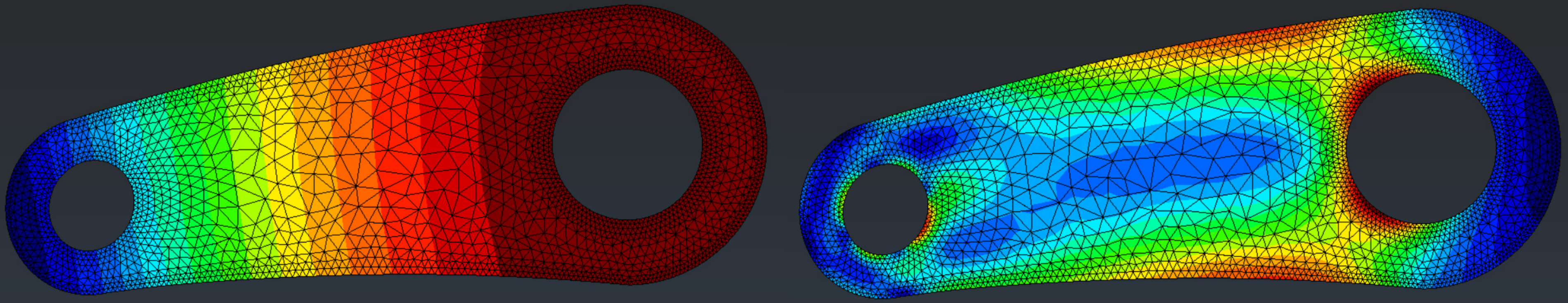
ELEMENTOS DE MÁQUINAS
ME5500



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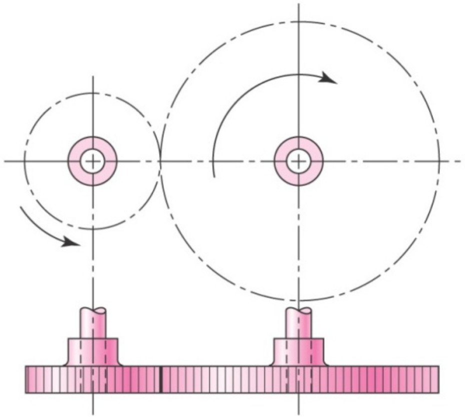
UNIVERSIDAD DE CHILE

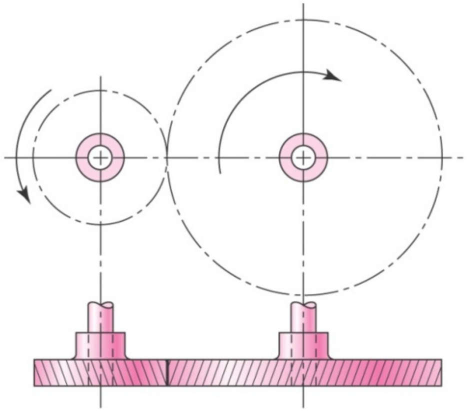
CÁLCULO DE ENGRANAJES

Contenidos

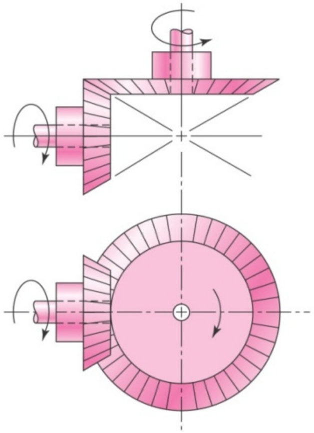
Tipos de Engranajes



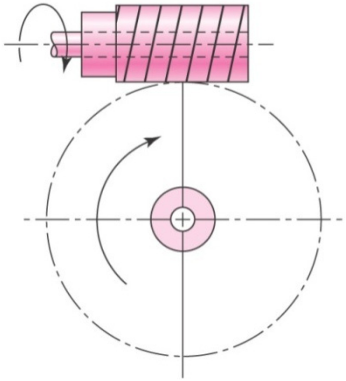
Rectos: transmiten movimiento de rotación entre ejes paralelos



Helicoidales: transmiten movimiento de rotación entre ejes paralelos y no paralelos



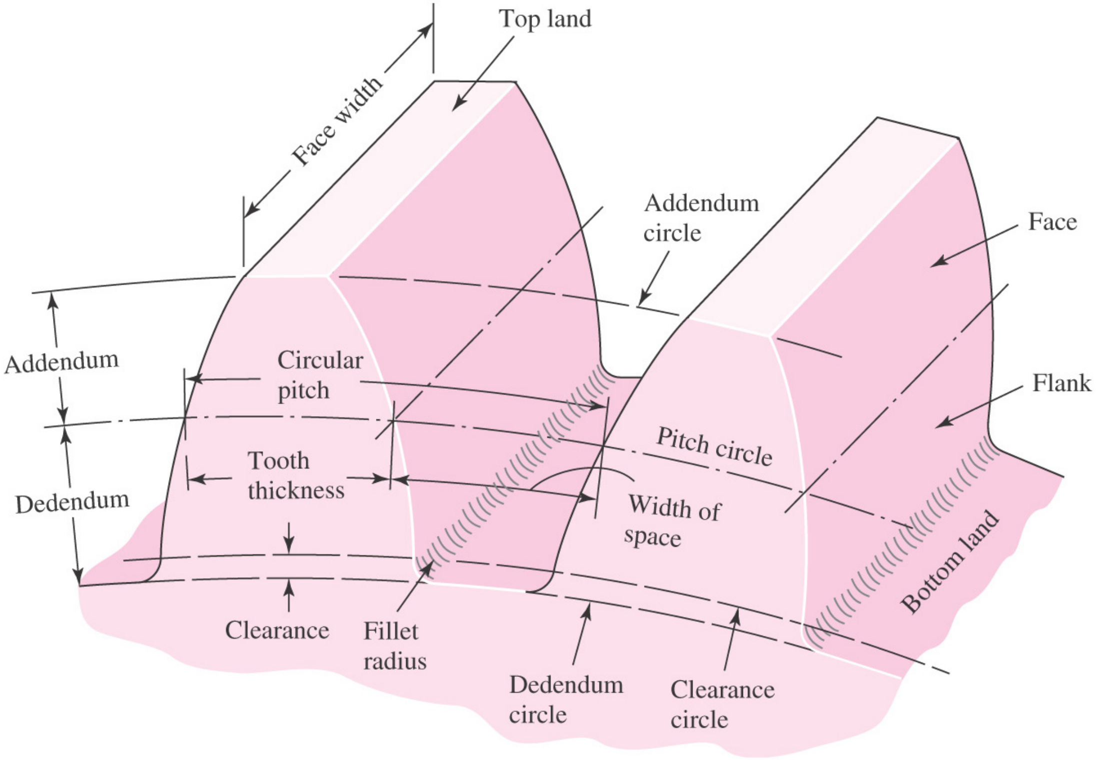
Cónicos: transmiten movimiento de rotación entre ejes que se intersectan



Tornillos sinfín: transmiten movimiento de rotación entre ejes no paralelos que no se intersectan

Tipos de Engranajes (Cont.)

Nomenclatura



Top land = Cresta
Bottom land = Valle
Face = Cara
Flank = Flanco
Addendum = Cabeza
Addendum circle = Círculo de la cabeza
Dedendum = Raíz
Dedendum circle = Círculo de la raíz
Face width = Ancho de la cara
Circular pitch = Paso circular
Pitch circle = Círculo del paso
Tooth thickness = Espesor de dientes
Width of space = Ancho del espacio
Clearance = Claro
Clearance circle = Círculo del claro
Fillet radius = Radio del entalle

Sistemas de Dientes Estandarizados

$$P = \frac{N}{d}$$

$$m = \frac{d}{N}$$

$$p = \frac{\pi d}{N} = \pi m$$

$$pP = \pi$$

Donde

P = paso diametral, dientes por pulgada
N = número de dientes
d = diámetro de paso, in o mm
m = módulo
p = paso circular

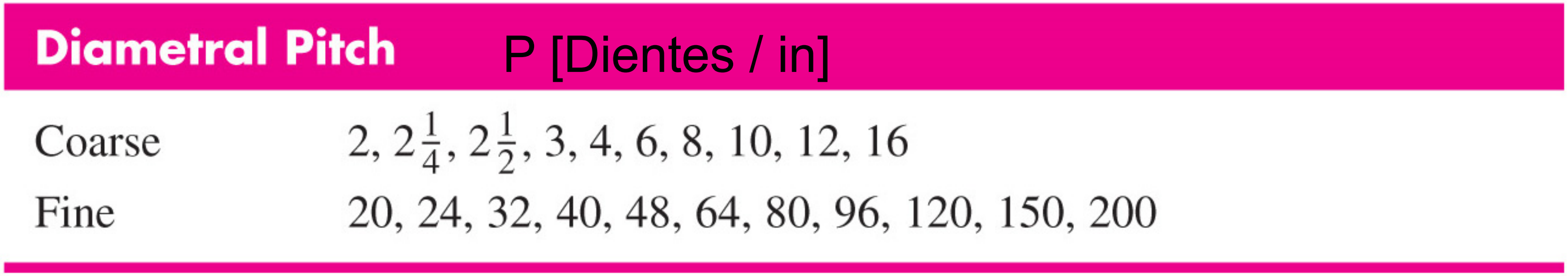
Sistema “Diametral Pitch”: *P* en [1/in]

Sistema “Módulo Métrico”: *m* en [mm]

Relación ambos sistemas: $m = 1/\underset{\overset{\nwarrow}{[1/\text{mm}]}}{P} = 25.4/\overset{\underset{\swarrow}{[1/\text{in}]}}{P}$

Sistemas de Dientes Estandarizados (Cont.)

Sistema "Diametral Pitch"



Diametral Pitch	P [Dientes / in]
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Sistema "Módulo Métrico"

Modules	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Sistemas de Dientes Estandarizados (Cont.)

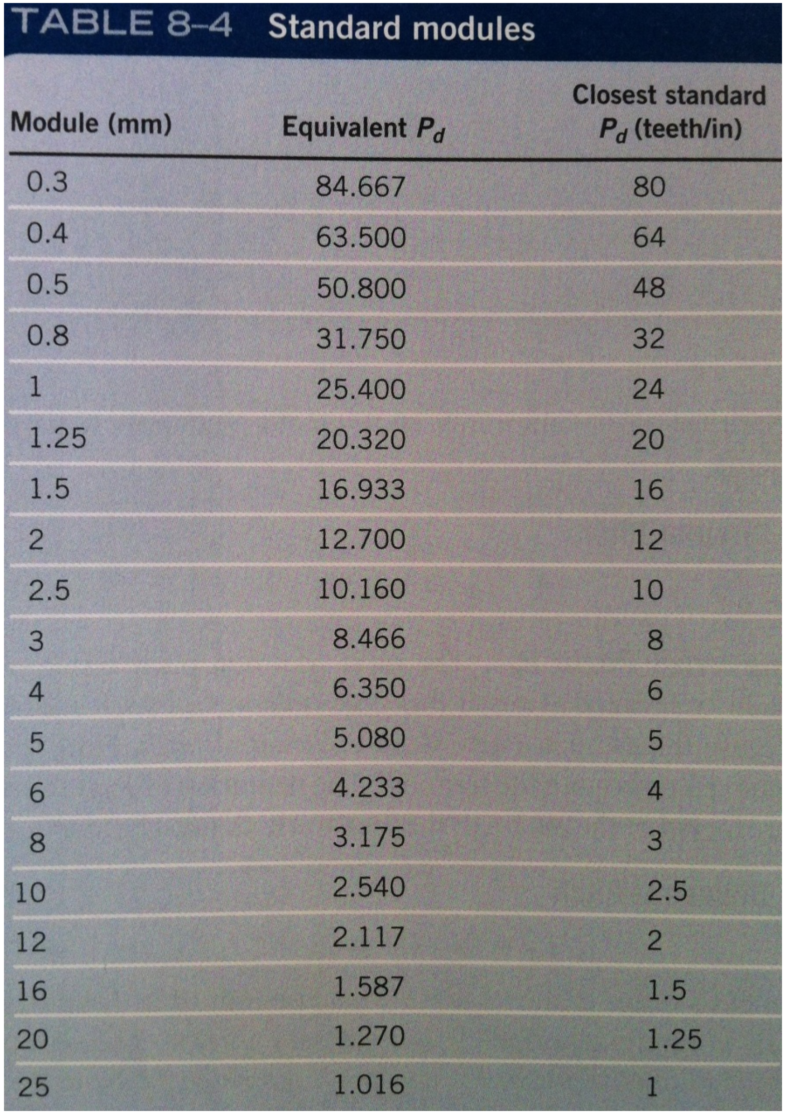


TABLE 8–4 **Standard modules**

Module (mm)	Equivalent P_d	Closest standard P_d (teeth/in)
0.3	84.667	80
0.4	63.500	64
0.5	50.800	48
0.8	31.750	32
1	25.400	24
1.25	20.320	20
1.5	16.933	16
2	12.700	12
2.5	10.160	10
3	8.466	8
4	6.350	6
5	5.080	5
6	4.233	4
8	3.175	3
10	2.540	2.5
12	2.117	2
16	1.587	1.5
20	1.270	1.25
25	1.016	1

Sistemas de Dientes Estandarizados (Cont.)

		Cabeza	Raíz
Tooth System	**Pressure Angle ϕ, deg**	**Addendum a**	**Dedendum b**
Full depth (Diente largo)	20	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	$22\frac{1}{2}$	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
	25	$1/P_d$ or $1m$	$1.25/P_d$ or $1.25m$
			$1.35/P_d$ or $1.35m$
Stub (Diente corto)	20	$0.8/P_d$ or $0.8m$	$1/P_d$ or $1m$

Sistemas de Dientes Estandarizados (Cont.)

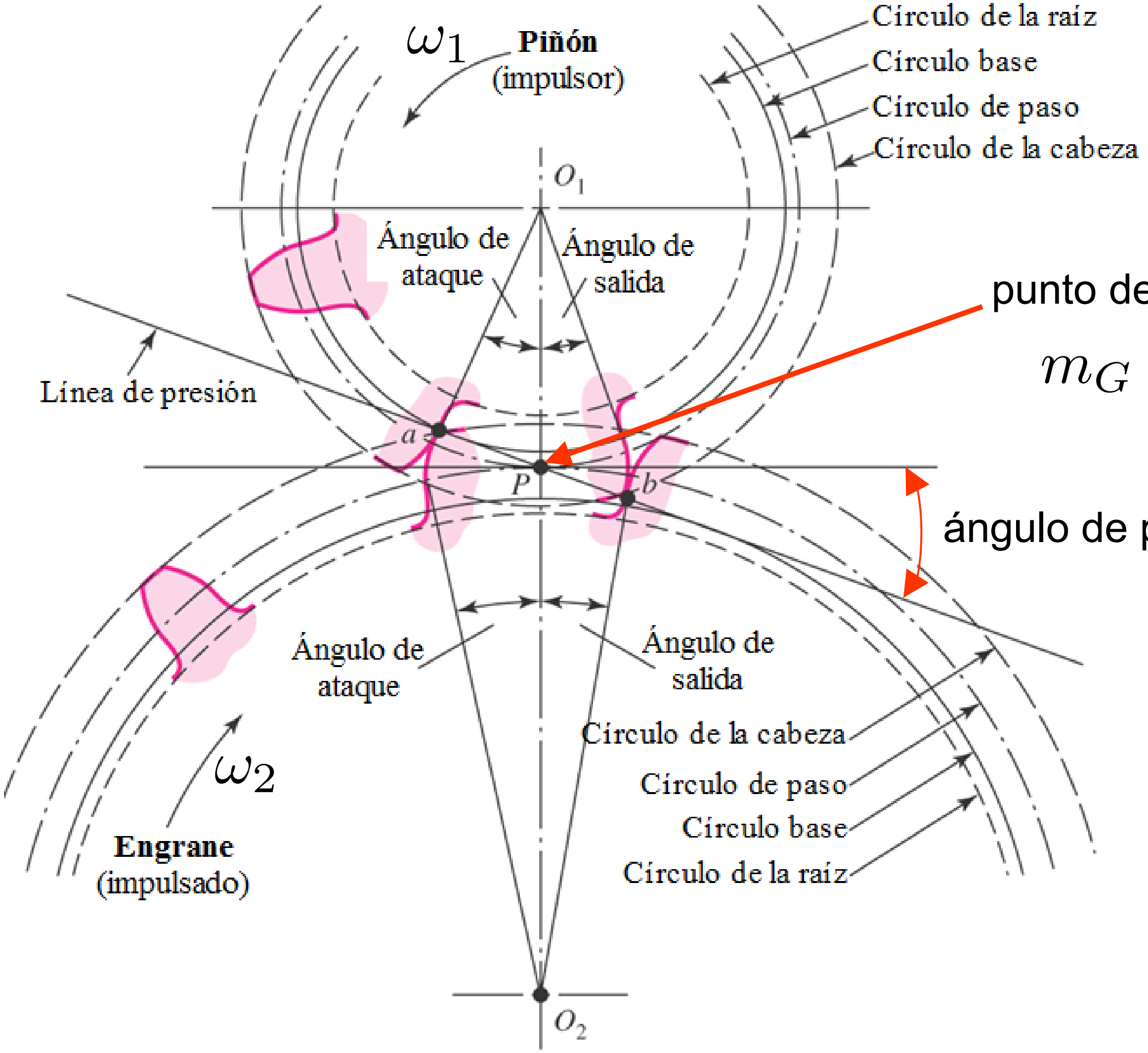
- Ángulos de presión típicos *f*: 20 y 25º
- Ángulo de presión antiguo: 14 ½ º
- Anchos de cara típico:

$$3p < F < 5p$$

$$p = \frac{\pi}{P}$$

$$\frac{3\pi}{P} < F < \frac{5\pi}{P}$$

Acción del Diente



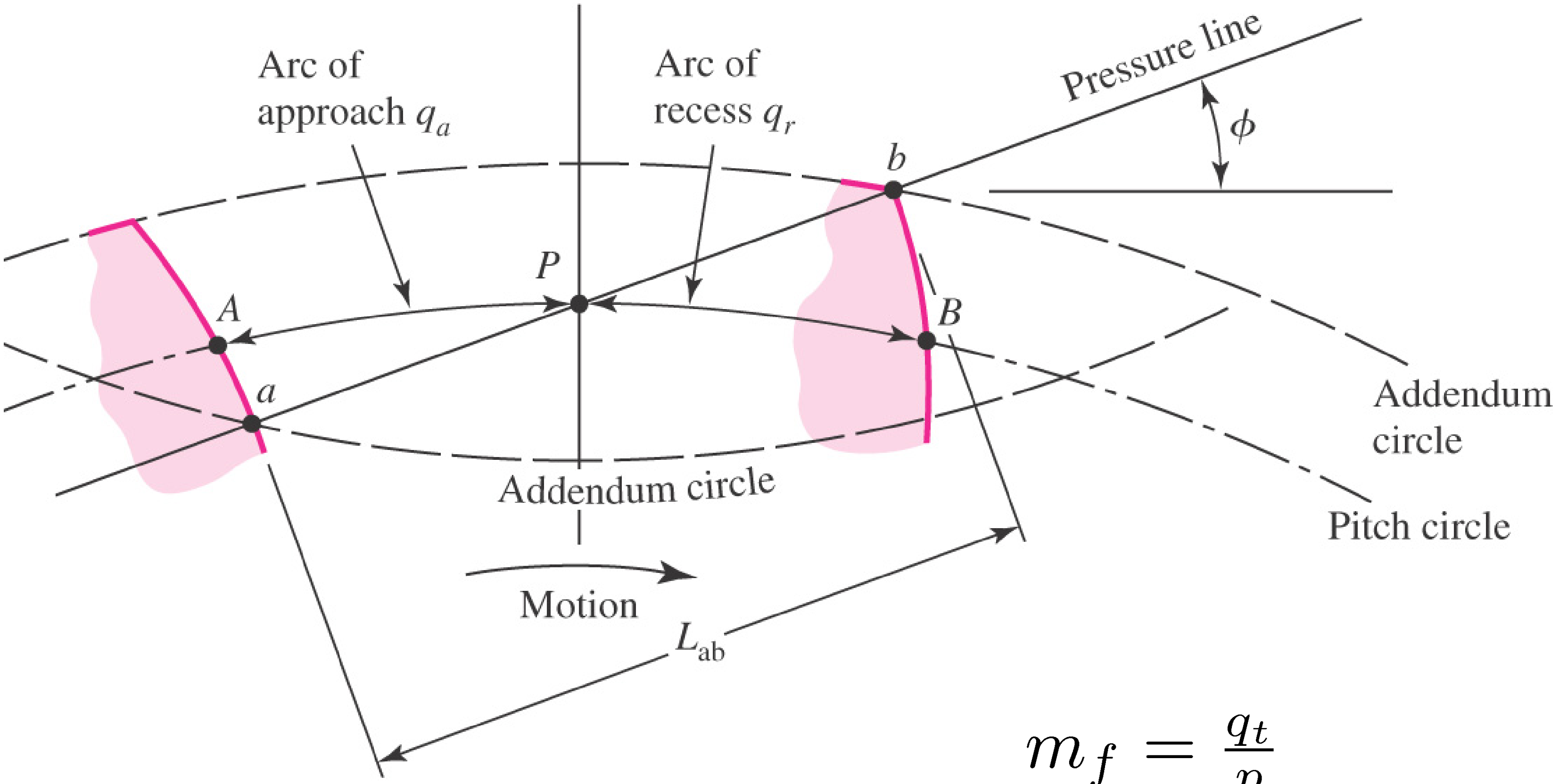
punto de paso:

$$m_G = \left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} = \frac{N_2}{N_1}$$

ángulo de presión

Relación de Contacto

$$q_t = q_a + q_r$$



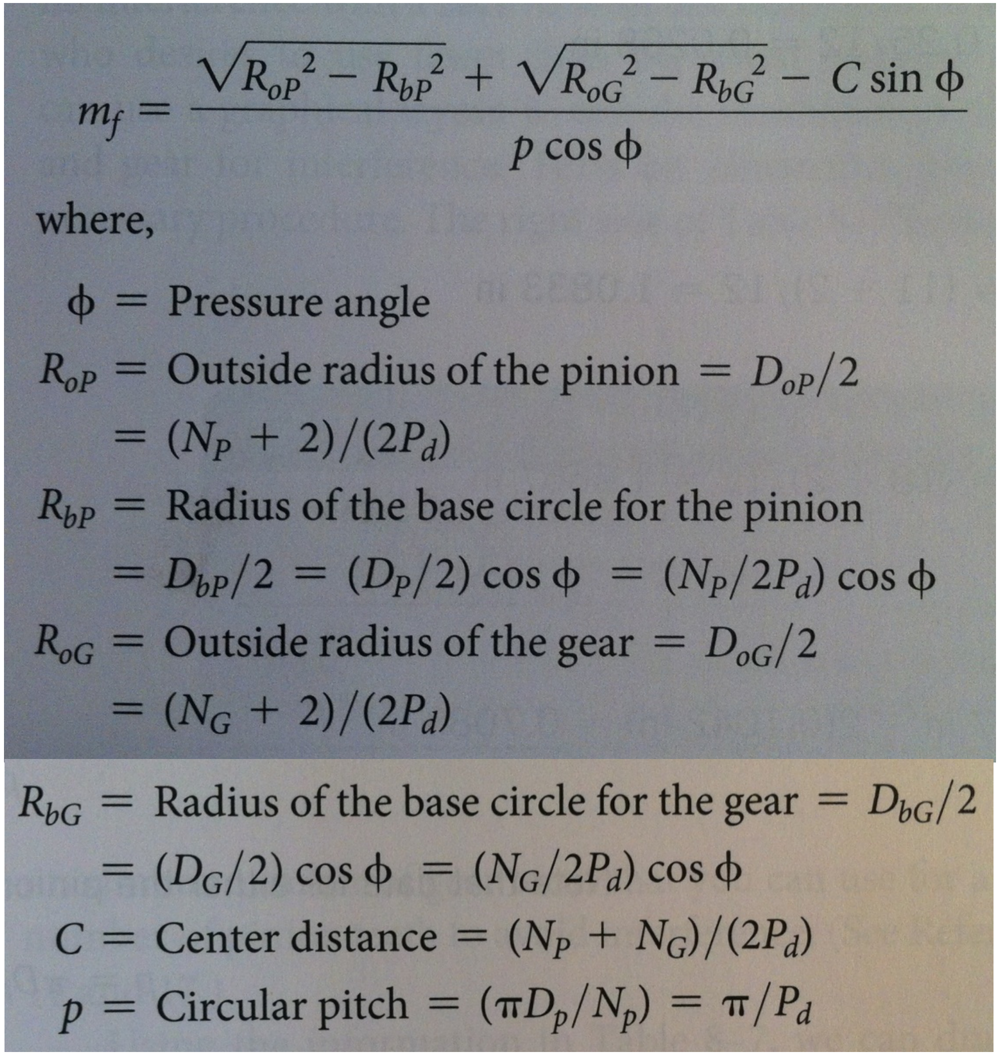
$$m_f = \frac{q_t}{p}$$

Relación de Contacto (Cont.)

Para sistema
diametral pitch

$$q_t = q_a + q_r$$

$$m_f = \frac{q_t}{p}$$



$$m_f = \frac{\sqrt{R_{oP}{}^2 - R_{bP}{}^2} + \sqrt{R_{oG}{}^2 - R_{bG}{}^2} - C \sin \phi}{p \cos \phi}$$

where,

ϕ = Pressure angle

R_{oP} = Outside radius of the pinion $= D_{oP}/2$
$= (N_P + 2)/(2P_d)$

R_{bP} = Radius of the base circle for the pinion
$= D_{bP}/2 = (D_P/2) \cos \phi \ = (N_P/2P_d) \cos \phi$

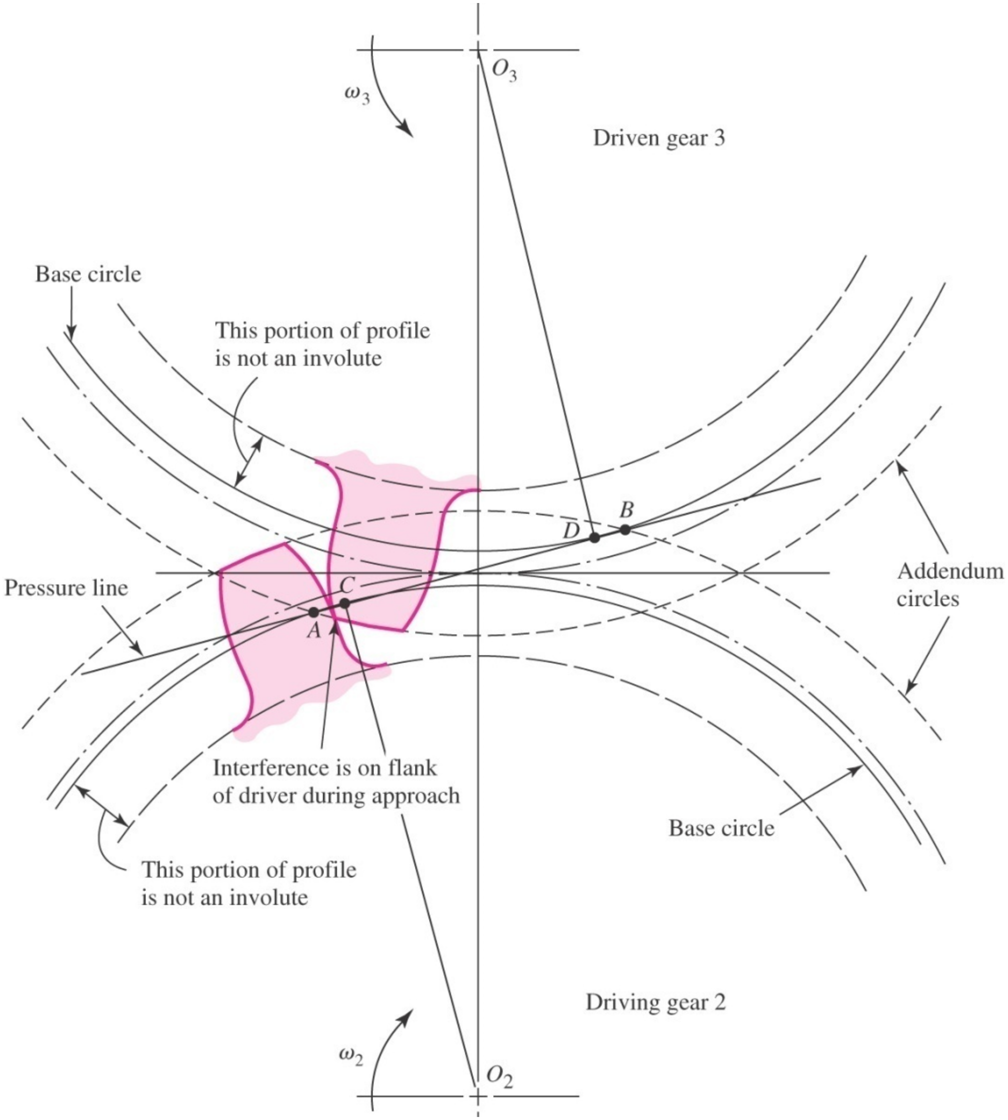
R_{oG} = Outside radius of the gear $= D_{oG}/2$
$= (N_G + 2)/(2P_d)$

R_{bG} = Radius of the base circle for the gear $= D_{bG}/2$
$= (D_G/2) \cos \phi \ = (N_G/2P_d) \cos \phi$

C = Center distance $= (N_P + N_G)/(2P_d)$

p = Circular pitch $= (\pi D_p/N_p) = \pi/P_d$

Interferencia



Eliminando la Interferencia

“Corona con infinitos dientes”

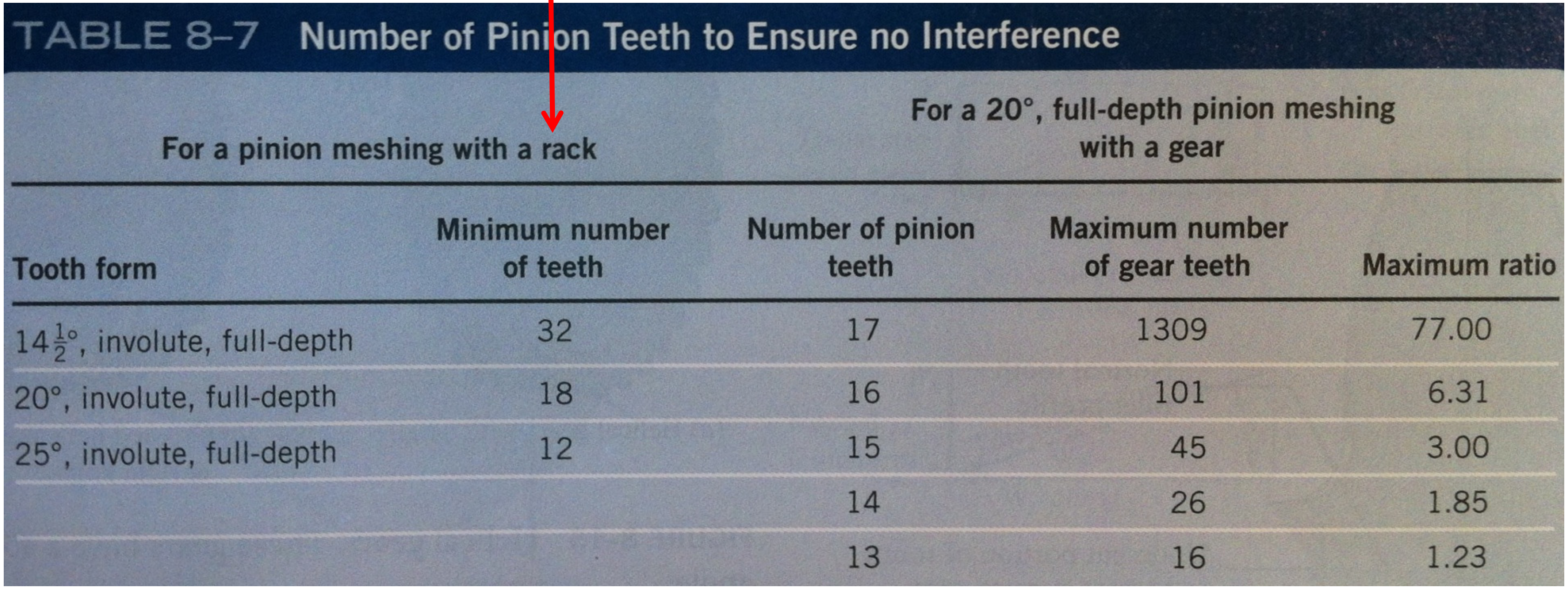
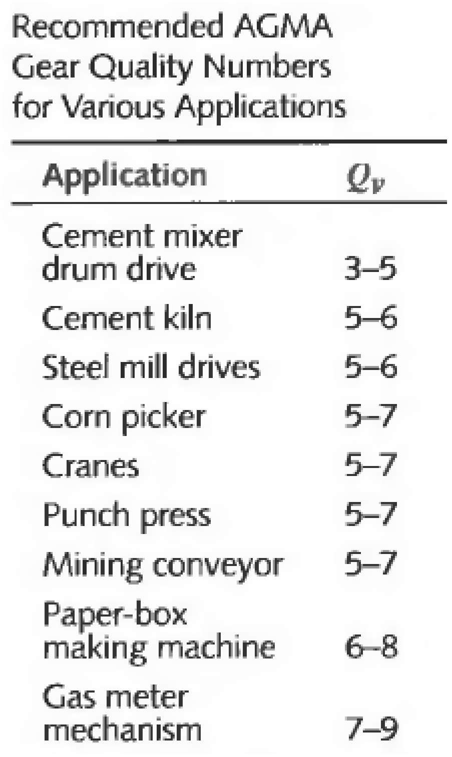


TABLE 8–7 Number of Pinion Teeth to Ensure no Interference

For a pinion meshing with a rack		For a 20°, full-depth pinion meshing with a gear		
Tooth form	Minimum number of teeth	Number of pinion teeth	Maximum number of gear teeth	Maximum ratio
$14\frac{1}{2}$°, involute, full-depth	32	17	1309	77.00
20°, involute, full-depth	18	16	101	6.31
25°, involute, full-depth	12	15	45	3.00
		14	26	1.85
		13	16	1.23

Para ángulo de presión 20º, N >= 18 no existirá interferencia

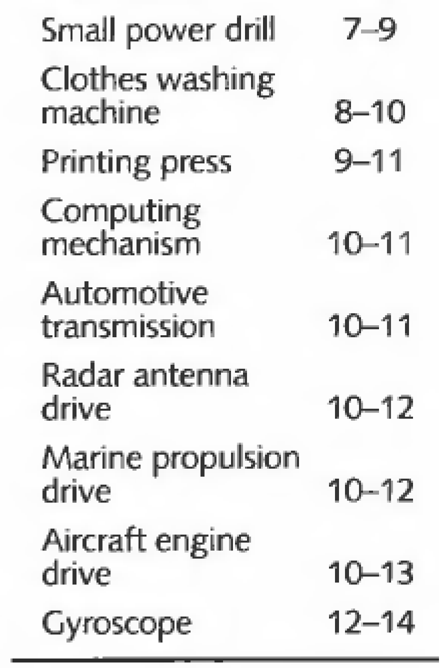
Número de Calidad AGMA



Recommended AGMA Gear Quality Numbers for Various Applications

Application	Q_v
Cement mixer drum drive	3–5
Cement kiln	5–6
Steel mill drives	5–6
Corn picker	5–7
Cranes	5–7
Punch press	5–7
Mining conveyor	5–7
Paper-box making machine	6–8
Gas meter mechanism	7–9

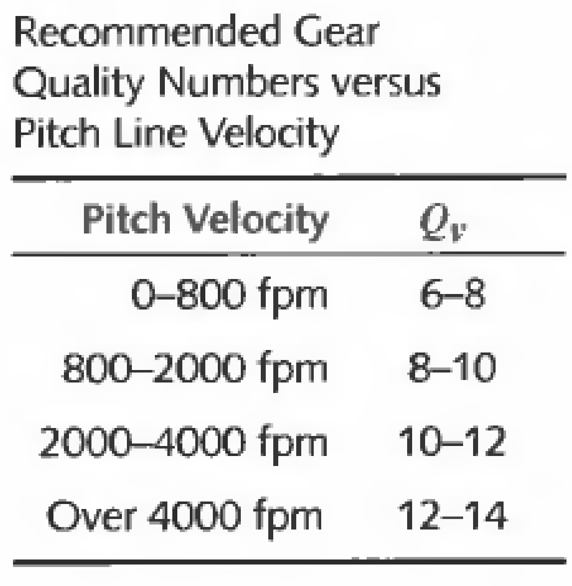
Precisión



Small power drill	7–9
Clothes washing machine	8–10
Printing press	9–11
Computing mechanism	10–11
Automotive transmission	10–11
Radar antenna drive	10–12
Marine propulsion drive	10–12
Aircraft engine drive	10–13
Gyroscope	12–14

Precisión

Número de Calidad AGMA (Cont.)



Recommended Gear Quality Numbers versus Pitch Line Velocity

Pitch Velocity	Q_v
0–800 fpm	6–8
800–2000 fpm	8–10
2000–4000 fpm	10–12
Over 4000 fpm	12–14

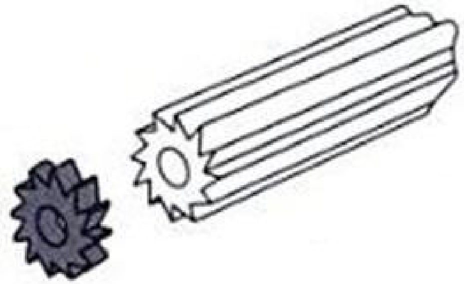
Precisión

1 fpm = 0.00508 m/s

1000 fpm = 5.08 m/s

Número de Calidad AGMA (Cont.)

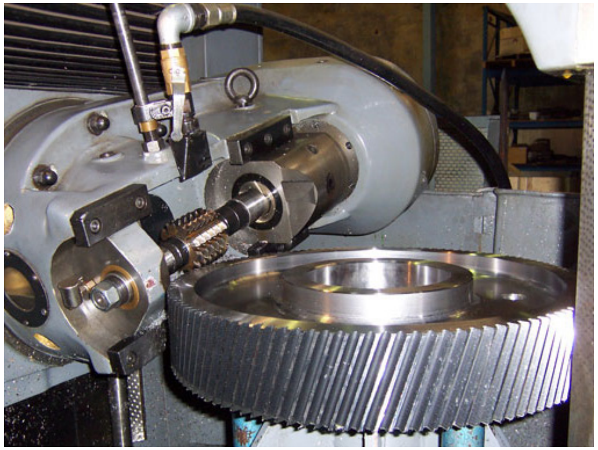
Extrusión en frío



Fundición

Qv = 3 a 4

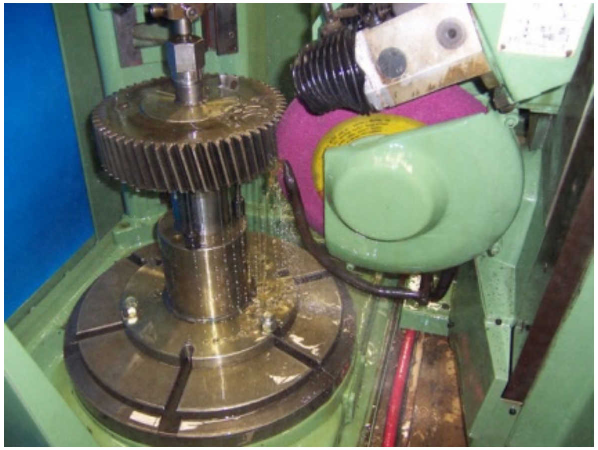
Mecanizado en metales forjados, fundidos o laminados en caliente



Qv = 5 a 7

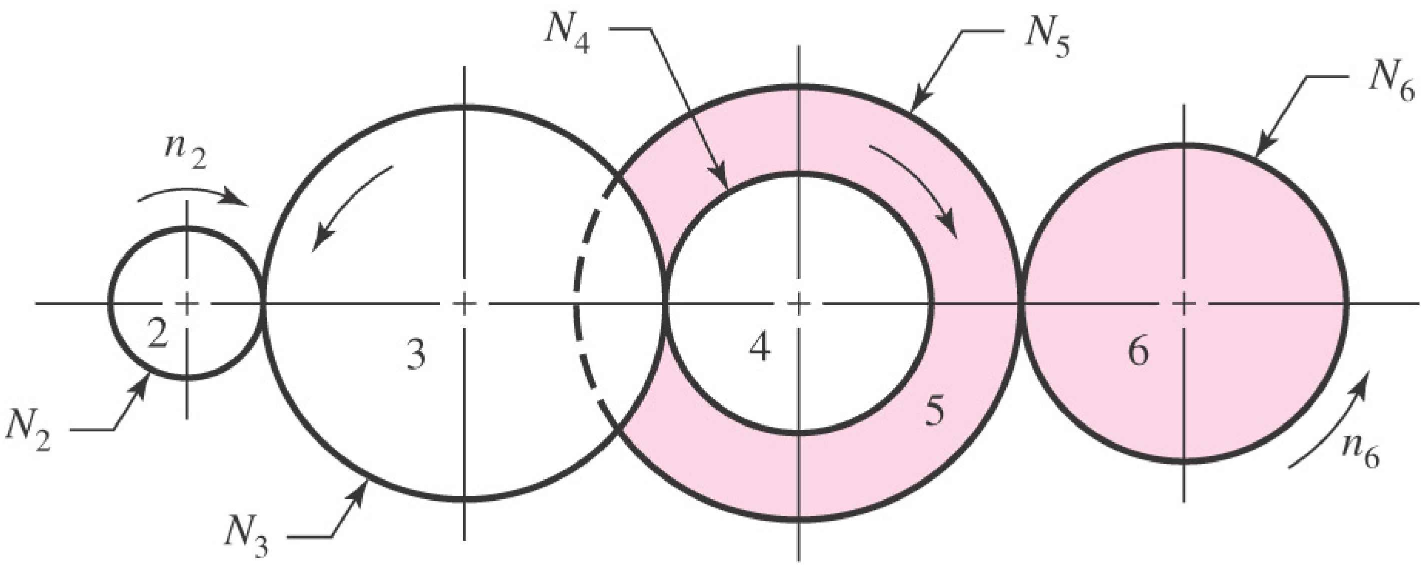
Número de Calidad AGMA (Cont.)

Esmerilado: Qv = 8 a 11



Lapeado / honing: Qv > 11

Trenes de Engranaje



Valor del tren. Signo depende del sentido de rotación del engrane inicial y final

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6}n_2$$

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

Velocidad del último engrane

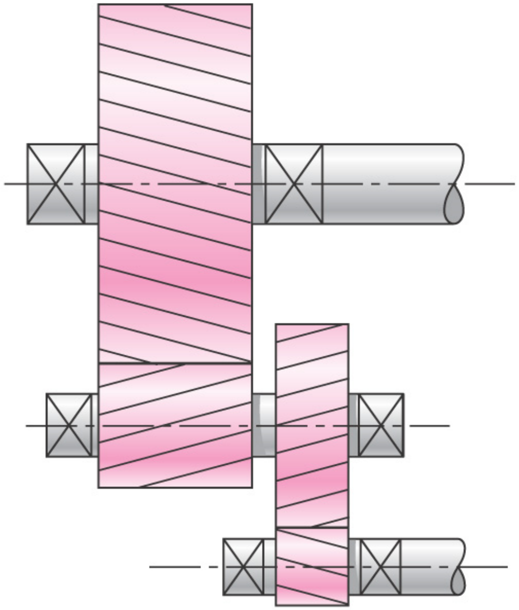
$$n_L = e n_F$$

Velocidad del primer engrane

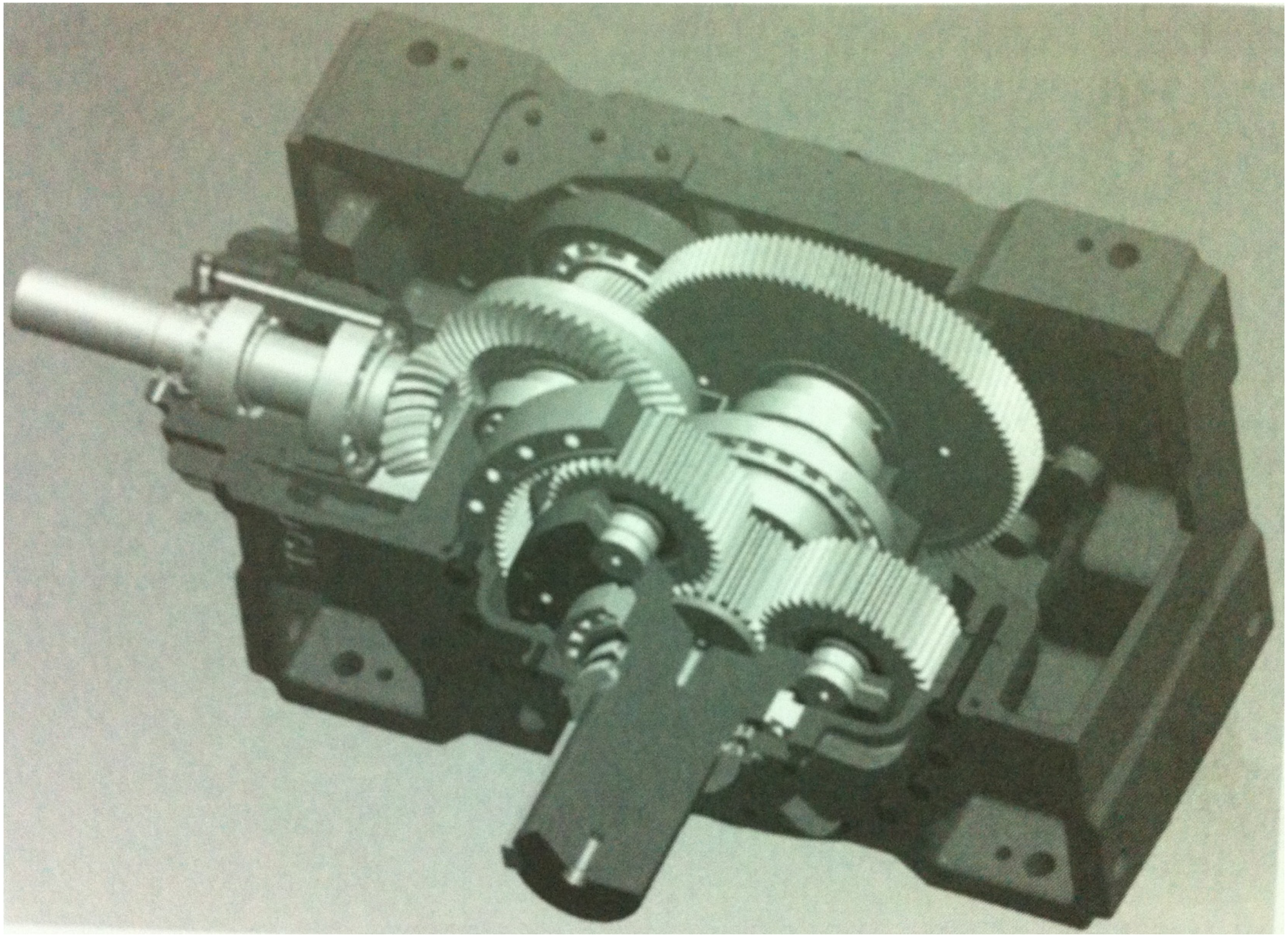
Trenes de Engranaje (Cont.)

Reducción de espacio, vibraciones, ruido:

- Relación 10 a 1 con un par de engranes
- Relación 100 a 1, usar dos ruedas en un mismo eje



Reductores



Reductores (Cont.)

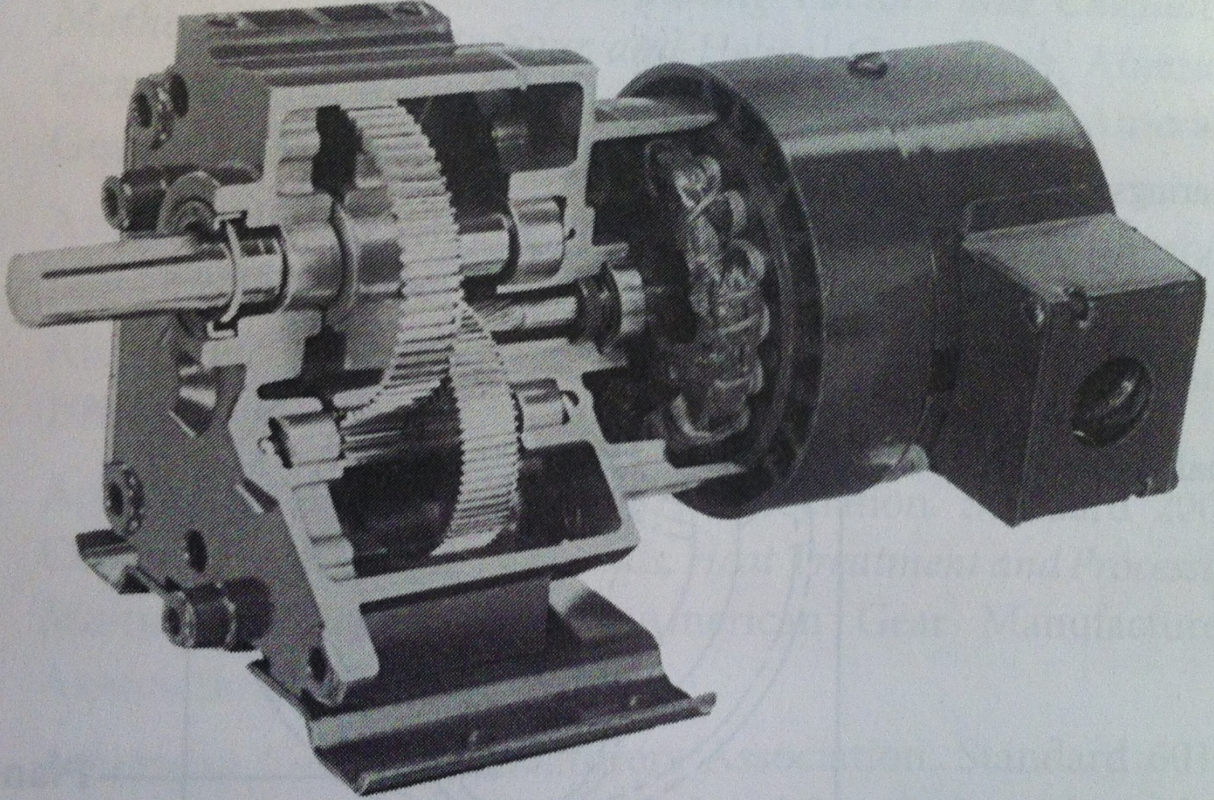


FIGURE 9–31 Double-reduction spur gear reducer (Bison Gear & Engineering Corporation, St. Charles, IL)

Reductores (Cont.)

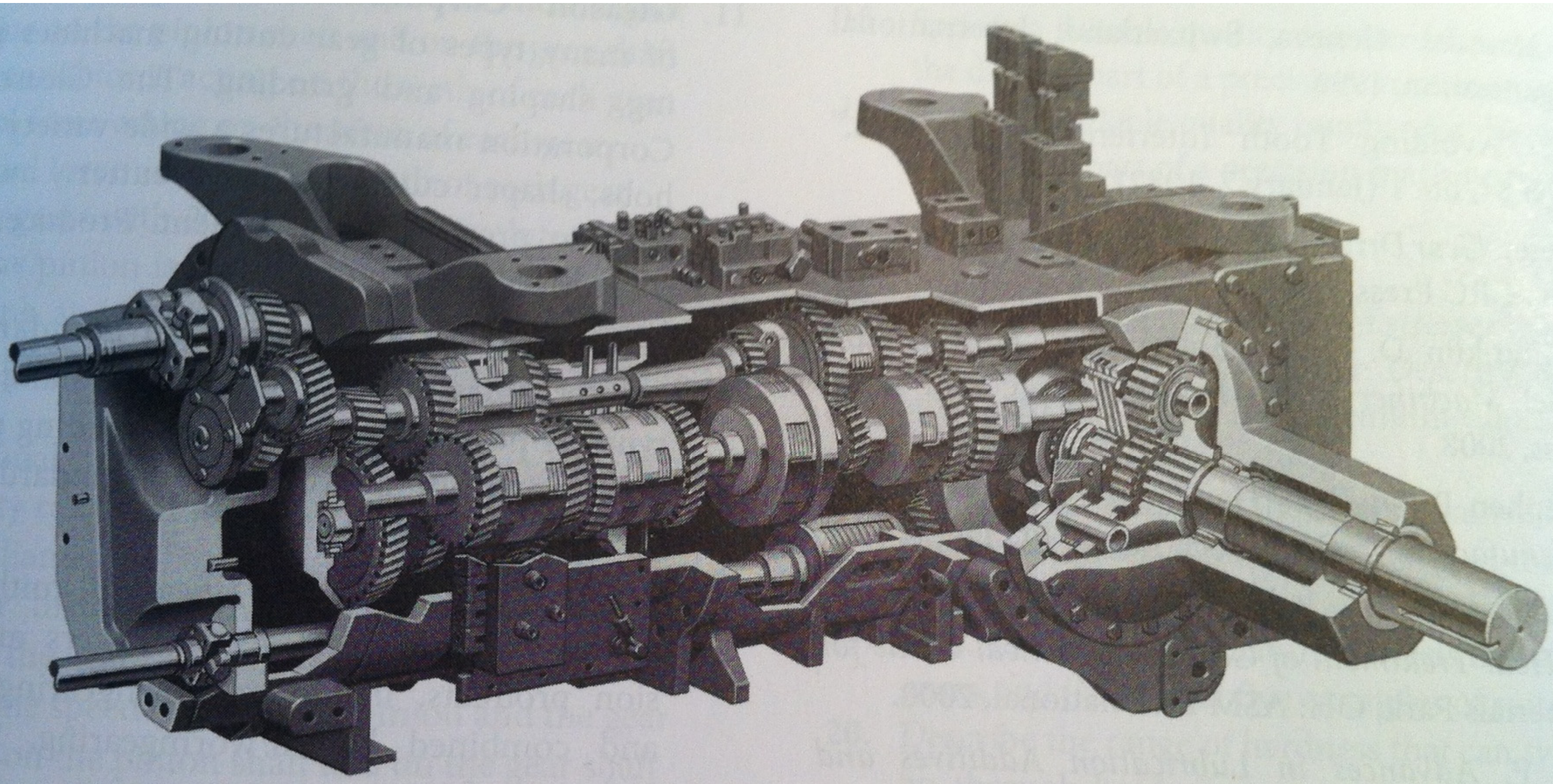
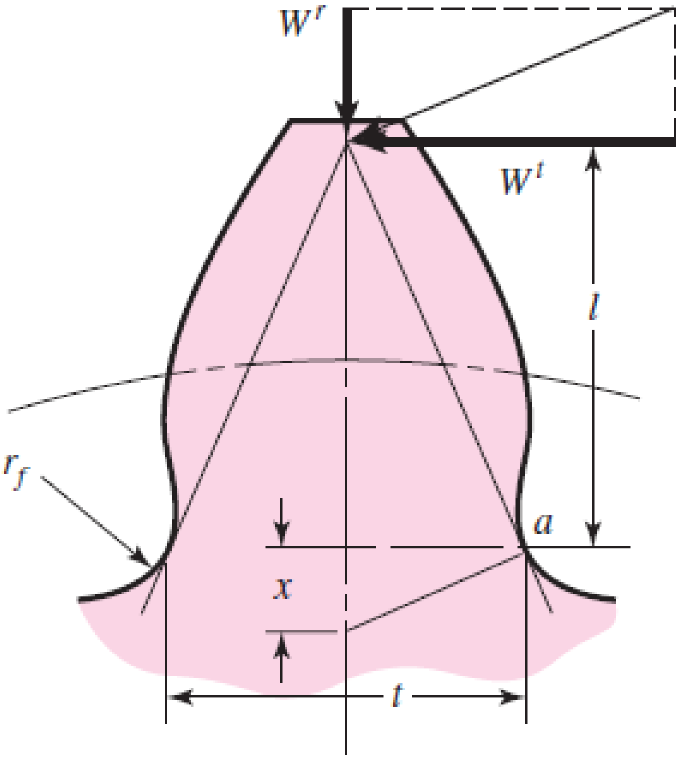


FIGURE 9–35 Eight-speed tractor transmission (Case IH, Racine, WI)

Esfuerzo de Lewis



$$\sigma = \frac{M}{I/c} = \frac{6W^t l}{Ft^2}$$

$$\sigma = \frac{W^t P}{FY}$$

Factor de Forma de Lewis

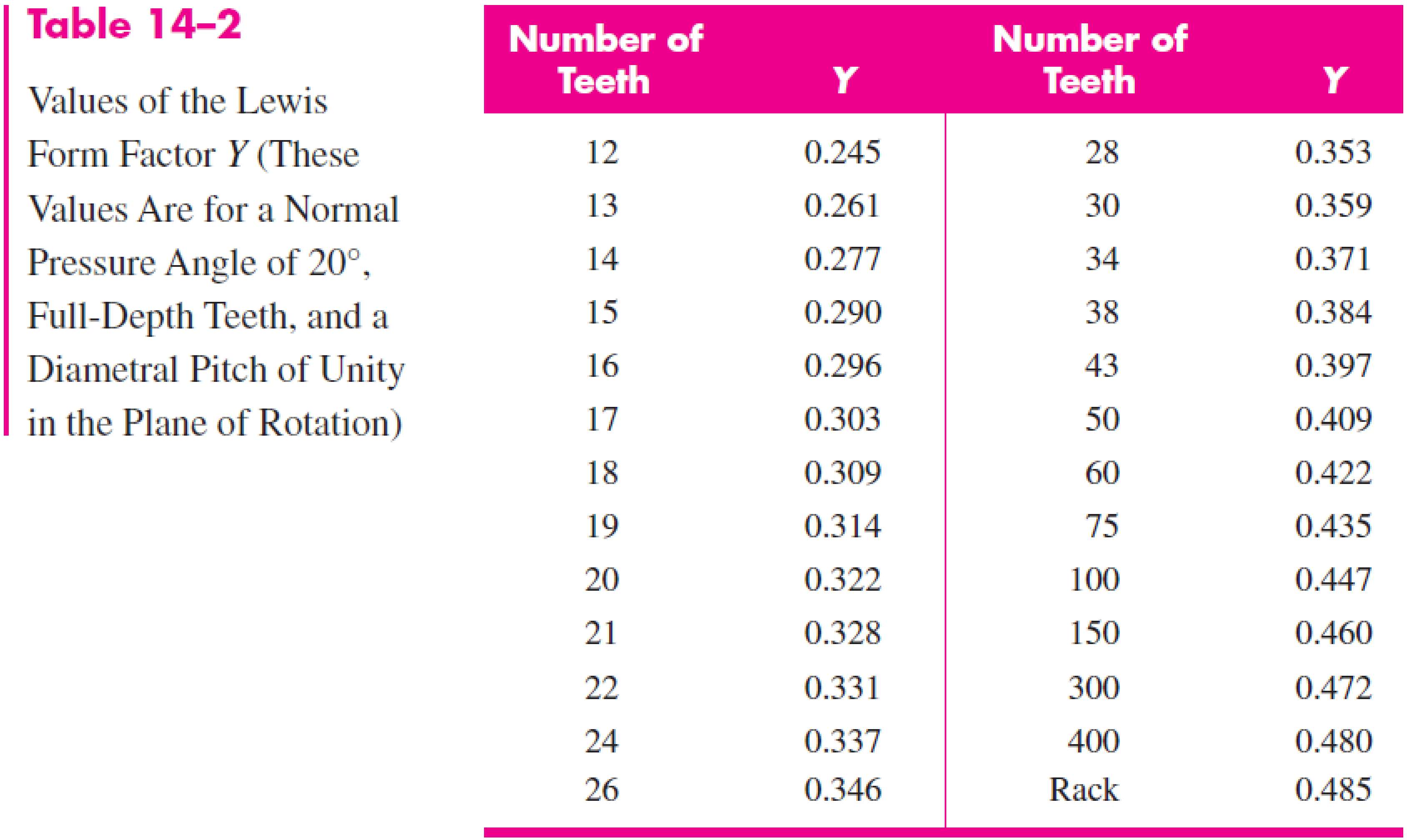


Table 14–2

Values of the Lewis Form Factor *Y* (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Esfuerzo de Flexión Según AGMA

ancho en [in]

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

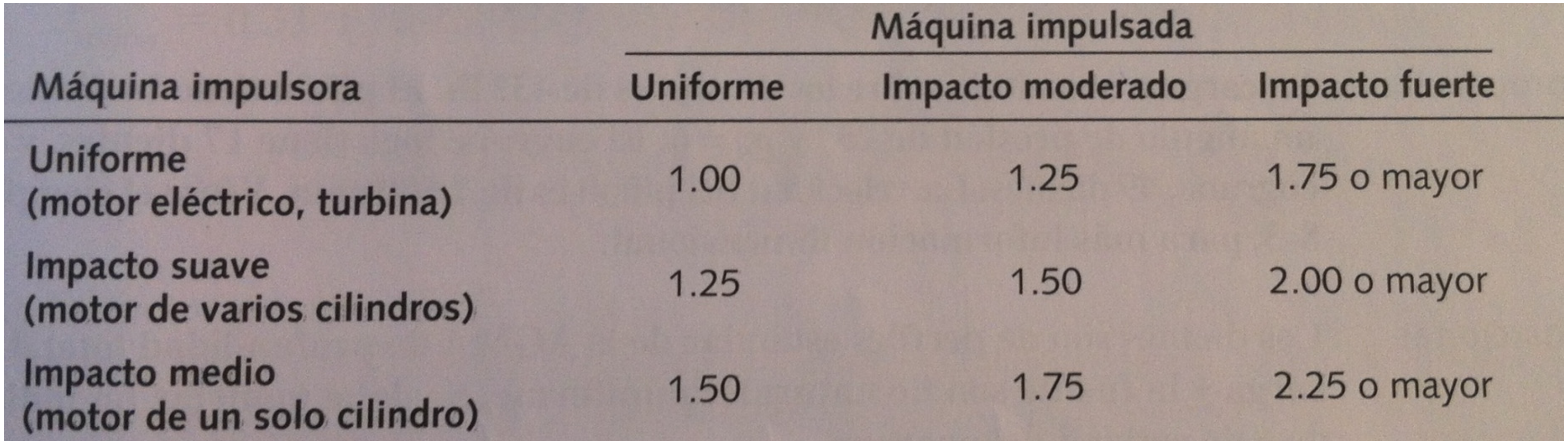
Módulo métrico [mm]

ancho en [mm]

Lewis: $\sigma = \dfrac{W^t P}{F Y}$

Factor de Carga

$$\sigma = W^t \boxed{K_o} K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$



Máquina impulsora	Máquina impulsada Uniforme	Impacto moderado	Impacto fuerte
Uniforme (motor eléctrico, turbina)	1.00	1.25	1.75 o mayor
Impacto suave (motor de varios cilindros)	1.25	1.50	2.00 o mayor
Impacto medio (motor de un solo cilindro)	1.50	1.75	2.25 o mayor

Factor Dinámico

$$\sigma = W^t K_o \boxed{K_v} K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A} \right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A} \right)^B & V \text{ in m/s} \end{cases}$$

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

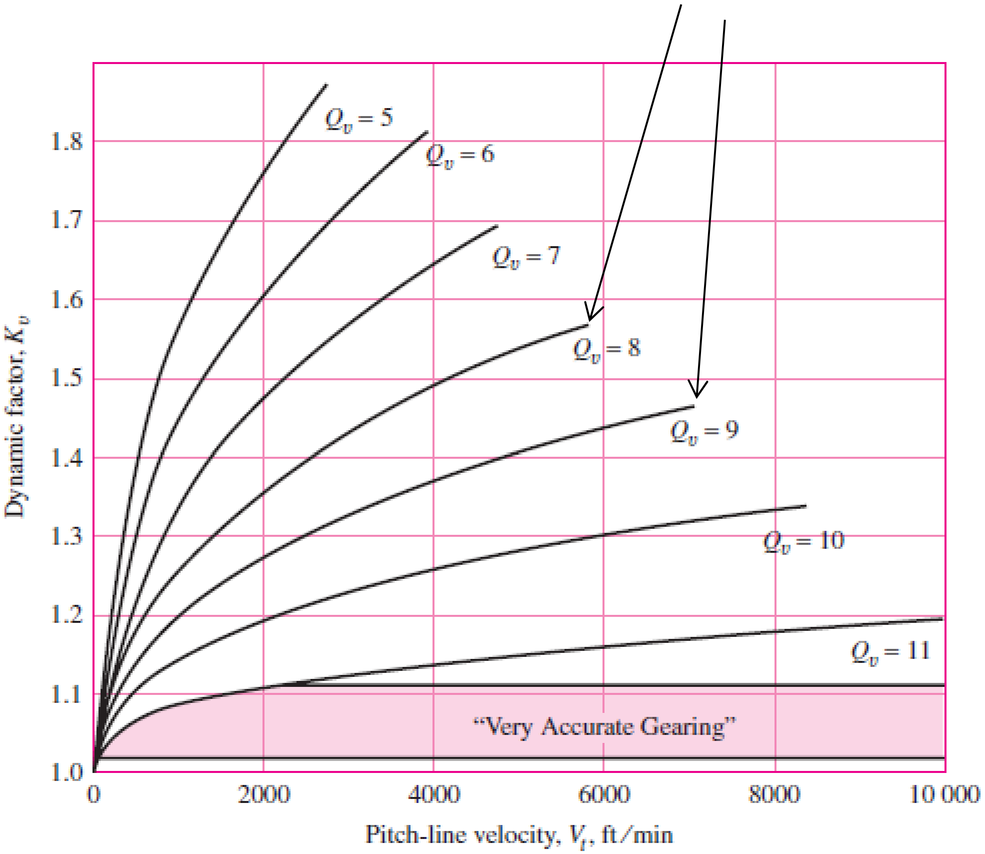
Factor Dinámico (Cont.)

$$\sigma = W^t K_o \boxed{K_v} K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases}$$

Figure 14–9

Dynamic factor K_v. The equations to these curves are given by Eq. (14–27) and the end points by Eq. (14–29). *(ANSI/AGMA 2001-D04, Annex A)*



Factor de Tamaño

$$\sigma = W^t K_o K_v \boxed{K_s} \frac{P_d}{F} \frac{K_m K_B}{J}$$

Ancho del diente [in]

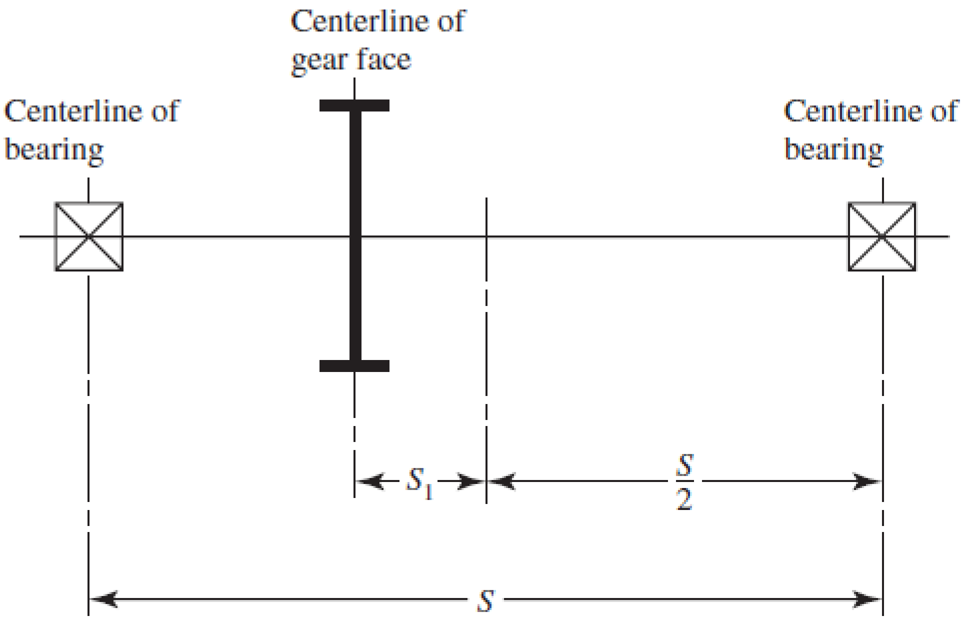
$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} \geq 1.0$$

Factor de Lewis

Paso diametral [dientes/in]

Factor de Distribución de Carga

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{\boxed{K_m} K_B}{J}$$



Método aplicable cuando:

- F/d ≤ 2
- F ≤ 40 [in]

Factor de Distribución de Carga (Cont.)

$$\boxed{K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)}$$

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$

$$C_{pf} = \begin{cases} \dfrac{F}{10d} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

Factor de Distribución de Carga (Cont.)

$$\boxed{K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)}$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \geq 0.175 \end{cases}$$

$$C_{ma} = A + BF + CF^2$$

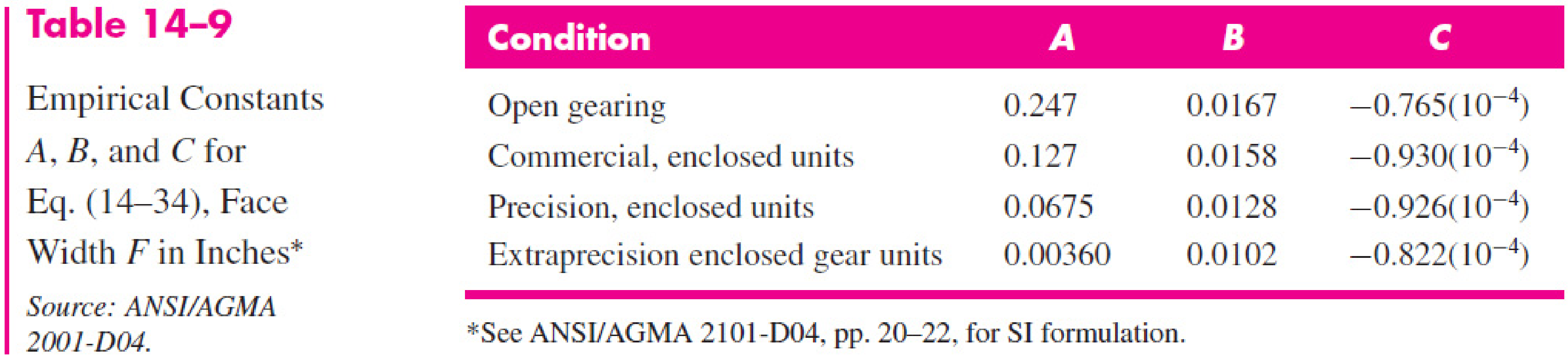


Table 14–9

Empirical Constants A, B, and C for Eq. (14–34), Face Width F in Inches*

Source: ANSI/AGMA 2001-D04.

Condition	**A**	**B**	**C**
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

*See ANSI/AGMA 2101-D04, pp. 20–22, for SI formulation.

Factor de Distribución de Carga (Cont.)

Gráficamente: $C_{ma} = A + BF + CF^2$

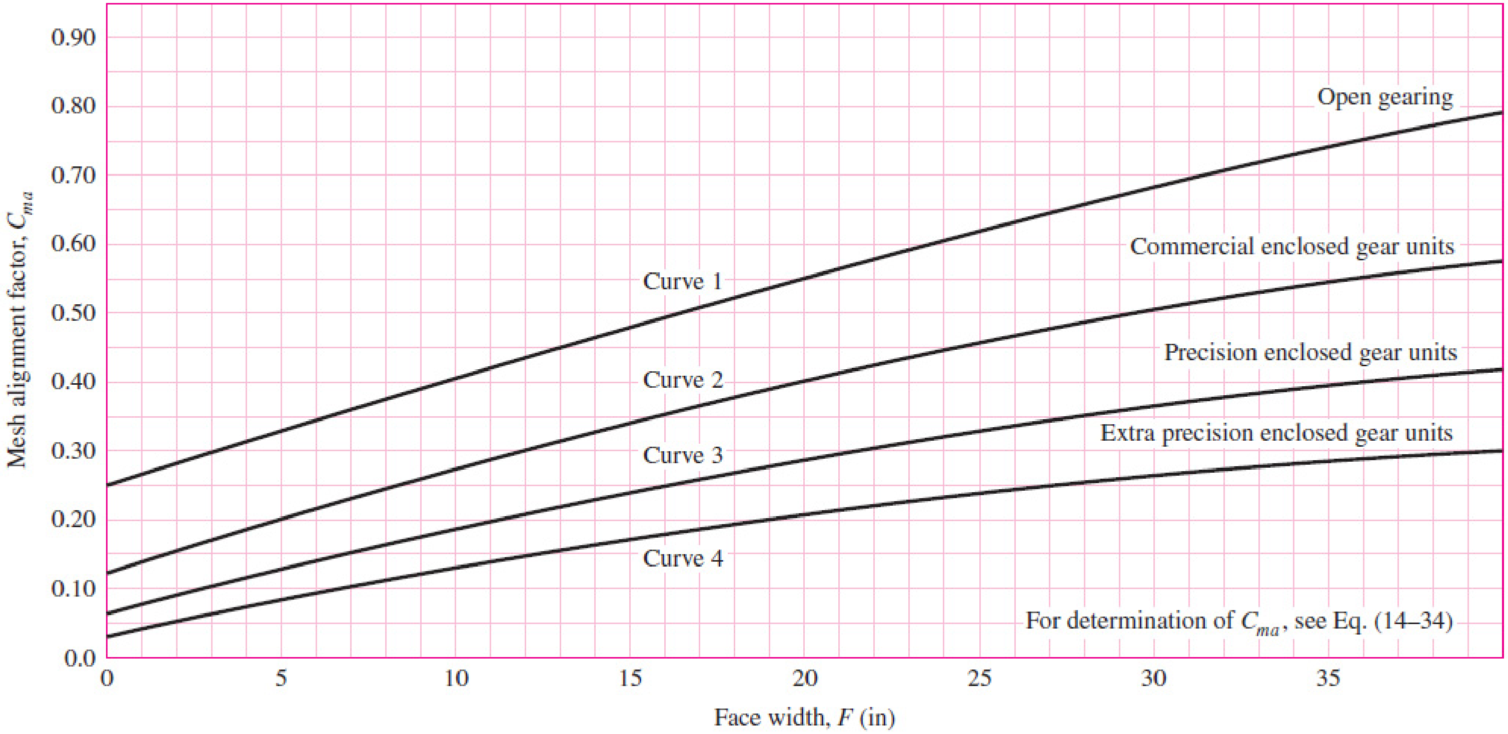


Figure 14–11

Mesh alignment factor C_{ma}. Curve-fit equations in Table 14–9. *(ANSI/AGMA 2001-D04.)*

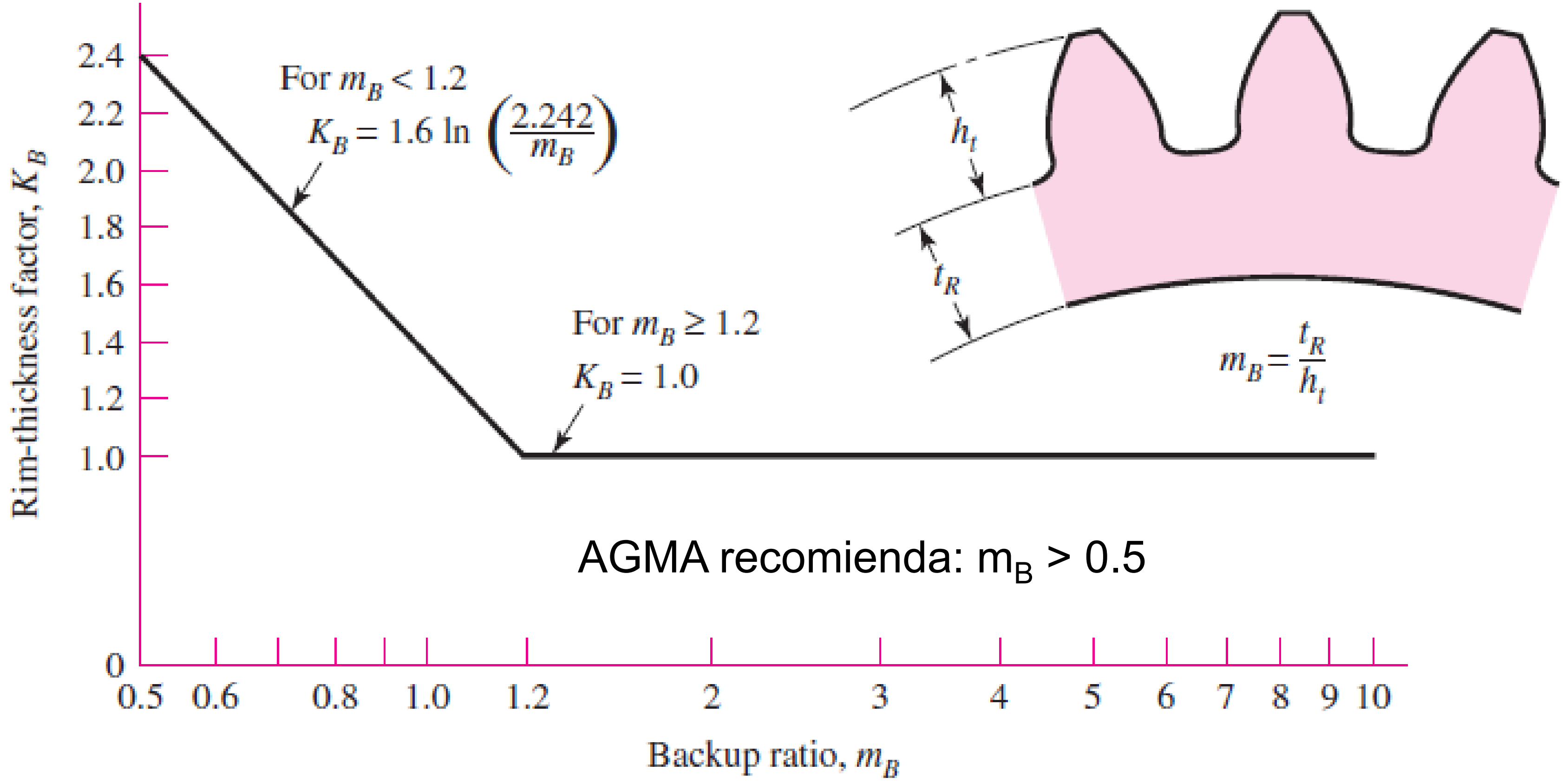
Factor de Distribución de Carga (Cont.)

$$\boxed{K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)}$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases}$$

Factor de Espesor del Aro

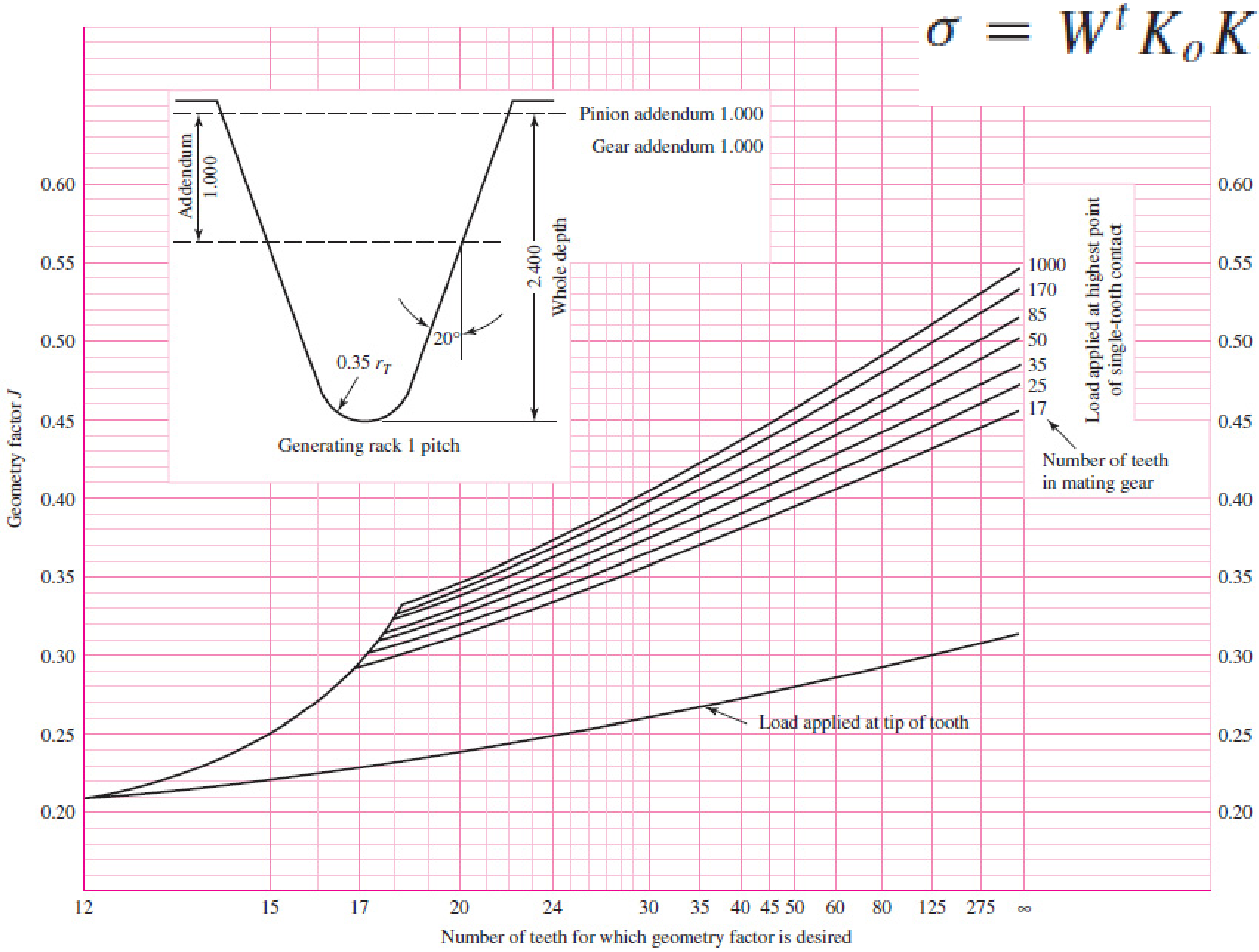
$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m \boxed{K_B}}{J}$$



AGMA recomienda: $m_B > 0.5$

Factor de Forma J (Y_J)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{\boxed{J}}$$



Para diente de profundidad completa y ángulo de presión 20°

Esfuerzo de Contacto Según AGMA

Diámetro de paso del piñón en [in]

$C_f = 1$

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

Diámetro de paso del piñón [mm]

$$C_p = \left[\frac{1}{\pi \left(\dfrac{1 - \nu_P^2}{E_P} + \dfrac{1 - \nu_G^2}{E_G} \right)} \right]^{1/2}$$

(1900 – 2300 $\text{psi}^{1/2}$)

(158 – 191 $\text{MPa}^{1/2}$)

Factor Geométrico a Picadura

$$I = \begin{cases} \dfrac{\cos\phi \ \sin\phi}{2} \dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi \ \sin\phi}{2} \dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases}$$

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P}$$

Esfuerzos Admisibles Según AGMA

K_T = 1.0 para T ≤ 250º F (120º C)

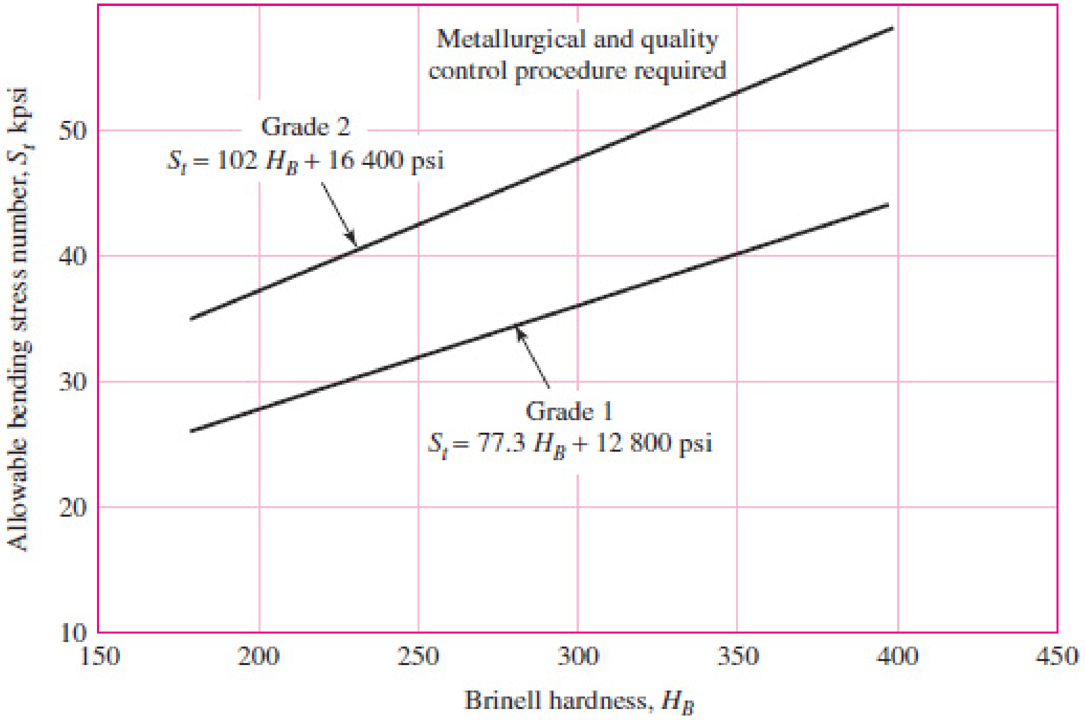
$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Esfuerzo de Flexión Permisible, St

Figure 14–2

Allowable bending stress number for through-hardened steels. The SI equations are $S_t = 0.533H_B + 88.3$ MPa, grade 1, and $S_t = 0.703H_B + 113$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*



Esfuerzo de Contacto Permisible, Sc

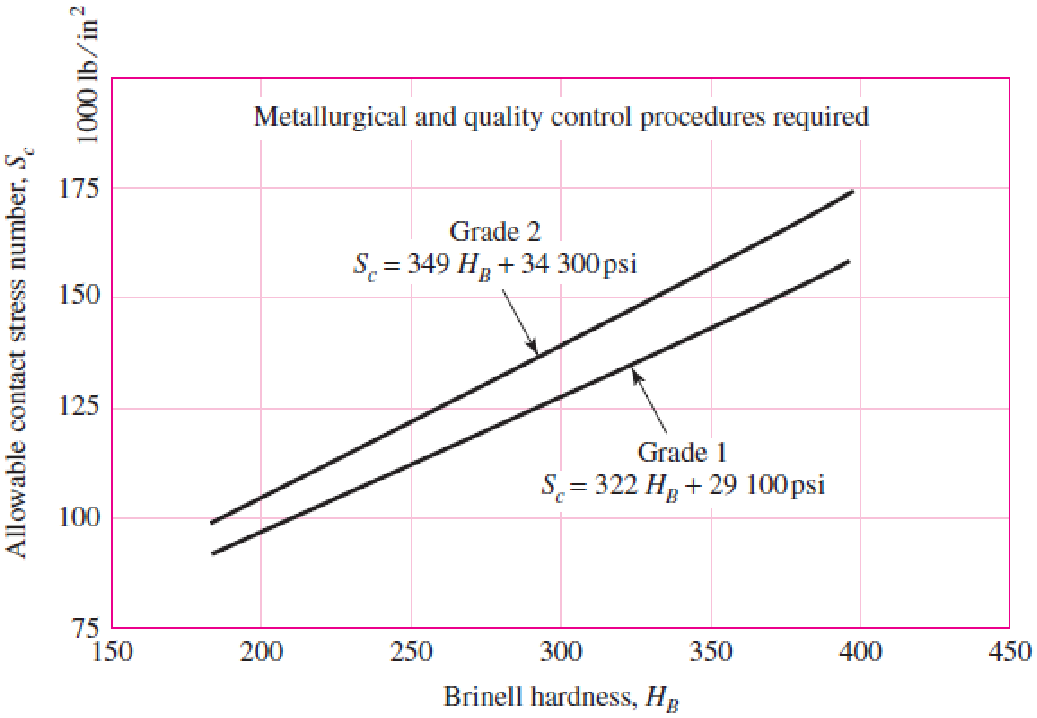


Figure 14–5

Contact-fatigue strength S_c at 10^7 cycles and 0.99 reliability for through-hardened steel gears. The SI equations are $S_c = 2.22H_B + 200$ MPa, grade 1, and $S_c = 2.41H_B + 237$ MPa, grade 2. *(Source: ANSI/AGMA 2001-D04 and 2101-D04.)*

Factor de Confiabilidad

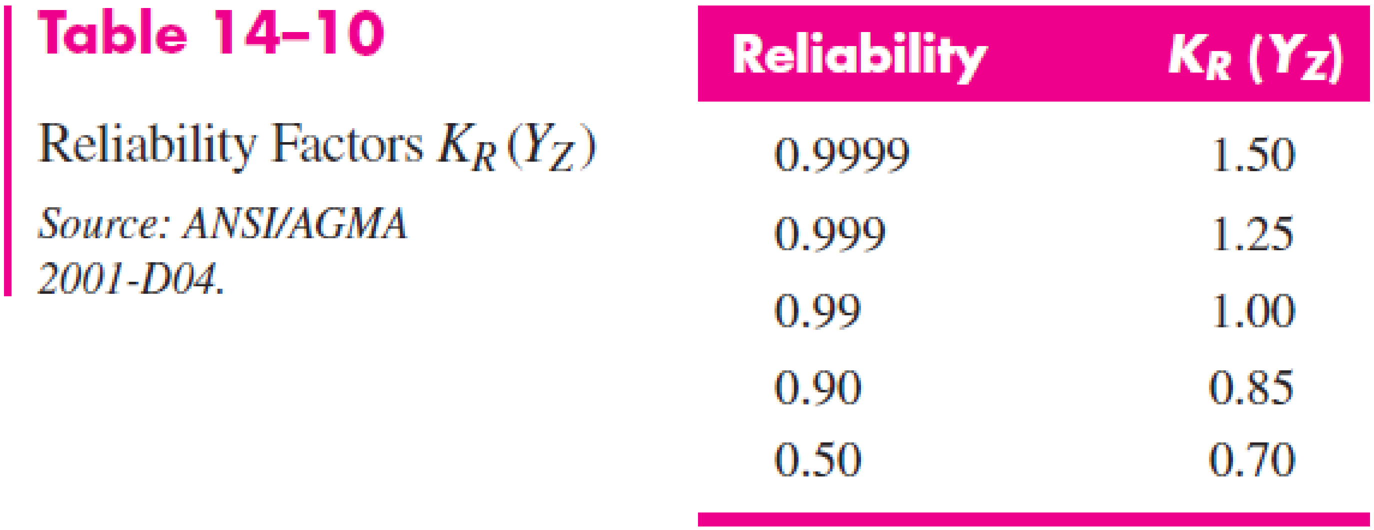


Table 14–10

Reliability Factors K_R (Y_Z)

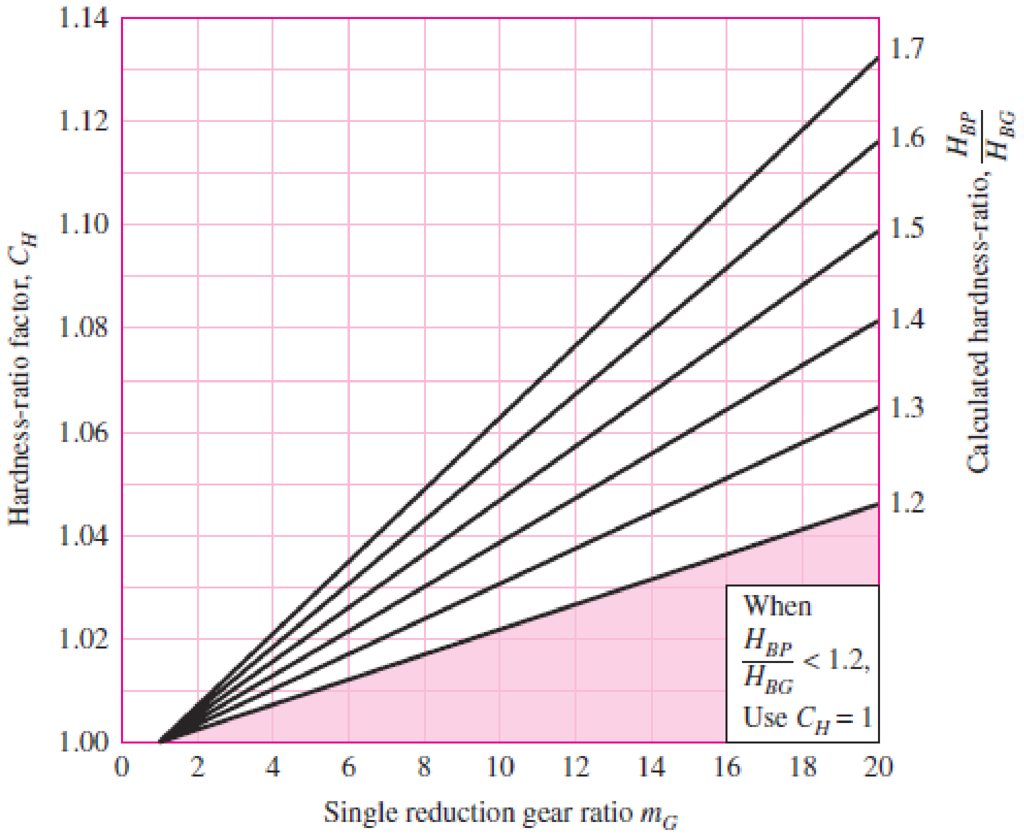
Source: ANSI/AGMA 2001-D04.

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

Relación de Dureza

Figure 14–12

Hardness-ratio factor C_H (through-hardened steel). *(ANSI/AGMA 2001-D04.)*



Se usa solo para la corona

Factor de Ciclaje YN

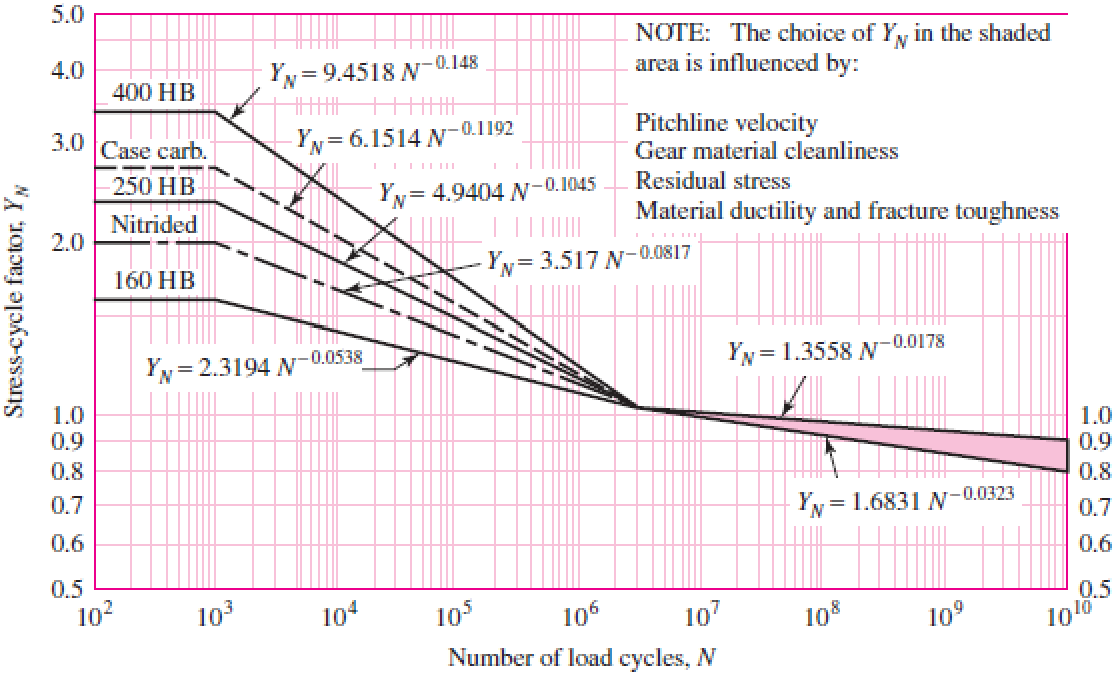


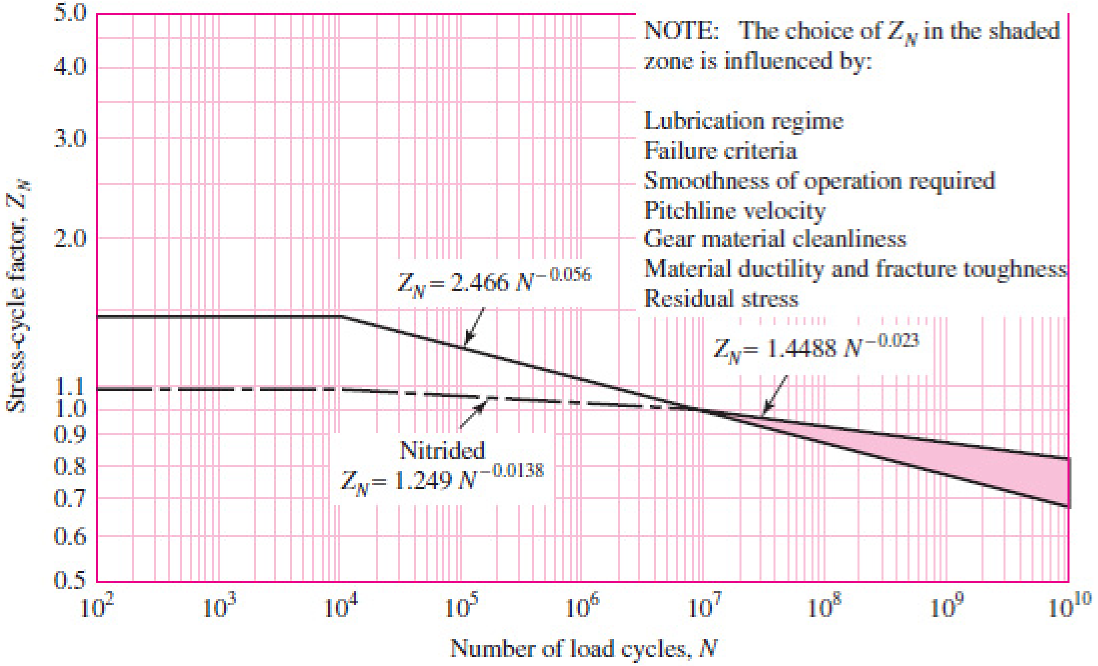
Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

Factor de Ciclaje ZN

Figure 14–15

Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*



NOTE: The choice of Z_N in the shaded zone is influenced by:

- Lubrication regime
- Failure criteria
- Smoothness of operation required
- Pitchline velocity
- Gear material cleanliness
- Material ductility and fracture toughness
- Residual stress

Factores de Seguridad

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{\text{fully corrected bending strength}}{\text{bending stress}}$$

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{\text{fully corrected contact strength}}{\text{contact stress}}$$

Factor de seguridad:

Comparar S_F con $S_H^2 \; o \; S_H^3$

Dientes no coronados

Dientes coronados