Pauta Auxiliar 13

$\boxed{P1}$ Reemplazando $u(x,t) = X_{(x)} \cdot T_{(t)}$

$$T'(t) \cdot X(x) + e^2 X''_{(x)} \cdot T_{(t)} = 0$$

$$\Rightarrow \frac{T'(t)}{e^2 T(t)} = \frac{X''(x)}{-X(x)} = \alpha$$

$$\Rightarrow X''(x) + \alpha X_{(x)} = 0 \ , \ X_{(0)} = 0 = X_{(L)}$$

b) $\boxed{\alpha = 0}$ $\Rightarrow$

$$X''(x) = 0 \Rightarrow X_{(x)} = Ax + B \Rightarrow X_{(0)} = \boxed{0 = B}$$

$$X(L) = AL = 0$$

$$\Rightarrow \boxed{A = 0}$$

$$\Rightarrow \boxed{X(x) = 0}$$

c)

$$\underbrace{\int_0^L X''(x) \cdot X(x) dx}_{\text{por partes}} + \alpha \int_0^L X_{(x)}^2 dx = 0$$

$$\Rightarrow -\int_0^L X'(x)^2 dx + \alpha \int_0^L X^2(x) dx = 0$$

((es eso error enunciado))

d) Si $\alpha < 0 \Rightarrow$ Negativo + Negativo $= 0$

$\Rightarrow$ Ambos son 0

$$\Rightarrow \int_0^L X^2(x)\,dx = 0 \Rightarrow \boxed{X(x) = 0}$$

e) Usando $\lambda = \alpha^2 > 0$

$$\Rightarrow X''(x) + a^2 X(x) = 0 \Rightarrow X(x) = A\cos(ax) + B\sin(ax)$$

$$X(0) = 0 \Rightarrow \boxed{A = 0}$$

$$X(L) = 0 \Rightarrow B\sin(aL) = 0 \Rightarrow aL = N\pi$$

$$\Rightarrow \boxed{a = \frac{N\pi}{L}}$$

$$\Rightarrow X_N(x) = B_N \sin\left(\frac{N\pi}{L}x\right)$$

f) ~~$U_N(x,t) =$~~ $\dfrac{T_N'(t)}{c^2 T_N(t)} = \alpha \Rightarrow T_N'(t) - \alpha c^2 T_N(t) = 0$

$$T_N(t) = e^{\alpha c^2 \cdot t} \cdot D_N$$

$$\Rightarrow T_N(t) = D_N\, e^{\left(\frac{N\pi c}{L}\right)^2 t}$$

$$u_n(x,t) = C_n \cdot e^{\left(\frac{n\pi c}{L}\right)^2 \cdot t} \sin\left(\frac{n\pi}{L} x\right)$$

g) $u(x,t) = \sum_{n=1}^{\infty} u_n(x,t)$

$$u(x,0) = \sum_{n=1}^{\infty} C_n \cancel{e^{\left(\frac{n\pi c}{L}\right)^2 \cdot 0}}^{\,1} \sin\left(\frac{n\pi}{L} x\right)$$

$$\overset{CI}{=} 5 \sin\left(\frac{4\pi}{L} x\right)$$

$\Rightarrow C_n = 0 \quad \forall n \neq 4$ y $C_4 = 5$ (Iguales Termino a Termino)

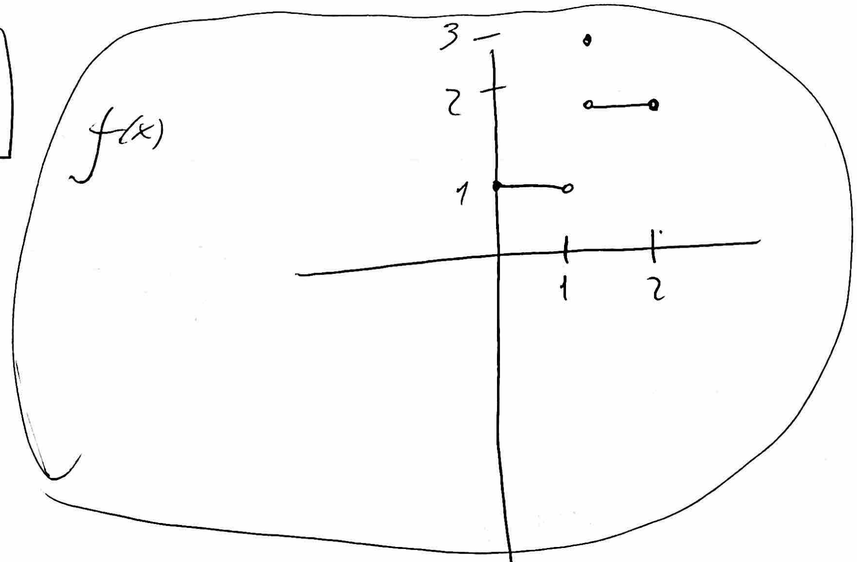
$$\Rightarrow \boxed{u(x,t) = 5 e^{\left(\frac{4\pi c}{L}\right)^2 \cdot t} \sin\left(\frac{4\pi}{L} x\right)}$$

h) $f(x)$ en Serie de Senos es

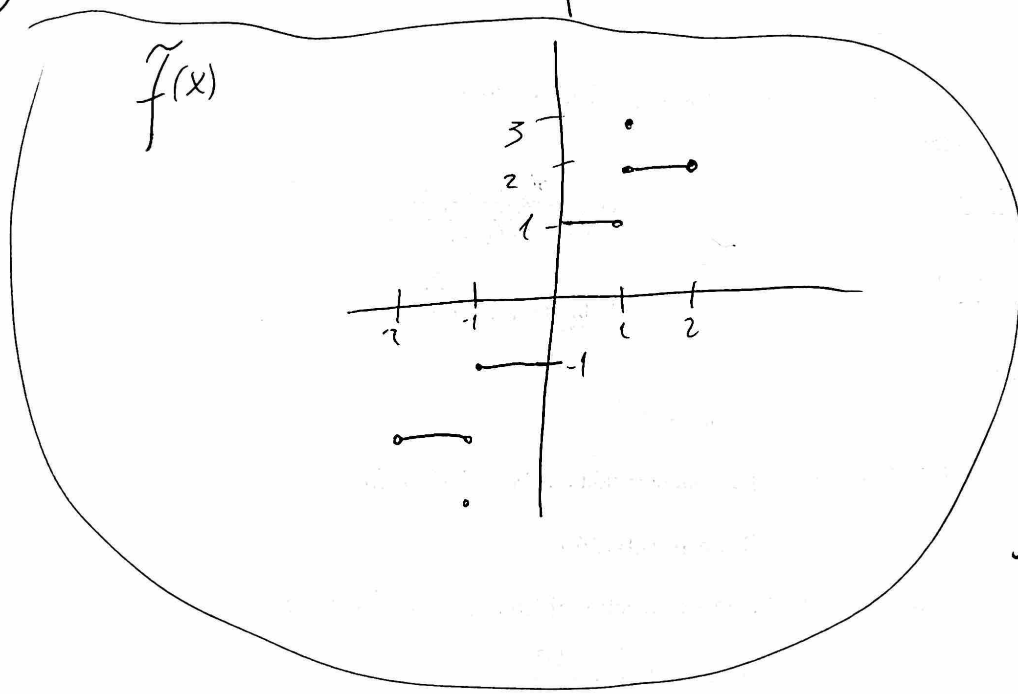
$a_n = 0$ y $b_n = \frac{1}{L} \int_{-L}^{L} \tilde{f}(x) \cdot \sin\left(\left(\frac{n\pi}{L}\right) x\right) dx$, con $\tilde{f}(x)$ expanción impar

$$\Rightarrow \boxed{b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi}{L} \cdot x\right)} \Rightarrow \boxed{C_n = b_n}$$

P2



a)



Obs:
Si no lo piden, no es necesario saber escribir $\tilde{f}(x)$. Solo que es impar

$a_n = 0$

(Propuesto)

Periodo $2 \cdot 2 = 4$

$$b_n = \frac{1}{2}\int_{-2}^{2} \tilde{f}(x) \cdot \sin\left(\frac{n\pi}{2}x\right) dx$$

Paridad

$$= \int_0^2 f(x) \cdot \sin\left(\frac{n\pi}{2}x\right) dx = \int_0^1 \sin\left(\frac{n\pi}{2}x\right) dx + 2\int_1^2 \sin\left(\frac{n\pi}{2}x\right) dx$$

$$b_N = \frac{-\cos\left(\frac{N\pi}{2}x\right)}{\left(\frac{N\pi}{2}\right)}\Bigg|_0^1 + (2)\cdot\frac{\cos\left(\frac{N\pi}{2}x\right)}{\left(\frac{N\pi}{2}\right)}\Bigg|_1^2$$

$$= \frac{1-\cos\left(\frac{N\pi}{2}\right)}{\left(\frac{N\pi}{2}\right)} + 2\,\frac{\cos\left(\frac{N\pi}{2}\right)-\cos(N\pi)}{\left(\frac{N\pi}{2}\right)}$$

$$= \left(\frac{2}{N\pi}\right)\left[1+\cos\left(\frac{N\pi}{2}\right)-2(-1)^N\right]$$

$$\Rightarrow S_f(x) = \sum_{N=1}^{\infty}\left(\frac{2}{N\pi}\right)\left[1+\cos\left(\frac{N\pi}{2}\right)-2(-1)^N\right]\sin\left(\frac{N\pi}{2}x\right)$$

b) Para discontinuidades $S_f(\bar{x}) = \dfrac{f(\bar{x}^-)+f(\bar{x}^+)}{2}$ (Punto medio)

$$S_f(1) = \frac{\overset{1}{f(1^-)}+\overset{2}{f(1^+)}}{2} = \frac{3}{2}$$

$$\Rightarrow \frac{3}{2} = \sum_{N=1}^{\infty}\left(\frac{2}{N\pi}\right)\left[1+\cos\left(\frac{N\pi}{2}\right)-2(-1)^N\right]\sin\left(\frac{N\pi}{2}\right)$$

$$\Rightarrow \frac{3}{2} = \sum_{K=1}^{\infty}\frac{2}{(2K-1)\pi}\cdot\left[1+\cos\left(\frac{(2K-1)\pi}{2}\right)-2(-1)^{2K-1}\right]\cdot\sin\left(\frac{(2K-1)\pi}{2}\right)$$

(Los pares $\uparrow$ dan 0)

Creado con Tiny Scanner

$$\Rightarrow \frac{\cancel{\pi}}{2} = \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)}\cdot\left[1 \cancel{+0+2}\right](-1)^{k+1}$$

$$\Rightarrow \frac{\pi}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k-1} \Rightarrow \boxed{-\frac{\pi}{4} = \sum_{k=1}^{\infty}\frac{(-1)^{k}}{(2k-1)}}$$