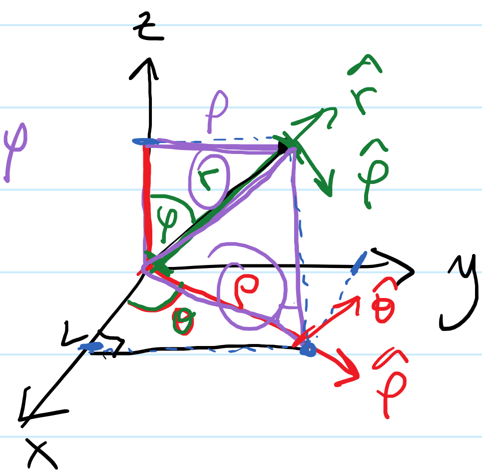
Auxiliar 2

$$\psi(x,y,z) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-\sqrt{x^2+y^2+z^2}/a_0}$$

$$\langle x^2 \rangle \equiv \iiint x^2 \psi^2 \, dx\, dy\, dz$$

$$= \iiint x^2 \cdot \frac{1}{\pi a_0^3} e^{-2\sqrt{x^2+y^2+z^2}/a_0} \, dx\, dy\, dz$$

$\rho = r \sin\varphi$



$(x, y, z) \to \{\hat{x}, \hat{y}, \hat{z}\}$

$(\rho, \theta, z) \to \{\hat{\rho}, \hat{\theta}, \hat{z}\}$

$(r, \varphi, \theta) \to \{\hat{r}, \hat{\varphi}, \hat{\theta}\}$

$(q_1, q_2, q_3) \to \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\hat{\rho} \equiv \frac{\partial \vec{r}}{\partial \rho} \Big/ \left\| \frac{\partial \vec{r}}{\partial \rho} \right\|$$

$$\hat{e}_i \equiv \frac{\partial \vec{r}}{\partial q_i} \Big/ \left\| \frac{\partial \vec{r}}{\partial q_i} \right\|$$

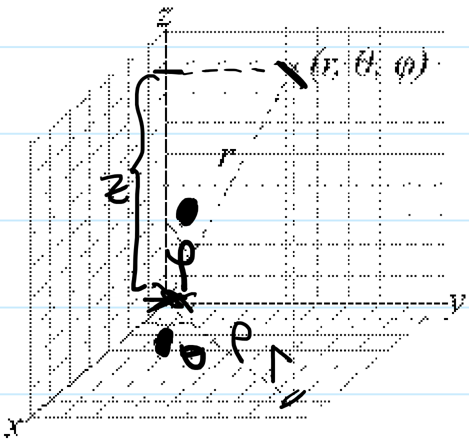
$x = \rho \cos\theta$

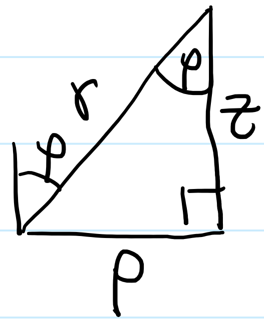
$y = \rho \sin\theta$

$z = z$

$x^2 + y^2 = \rho^2$

Auxiliar 2





$x = \rho \cos\theta$

$y = \rho \sin\theta$

$z = z$

$\rho = r \operatorname{sen}\varphi$

$z = r\cos\varphi$

$x = r \operatorname{sen}\varphi \cos\theta$

$y = r \operatorname{sen}\varphi \operatorname{sen}\theta$

$z = r\cos\varphi$

$x^2 + y^2 + z^2 = r^2$

$$\begin{aligned}
x^2+y^2+z^2 &= r^2\operatorname{sen}^2\varphi\cos^2\theta + r^2\operatorname{sen}^2\varphi\operatorname{sen}^2\theta + r^2\cos^2\varphi \\
&= r^2\left(\underline{\operatorname{sen}^2\varphi\cos^2\theta} + \operatorname{sen}^2\varphi\operatorname{sen}^2\theta + \cos^2\varphi\right) \\
&= r^2\left(\operatorname{sen}^2\varphi\left(\underbrace{\cos^2\theta + \operatorname{sen}^2\theta}_{1}\right) + \cos^2\varphi\right) \\
&= r^2\left(\operatorname{sen}^2\varphi + \cos^2\varphi\right)
\end{aligned}$$

$x^2+y^2+z^2 = r^2$

Auxiliar 2

$$\langle x^2 \rangle = \iiint_{\mathbb{R}^3} x^2 \cdot \frac{1}{\pi a_0^3} e^{-2\sqrt{x^2+y^2+z^2}/a_0} \, dx\,dy\,dz$$

$$\frac{1}{\pi a_0^3} \iiint r^2 \operatorname{sen}^2\varphi \cos^2\theta \; e^{-2r/a_0} \, |J| \, dr\,d\varphi\,d\theta$$

$x = r \operatorname{sen}\varphi \cos\theta$

$$\frac{\partial x}{\partial r} = \operatorname{sen}\varphi\cos\theta \,, \quad \frac{\partial x}{\partial \varphi} = r\cos\varphi\cos\theta \,, \quad \frac{\partial x}{\partial \theta} = -r\operatorname{sen}\varphi\operatorname{sen}\theta$$

$y = r \operatorname{sen}\varphi \operatorname{sen}\theta$

$$\frac{\partial y}{\partial r} = \operatorname{sen}\varphi\operatorname{sen}\theta \,, \quad \frac{\partial y}{\partial \varphi} = r\cos\varphi\operatorname{sen}\theta \,, \quad \frac{\partial y}{\partial \theta} = r\operatorname{sen}\varphi\cos\theta$$

$z = r\cos\varphi$

$$\frac{\partial z}{\partial r} = \cos\varphi \,, \quad \frac{\partial z}{\partial \varphi} = -r\operatorname{sen}\varphi \,, \quad \frac{\partial z}{\partial \theta} = 0$$

$$J = \begin{pmatrix} \operatorname{sen}\varphi\cos\theta & r\cos\varphi\cos\theta & -r\operatorname{sen}\varphi\operatorname{sen}\theta \\ \operatorname{sen}\varphi\operatorname{sen}\theta & r\cos\varphi\operatorname{sen}\theta & r\operatorname{sen}\varphi\cos\theta \\ \cos\varphi & -r\operatorname{sen}\varphi & 0 \end{pmatrix}$$

$$\begin{pmatrix} \cos\varphi & -r\sin\varphi & 0 \end{pmatrix}$$

Auxiliar 2

$$J = \begin{pmatrix} \operatorname{sen}\varphi\cos\theta & r\cos\varphi\cos\theta & -r\operatorname{sen}\varphi\operatorname{sen}\theta \\ \operatorname{sen}\varphi\operatorname{sen}\theta & r\cos\varphi\operatorname{sen}\theta & r\operatorname{sen}\varphi\cos\theta \\ \cos\varphi & -r\operatorname{sen}\varphi & 0 \end{pmatrix}$$

$$|J| = r^2 \operatorname{sen}\varphi$$

$$\langle x^2 \rangle = \frac{1}{\pi a_0^3}\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} r^2 \operatorname{sen}^2\varphi \cos^2\theta \, e^{-2r/a_0} r^2 \operatorname{sen}\varphi \, dr\, d\varphi\, d\theta$$

$$= \frac{1}{\pi a_0^3}\underbrace{\left(\int_0^{\infty} r^4 e^{-2r/a_0}\, dr\right)}_{I_3 \,\to\, \frac{3}{4}a_0^5}\underbrace{\left(\int_0^{\pi}\operatorname{sen}^3\varphi\, d\varphi\right)}_{I_2 \,\to\, 4/3}\underbrace{\left(\int_0^{2\pi}\cos^2\theta\, d\theta\right)}_{I_1 \,\to\, \pi}$$

$$I_2 = \int_0^{\pi}\operatorname{sen}^3\varphi\, d\varphi$$

$$\operatorname{sen}^2\varphi = 1 - \cos^2\varphi = 1 - u^2$$

$$\to u = \cos\varphi$$

$$du = -\operatorname{sen}\varphi\, d\varphi$$

$$= \int_0^{\pi}\operatorname{sen}^2\varphi \cdot \operatorname{sen}\varphi\, d\varphi$$

$$= -\int_1^{-1}(1-u^2)\, du = \int_{-1}^{+1}(1-u^2)\, du$$

$$= 2 - 2\cdot\frac{1}{3} = \frac{4}{3}$$

Auxiliar 2

$$I_3 = \int_0^\infty r^4 e^{-2r/a_0}\, dr \quad , \quad u = \frac{2r}{a_0}$$

$$= \int_0^\infty \frac{a_0^4}{2^4} u^4 e^{-u} \frac{a_0}{2}\, du \qquad dr = \frac{a_0}{2}\, du$$

$$r^4 = \frac{a_0^4}{2^4} u^4$$

$$= \frac{a_0^5}{2^5} \overbrace{\int_0^\infty u^4 e^{-u}\, du}^{I} = \frac{24\, a_0^5}{2^5} = \frac{3}{4} a_0^5$$

Derivada		integral
u^4	$+$	e^{-u}
$4u^3$	$-$	$-e^{-u}$
$12u^2$	$+$	e^{-u}
$24u$	$-$	$-e^{-u}$
24	$+$	e^{-u}
0		$-e^{-u}$

$$I = \cancel{u^4 \cdot -e^{-u}\Big|_0^\infty} - \cancel{4u^3 e^{-u}\Big|_0^\infty} + \cancel{12u^2 \cdot -e^{-u}\Big|_0^\infty}$$
$$- \cancel{24u \cdot e^{-u}\Big|_0^\infty} + 24 \cdot -e^{-u}\Big|_0^\infty$$

$$= -24\left(e^{-\infty} - e^{-0}\right) = -24(0 - 1)$$

$= 24$

Auxiliar 2

$$(\Delta x)^2 = \langle x^2 \rangle = a_0^2$$

$$(\Delta p)^2 = \frac{\hbar^2}{3 a_0^2}$$

$$\Delta x \Delta p = \frac{\hbar}{\sqrt{3}} \geq \frac{\hbar}{2} \quad \checkmark\checkmark$$

Auxiliar 2

$$\vec{E}(x,y,z) = \frac{q}{4\pi\varepsilon_0} \frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}}$$

$$x\hat{x} + y\hat{y} = \rho\hat{\rho}$$

$$x^2+y^2 = \rho^2$$

$$\vec{E}(\rho,\theta,z) = \frac{q}{4\pi\varepsilon_0}\left(\frac{\rho\hat{\rho} + z\hat{z}}{(\rho^2+z^2)^{3/2}}\right)$$