Falta orientación Usar Antihorario

P1 $r(t) = (5\cos(t)+1,\ 5\sin(t)+1)$, $t \in [0, 2\pi]$

a) $$\int_0^{2\pi} (5\sin(t)+1,\ 5\cos(t)+1) \cdot (-5\sin(t),\ 5\cos(t))\, dt$$

$$= \int_0^{2\pi} -25\sin(t)^2 - \cancel{5\sin(t)}^{0} + 25\cos(t)^2 + \cancel{5\cos(t)}^{0}\, dt$$

$$= \int_0^{2\pi} \cos(2t) \cdot 25\, dt = \sin(2t) \cdot \frac{25}{2} \Big|_0^{2\pi} = 0$$

b) No Tomando que Γ es cerrada, veamos si $F(x,y) = (1, y^2 - y)$ es conservativo

$$\frac{\partial g(x,y)}{\partial y} = y^2 - y \overset{\int dy}{\Longrightarrow} g(x,y) = \frac{y^3}{3} - \frac{y^2}{2} + C(x)$$

$$\frac{\partial g}{\partial x} = C'(x) \overset{!}{=} 1 \overset{\int dx}{\Longrightarrow} C(x) = x + K$$

$$\Rightarrow \boxed{g(x,y) = x + \frac{y^3}{3} - \frac{y^2}{2} + K}$$ cumple $\nabla g = F$ (Basta usar usemos $K=0$)

$\Rightarrow F$ es conservativo $\Longrightarrow \oint F \cdot \frac{dr}{dt}\, dt = 0$, para cualquier curva

Por lo Tanto es 0

*** Nota**: También se puede hacer para a pues $g(x,y) = xy$

$\nabla g = (y, x)$

Propuesta: Ambos son conservativos $\Rightarrow \oint F \cdot \frac{dr}{dt} = 0$, Ambos serán cero

P2 Notando que $g(x,y) = xy \Rightarrow \nabla g = F \Rightarrow$ es conservativo

$$\Rightarrow \int_{C_1} F \cdot \frac{dr}{dt} dt = \int_{C_2} F \cdot \frac{dr}{dt} dt = \int_{C_3} F \cdot \frac{dr}{dt} dt,$$ pues parten y terminan en $(-1,0)$ a $(1,0)$

Pero calculemos C_1, ~~...~~ C_2 y C_3 igual

a) $C_1 \Rightarrow r_{C_1} = (-\cos(t), \sin(t))$, $t \in [0, \pi]$

$C_2 \Rightarrow r_{C_2} = (t, 0)$, $t \in [-1, 1]$

$C_3 \Rightarrow r_{C_3} = (\cos(t), \sin(t))$ $t \in [\pi, 2\pi]$

$\boxed{C_1}$

$$\int_0^\pi (\sin(t), -\cos(t)) \cdot (\sin(t), \cos(t)) dt = \int_0^\pi -\cos(2t)\, dt = -\frac{\sin(2t)}{2}\Big|_0^\pi = \boxed{0}$$

Todas deberían ser 0

$\boxed{C_2}$
$$\int_{-1}^{1} (0, t) \cdot (1, 0)\, dt = \int_{-1}^{1} 0 = \boxed{0}$$

$\boxed{C_3}$
$$\int_\pi^{2\pi} (\sin(t), \cos(t)) \cdot (-\sin(t), \cos(t))\, dt = \int_\pi^{2\pi} \cos(2t) = \frac{\sin(2t)}{2}\Big|_\pi^{2\pi} = \boxed{0}$$

b) $\boxed{C_1}$
$$\int_0^\pi (-\sin(t), -\cos(t)) \cdot (\sin(t), \cos(t))\, dt = -\int_0^\pi 1\, dt = \boxed{-\pi}$$

$\boxed{C_2}$ $\int_{-1}^{1} (0, t)\cdot(1,0)\,dt = \boxed{0}$

$\boxed{C_3}$ $\int_{\pi}^{2\pi} (-\sin(t), \cos(t))\cdot(-\sin(t), \cos(t))\,dt$

$= \int_{\pi}^{2\pi} 1\,dt = \boxed{\pi}$

$\Rightarrow$ El segundo F no es conservativo

$\boxed{P3}$ $\frac{\partial g}{\partial x} = y^2\cos(x) + z^3 \xRightarrow{\int dx} y^2\sin(x) + z^3x + C(y,z)$

$\Rightarrow \frac{\partial g}{\partial y} = 2y\sin(x) + C'(y,z) = 2y\sin(x) - 4 \Rightarrow C'(y,z) = -4$

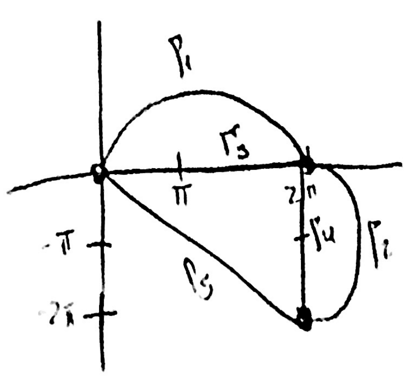
$\xRightarrow{\int dy} C(y,z) = -4y + K(z)$

$\Rightarrow \frac{\partial g}{\partial z} = 3z^2x + K'(z) = 3xz^2 + 2z \Rightarrow K'(z) = 2z \Rightarrow K(z) = z^2$

$\Rightarrow \boxed{g(x,y,z) = y^2\sin(x) + z^3x - 4y + z^2} \Rightarrow$ Es conservativo

$\boxed{P4}$ Error enunciado $\Gamma_2 = (\pi(2+\sin(t)), \pi(-\cos(t)-1))$

Grafico



$\Gamma_1 : r_{\Gamma_1}(t) (\pi(1-\cos(t)), \pi\sin(t))$ $t \in [0,\pi]$

$\Gamma_2 : r_{\Gamma_2}(t) = (\pi(2+\sin(t)), \pi(-\cos(t)-1))$ $t \in [0,2\pi]$

$\Gamma_3 : r_{\Gamma_3}(t) = (t, 0)$ $t \in [0, 2\pi]$

$\Gamma_4 : r_{\Gamma_4}(t) = (2\pi, -t)$ $t \in [0, 2\pi]$

$\Gamma_5 : r_{\Gamma_5}(t) = (t, -t)$ $t \in [0, 2\pi]$

Como $W_1 = \int_0^{\pi} \left(\sin(\pi(1-\cos(t)))\, \pi \sin(t),\ \sin(\pi \sin(t)) - \cos(\pi(1-\cos(t))) \right) \cdot (\pi \sin(t), \pi \cos ...$

Es super feo

Usar que si F conservativo $W_1 = \int_{\Gamma_1} F \cdot dr = \int_{\Gamma_3} F \cdot dr$

$$W_2 = \int_{\Gamma_2} F \cdot dr = \int_{\Gamma_4} F \cdot dr$$

$$W_1 + W_2 = \int_{\Gamma_3} F \cdot dr + \int_{\Gamma_4} F \cdot dr = \int_{\Gamma_5} F \cdot dr$$

<u>Ejercicio</u> Encontrar g tal que $\nabla g = F$, usando Tecnica de P_3

$$\boxed{R: \quad g(x,y) = -\cos(x)y - \cos(y) + K} \implies F \text{ es conservativo}$$

$$W_1 = \int_0^{2\pi} (\sin(t) \cdot 0,\ \sin(0) - \cos(t)) \cdot (1,0)\, dt = \int_0^{2\pi} 0\, dt = \boxed{0}$$

$$W_2 = \int_0^{2\pi} (\sin(2\pi) \cdot (-t),\ \sin(-t) - \cos(2\pi)) \cdot (0,-1)\, dt$$

$$= \int_0^{2\pi} \sin(t) + 1\, dt = \boxed{2\pi}$$

$\implies$ El Trabajo es $\boxed{2\pi}$

<u>otra forma</u> $W = \int_0^{2\pi} (\sin(t)(-t),\ \sin(t) - \cos(t))(1,-1)\, dt$

$$= \int_0^{2\pi} (-\sin(t)\, t)\, dt = \boxed{2\pi}$$

Ya $\int_0^{2\pi} \sin(t)\, dt = \int_0^{2\pi} \cos(t)\, dt = 0$