Debemos resolver la siguiente integral:

$$\int \frac{\text{sen}(x)}{1+\text{sen}(x)}\,dx$$ // multiplicamos por 1

$$= \int \frac{\text{sen}(x)}{1+\text{sen}(x)} \cdot \frac{1-\text{sen}(x)}{1-\text{sen}(x)}\,dx$$

$$= \int \frac{\text{sen}(x)-\text{sen}^2(x)}{1-\cancel{\text{sen}(x)}+\cancel{\text{sen}(x)}-\text{sen}^2(x)}\,dx$$

$$= \int \frac{\text{sen}(x)-\text{sen}^2(x)}{1-\text{sen}^2(x)}\,dx$$ // $1-\text{sen}^2(x)=\cos^2(x)$

$$= \int \frac{\text{sen}(x)-\text{sen}^2(x)}{\cos^2(x)}\,dx = \underbrace{\int \frac{\text{sen}(x)}{\cos^2(x)}\,dx}_{1} - \underbrace{\int \frac{\text{sen}^2(x)}{\cos^2(x)}\,dx}_{2}$$

1.)

$$\int \frac{\text{sen}(x)\,dx}{\cos^2(x)} = \int \frac{\text{sen}(x)}{\cos(x)} \cdot \frac{1}{\cos(x)}\,dx$$

$\text{sen}/\cos = \tan(x)$, $1/\cos(x) = \sec(x)$

$\int \tan(x)\sec(x)\,dx$, usemos sustitución

$t=\sec(x)$, $t' = \sec(x)\tan(x)$

$$dx = \frac{1}{t'}\,dt$$

$$\int \tan(x)\sec(x)\,dx = \int \frac{\cancel{\sec(x)}\,\cancel{\tan(x)}\,dt}{\cancel{\sec(x)}\,\cancel{\tan(x)}}$$

$$= \int 1\,dt = t + c, \quad t = \sec(x) \Rightarrow$$

$$\int \tan(x)\sec(x)\,dx = \sec(x) + C_1, \; C_1 \in \mathbb{R}$$

2.-) $$\int \frac{\operatorname{sen}^2(x)}{\cos^2(x)}\,dx = \int \tan^2(x)\,dx,$$

$$\tan^2(x) = -1 + \sec^2(x) \Rightarrow$$

$$= \int -1 + \sec^2(x)\,dx = \int -1\,dx + \int \sec^2(x) \Rightarrow$$

$$= -x + C_2 + \int \sec^2(x) \text{ (integral conocida)}$$

$$= -x + C_2 + \tan^2(x) + C_2, \quad C_1 + C_2 = K$$

$$\int \frac{\operatorname{sen}^2(x)}{\cos^2(x)}\,dx = -x + \tan^2(x) + K, \; K \in \mathbb{R}$$

en conclusión y uniendo:

$$\int \frac{\operatorname{sen}(x)}{1 + \operatorname{sen}(x)} = \sec(x) + \tan^2(x) - x, \; \bar{K} \in \mathbb{R}$$