Paula Aguilar 12

P1 Aplicando Transformada, $\mathcal{F}(u)(\nu,0) = \mathcal{F}(f)(\nu)$

$$i\frac{d}{dt}\mathcal{F}(u)(\nu,t) + \mathcal{F}(u_{xx})(\nu,t) = 0$$

$$\Rightarrow i\frac{d}{dt}\mathcal{F}(u)(\nu,t) + (i\nu)^2\mathcal{F}(u)(\nu,t) = 0$$

$$i\frac{d}{dt}\mathcal{F}(u)(\nu,t) + (-\nu^2)\mathcal{F}(u)(\nu,t) = 0$$

$$\frac{d}{dt}\mathcal{F}(u)(\nu,t) + i\nu^2\mathcal{F}(u)(\nu,t) = 0$$

$$\Rightarrow \mathcal{F}(u)(\nu,t) = e^{-i\nu^2 t} A(\nu)$$

$$\Rightarrow \mathcal{F}(u)(\nu,0) = A(\nu) = \mathcal{F}(f)(\nu)$$

$$\Rightarrow \mathcal{F}(u)(\nu,t) = \mathcal{F}(f)(\nu)\, e^{-i\nu^2 t}$$

P2

$$v(x,t) = \begin{cases} u(x,t) & \text{si } x \geq 0 \\ u(-x,t) & \text{si } x < 0 \end{cases}$$

a) Propuesto verificar que cumple todo

b) Ahora que está definido para $x \in \mathbb{R}$

$$v_{tt} + a^2 v_{xxxx} = 0 \Rightarrow \frac{d^2}{dt^2} F(v) + a^2 (is)^4 F(v) = 0$$

$$\Rightarrow \frac{d}{dt^2} F(v) + (as^2)^2 F(v) = 0$$

$$\Rightarrow F(v)(s,t) = A\cos(as^2 t) + B\sin(as^2 t)$$

$$F(v)(s,0) = A = F(f)(s) = \sqrt{\tfrac{\pi}{2}}\, e^{-|s|}$$

$$\Rightarrow F(v)(s,t) = \sqrt{\tfrac{\pi}{2}}\, e^{-|s|} \cdot \cos(as^2 t) + B \sin(as^2 t)$$

$$\Rightarrow \frac{d}{dt} F(v)(s,t) = -as^2 \sqrt{\tfrac{\pi}{2}}\, e^{-|s|} \cdot \sin(as^2 t) + Bas^2 \cos(as^2 t)$$

$$0 = \frac{d}{dt} F(v)(s,0) = -as^2 \sqrt{\tfrac{\pi}{2}}\, e^{-|s|} \sin(0) + Bas^2 \Rightarrow \boxed{B = 0}$$

$$\Rightarrow \boxed{F(v)(s,t) = \sqrt{\tfrac{\pi}{2}} \cdot e^{-|s|} \cdot \cos(as^2 t)}$$

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c) $v(t,x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{\pi}{2}}\, e^{-|s|} \cdot \cos(as^2 t) \cdot e^{isx}\, ds$

$$= \frac{1}{2} \int_{-\infty}^{\infty} e^{-|s|} \cdot \cos(as^2 t) \cdot [\cos(sx) + i \underbrace{\cancel{\sin(sx)}}_{\text{Impar}=0}]\, ds$$

$$\overset{\text{Paridad}}{=} \int_{0}^{\infty} e^{-|s|} \cos(as^2 t) \cdot \cos(sx)\, ds$$

$$= \int_0^{\infty} e^{-s} \cos(as^2 t)\, \cos(sx)\, ds$$

$$\Rightarrow \boxed{u(t,x) = \int_0^{\infty} e^{-s} \cos(as^2 t) \cdot \cos(sx)\, ds}$$