Auxiliar 1

1

Calculo vectorial

$\phi \in \mathbb{R}^3 \to \mathbb{R}$

$\vec{F} \in \mathbb{R}^3 \to \mathbb{R}^3$

$\nabla\phi$ $\quad \nabla\times\vec{F}$

$\nabla\cdot\vec{F}$ $\quad \nabla\cdot\nabla\times(\vec{F}\phi)$

$\iint \vec{F}\cdot d\vec{A}$

$\iiint \nabla\cdot\vec{F}\, dV$

$$\boxed{\begin{array}{l} TG \to \text{div} \\ TS \to \text{rot} \end{array}}$$

2

Calculo complejo

$f : \mathbb{C} \to \mathbb{C}$

$z \in \mathbb{C}$

$f(z) \in \mathbb{C}$

$\dfrac{df}{dz}$

$$\boxed{\oint f(z)\,dz}$$

3

Modelacion matematica

T.F.

S.F.

EDP /

$\dfrac{\partial T}{\partial t} = K\nabla^2 T$

$\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$

$i\hbar\partial_t\psi = \hat{H}\psi$

Auxiliar 1

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$A(\Omega) = \int_{\Omega} dA = \iint dx\,dy$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

$$x \in (-a, +a)$$

$$y \in \left(-b\sqrt{1-\frac{x^2}{a^2}},\ b\sqrt{1-\frac{x^2}{a^2}}\right)$$

$$A(\Omega) = \int_{-a}^{+a} \int_{-b\sqrt{1-\frac{x^2}{a^2}}}^{b\sqrt{1-\frac{x^2}{a^2}}} 1 \cdot dy\, dx$$

$$= \int_{-a}^{+a} 2b\sqrt{1-\frac{x^2}{a^2}}\, dx$$

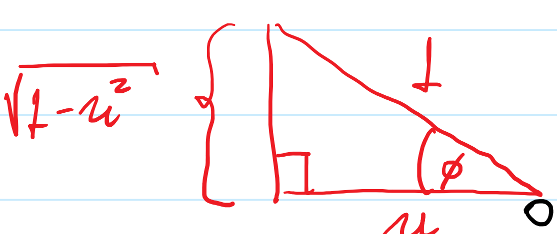
$$= 2b\int_{-a}^{+a} \sqrt{1-\frac{x^2}{a^2}}\, dx$$

$$u = \frac{x}{a}$$

$$du = \frac{dx}{a}$$

$$= 2ba\int_{-1}^{+1} \sqrt{1-u^2}\, du$$

$$dx = a\,du$$



$$\cos\phi = u$$

$$\operatorname{sen}\phi = \sqrt{1-u^2}$$

$$-\operatorname{sen}\phi\, d\phi = du$$

$$A(\Omega) = 2 \cdot b \cdot a \cdot -\int_{\pi} \operatorname{sen}^2\phi\, d\phi$$

Auxiliar 1

$$A(x) = 2 \cdot a \cdot b \cdot \int_0^{\pi} \operatorname{sen}^2\phi \, d\phi$$

$$\left.\begin{aligned} \cos(2\phi) &= \cos^2\phi - \operatorname{sen}^2\phi \\ 1 &= \cos^2\phi + \operatorname{sen}^2\phi \end{aligned}\right\} \quad \operatorname{sen}^2\phi = \frac{1-\cos(2\phi)}{2}$$

$$A(x) = 2 \cdot a \cdot b \int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos(2\phi) \, d\phi$$

$$= 2 \cdot a \cdot b \left(\int_0^{\pi} \frac{1}{2} \, d\phi - \frac{1}{2}\int_0^{\pi} \cos(2\phi) \, d\phi \right)$$

$$= 2 \cdot a \cdot b \left(\frac{\pi}{2} - \cancel{\frac{1}{2} \frac{\operatorname{sen}(2\phi)}{2} \Big|_0^{\pi}} \right)$$

$$A(x) = a \cdot b \cdot \pi$$

Auxiliar 1

$$J = \frac{\partial(x,y)}{\partial(u,v)} \equiv \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}$$

$x = au \qquad \frac{\partial x}{\partial u} = a \qquad \frac{\partial x}{\partial v} = 0$

$y = bv \qquad \frac{\partial y}{\partial u} = 0 \qquad \frac{\partial y}{\partial v} = b$

$$J = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$A(\Omega) = \int_\Omega 1 \cdot dx\,dy = \iint |J| \, du\,dv$$

$$= \iint_\Omega a \cdot b \, du\,dv$$

$|J| = a \cdot b$

$$A(\Omega) = a \cdot b \boxed{\iint_\Omega \cdot \, du\,dv} \quad \Bigg| \begin{array}{l} x = au \\ y = bv \end{array}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \iff u^2 + v^2 = 1$$

$$A(\Omega) = a \cdot b \cdot \pi$$

Auxiliar 1

$x = r\cos\phi$ $\qquad T(r,\phi) = (x, y)$

$y = r\,\text{sen}\,\phi$ $\qquad \left|\dfrac{\partial(x,y)}{\partial(r,\phi)}\right| = r$

Auxiliar 1

$$\iint \phi(x,y)\,dx\,dy \quad , \quad \phi(x,y) = f(x)\,g(y)$$

$$\left(\int f(x)\,dx\right)\left(\int g(y)\,dy\right)$$

Auxiliar 1

$$\underbrace{\int_{-\infty}^{+\infty} e^{-x^2}\,dx}_{I} = \int_{-\infty}^{+\infty} e^{-y^2}\,dy = \int_{-\infty}^{+\infty} e^{-u^2}\,du$$

$$I^2 = \left(\int_{-\infty}^{+\infty} e^{-x^2}\,dx\right)\left(\int_{-\infty}^{+\infty} e^{-y^2}\,dy\right)$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-x^2}\cdot e^{-y^2}\,dx\,dy$$

$$I = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-(x^2+y^2)}\,dx\,dy$$

$$x = r\cos\phi \qquad \frac{\partial x}{\partial r} = \cos\phi \quad \Big| \quad \frac{\partial x}{\partial \phi} = -r\,\mathrm{sen}\,\phi$$

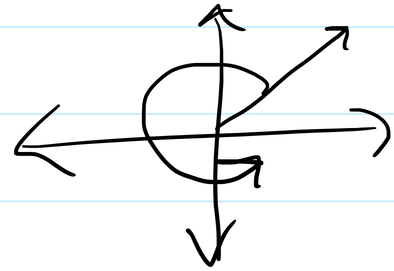
$$y = r\,\mathrm{sen}\,\phi \qquad \frac{\partial y}{\partial r} = \mathrm{sen}\,\phi \quad \Big| \quad \frac{\partial y}{\partial \phi} = r\cos\phi$$

$$|J| = \left|\begin{pmatrix} \cos\phi & -r\,\mathrm{sen}\,\phi \\ \mathrm{sen}\,\phi & r\cos\phi \end{pmatrix}\right| = r\cos^2\phi + r\,\mathrm{sen}^2\phi$$

$$= r$$

Auxiliar 1

$$x^2+y^2=r^2$$



$$I^2 = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-(x^2+y^2)}\,dx\,dy$$

$$I^2 = \int_0^{2\pi}\int_0^{\infty} e^{-r^2}\cdot r\cdot dr\,d\phi$$

$$= \left(\int_0^{2\pi} 1\,d\phi\right)\left(\int_0^{\infty} e^{-r^2}\, r\,dr\right)$$

$u=r^2$

$du = 2r\,dr$

$$= \cancel{2}\pi\left(\int_0^{\infty} e^{-u}\,\frac{du}{\cancel{2}}\right)$$

$$= \pi\left(\int_0^{\infty} e^{-u}\,du\right)$$

$$I^2 = \pi\left(-e^{-u}\Big|_0^{\infty}\right) = \pi$$

$$I = \sqrt{\pi}$$

Auxiliar 1

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$