta} + c = -\frac{\operatorname{cosec}^3 \theta}{27} + c$$

$$= -\frac{1}{27} \left( \frac{\sqrt{x^2 + 9}}{x} \right)^3 + c = -\frac{x^2 + 9}{27 x^3} \sqrt{x^2 + 9} + c$$

**79.-**  $\int x^2 \operatorname{sen}^5 x^3 \cos x^3 dx$

Solución.- Sea:  $u = \operatorname{sen} x^3, du = 3x^2 \cos x^3 dx$

$$\int x^2 \operatorname{sen}^5 x^3 \cos x^3 dx = \frac{1}{3} \int u^5 du = \frac{1}{3} \frac{u^6}{6} + c = \frac{u^6}{18} + c = \frac{\operatorname{sen}^6 x^3}{18} + c$$

**80.-**  $\int \frac{xdx}{\sqrt{5x^2 + 7}}$

Solución.- Sea:  $u = 5x^2 + 7, du = 10x dx$

$$\int \frac{xdx}{\sqrt{5x^2 + 7}} = \frac{1}{10} \int \frac{du}{u^{\frac{1}{2}}} = \frac{1}{10} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{u^{\frac{1}{2}}}{5} + c = \frac{(5x^2 + 7)^{\frac{1}{2}}}{5} + c = \frac{\sqrt{5x^2 + 7}}{5} + c$$

**81.-**  $\int \frac{x^3 dx}{x^2 - x - 6}$

Solución.-

$$\begin{aligned} \int \frac{x^3 dx}{x^2 - x - 6} &= \int \left( x + 1 + \frac{7x + 6}{x^2 - x - 6} \right) dx = \int x dx + \int dx + \int \frac{(7x + 6) dx}{(x - 3)(x + 2)} \\ &= \frac{x^2}{2} + x + \int \frac{(7x + 6) dx}{(x - 3)(x + 2)} (*) \end{aligned}$$

$$\frac{(7x+6)}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \Rightarrow 7x+6 = A(x+2) + B(x-3)$$

De donde: 
$$\begin{cases} x = -2 \Rightarrow -8 = -5B \Rightarrow B = 8/5 \\ x = 3 \Rightarrow 27 = 5A \Rightarrow A = 27/5 \end{cases}$$

$$\begin{aligned} (*) &= \frac{x^2}{2} + x + \int \frac{A dx}{x-3} + \int \frac{B dx}{x+2} = \frac{x^2}{2} + x + \frac{27}{5} \int \frac{dx}{x-3} + \frac{8}{5} \int \frac{dx}{x+2} \\ &= \frac{x^2}{2} + x + \frac{27}{5} \ell \eta |x-3| + \frac{8}{5} \ell \eta |x+2| + c \end{aligned}$$

**82.-**  $\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2 \theta} d\theta$

Solución.- Sea:  $u = \operatorname{sen}^2 \theta, du = 2 \operatorname{sen} \theta \cos \theta d\theta$

$$\int \operatorname{sen} 2\theta e^{\operatorname{sen}^2 \theta} d\theta = \int 2 \operatorname{sen} \theta \cos \theta e^{\operatorname{sen}^2 \theta} d\theta = \int e^u du = e^u + c = e^{\operatorname{sen}^2 \theta} + c$$

**83.-**  $\int \frac{dx}{e^x - 9e^{-x}}$

Solución.- Sea:  $u = e^x, du = e^x dx$

$$\int \frac{dx}{e^x - 9e^{-x}} = \int \frac{e^x dx}{e^{2x} - 9} = \int \frac{e^x dx}{(e^x)^2 - 9} = \int \frac{du}{u^2 - 9} = \frac{1}{6} \ell \eta \left| \frac{u-3}{u+3} \right| + c = \frac{1}{6} \ell \eta \left| \frac{e^x - 3}{e^x + 3} \right| + c$$

**84.-**  $\int \frac{dw}{1 + \cos w}$

Solución.-

$$\begin{aligned} \int \frac{dw}{1 + \cos w} &= \int \frac{(1 - \cos w) dw}{1 - \cos^2 w} = \int \frac{(1 - \cos w) dw}{\operatorname{sen}^2 w} = \int \cos ec^2 w dw - \int \frac{\cos w dw}{\operatorname{sen}^2 w} \\ &= -\cot gw - \frac{(\operatorname{sen} w)^{-1}}{-1} + c = -\cot gw + \frac{1}{\operatorname{sen} w} + c = -\cot gw + \operatorname{cosec} w + c \end{aligned}$$

**Nota:** Este ejercicio esta desarrollado diferente en el capitulo 8.

**85.-**  $\int e^{\left(\frac{1 - \operatorname{sen}^2 x/2}{3}\right)^2} (\cos^3 x/2 \operatorname{sen} x/2) dx$

Solución.- Sea:  $u = \left(\frac{1 - \operatorname{sen}^2 x/2}{3}\right)^2, du = -\frac{2}{9} \cos^3 \frac{x}{2} \operatorname{sen} \frac{x}{2} dx$

$$\int e^{\left(\frac{1 - \operatorname{sen}^2 x/2}{3}\right)^2} (\cos^3 x/2 \operatorname{sen} x/2) dx = -\frac{9}{2} \int e^u du = -\frac{2}{9} e^u + c = -\frac{2}{9} e^{\left(\frac{1 - \operatorname{sen}^2 x/2}{3}\right)^2} + c$$

**86.-**  $\int \frac{x^3 dx}{\sqrt{19 - x^2}}$

Solución.- Sea:  $x = \sqrt{19} \operatorname{sen} \theta, dx = \sqrt{19} \cos \theta d\theta, \sqrt{19 - x^2} = \sqrt{19} \cos \theta$

$$\int \frac{x^3 dx}{\sqrt{19 - x^2}} = \int \frac{(\sqrt{19})^3 \operatorname{sen}^3 \theta \cancel{\sqrt{19} \cos \theta} d\theta}{\cancel{\sqrt{19} \cos \theta}} = 19\sqrt{19} \int \operatorname{sen} \theta (1 - \cos^2 \theta) d\theta$$

$$\begin{aligned}
&= 19\sqrt{19} \int \operatorname{sen} \theta d\theta - 19\sqrt{19} \int \operatorname{sen} \theta \cos^2 \theta d\theta = -19\sqrt{19} \cos \theta + \frac{19\sqrt{19}}{3} \cos^3 \theta + c \\
&= -19\sqrt{19} \frac{\sqrt{19-x^2}}{\sqrt{19}} + \frac{19\sqrt{19}}{3} \frac{\sqrt{(19-x^2)^3}}{(\sqrt{19})^3} + c = -19\sqrt{19-x^2} + \sqrt{(19-x^2)^3} + c
\end{aligned}$$

**87.-**  $\int \frac{\operatorname{sen} \varphi d\varphi}{\cos^{\frac{1}{2}} \varphi}$

Solución.- Sea:  $u = \cos \varphi, du = -\operatorname{sen} \varphi d\varphi$

$$\int \frac{\operatorname{sen} \varphi d\varphi}{\cos^{\frac{1}{2}} \varphi} = -\int \frac{du}{u^{\frac{1}{2}}} = -\int u^{-\frac{1}{2}} du = -\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = -2u^{\frac{1}{2}} + c = -2\sqrt{\cos \varphi} + c$$

**88.-**  $\int (\sec \varphi + \tau g \varphi)^2 d\varphi$

Solución.-

$$\begin{aligned}
\int (\sec \varphi + \tau g \varphi)^2 d\varphi &= \int (\sec^2 \varphi + 2 \sec \varphi \tau g \varphi + \tau g^2 \varphi) d\varphi \\
&= \int (\sec^2 \varphi + 2 \sec \varphi \tau g \varphi + \sec^2 \varphi - 1) d\varphi = \int (2 \sec^2 \varphi + 2 \sec \varphi \tau g \varphi - 1) d\varphi \\
&= 2 \int \sec^2 \varphi d\varphi + 2 \int \sec \varphi \tau g \varphi d\varphi - \int d\varphi = 2 \tau g \varphi + 2 \sec \varphi - \varphi + c
\end{aligned}$$

**89.-**  $\int \frac{dt}{t(4 + \ell \eta^2 t)^{\frac{1}{2}}}$

Solución.- Sea:  $u = \ell \eta t, du = \frac{dt}{t}$ , además:  $u = 2 \tau g \theta, du = 2 \sec^2 \theta d\theta, \sqrt{4+u^2} = 2 \sec \theta$

$$\begin{aligned}
\int \frac{dt}{t(4 + \ell \eta^2 t)^{\frac{1}{2}}} &= \int \frac{du}{\sqrt{4+u^2}} = \int \frac{2 \sec^2 \theta d\theta}{2 \sec \theta} = \int \sec \theta d\theta = \ell \eta |\sec \theta + \tau g \theta| + c \\
&= \ell \eta \left| \frac{\sqrt{4+u^2}}{2} + \frac{u}{2} \right| + c = \ell \eta \left| \frac{\sqrt{4+u^2} + u}{2} \right| + c = \ell \eta \left| \frac{\sqrt{4 + \ell \eta^2 t} + \ell \eta t}{2} \right| + c
\end{aligned}$$

**90.-**  $\int a^\theta b^{2\theta} c^{3\theta} d\theta$

Solución.- Sea:  $ab^2c^3 = k$ ,

$$\int a^\theta b^{2\theta} c^{3\theta} d\theta = \int a^\theta (b^2)^\theta (c^3)^\theta d\theta = \int (ab^2c^3)^\theta d\theta = \int k^\theta d\theta = \frac{k^\theta}{\ell \eta |k|} + c = \frac{(ab^2c^3)^\theta}{\ell \eta |(ab^2c^3)|} + c$$

**91.-**  $\int \operatorname{sen}^{\frac{1}{2}} \varphi \cos^3 \varphi d\varphi$

Solución.-

$$\begin{aligned}
\int \operatorname{sen}^{\frac{1}{2}} \varphi \cos^3 \varphi d\varphi &= \int \operatorname{sen}^{\frac{1}{2}} \varphi \cos^2 \varphi \cos \varphi d\varphi = \int \operatorname{sen}^{\frac{1}{2}} \varphi (1 - \operatorname{sen}^2 \varphi) \cos \varphi d\varphi \\
&= \int \operatorname{sen}^{\frac{1}{2}} \varphi \cos \varphi d\varphi - \int \operatorname{sen}^{\frac{5}{2}} \varphi \cos \varphi d\varphi = \frac{\operatorname{sen}^{\frac{3}{2}} \varphi}{\frac{3}{2}} - \frac{\operatorname{sen}^{\frac{7}{2}} \varphi}{\frac{7}{2}} + c \\
&= \frac{2 \operatorname{sen}^{\frac{3}{2}} \varphi}{3} - \frac{2 \operatorname{sen}^{\frac{7}{2}} \varphi}{7} + c
\end{aligned}$$

$$92.- \int \frac{\sec^2 \theta d\theta}{9 + \tau g^2 \theta}$$

Solución.- Sea:  $u = \tau g \theta, du = \sec^2 \theta d\theta$

$$\int \frac{\sec^2 \theta d\theta}{9 + \tau g^2 \theta} = \int \frac{du}{9 + u^2} = \frac{1}{3} \arctan \tau g \frac{u}{3} + c = \frac{1}{3} \arctan \tau g \frac{(\tau g \theta)}{3} + c$$

$$93.- \int \frac{dx}{\sqrt{e^{2x} - 16}}$$

Solución.- Sea:  $u = e^x, du = e^x dx \Rightarrow dx = \frac{du}{u}$

Además:  $u = 4 \sec \theta, du = 4 \sec \theta \tau g \theta d\theta, \sqrt{u^2 - 16} = 4 \tau g \theta$

$$\begin{aligned} \int \frac{dx}{\sqrt{e^{2x} - 16}} &= \int \frac{\frac{du}{u}}{\sqrt{u^2 - 16}} = \int \frac{du}{u \sqrt{u^2 - 16}} = \int \frac{\cancel{4 \sec \theta} \tau g \theta d\theta}{\cancel{4 \sec \theta} 4 \tau g \theta} = \frac{1}{4} \int d\theta = \frac{1}{4} \theta + c \\ &= \frac{1}{4} \arcsin \frac{u}{4} + c = \frac{1}{4} \arcsin \frac{e^x}{4} + c \end{aligned}$$

$$94.- \int (e^{2s} - 1)(e^{2s} + 1) ds$$

Solución.-

$$\int (e^{2s} - 1)(e^{2s} + 1) ds = \int [(e^{2s})^2 - 1] ds = \int e^{4s} ds - \int ds = \frac{1}{4} e^{4s} + s + c$$

$$95.- \int \frac{dx}{5x^2 + 8x + 5}$$

Solución.-

$$\int \frac{dx}{5x^2 + 8x + 5} = \int \frac{dx}{5(x^2 + \frac{8}{5}x + 1)} = \frac{1}{5} \int \frac{dx}{x^2 + \frac{8}{5}x + 1} (*), \text{ completando cuadrados:}$$

$$x^2 + \frac{8}{5}x + 1 = (x^2 + \frac{8}{5}x + \frac{16}{25}) + 1 - \frac{16}{25} = (x + \frac{4}{5})^2 + \frac{9}{25} = (x + \frac{4}{5})^2 + (\frac{3}{5})^2$$

$$(*) = \frac{1}{5} \int \frac{dx}{(x + \frac{4}{5})^2 + (\frac{3}{5})^2} = \frac{1}{5} \cdot \frac{1}{\cancel{3} \frac{3}{\cancel{5}}} \arctan \tau g \frac{x + \frac{4}{5}}{\frac{3}{5}} + c = \frac{1}{3} \arctan \tau g \frac{5x + 4}{3} + c$$

$$96.- \int \frac{x^3 + 1}{x^3 - x} dx$$

Solución.-

$$\int \frac{x^3 + 1}{x^3 - x} dx = \int \left( 1 + \frac{x + 1}{x^3 - x} \right) dx = \int dx + \int \frac{x + 1}{x^3 - x} dx = x + \int \frac{(x + 1) dx}{x(x^2 - 1)}$$

$$= x + \int \frac{\cancel{(x + 1)} dx}{x \cancel{(x + 1)} (x - 1)} = x + \int \frac{dx}{x(x - 1)} = x + \int \frac{A dx}{x} + \int \frac{B dx}{x - 1} (*)$$

$$\frac{1}{x(x - 1)} = \frac{A}{x} + \frac{B}{x - 1} \Rightarrow 1 = A(x - 1) + Bx$$

De donde:  $\begin{cases} x=0 \Rightarrow 1=-A \Rightarrow A=-1 \\ x=1 \Rightarrow 1=B \Rightarrow B=1 \end{cases}$

$$(*) = x - \int \frac{dx}{x} + \int \frac{dx}{x-1} = x - \ell \eta |x| + \ell \eta |x-1| + c = x + \ell \eta \left| \frac{x-1}{x} \right| + c$$

**97.-**  $\int (\arcsen \sqrt{1-x^2})^0 dx$

Solución.-

$$\int (\arcsen \sqrt{1-x^2})^0 dx = \int dx = x + c$$

**98.-**  $\int \frac{3dy}{1+\sqrt{y}}$

Solución.-Sea:  $y^{1/2} = t \Rightarrow y = t^2, dy = 2tdt$

$$\begin{aligned} \int \frac{3dy}{1+\sqrt{y}} &= 3 \int \frac{dy}{1+\sqrt{y}} = 3 \int \frac{2tdt}{1+t} = 6 \int \frac{tdt}{1+t} = 6 \int \left( 1 - \frac{1}{1+t} \right) dt = 6 \int dt - 6 \int \frac{dt}{1+t} \\ &= 6t - 6\ell \eta |1+t| + c = 6\sqrt{y} - 6\ell \eta |1+\sqrt{y}| + c = 6(\sqrt{y} - \ell \eta |1+\sqrt{y}|) + c \end{aligned}$$

**99.-**  $\int x(1+x)^{1/5} dx$

Solución.-Sea:  $u = 1+x \Rightarrow x = u-1, du = dx$

$$\begin{aligned} \int x(1+x)^{1/5} dx &= \int (u-1)u^{1/5} du = \int (u^{6/5} - u^{1/5}) du = \int u^{6/5} du - \int u^{1/5} du = \frac{u^{11/5}}{11/5} - \frac{u^{6/5}}{6/5} + c \\ &= \left( \frac{5u^2}{11} - \frac{5u}{6} \right) u^{1/5} + c = \left( \frac{5(1+x)^2}{11} - \frac{5(1+x)}{6} \right) (1+x)^{1/5} + c \end{aligned}$$

**100.-**  $\int \frac{d\varphi}{a^2 \sen^2 \varphi + b^2 \cos^2 \varphi}$

Solución.-Sea:  $u = \tau g \varphi, du = \sec^2 \varphi d\varphi$

$$\begin{aligned} \int \frac{d\varphi}{a^2 \sen^2 \varphi + b^2 \cos^2 \varphi} &= \int \frac{\sen^4 \varphi d\varphi}{\frac{1}{\cos^2 \varphi} (a^2 \tau g^2 \varphi + b^2)} = \int \frac{\sen^2 \varphi d\varphi}{(a^2 \tau g^2 \varphi + b^2)} = \int \frac{du}{(a^2 u^2 + b^2)} \\ &= \frac{1}{a^2} \int \frac{du}{u^2 + (b/a)^2} = \frac{1}{a^2} \frac{1}{b/a} \arctan \tau g \frac{u}{b/a} + c = \frac{1}{ab} \arctan \tau g \frac{au}{b} + c = \frac{1}{ab} \arctan \tau g \left( \frac{a \tau g \varphi}{b} \right) + c \end{aligned}$$

**101.-**  $\int \frac{tdt}{(2t+1)^{1/2}}$

Solución.-

Sea:  $\begin{array}{ll} u = t & dv = \frac{dt}{\sqrt{2t+1}} \\ du = dt & v = \sqrt{2t+1} \end{array}$

$$\int \frac{tdt}{(2t+1)^{3/2}} = t\sqrt{2t+1} - \int \sqrt{2t+1} dt = t\sqrt{2t+1} - \frac{1}{2} \frac{(2t+1)^{3/2}}{3/2} + c = t\sqrt{2t+1} - \frac{(2t+1)^{3/2}}{3} + c$$

$$= \sqrt{2t+1} \left( t - \frac{2t+1}{3} \right) + c = \frac{\sqrt{2t+1}}{3} (t-1) + c$$

**102.-**  $\int \frac{s\ell\eta|s|ds}{(1-s^2)^{3/2}}$

Solución.-

Sea:  $u = \ell\eta|s|$   $dv = \frac{sds}{(1-s^2)^{3/2}}$ , además:  $s = \text{sen } \theta, ds = \cos \theta, \sqrt{1-s^2} = \cos \theta$

$du = \frac{ds}{s}$   $v = -(1-s^2)^{-1/2}$

$$\int \frac{s\ell\eta|s|ds}{(1-s^2)^{3/2}} = -\sqrt{1-s^2}\ell\eta|s| + \int \frac{\sqrt{1-s^2}}{s} ds = -\sqrt{1-s^2}\ell\eta|s| + \int \frac{\cos \theta \cos \theta d\theta}{\text{sen } \theta}$$

$$= -\sqrt{1-s^2}\ell\eta|s| + \int \frac{(1-\text{sen}^2 \theta)d\theta}{\text{sen } \theta} = -\sqrt{1-s^2}\ell\eta|s| + \int \text{cosec } \theta d\theta - \int \text{sen } \theta d\theta$$

$$= -\sqrt{1-s^2}\ell\eta|s| + \ell\eta|\text{cosec } \theta - \text{cosen } \theta| + \cos \theta + c$$

$$= -\sqrt{1-s^2}\ell\eta|s| + \ell\eta \left| \frac{1-\sqrt{1-s^2}}{s} \right| + \sqrt{1-s^2} + c$$

**103.-**  $\int (2\cos \alpha \text{sen } \alpha - \text{sen } \alpha) d\alpha$

Solución.-

$$\int (2\cos \alpha \text{sen } \alpha - \text{sen } 2\alpha) d\alpha = \int (\text{sen } 2\alpha - \text{sen } 2\alpha) d\alpha = \int 0 d\alpha = c$$

**104.-**  $\int t^4 \ell \eta^2 t dt$

Sea:  $u = \ell \eta^2 t$   $dv = t^4 dt$

$du = 2\ell \eta t \frac{dt}{t}$   $v = \frac{t^5}{5}$

$$\int t^4 \ell \eta^2 t dt = \frac{t^5}{5} \ell \eta^2 t - \frac{2}{5} \int t^4 \ell \eta t dt (*)$$
 , trabajando por partes nuevamente:

Sea:  $u = \ell \eta t$   $dv = t^4 dt$

$du = \frac{dt}{t}$   $v = \frac{t^5}{5}$

$$(*) = \frac{t^5}{5} \ell \eta^2 t - \frac{2}{5} \left( \frac{t^5}{5} \ell \eta t - \frac{1}{5} \int t^4 dt \right) = \frac{t^5}{5} \ell \eta^2 t - \frac{2t^5}{25} \ell \eta t + \frac{2}{25} \frac{t^5}{5} + c$$

$$= \frac{t^5}{5} \ell \eta^2 t - \frac{2t^5}{25} \ell \eta t + \frac{2t^5}{125} + c$$

**105.-**  $\int u^2(1+v)^{11} dx$

Solución.-

$$\int u^2(1+v)^{11} dx = u^2(1+v)^{11} \int dx = u^2(1+v)^{11} x + c$$

$$106.- \int \frac{(\varphi + \operatorname{sen} 3\varphi) d\varphi}{3\varphi^2 - 2 \cos 3\varphi}$$

Solución.-Sea:  $u = 3\varphi^2 - 2 \cos 3\varphi, du = 6(\varphi + \operatorname{sen} 3\varphi) d\varphi$

$$\int \frac{(\varphi + \operatorname{sen} 3\varphi) d\varphi}{3\varphi^2 - 2 \cos 3\varphi} = \frac{1}{6} \int \frac{du}{u} = \frac{1}{6} \ell \eta |u| + c = \frac{1}{6} \ell \eta |3\varphi^2 - 2 \cos 3\varphi| + c$$

$$107.- \int \frac{(y^{\frac{1}{2}} + 1) dy}{y^{\frac{1}{2}}(y+1)}$$

Solución.-Sea:  $y^{\frac{1}{2}} = t \Rightarrow y = t^2, dy = 2t dt$

$$\begin{aligned} \int \frac{(y^{\frac{1}{2}} + 1) dy}{y^{\frac{1}{2}}(y+1)} &= \int \frac{(t+1) 2t dt}{t(t^2+1)} = 2 \int \frac{(t+1) dt}{(t^2+1)} = \int \frac{2t dt}{(t^2+1)} + \int \frac{dt}{(t^2+1)} = \ell \eta |t^2+1| + 2 \operatorname{arc} \tau g t + c \\ &= \ell \eta |y+1| + 2 \operatorname{arc} \tau g \sqrt{y} + c \end{aligned}$$

$$108.- \int \frac{ds}{s^3(s^2-4)^{\frac{1}{2}}}$$

Solución.-Sea:  $s = 2 \sec \theta, ds = 2 \sec \theta \tau g \theta d\theta$

$$\begin{aligned} \int \frac{ds}{s^3(s^2-4)^{\frac{1}{2}}} &= \int \frac{2 \sec \theta \tau g \theta d\theta}{8 \sec^3 \theta \sqrt{4 \tan^2 \theta}} = \frac{1}{8} \int \frac{d\theta}{\sec^2 \theta} = \frac{1}{8} \int \cos^2 \theta d\theta = \frac{1}{16} \int (1 + \cos 2\theta) d\theta \\ &= \frac{1}{16} \theta + \frac{1}{32} \operatorname{sen} 2\theta + c = \frac{1}{16} \left( \theta + \frac{\operatorname{sen} 2\theta}{2} \right) + c = \frac{1}{16} (\theta + \operatorname{sen} \theta \cos \theta) + c \\ &= \frac{1}{16} \left( \operatorname{arc} \sec \frac{s}{2} + \frac{2\sqrt{s^2-4}}{s^2} \right) + c \end{aligned}$$

$$109.- \int \sqrt{u}(1+u^2)^2 du$$

Solución.-

$$\begin{aligned} \int \sqrt{u}(1+u^2)^2 du &= \int \sqrt{u}(1+2u^2+u^4) du = \int u^{\frac{1}{2}} du + 2 \int u^{\frac{5}{2}} du + \int u^{\frac{9}{2}} du \\ &= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{u^{\frac{7}{2}}}{\frac{7}{2}} + \frac{u^{\frac{11}{2}}}{\frac{11}{2}} + c = \frac{2u^{\frac{3}{2}}}{3} + \frac{4u^{\frac{7}{2}}}{7} + \frac{2u^{\frac{11}{2}}}{11} + c = \frac{2u\sqrt{u}}{3} + \frac{4u^3\sqrt{u}}{7} + \frac{2u^5\sqrt{u}}{11} + c \\ &= \sqrt{u} \left( \frac{2u}{3} + \frac{4u^3}{7} + \frac{2u^5}{11} \right) + c \end{aligned}$$

$$110.- \int \frac{(x^3 + x^2) dx}{x^2 + x - 2}$$

Solución.-

$$\int \frac{(x^3 + x^2) dx}{x^2 + x - 2} = \int \left( x + \frac{2x}{x^2 + x - 2} \right) dx = \int x dx + \int \frac{2x dx}{(x+2)(x-1)} = \frac{x^2}{2} + \int \frac{2x dx}{(x+2)(x-1)}$$

$$= \frac{x^2}{2} + \int \frac{2x dx}{(x+2)(x-1)} = \frac{x^2}{2} + \int \frac{A dx}{x+2} + \int \frac{B dx}{x-1} (*)$$

$$\frac{2x}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1} \Rightarrow 2x = A(x-1) + B(x+2)$$

$$\text{De donde: } \begin{cases} x=1 \Rightarrow 2=3B \Rightarrow B=\frac{2}{3} \\ x=-2 \Rightarrow -4=-3A \Rightarrow A=\frac{4}{3} \end{cases}$$

$$(*) = \frac{x^2}{2} + \frac{4}{3} \int \frac{dx}{x+2} + \frac{2}{3} \int \frac{dx}{x-1} = \frac{x^2}{2} + \frac{4}{3} \ell \eta |x+2| + \frac{2}{3} \ell \eta |x-1| + c$$

$$= \frac{x^2}{2} + \frac{2}{3} \ell \eta |(x+2)^2(x-1)| + c$$

$$111.- \int a db$$

Solución.-

$$\int a db = a \int db = ab + c$$

$$112.- \int \frac{dx}{\sqrt{x^2 - 2x - 8}}$$

Solución.-

Completando cuadrados se tiene:  $x^2 - 2x - 8 = (x^2 - 2x + 1) - 9 = (x-1)^2 - 3^2$

Sea:  $x-1 = 3 \sec \theta$ ,  $dx = 3 \sec \theta \tau g \theta d\theta$ ,  $\sqrt{(x-1)^2 - 3^2} = 3 \tau g \theta$ , luego:

$$\int \frac{dx}{\sqrt{x^2 - 2x - 8}} = \int \frac{dx}{\sqrt{(x-1)^2 - 3^2}} = \int \frac{\cancel{3} \sec \theta \cancel{\tau g \theta} d\theta}{\cancel{3} \cancel{\tau g \theta}} = \int \sec \theta d\theta = \ell \eta |\sec \theta + \tau g \theta| + c$$

$$= \ell \eta \left| \frac{x-1}{3} + \frac{\sqrt{x^2 - 2x - 8}}{3} \right| + c$$

$$113.- \int \frac{(x+1)dx}{\sqrt{2x-x^2}}$$

Solución.-

Completando cuadrados se tiene:

$$2x - x^2 = -(x^2 - 2x) = -(x^2 - 2x + 1 - 1) = -(x^2 - 2x + 1) + 1 = 1 - (x^2 - 1)$$

Sea:  $x-1 = \sec \theta$ ,  $dx = \cos \theta d\theta$ ,  $\sqrt{1 - (x-1)^2} = \cos \theta$ , luego:

$$\int \frac{(x+1)dx}{\sqrt{2x-x^2}} = -\frac{1}{2} \int \frac{(2-2x)-4}{\sqrt{2x-x^2}} dx = -\frac{1}{2} \int \frac{(2-2x)dx}{\sqrt{2x-x^2}} + 2 \int \frac{dx}{\sqrt{2x-x^2}}$$

$$= -\sqrt{2x-x^2} + 2 \int \frac{dx}{\sqrt{2x-x^2}} = -\sqrt{2x-x^2} + 2 \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

$$= -\sqrt{2x-x^2} + 2 \int \frac{\cancel{\cos \theta} d\theta}{\cancel{\cos \theta}} = -\sqrt{2x-x^2} + 2\theta + c = -\sqrt{2x-x^2} + 2 \arcsen(x-1) + c$$

$$114.- \int f(x) f'(x) dx$$



Solución.- Sea:  $u = f(x)$ ,  $du = f'(x)dx$

$$\int f(x)f'(x)dx = \int udu = \frac{u^2}{2} + c = \frac{[f(x)]^2}{2} + c$$

**115.-**  $\int \frac{x^3 + 7x^2 - 5x + 5}{x^2 + 2x - 3} dx$

Solución.-

$$\int \frac{x^3 + 7x^2 - 5x + 5}{x^2 + 2x - 3} dx = \int \left( x + 5 + \frac{20 - 12x}{x^2 + 2x - 3} \right) dx = \int x dx + 5 \int dx + \int \frac{(20 - 12x)dx}{x^2 + 2x - 3}$$

$$\int x dx + 5 \int dx + \int \frac{(20 - 12x)dx}{(x+3)(x-1)} = \frac{x^2}{2} + 5x + \int \frac{A dx}{x+3} + \int \frac{B}{x-1} (*)$$

$$20 - 12x = A(x-1) + B(x+3)$$

De donde:  $\begin{cases} x=1 \Rightarrow 8 = 4B \Rightarrow B=2 \\ x=-3 \Rightarrow 56 = -4A \Rightarrow A=-14 \end{cases}$

$$(*) = \frac{x^2}{2} + 5x - 14 \int \frac{dx}{x+3} + 2 \int \frac{dx}{x-1} = \frac{x^2}{2} + 5x + 14 \ell \eta |x+3| + 2 \ell \eta |x-1| + c$$

**116.-**  $\int e^{\ell \eta |1+x+x^2|} dx$

Solución.-

$$\int e^{\ell \eta |1+x+x^2|} dx = \int (1+x+x^2) dx = x + \frac{x^2}{2} + \frac{x^3}{3} + c$$

**117.-**  $\int \frac{(x-1)dx}{\sqrt{x^2 - 4x + 3}}$

Solución.-

Completando cuadrados se tiene:  $x^2 - 4x + 3 = x^2 - 4x + 4 - 1 = (x-2)^2 - 1$

Sea:  $x-2 = \sec \theta$ ,  $dx = \sec \theta \tau g \theta d\theta$ ,  $\sqrt{(x-2)^2 - 1} = \tau g \theta$ , luego:

$$\int \frac{(x-1)dx}{\sqrt{x^2 - 4x + 3}} = \frac{1}{2} \int \frac{(2x-4)+2}{\sqrt{x^2 - 4x + 3}} dx = \frac{1}{2} \int \frac{(2x-4)dx}{\sqrt{x^2 - 4x + 3}} + \int \frac{dx}{\sqrt{x^2 - 4x + 3}}$$

$$= \sqrt{x^2 - 4x + 3} + \int \frac{dx}{\sqrt{x^2 - 4x + 3}} = \sqrt{x^2 - 4x + 3} + \int \frac{dx}{\sqrt{(x-2)^2 - 1}}$$

$$= \sqrt{x^2 - 4x + 3} + \int \frac{\sec \theta \cancel{\tau g \theta} d\theta}{\cancel{\tau g \theta}} = \sqrt{x^2 - 4x + 3} + \int \sec \theta d\theta$$

$$= \sqrt{x^2 - 4x + 3} + \ell \eta |\sec \theta + \tau g \theta| + c$$

$$= \sqrt{x^2 - 4x + 3} + \ell \eta |x-2 + \sqrt{x^2 - 4x + 3}| + c$$

**118.-**  $\int \frac{x dx}{\sqrt{x^2 + 4x + 5}}$

Solución.-

Completando cuadrados se tiene:  $x^2 + 4x + 5 = x^2 + 4x + 4 + 1 = (x + 2)^2 + 1$

Sea:  $x + 2 = \tau g \theta$ ,  $dx = \sec^2 \theta d\theta$ ,  $\sqrt{(x + 2)^2 + 1} = \sec \theta$ , luego:

$$\begin{aligned} \int \frac{x dx}{\sqrt{x^2 + 4x + 5}} &= \int \frac{x dx}{\sqrt{(x + 2)^2 + 1}} = \int \frac{(\tau g \theta - 2) \sec^2 \theta d\theta}{\sec \theta} = \int \tau g \theta \sec \theta d\theta - 2 \int \sec \theta d\theta \\ &= \sec \theta - 2 \ell \eta |\sec \theta + \tau g \theta| + c = \sqrt{x^2 + 4x + 5} - 2 \ell \eta \left| \sqrt{x^2 + 4x + 5} + x + 2 \right| + c \end{aligned}$$

**119.-**  $\int \frac{4dx}{x^3 + 4x}$

Solución.-

$$\begin{aligned} \int \frac{4dx}{x^3 + 4x} &= \int \frac{(3x^2 + 4) - 3x^2}{x^3 + 4x} dx = \int \frac{(3x^2 + 4)dx}{x^3 + 4x} - 3 \int \frac{x^2 dx}{x^3 + 4x} \\ &= \ell \eta |x^3 + 4x| - \frac{3}{2} \int \frac{2x dx}{x^2 + 4} = \ell \eta |x^3 + 4x| - \frac{3}{2} \ell \eta |x^2 + 4| + c \\ &= \ell \eta \left| \frac{x(x^2 + 4)}{(x^2 + 4)^{3/2}} \right| + c = \ell \eta \left| \frac{x}{\sqrt{x^2 + 4}} \right| + c \end{aligned}$$

**120.-**  $\int \frac{\cos x dx}{\ell \eta |\sin x|}$

Solución.- Sea:  $u = \ell \eta |\sin x|$ ,  $du = \cos x dx$

$$\int \frac{\cos x dx}{\ell \eta |\sin x|} = \int \frac{du}{u} = \ell \eta |u| + c = \ell \eta |\ell \eta |\sin x|| + c$$

**121.-**  $\int \ell \eta \exp \sqrt{x-1} dx$

Solución.-

$$\int \ell \eta \exp \sqrt{x-1} dx = \int \sqrt{x-1} dx = \frac{(x-1)^{3/2}}{3/2} + c = \frac{2(x-1)\sqrt{x-1}}{3} + c$$

**122.-**  $\int \frac{\sqrt{1+x^3}}{x} dx$

Solución.- Sea:  $\sqrt{1+x^3} = t \Rightarrow t^2 = 1+x^3 \Rightarrow x = \sqrt[3]{t^2-1}$ ,  $dx = \frac{2tdt}{3(t^2-1)^{2/3}}$

$$\begin{aligned} \int \frac{\sqrt{1+x^3}}{x} dx &= \int \frac{t \cdot \frac{2tdt}{3(t^2-1)^{2/3}}}{(\frac{t^2-1}{t^2})^{1/3}} = \frac{2}{3} \int \frac{t^2 dt}{t^2-1} = \frac{2}{3} \int \left( 1 + \frac{1}{t^2-1} \right) dt = \frac{2}{3} \int dt + \frac{2}{3} \int \frac{dt}{t^2-1} \\ &= \frac{2}{3} t + \frac{1}{3} \ell \eta \left| \frac{t-1}{t+1} \right| + c = \frac{2}{3} \sqrt{1+x^3} + \frac{1}{3} \ell \eta \left| \frac{\sqrt{1+x^3}-1}{\sqrt{1+x^3}+1} \right| + c \end{aligned}$$

**123.-**  $\int \sqrt{\frac{x-1}{x+1}} \frac{1}{x} dx$

Solución.- Sea:  $\sqrt{\frac{x-1}{x+1}} = t \Rightarrow t^2 = \frac{x-1}{x+1} \Rightarrow x(1-t^2) = t^2 \Rightarrow x = \frac{1+t^2}{1-t^2}, dx = \frac{4t dt}{(1-t^2)^2}$

$$\int \sqrt{\frac{x-1}{x+1}} \frac{1}{x} dx = \int t \frac{(1-t^2)}{(1+t^2)} \frac{4t dt}{(1-t^2)^2} = 4 \int \frac{t^2 \cancel{(1-t^2)} dt}{(1+t^2)(1-t^2)^2} = 4 \int \frac{t^2 dt}{(1+t^2)(1-t^2)}$$

$$= 4 \int \frac{t^2 dt}{(1+t)(1-t)(1+t^2)} = 4 \left[ \int \frac{A dt}{1+t} + \int \frac{B dt}{1-t} + \int \frac{Ct+D}{1+t^2} dt \right] (*)$$

$$\frac{t^2}{(1+t)(1-t)(1+t^2)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{Ct+D}{1+t^2}$$

$$\Rightarrow t^2 = A(1-t)(1+t^2) + B(1+t)(1+t^2) + (Ct+D)(1-t^2)$$

De donde: 
$$\begin{cases} t=1 \Rightarrow 1=4B \Rightarrow B=\frac{1}{4} \\ t=-1 \Rightarrow 1=4A \Rightarrow A=\frac{1}{4} \\ t=0 \Rightarrow 0=A+B+D \Rightarrow D=-\frac{1}{2} \end{cases}$$

$$t=2 \Rightarrow 4=-5A+15B+(2C+D)(-3) \Rightarrow C=0$$

$$(*) = 4 \left( \frac{1}{4} \int \frac{dt}{1+t} + \frac{1}{4} \int \frac{dt}{1-t} - \frac{1}{2} \int \frac{dt}{1+t^2} \right) = \int \frac{dt}{1+t} - \int \frac{dt}{1-t} - 2 \int \frac{dt}{1+t^2}$$

$$= \ell \eta |t+1| - \ell \eta |t-1| - 2 \operatorname{arc} \tau g t + c = \ell \eta \left| \frac{t+1}{t-1} \right| - 2 \operatorname{arc} \tau g t + c$$

$$= \ell \eta \left| \frac{\sqrt{\frac{x+1}{x-1}} + 1}{\sqrt{\frac{x+1}{x-1}} - 1} \right| - 2 \operatorname{arc} \tau g \sqrt{\frac{x+1}{x-1}} + c = \ell \eta \left| \frac{\sqrt{x-1} + \sqrt{x+1}}{\sqrt{x-1} - \sqrt{x+1}} \right| - 2 \operatorname{arc} \tau g \sqrt{\frac{x+1}{x-1}} + c$$

**124.-**  $\int \frac{\operatorname{sen} x dx}{1 + \operatorname{sen} x + \cos x}$

Solución.- Sea:  $\operatorname{sen} x = \frac{2z}{1+z^2}, \cos x = \frac{1-z^2}{1+z^2}, z = \tau g \frac{x}{2}, dx = \frac{2dz}{1+z^2}$

$$\int \frac{\operatorname{sen} x dx}{1 + \operatorname{sen} x + \cos x} = \int \frac{\left( \frac{2z}{1+z^2} \right) \left( \frac{2}{1+z^2} \right)}{1 + \left( \frac{2z}{1+z^2} \right) \left( \frac{1-z^2}{1+z^2} \right)} dz = \int \frac{\frac{4z}{1+z^2} dz}{1+z^2+2z+1-z^2}$$

$$\int \frac{4z dz}{(1+z^2)(2+2z)} = \int \frac{2z dz}{(1+z)(1+z^2)} = \int \frac{A dz}{1+z} + \int \frac{Bz+C}{1+z^2} dz (*)$$

$$\frac{2z}{(1+z)(1+z^2)} = \frac{A}{1+z} + \frac{Bz+C}{1+z^2}$$

De donde: 
$$\begin{cases} z=-1 \Rightarrow -2=2A \Rightarrow A=-1 \\ z=0 \Rightarrow 0=A+C \Rightarrow C=1 \\ z=1 \Rightarrow 2=2A+2B+2C \Rightarrow B=1 \end{cases}$$

$$\begin{aligned}
(*) &= -\int \frac{dz}{1+z} + \int \frac{z+1}{1+z^2} dz = -\ell \eta |1+z| + \frac{1}{2} \int \frac{2z dz}{z^2+1} + \int \frac{dz}{z^2+1} \\
&= -\ell \eta |1+z| + \frac{1}{2} \ell \eta |z^2+1| + \operatorname{arc} \tau g z + c = \ell \eta \left| \frac{\sqrt{z^2+1}}{z+1} \right| + \operatorname{arc} \tau g z + c \\
&= \ell \eta \left| \frac{\sqrt{\tau g^2 \frac{x}{2} + 1}}{\tau g \frac{x}{2} + 1} \right| + \operatorname{arc} \tau g z + c
\end{aligned}$$

**125.-**  $\int \frac{dx}{3+2\cos x}$

Solución.- Sea:  $\operatorname{sen} x = \frac{2z}{1+z^2}$ ,  $\cos x = \frac{1-z^2}{1+z^2}$ ,  $z = \tau g \frac{x}{2}$ ,  $dx = \frac{2dz}{1+z^2}$

$$\begin{aligned}
\int \frac{dx}{3+2\cos x} &= \int \frac{\frac{2z}{1+z^2}}{3+2\left(\frac{1-z^2}{1+z^2}\right)} dz = \int \frac{2dz}{3+3z^2+2-2z^2} = 2 \int \frac{dz}{5+z^2} = \frac{2}{\sqrt{5}} \operatorname{arc} \tau g \frac{z}{\sqrt{5}} + c \\
&= \frac{2\sqrt{5}}{5} \operatorname{arc} \tau g \left( \frac{\sqrt{5}}{5} \tau g \frac{x}{2} \right) + c
\end{aligned}$$

**126.-**  $\int \frac{x dx}{\sqrt{x^2-2x+5}}$

Solución.-

Completando cuadrados se tiene:  $x^2-2x+5 = x^2-2x+1+4 = (x-1)^2+2^2$ ,

Sea:  $x-1 = 2\tau g \theta$ ,  $dx = 2\sec^2 \theta d\theta$ ,  $\sqrt{(x-1)^2+2^2} = 2\sec \theta$ , luego:

$$\begin{aligned}
\int \frac{x dx}{\sqrt{x^2-2x+5}} &= \frac{1}{2} \int \frac{(2x-2+2)dx}{\sqrt{x^2-2x+5}} = \frac{1}{2} \int \frac{(2x-2)dx}{\sqrt{x^2-2x+5}} + \int \frac{dx}{\sqrt{x^2-2x+5}} \\
&= \sqrt{x^2-2x+5} + \int \frac{dx}{\sqrt{x^2-2x+5}} = \sqrt{x^2-2x+5} + \int \frac{dx}{\sqrt{(x-1)^2+2^2}} \\
&= \sqrt{x^2-2x+5} + \int \frac{\cancel{2} \sec^2 \theta d\theta}{2\sec \theta} = \sqrt{x^2-2x+5} + \int \sec \theta d\theta \\
&= \sqrt{x^2-2x+5} + \ell \eta |\sec \theta + \tau g \theta| + c
\end{aligned}$$

**127.-**  $\int \frac{(1+\operatorname{sen} x)dx}{\operatorname{sen} x(2+\cos x)}$

Solución.- Sea:  $\operatorname{sen} x = \frac{2z}{1+z^2}$ ,  $\cos x = \frac{1-z^2}{1+z^2}$ ,  $z = \tau g \frac{x}{2}$ ,  $dx = \frac{2dz}{1+z^2}$

$$\int \frac{(1+\operatorname{sen} x)dx}{\operatorname{sen} x(2+\cos x)} = \int \frac{\left(1+\frac{2z}{1+z^2}\right) \frac{dz}{1+z^2}}{\frac{dz}{1+z^2} \left(2+\frac{1-z^2}{1+z^2}\right)} dz = \int \frac{(1+z^2+2z)dz}{2z(1+z^2)+z(1-z^2)}$$

$$= \int \frac{(z^2+2z+1)dz}{z^3+3z} = \int \frac{(z^2+2z+1)dz}{z(z^2+3)} = \int \frac{Adz}{z} + \int \frac{Bz+C}{(z^2+3)} dz(*)\}$$

$$\frac{(z^2+2z+1)}{z(z^2+3)} = \frac{A}{z} + \frac{Bz+C}{(z^2+3)} \Rightarrow z^2+2z+1 = A(z^2+3) + (Bz+C)z$$

$\Rightarrow Az^2 + 3A + Bz^2 + Cz \Rightarrow (A+B)z^2 + Cz + 3A$ , igualando coeficientes se tiene:

$$\begin{pmatrix} A+B & =1 \\ C & =2 \\ 3A & =1 \end{pmatrix} \Rightarrow A = \frac{1}{3}, B = \frac{2}{3}, C = 2$$

$$(*) = \frac{1}{3} \int \frac{dz}{z} + \int \frac{\frac{2}{3}z+2}{(z^2+3)} dz = \frac{1}{3} \int \frac{dz}{z} + \frac{1}{3} \int \frac{2zdz}{(z^2+3)} + 2 \int \frac{dz}{(z^2+3)}$$

$$= \frac{1}{3} \ell \eta \left| \tau g \frac{x}{2} \right| + \frac{1}{3} \ell \eta \left| \tau g^2 \frac{x}{2} + 3 \right| + \frac{2}{\sqrt{3}} \operatorname{arc} \tau g \left( \frac{\tau g^2 \frac{x}{2}}{\sqrt{3}} \right) + c$$

**128.-**  $\int \frac{dx}{x^4+4}$

Solución.- Sea:  $x^4+4 = x^4+4x^2+4-4x^2 = (x^2+2)^2 - (2x)^2 = (x^2+2x+2)(x^2-2x+2)$

$$\int \frac{dx}{x^4+4} = \int \frac{dx}{(x^2+2x+2)(x^2-2x+2)} = \int \frac{(Ax+B)dx}{(x^2+2x+2)} + \int \frac{(Cx+D)dx}{(x^2-2x+2)} (*)$$

$$\frac{1}{(x^4+4)} = \frac{(Ax+B)}{(x^2+2x+2)} + \frac{(Cx+D)}{(x^2-2x+2)}$$

$$1 = (Ax+B)(x^2-2x+2) + (Cx+D)(x^2+2x+2)$$

$$1 = (A+C)x^3 + (-2A+B+2C+D)x^2 + (2A-2B+2C+2D)x + (2B+2D)$$

Igualando coeficientes se tiene:

$$\begin{pmatrix} A+C & =0 \\ -2A+B+2C+D & =0 \\ 2A-2B+2C+2D & =0 \\ 2B+2D & =1 \end{pmatrix} \Rightarrow A = \frac{1}{8}, B = \frac{1}{4}, C = -\frac{1}{8}, D = \frac{1}{4}$$

$$(*) = \frac{1}{8} \int \frac{(x+2)dx}{(x^2+2x+2)} - \frac{1}{8} \int \frac{(x-2)dx}{(x^2-2x+2)}$$

$$= \frac{1}{8} \int \frac{(x+1)dx}{(x+1)^2+1} + \frac{1}{8} \int \frac{dx}{(x+1)^2+1} - \frac{1}{8} \int \frac{(x-1)dx}{(x-1)^2+1} + \frac{1}{8} \int \frac{dx}{(x-1)^2+1}$$

$$= \frac{1}{16} \ell \eta \left| x^2+2x+2 \right| + \frac{1}{8} \operatorname{arc} \tau g(x+1) - \frac{1}{16} \ell \eta \left| x^2-2x+2 \right| + \frac{1}{8} \operatorname{arc} \tau g(x-1) + c$$

$$= \frac{1}{16} \ell \eta \left| \frac{x^2+2x+2}{x^2-2x+2} \right| + \frac{1}{8} [\operatorname{arc} \tau g(x+1) + \operatorname{arc} \tau g(x-1)] + c$$