P1

Analicemos el comportamiento, hagamos un cambio de variable:

$$x = -u \implies$$

$$\frac{1}{1+u} = \sum_{n=0}^{\infty} (-u)^n \implies$$

$$\frac{1}{1+u} = \sum_{n=0}^{\infty} (-1)^n (u)^n$$

¿cómo se comporta?

$$\frac{1}{1+u} = 1 - u + u^2 - u^3 + u^4 - u^5 + \dots$$

$$\implies$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n (x)^n$$

$$\iff [(1+x)^{-1}]' = \left[\sum_{n=0}^{\infty} (-1)^n (x)^n\right]'$$

$$\implies -(1+x)^{-2} = -1 + 2x - 3x^2 + 4x^3 - 5x^4 + \dots$$

$$\implies \frac{-1}{(1+x)^2} = \sum_{n=1}^{\infty} (-1)^n n\, x^{n-1}$$

$$\implies (-1)\frac{-1}{(1+x)^2} = (-1)\sum_{n=1}^{\infty} (-1)^n n\, x^{n-1}$$

$$\implies \frac{1}{(1+x)^2} = \sum_{n=1}^{\infty} (-1)^{n+1} n\, x^{(n-1)} \quad / (2x)$$

$$(2x)\frac{1}{(1+x)^2} = 2x\sum_{n=1}^{\infty}(-1)^{n+1} n\, x^{(n-1)}$$

$$\Rightarrow \frac{2x}{(1+x)^2} = \sum_{n=1}^{\infty}(-1)^{n+1} 2n\, x^n \qquad \square$$

P2) Aplicamos el criterio de D'Alembert:

$$f(x) = \sum_{n=0}^{\infty}\frac{x^n}{2^n n!}$$

$$\Rightarrow \lim_{n\to\infty}\frac{\left|\frac{x^{n+1}}{2^{n+1}(n+1)!}\right|}{\left|\frac{x^n}{2^n n!}\right|} < 1$$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{x^{n+1}}{2^{n+1}(n+1)!}\right)\cdot\left(\frac{2^n n!}{x^n}\right)$$

$$\Rightarrow \frac{x^n\cdot x}{\cancel{2^n}\,2(n+1)!}\cdot\frac{\cancel{2^n}\cdot n!}{x^n}$$

$$\Rightarrow \frac{x\,n!}{2(n+1)!} = \frac{x\,\cancel{n!}}{2\cancel{(n)!}\cdot(n+1)}$$

$$\Rightarrow \frac{x}{2(n+1)}$$ aplicamos el límite

$$\Rightarrow \lim_{n\to 0}\left|\frac{x}{\underbrace{2(n+1)}_{\text{crece rápido}}}\right| = 0,\ \forall x\in\mathbb{R}$$

$\downarrow$

converge.

$\cdot)\ f'(x) = \left(\sum_{n=0}^{\infty} x^n / 2^n n!\right)' = \sum_{n=1}^{\infty} n x^{n-1} / 2^n n! = \sum_{n=1}^{\infty} x^{n-1} / 2 2^{n-1} (n-1)!$

$$= \sum_{n=0}^{\infty} x^n / 2 2^n n! = 1/2\, f(x) \qquad \square$$