Pauta C3 - P1 MA3403-2 Primavera 2022

P1 a) (2pts) $X \sim N(180,64) \wedge Y \sim N(170,25)$

con $Z \sim N(0,1) \Rightarrow \bar{X} \sim N(180, \frac{32}{3}) \wedge \bar{Y} \sim N(170, \frac{25}{2})$ +0,5

tenemos que

$$\Rightarrow P(\bar{x} < \bar{y}) = P(\bar{x} - \bar{y} < 0) \overset{!}{=} P\left(Z < \frac{0-10}{\sqrt{\frac{32}{3} + \frac{25}{2}}}\right) = P(Z < -2{,}077) \leftarrow +0{,}5$$

$$= 1 - P(Z \leq 2{,}07) = 1 - 0{,}9808 = 0{,}0192 = 1{,}92\%$$ +0,5 +0,5

b) (1pts) $P(X < c) = P(Y > c)$

$$\Rightarrow P\left(Z < \frac{c-180}{8}\right) = P\left(Z > \frac{c-170}{5}\right)$$

con $Z \sim N(0,1)$ y por simetría $\Rightarrow \frac{c-180}{8} = \frac{-c+170}{5}$ +0,5

$$\Rightarrow 5c - 900 = -8c + 1360 \iff 13c = 2260 \iff c = 173{,}85$$ +0,5

c) (3 pts)

i) $P(|\bar{x} - \mu| < 2) = 0{,}9$

$$\rightarrow P(|\bar{x} - \mu| < 2) = P\left(|Z| < \frac{2-0}{8/\sqrt{n}}\right) = P(|Z| < \sqrt{n}/4)$$ +0,5 / con $Z \sim N(0,1)$

$$= 1 - 2P(Z > \sqrt{n}/4) \overset{!}{=} 0{,}9 \Rightarrow P(Z > \sqrt{n}/4) = 0{,}05$$ +0,75 +0,75

$$\Rightarrow \sqrt{n}/4 = 1{,}64 \quad \therefore \quad n = 43{,}29$$ +0,75

ii) • $\bar{x} = 180 \quad Z_{1-\frac{\alpha}{2}} = 1{,}96 \quad \sigma = 8 \quad n = 50$

$$\Rightarrow IC(\mu)_{95\%} = \left[180 - 1{,}96 \cdot \frac{8}{\sqrt{50}} ; 180 + 1{,}96 \cdot \frac{8}{\sqrt{50}}\right]$$ +0,5

$$= [177{,}78 ; 182{,}22]$$ +0,5