P1 a)

$$\mathrm{Div}(F) = 2x + 2z$$

$$\Rightarrow \underbrace{\int_0^1\int_0^1\int_0^1}_{\text{Cubo}} 2x+2z\, dx\,dy\,dz = \int_0^1\int_0^1 x^2 + 2zx\Big|_0^1 dy\,dz$$

$$= \int_0^1\int_0^1 1 + 2z\, dy\,dz$$

$$= \int_0^1 1+2z\, dz = z + z^2\Big|_0^1 = \boxed{2}$$

S_1 $\quad x,y \in [0,1] \quad z=0 \Rightarrow \hat{n} = \begin{pmatrix}0\\0\\-1\end{pmatrix}$ (es la exterior)

$$\iint_{S_1} \vec{F}\cdot d\vec{S} = \int_0^1\int_0^1 \begin{pmatrix}x^2\\x^2\\0\end{pmatrix}\cdot\begin{pmatrix}0\\0\\-1\end{pmatrix} dy\,dz = \boxed{0}$$

S_2 $\quad x,y \in [0,1] \quad z=1 \Rightarrow \hat{n} = \begin{pmatrix}0\\0\\1\end{pmatrix}$ (es la exterior)

$$\iint_{S_2} \vec{F}\cdot d\vec{S} = \int_0^1\int_0^1 \begin{pmatrix}x^2\\x^2\\1\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix} dy\,dz = \int_0^1\int_0^1 dy\,dz = 1$$

S_3 $\quad x,z \in [0,1] \quad y=0 \Rightarrow \hat{n} = \begin{pmatrix}0\\-1\\0\end{pmatrix}$ (exterior)

$$\iint_{S_3} \vec{F}\cdot d\vec{S} = \int_0^1\int_0^1 \begin{pmatrix}x^2\\x^2\\z^2\end{pmatrix}\cdot\begin{pmatrix}0\\-1\\0\end{pmatrix} dx\,dz = \int_0^1\int_0^1 -x^2 dx\,dy = \boxed{-\frac{1}{3}}$$

Creado con Tiny Scanner

$\boxed{S_4}$ $x, z \in [0,1]$, $y = 1 \Rightarrow n = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ (exterior)

$$\iint_{S_4} \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^1 \begin{pmatrix} x^2 \\ x^2 \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} dx\,dz = \int_0^1 \int_0^1 x^2\,dx\,dz = \boxed{\frac{1}{3}}$$

$\boxed{S_5}$ $y, z \in [0,1]$, $x = 0 \Rightarrow \hat{n} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$$\iint_{S_5} \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^1 \begin{pmatrix} 0 \\ 0 \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} dy\,dz = 0$$

$\boxed{S_6}$ $y, z \in [0,1]$, $x = 1 \Rightarrow \hat{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\iint_{S_6} \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^1 \begin{pmatrix} 1 \\ 1 \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} dy\,dz = \int_0^1 \int_0^1 dy\,dz = \boxed{1}$$

$$\Rightarrow \iint_{\bigcup_{i=1}^{6} S_i} \vec{F} \cdot d\vec{S} = 1 + \left(\frac{1}{3}\right) + \left(-\frac{1}{3}\right) + 1 = \boxed{2}$$

Se verifica Gauss

P4 $\quad Div(F) = 0 \Rightarrow \iiint_{\Omega} Div(F)\, dV = 0$

Con Ω el cono invertido, por Gauss

$$\Rightarrow 0 = \iint_{\partial\Omega} \vec{F}\cdot d\vec{S} = \underbrace{\iint_{\text{Manto}} \vec{F}\cdot \underset{\text{exterior}}{\hat{n}}\, dS}_{\text{Pedido}} + \underbrace{\iint_{\text{Tapa}} \vec{F}\cdot \underset{\text{exterior}}{\hat{n}}\, dS}_{\text{Más Fácil}}$$

En la Tapa $\quad dS = r\,dr\,d\theta$, $\quad \hat{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\left.\right\}$ $r \in [0, a]$, $\theta \in [0, 2\pi]$

$$\Rightarrow \iint_{\text{Tapa}} \vec{F}\cdot d\vec{S} = \int_0^a \int_0^{2\pi} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r\, d\theta\, dr = -2\pi \int_0^a r\, dr$$

$$= \boxed{-\pi a^2} \Longrightarrow \boxed{\iint_{\text{Manto}} \vec{F}\cdot d\vec{S} = \pi a^2}$$

$\boxed{P3}$ $\vec{F}(x,y,z) = 4xz\,\hat{\imath} + xyz\,\hat{\jmath} + 3z\,\hat{k}$

$$\mathrm{Div}(\vec{F}) = 4z + xz + 3$$

Definir parametrización en polares (Cilíndricas)

$$\varphi(r,\theta,h) = (r\cos\theta,\ r\sin\theta,\ h)$$

$r \in [0,h]$ (1)

$r\sin\theta \in [0,\infty)$ (2)

$h \in [0,4]$ (3)

$\theta \in [0,2\pi)$ (4)

$$(4) \text{ y } (2) \iff \sin(\theta) \geq 0 \text{ y } \theta \in [0,2\pi)$$
$$\iff \theta \in [0,\pi]$$

Finalmente $V = \begin{cases} r \in [0,h] \\ \theta \in [0,\pi] \\ h \in [0,4] \end{cases} \implies \varphi(V) = \Omega$

$$dV = r\,dr\,d\theta\,dh$$

$$\implies \iiint_\Omega \mathrm{Div}(F)\,dV = \int_0^4 \int_0^h \int_0^\pi (4h + rh\cos\theta + 3)\,r\,d\theta\,dr\,dh$$

$$= \int_0^4 \int_0^h \left[4\theta h + rh \cancel{\sin\theta} + 3\theta \right] r \Big|_0^{\pi} dr\, dh$$

$$= \int_0^4 \int_0^h 4\pi r h + 3\pi r \, dr\, dh$$

$$= \int_0^4 \left[2\pi r^2 h + \frac{3}{2}\pi r^2 \right]_0^h dh$$

$$= \int_0^4 2\pi h^3 + \frac{3}{2}\pi h^2 \, dh$$

$$= \frac{\pi h^4}{2} + \frac{\pi h^3}{2} \Big|_0^4 = \cancel{4^3\pi\left(\frac{2}{3}+\frac{1}{2}\right) = \left[\frac{224}{3}\pi\right]}$$

Creo

$$\boxed{160\pi}$$

Vamos a calcular cada flujo

$\boxed{S_3}$ $\varphi(x,z) = (x, 0, z)$, con $x \in [-z, z]$, $z \in [0, 4]$

$$\left.\begin{aligned} \frac{\partial \varphi}{\partial x} &= (1,0,0) \\ \frac{\partial \varphi}{\partial z} &= (0,0,1) \end{aligned}\right\} \Rightarrow \frac{\partial \varphi}{\partial x} \times \frac{\partial \varphi}{\partial z} = (0,-1,0)$$

(orientado cambiar

$$\Rightarrow \iint_{S_3} \vec{F} \cdot d\vec{S} = \int_0^4 \int_{-z}^{z} \begin{pmatrix} \text{Algo} \\ 0 \\ \text{Algo} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} dx\, dz = 0$$

$\boxed{S_2}$ $\varphi(r,\theta) = (r\cos\theta, r\sin\theta, 4)$ $r \in [0,4]$, $\theta \in [0,\pi]$

$$\left.\begin{aligned} \frac{\partial \varphi}{\partial r} &= (\cos\theta, \sin\theta, 0) \\ \frac{\partial \varphi}{\partial \theta} &= (-r\sin\theta, r\cos\theta, 0) \end{aligned}\right\} \frac{\partial \varphi}{\partial r} \times \frac{\partial \varphi}{\partial \theta} = (0,0,r) \ \left(\begin{matrix}\text{Apunta} \\ \text{exterior}\end{matrix}\right)$$

$$\Rightarrow \iint_{S_2} \vec{F} \cdot \vec{dS} = \int_0^4 \int_0^{\pi} \begin{pmatrix} \text{Algo} \\ \text{Algo} \\ 12 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix} d\theta\, dr$$

$$= \int_0^4 \int_0^{\pi} 12 r \, d\theta\, dr = \int_0^4 6\pi \cdot 2r\, dr = 6\pi\, 4^2$$

$$= \boxed{96\pi}$$

$\boxed{S_1}$ $\varphi(\theta,h) = (h\cos\theta, h\sin\theta, h)$ $h \in [0,4]$, $\theta \in [0,\pi]$

$$\left.\begin{aligned} \frac{\partial \varphi}{\partial \theta} &= (-h\sin\theta, h\cos\theta, 0) \\ \frac{\partial \varphi}{\partial h} &= (\cos\theta, \sin\theta, 1) \end{aligned}\right\} \frac{\partial \varphi}{\partial \theta} \times \frac{\partial \varphi}{\partial h} = (h\cos\theta, h\sin\theta, -h) \ \left(\begin{matrix}\text{es el} \\ \text{exterior}\end{matrix}\right)$$

$$\Rightarrow \iint_{S_1} \vec{F} \cdot \vec{dS} = \int_0^4 \int_0^{\pi} \begin{pmatrix} 4h^2\cos\theta \\ h^3 \sin\theta\cos\theta \\ 3h \end{pmatrix} \cdot \begin{pmatrix} h\cos\theta \\ h\sin\theta \\ -h \end{pmatrix} d\theta\, dh$$

$$= \int_0^4 \int_0^\pi \underbrace{4h^3 \cos^2\theta}_{\frac{1+\cos(2\theta)}{2}} + h^4 \cos\theta \sin\theta^2 - 3h^2 \, d\theta \, dh$$

$$= \int_0^4 \int_0^\pi 2h^3 - 3h^2 \, d\theta \, dh + \int_0^4 \int_0^\pi \overset{0}{\cancel{\cos(2\theta) \cdot 2h^3}} + \overset{0}{\cancel{h^4 \cos\theta \sin\theta^2}} \, d\theta \, dh$$

$$= \int_0^4 2\pi h^3 - 3h^2 \pi \, dh = \left[\frac{\pi h^4}{2} - h^3 \pi \right]_0^4 = \pi \cdot 4^3 (2-1)$$

$$= \boxed{64\pi}$$

$$\Longrightarrow \underbrace{\iiint_\Omega \operatorname{Div}(F) \cdot dV}_{160\pi} = \underbrace{\iint_{\partial\Omega} \vec{F} \cdot d\vec{S}}_{96\pi + 0 + 64\pi = 160\pi}$$

✓

Verificado

<u>Propuesto</u> / Puede que lo hagamos el Miercoles