Pauta Auxiliar 13

P1 a) $\lim_{x\to\infty} \frac{x^2(e^{\frac{2}{x}}-1)}{x+1} = \lim_{x\to\infty} \frac{\boxed{x}}{x+1} \cdot \frac{(e^{\frac{2}{x}}-1)}{\boxed{\frac{1}{x}}\cdot 2} \cdot 2$

Usando que:

$$\frac{x}{x+1} \xrightarrow{x\to\infty} 1$$

$$\frac{e^u - 1}{u} \xrightarrow{u\to 0} 1 \quad , \text{con } u = \frac{2}{x} \xrightarrow[x\to\infty]{} 0$$

y Algebra de límites

$$\Rightarrow \lim_{x\to\infty} \frac{x^2(e^{\frac{2}{x}}-1)}{x+1} = 1 \cdot 1 \cdot 2 = \boxed{2}$$

b) $\lim_{x\to\infty} \left(\sqrt{x^2+1} + 3x + 1\right) \cdot \sin\left(\frac{1}{x}\right)$

$$= \lim_{x\to\infty} \left(\sqrt{x^2+1} + 3x+1\right) \cdot \frac{\sin\left(\frac{1}{x}\right) \cdot \frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \left(\sqrt{1+\frac{1}{x^2}} + 3 + \frac{1}{x}\right) \cdot \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \left(\sqrt{1+0} + 3 + 0\right) \cdot 1$$

Usando Algebra de Limites y $\lim_{u\to 0} \frac{\sin(u)}{u} = 1$, $u = \frac{1}{x} \xrightarrow{x\to\infty} 0$

$$= \boxed{4}$$

c) $\lim_{x \to -\infty} \sqrt{-x} \cdot e^{2022x} = \lim_{\substack{u \to \infty \\ u=-x}} \sqrt{u} \cdot e^{-2022u}$

$= \lim_{u \to \infty} \frac{\sqrt{u}}{e^{2022u}} \longrightarrow 0$, pues

$e^u > u^k$

para u grande

d) $\lim_{x \to \infty} \frac{\ln(x)}{x} = 0$, idea: $\ln(x) < x^k$ para x grande

Dem: $\ln(x) < x-1 < x$

$\Rightarrow \ln(x^\alpha) < x^\alpha$

$\Rightarrow \alpha \cdot \ln(x) < x^\alpha$

$\Rightarrow \ln(x) < \frac{x^\alpha}{\alpha}$

En particular $\ln(x) < 2\sqrt{x}$, $\alpha = \frac{1}{2}$

$\Rightarrow 0 < \frac{\ln(x)}{x} < \frac{x^{\frac{1}{2}}}{x} = \frac{1}{x^{\frac{1}{2}}} \xrightarrow{x \to \infty} 0$

$\Rightarrow$ Por Sandwich $\boxed{\lim_{x \to \infty} \frac{\ln(x)}{x} = 0}$

e) $\lim_{x\to\infty} \dfrac{x+\frac{1}{x^2}}{2x^2+\sqrt{x}+\frac{1}{x}} = \lim_{x\to\infty} \dfrac{1/x+\frac{1}{x^4}}{2+\frac{1}{x^{\frac{3}{2}}}+\frac{1}{x^3}} \overset{\text{Algebra}}{=} \dfrac{0+0}{2+0+0} = 0$

f) $\lim_{x\to\infty} \dfrac{\sin(\tan(\frac{1}{2x}))}{\sin(\frac{1}{x})} = \lim_{x\to\infty} \dfrac{\sin(\tan(\frac{1}{2x}))}{\tan(\frac{1}{2x})} \cdot \dfrac{\tan(\frac{1}{2x})}{\sin(\frac{1}{x})}$

$= \lim_{x\to\infty} \dfrac{\sin(\tan(\frac{1}{2x}))}{\tan(\frac{1}{2x})} \cdot \dfrac{\sin(\frac{1}{2x})}{\cos(\frac{1}{2x})} \cdot \dfrac{1}{\sin(\frac{1}{x})}$

$= \lim_{x\to\infty} \dfrac{\sin(\tan(\frac{1}{2x}))}{\tan(\frac{1}{2x})} \cdot \dfrac{\sin(\frac{1}{2x})}{\frac{1}{2x}} \cdot \dfrac{1}{2} \cdot \dfrac{\frac{1}{x}}{\sin(\frac{1}{x})} \cdot \dfrac{1}{\cos(\frac{1}{2x})}$

Usando los limites conocidos y Algebra de limites

$\Rightarrow 1 \cdot 1 \cdot \frac{1}{2} \cdot 1 \cdot 1 = \boxed{\frac{1}{2}}$

$$\lim_{x \to \infty} x \ln\left(\frac{x+a}{x-a}\right) = \lim_{x \to \infty} \frac{\ln\left(\frac{x+a}{x-a}\right)}{\left(\frac{x+a}{x-a}\right) - 1} \cdot \frac{\left(\frac{x+a}{x-a}\right) - 1}{1} \cdot x$$

$$= \lim_{x \to \infty} \underbrace{\frac{\ln\left(\frac{x+a}{x-a}\right)}{\frac{2a}{x-a}}}_{\to 1} \cdot \underbrace{\frac{2a}{x-a} \cdot x}_{\to 2a}$$

$$= \boxed{2a}$$

11) $$\lim_{x \to \infty} \frac{\sin(x)}{x^2 \sin\left(\frac{2}{x}\right)} = \lim_{x \to \infty} \underbrace{\frac{\frac{2}{x}}{\sin\left(\frac{2}{x}\right)}}_{\to 1} \cdot \frac{1}{2} \cdot \underbrace{\underbrace{\frac{1}{x}}_{\text{Nulo}} \cdot \underbrace{\sin(x)}_{\text{Acotada}}}_{\text{Nulo}}$$

$$= 0$$

P2 a) Dom $\mathbb{R} - \{-1\}$

$$\lim_{x\to -1} \frac{x^3}{(x+1)^2} = \frac{-1}{0^2} = -\infty \implies -1 \text{ es Asintota Vertical}$$

$$\lim_{x\to\infty} \frac{x^3}{(x+1)^2} = \infty \quad y \quad \lim_{x\to -\infty} \frac{x^3}{(x+1)^2} = -\infty$$

<u>No hay Asintotas horizontales</u>

$$\lim_{x\to\infty} \frac{x^{\cancel{3}\,2}}{(x+1)^2 \cdot \cancel{x}} = \lim_{x\to\infty} \frac{x^2}{x^2+2x+1} = 1 = m_{\infty}$$

$$\lim_{x\to -\infty} \frac{x^2}{(x+1)^2} = 1 = m_{-\infty}$$

$$\lim_{x\to\infty} \frac{x^3}{(x+1)^2} - x = \lim_{x\to\infty} \frac{\cancel{x^3} - \cancel{x^3} - 2x^2 - x}{x^2+2x+1} = \lim_{x\to\infty} \frac{-2x^2-x}{x^2+2x+1} = \boxed{-2}$$

$$\boxed{N_{\infty} = -2}$$

$$\lim_{x\to -\infty} \frac{x^3}{(x+1)^2} - x = \lim_{x\to\infty} \frac{-2x^2-x}{x^2+2x+1} = \boxed{-2} = N_{\infty}$$

$\implies$ hacia ambos infinitos tiene la asintota $\boxed{y = x - 2}$

$f(x) = x^2 \sin\left(\frac{1}{x}\right)$ $\quad$ Dom $= \mathbb{R} - \{0\}$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \underbrace{x^2}_{\text{Nula}} \cdot \underbrace{\sin\left(\frac{1}{x}\right)}_{\text{Acotada}} = 0 \Rightarrow \text{No hay Verticales}$$

$\sin(y) \in [-1, 1]$

$$\lim_{x \to \infty} x^2 \sin\left(\frac{1}{x}\right) = \lim_{x \to \infty} x \cdot \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = \infty$$

Similar con $-\infty$ $\Rightarrow$ No hay horizontales

$$\lim_{x \to \infty} x \cdot \sin\left(\frac{1}{x}\right) = \lim_{x \to \infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} = 1 = m_\infty$$

$$\lim_{x \to -\infty} x \sin\left(\frac{1}{x}\right) = 1 = m_{-\infty}$$

$$\lim_{x \to \infty} x^2 \sin\left(\frac{1}{x}\right) - x$$

(Solo se me ocurrio con l'hopital)

$$= \lim_{u \to 0^+} \frac{1}{u^2} \sin(u) - \frac{1}{u}$$

$u = \frac{1}{x}$

$$= \lim_{u \to 0^+} \frac{\sin(u) - u}{u^2} \overset{L'H}{=} \lim_{u \to 0^+} \frac{\cos(u) - 1}{2u} = \overset{\text{conocido}}{0}$$

El hacia lim $-\infty$ es Analoga

$\Rightarrow$ Tiene la Asintota oblicua a ambos infinitos de la forma $\boxed{y = x}$

P3 No Tiene Verticales $Dom = \mathbb{R}$

$$\lim_{x\to\infty} f(x) = (2-0)(\infty) = \infty$$

$$\lim_{x\to-\infty} f(x) = (-\infty)(-\infty) = +\infty$$

No hay Horizontales

Oblicuas

$$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} (2-e^{-x})\left(2+\frac{5}{x}\right)$$

$$= (2-0)(2+0) = 4 = m_\infty$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = (-\infty)(2+0) = -\infty$$ No hay Oblicua al $-\infty$

$$\lim_{x\to\infty} f(x) - m_\infty x = \lim_{x\to\infty} (2-e^{-x})(2x+5) - 4x$$

$$= \lim_{x\to\infty} \cancel{4x} + 10 - e^{-x}2x - 5e^{-x} - \cancel{4x}$$

$$= 10 - 0 - 0 = \boxed{10}$$

Tiene Asintota oblicua $y = 4x + 10$ al ∞