P1 El cilindro $= \{x^2+y^2=2\} \cap \underbrace{\{x^2+y^2+z^2 \le 4\}}_{\text{Dentro de la esfera}}$

$$= \{x^2+y^2=2\} \cap \{z^2 \le 2\} \quad (\text{Reemplazando})$$

$$= \{x^2+y^2=2\} \cap \{-\sqrt{2} \le z \le \sqrt{2}\} = S$$

Para integrar recomiendo buscar $\varphi(z,\theta)$ tal que $\varphi(D)=S$

y D sea mas facil de escribir

$$\varphi(z,\theta) = (\sqrt{2}\cos(\theta), \sqrt{2}\sin(\theta), z) \quad , \quad \begin{array}{l} z \in [-\sqrt{2}, \sqrt{2}] \\ \theta \in [0, 2\pi) \end{array}$$

Para buscar el vector Normal $\frac{\partial \varphi}{\partial z} \times \frac{\partial \varphi}{\partial \theta}$

$\frac{\partial \varphi}{\partial z} = (0,0,1)$ y $\frac{\partial \varphi}{\partial \theta} = (-\sqrt{2}\sin\theta, \sqrt{2}\cos\theta, 0)$

$$\Rightarrow \frac{\partial \varphi}{\partial z} \times \frac{\partial \varphi}{\partial \theta} = (-\sqrt{2}\cos\theta, -\sqrt{2}\sin\theta, 0) \Rightarrow \left\| \frac{\partial \varphi}{\partial z} \times \frac{\partial \varphi}{\partial \theta} \right\| = \sqrt{2}$$

Como $\left\| \frac{\partial \varphi}{\partial z} \times \frac{\partial \varphi}{\partial \theta} \right\| \neq 0 \Rightarrow$ es regular $\Rightarrow$ orientable

Para la dirección estudiemos $\theta = 0$ y $z = 0$

$\Rightarrow \varphi(0,0) = (\sqrt{2}, 0, 0)$ $\xrightarrow{\text{opción ①}} (1,0,0)$

$\xrightarrow{\text{opción ②}} (-1,0,0)$

$\boxed{\text{Normal interior } (-\cos\theta, -\sin\theta, 0)}$

Normal Exterior $(\cos\theta, \sin\theta, 0)$

$$\Rightarrow \iint_S V\cdot \hat{n}\, dS = \int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2\pi} \begin{pmatrix}\sqrt{2}\cos\theta - z\sqrt{2}\sin\theta\\ \sqrt{2}\sin\theta + z\sqrt{2}\cos\theta \\ z+2\cos\theta\sin\theta\end{pmatrix}\cdot\begin{pmatrix}-\cos\theta\\-\sin\theta\\0\end{pmatrix}\sqrt{2}\, d\theta\, dz$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2\pi} -\sqrt{2}\cos\theta^2 + \cancel{z\sqrt{2}\cos\theta\sin\theta - \sqrt{2}\cdot z\cos\theta\sin\theta} - \sqrt{2}\sin\theta^2\, d\theta\, dz$$

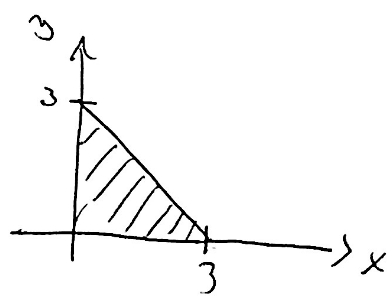
$$= \int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2\pi} (-\sqrt{2})\, d\theta\, dz = (2\pi)(2\sqrt{2})(-\sqrt{2}) = \boxed{-8\pi}$$

$\boxed{P2}$ Vamos a definir $\varphi(x,y) = (x, y, 6-2x-2y)$

Falta el rango de x e y para que esté en primer octante

$\Rightarrow \underbrace{x\geq 0,\ y\geq 0,\ 6-2x-2y\geq 0}_{\text{equivale}} = D$

$\underbrace{x\geq 0,\ y\geq 0,\ 3\geq x+y}_{\text{equivale}}$



(1) $0\leq x\leq 3,\quad 0\leq y\leq 3-x$ o

(2) $0\leq y\leq 3,\quad 0\leq x\leq 3-y$

Escogeremos (1) sin ningún motivo en especial

Falta normal, $\frac{\partial\varphi}{\partial x} = (1,0,-2)$, $\frac{\partial\varphi}{\partial y} = (0,1,-2)$

$\Rightarrow \frac{\partial\varphi}{\partial x}\times\frac{\partial\varphi}{\partial y} = (2,2,1) \Rightarrow \left\|\frac{\partial\varphi}{\partial x}\times\frac{\partial\varphi}{\partial y}\right\| = 3 \neq 0 \Rightarrow$ Regular

$\Rightarrow \vec{n} = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$, como $n_z = \left(\frac{1}{3}\right)$ Apunta hacia $\uparrow$

$$\Rightarrow \iint_S F \cdot d\vec{S} = \int_0^3 \int_0^{3-x} \begin{pmatrix} xy \\ -x^2 \\ 6-x-2y \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} dy\, dx$$

$$= \int_0^3 \int_0^{3-x} 2xy - 2x^2 + 6 - x - 2y \; dy\, dx$$

$$= \int_0^3 \Big[xy^2 - 2x^2 y + 6y - xy - y^2 \Big]_0^{3-x} dx$$

$$= \int_0^3 x(9-6x+x^2) - 2x^2(3-x) + 6(3-x) - x(3-x) - (3-x)^2 \, dx$$

$$= \int_0^3 (3-x)\left[3x - x^2 - 2x^2 + \cancel{6} - \cancel{x} - \cancel{3} + \cancel{x}\right] dx$$

$$= \int_0^3 (3-x)(3+3x-3x^2)\, dx$$

$$= 3\left[\int_0^3 (3+3x-3x^2) - x - x^2 + x^3 \, dx\right]$$

$$= 3\left[\int_0^3 3 + 2x - 4x^2 + x^3 \, dx\right] = 3\left[3x + x^2 - \frac{4}{3}x^3 + \frac{x^4}{4}\right]_0^3 = 3\left[9 + 9 - 9 \cdot 4 + \frac{27}{4}\right]$$

$$= \boxed{\frac{27}{4}}$$

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P3 Está dado que $\varphi(D) = S$, $D\begin{cases}\theta \in [0, 2\pi) \\ z \in [0, \frac{\pi}{2})\end{cases}$

Con $\varphi(\theta, z) = (\cos(z)\cdot\cos(\theta), \cos(z)\sin(\theta), z)$

a) Sacar $\left\|\frac{\partial\varphi}{\partial\theta} \times \frac{\partial\varphi}{\partial z}\right\|$

$$\frac{\partial\varphi}{\partial\theta} = (-\cos(z)\cdot\sin\theta, \cos(z)\cdot\cos(\theta), 0)$$

$$\frac{\partial\varphi}{\partial z} = (-\sin(z)\cos(\theta), -\sin(z)\sin(\theta), 1)$$

$$\frac{\partial\varphi}{\partial\theta} \times \frac{\partial\varphi}{\partial z} = (\cos(z)\cdot\cos(\theta), \cos(z)\cdot\sin(\theta), \cos(z)\sin(z))$$

$$\left\|\frac{\partial\varphi}{\partial\theta} \times \frac{\partial\varphi}{\partial z}\right\| = \sqrt{1 + \sin(z)^2}\cdot\cos(z)$$

$$\Rightarrow Area(S) = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sqrt{1+\sin(z)^2}\cdot\cos(z)\, dz\, d\theta$$

$u = \sin(z)$
$du = \cos(z)\,dz$

$$= \int_0^{2\pi}\int_0^{1} \sqrt{1+u^2}\, du\, d\theta$$

Extra

$$= \int_0^{2\pi}\int_0^{\operatorname{Argsh}(1)} \sqrt{1+\sinh(t)^2}\, \cosh(t)\, dt\, d\theta$$

$u = \sinh(t)$
$du = \cosh(t)\, dt$

$$= \int_0^{2\pi}\int_0^{\operatorname{Argsinh}(1)} \cosh^2(t)\, dt\, d\theta = \int_0^{2\pi}\int_0^{\operatorname{Arg\,sinh}(1)} \frac{1+\cosh(2t)}{2}\, dt\, d\theta$$

$$= \int_0^{2\pi} \frac{\text{Arg}\sinh(1)}{2} + \left[\left. \frac{\sinh(2t)}{4} \right|_0^{\text{Arg}\sinh(1)} \right] d\sigma$$

$$= \int_0^{2\pi} \frac{\text{Arg}\sinh(1)}{2} + \frac{\sinh(2\,\text{Arg}\sinh(1))}{4} \, d\sigma$$

$$= \boxed{2\pi \left[\frac{\text{Arg}\sinh(1)}{2} + \frac{\sinh(2\,\text{Arg}\sinh(1))}{4} \right]}$$

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6) i) De la parte anterior $\hat{n} = \left(\frac{\cos\sigma}{\sqrt{1+\sin(z)^2}}, \frac{\sin\sigma}{\sqrt{1+\sin(z)^2}}, \frac{\sin(z)}{\sqrt{1+\sin(z)^2}} \right)$

$$\vec{F} \cdot \hat{n} = \begin{pmatrix} \cos(z)\cdot\cos(\sigma) \\ \cos(z)\cdot\sin(\sigma) \\ \cancel{\cos(z)}\cdot 0 \end{pmatrix} \cdot \begin{pmatrix} \cos\sigma / \sqrt{1+\sin(z)^2} \\ \sin\sigma / \sqrt{1+\sin(z)^2} \\ \text{Algo} \end{pmatrix} = \frac{\cos(z)}{\sqrt{1+\sin(z)^2}}$$

$$\iint_S F \cdot \hat{n}\, dS = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos^2(z)\, dz\, d\sigma = \cancel{2}\pi \int_0^{\frac{\pi}{2}} \frac{1+\cos(2z)}{\cancel{2}}\, dz$$

$$= \pi \left[\frac{\pi}{2} + 0 \right] = \boxed{\frac{\pi^2}{2}}$$

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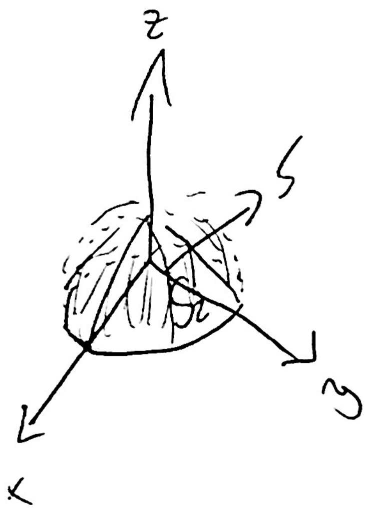
2) Usando $\nabla \cdot \vec{F}(r,\phi,\theta) = \frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\phi}\cdot\frac{\partial(\sin(\phi)F_\phi)}{\partial\phi}$

$$+ \frac{\partial(F_\theta)}{\partial\theta}$$

Como $F(r,\phi,\theta) = F_\theta = e^{r^3} + \cos(\phi) \Rightarrow \frac{\partial F_\theta}{\partial\theta} = 0$

$\Rightarrow Div(F) = 0$



, $S_2 = \{(x^2+y^2)=1 \text{ y } t=0\}$

$S \cup S_2 = \partial\Omega$, con

Ω La región entre S y el circulo

Por Gauss $\Rightarrow$

$$\underbrace{\iiint_\Omega div(F)\,dV}_{0} = \underbrace{\iint_S F\cdot d\vec{S}}_{\text{La pedida}} + \underbrace{\int_{S_1} F\cdot d\vec{S}}_{\text{Más Fácil}}$$

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S_1 $\quad \hat{n} = \begin{pmatrix}0\\0\\-1\end{pmatrix}$, $\quad dS = r\,dr\,d\theta$

↗ Es exterior a Ω

$$D = \begin{cases} r \in [0,1] \\ \theta \in [0,2\pi] \end{cases}$$

$$\iint_{S_1} F\cdot d\vec{S} = \int_0^1\int_0^{2\pi} \Big|\underline{\qquad\qquad\qquad}$$

Problema :(

$\vec{F}$ está en esferas y $\hat{n} = \begin{pmatrix}0\\0\\-1\end{pmatrix}$ en cartesianas

$\Rightarrow$ Pasaremos $\vec{F}$ a cartesianas con $z = 0 = r\cos\phi$

$\Rightarrow \boxed{\phi = \frac{\pi}{2}}$

$$\cancel{\hat{\theta} = (\cos\theta\cdot\cos\varphi\,\hat{i} + \cos\varphi\sin\theta\,\hat{j} - \sin\varphi\,\hat{k})}$$

$$\cancel{\overset{\varphi=\frac{\pi}{2}}{\Longrightarrow} \hat{\theta} = (0,0,-1)}$$

$$\cancel{\Rightarrow \iint_{S_1} F\cdot d\vec{S} = \int_0^1\int_0^{2\pi} \begin{pmatrix}0\\0\\-(e^{r^3}+\cos(\frac{\pi}{2}))\end{pmatrix}\cdot\begin{pmatrix}0\\0\\-1\end{pmatrix} r\,d\theta\,dr}$$

$\hat{\theta} = (-\operatorname{Sen}\theta\,\hat{\imath} + \operatorname{Cos}\theta\,\hat{\jmath})$

$\hat{\theta}\cdot(-\hat{k}) = 0$ (Se puede justificar de cilindricas)

$$\Longrightarrow \iint_{S_1} \vec{F}\cdot d\vec{S} = 0 \Longrightarrow \boxed{\iint_{S} \vec{F}\cdot d\vec{S} = 0}$$