Paula Auxiliar 14

P1 a) $z^5 = 1 \Rightarrow e^{i5\theta} = e^{i2\pi k}$

$\Rightarrow \theta = \frac{2}{5}k\pi$, $k \in \{0, 1, 2, 3, 4\}$

$\Rightarrow \boxed{z = e^{\frac{2}{5}k\pi}, \; k \in \{0, 1, 2, 3, 4\}}$

b) $z^2 - z + 1 = 0 \Rightarrow z = \frac{1 \pm \sqrt{1-4}}{2}$

$\boxed{z = \frac{1 \pm \sqrt{3}i}{2}}$

c) $z^3 = 8i \Rightarrow r = 2$

$e^{i3\theta} = e^{i2\pi k} \Rightarrow \theta = \frac{2\pi k}{3}$, $k \in \{0, 1, 2\}$

$\Rightarrow \boxed{z = 2e^{\frac{2\pi k}{3}}}$

P2 $f = u + iv$, tal que $u = x^2 - y^2 + 2x$, $f(i) = 2i - 1$

$$\frac{\partial u}{\partial x} = 2x + 2 \overset{CR}{=} \frac{\partial v}{\partial y} \Rightarrow v = 2xy + 2y + C(x)$$

$$\Rightarrow \frac{\partial v}{\partial x} = 2y + C'(x) = -\frac{\partial u}{\partial y} = 2y$$

$$\Rightarrow C'(x) = 0 \rightarrow C(x) = C$$

$$f(x,y) = x^2 - y^2 + 2x + i(2xy + 2y + C)$$

$$f(0,1) = 2i - 1 + C = 2i - 1 \Rightarrow C = 0$$

$$f(x,y) = x^2 - y^2 + 2x + i(2xy + 2y)$$

P3 f Holomorfa $\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ y $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\tilde{f}$ holomorfa $\Rightarrow \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$ y $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

$$\Rightarrow \frac{\partial u}{\partial x} = 0 = \frac{\partial v}{\partial y} \text{ y } \frac{\partial u}{\partial y} = 0 = \frac{\partial v}{\partial x}$$

$\Rightarrow f = C + iC$, pues todas las derivadas son 0

Pd $\frac{\partial u}{\partial x}(0,0) = \lim_{t\to 0} \frac{u((0,0)+t(1,0)) - u(0,0)}{t}$

$= \lim_{t\to 0} \frac{0-0}{t} = 0$

$\frac{\partial u}{\partial y}(0,0) = \lim_{t\to 0} \frac{u((0,0)+t(0,1)) - u(0,0)}{t} = \frac{0}{t} = 0$

Sim. Im $\frac{\partial v}{\partial x}(0,0) = \frac{\partial v}{\partial y} = (0) \Rightarrow C\text{-}R$ ✓

~~$x\to 0$~~ $\lim_{z\to 0} \frac{g(z) - g(0)}{z} = \nexists$

Camino 1 $x = y$

$\lim_{z\to 0} \frac{g(z)-g(0)}{z} = \lim_{x\to 0} \frac{g(x+ix)-0}{(x+ix)} = \lim_{x\to 0} \frac{x^3}{x^2+x^4} = 0$

$\lim_{z\to 0} \frac{g(z)-g(0)}{z} \overset{x=y^2}{=} \lim_{y\to 0} \frac{g(y^2+iy)-0}{y^2+iy} = \lim_{y\to 0} \frac{y^4}{2y^4} = \frac{1}{2}$ $\Big\}\neq$

Por lo Tanto el límite no existe

$\Rightarrow$ No es diferenciable