Pauta Auxiliar 12

P1 a) $\lim\limits_{x\to 1} \dfrac{\sin(x^2-1)}{x-1} = \lim\limits_{x\to 1} \dfrac{\sin(x^2-1)}{x^2-1}\cdot(x+1)$

$$= \lim_{x\to 1} \frac{\sin(x^2-1)}{x^2-1} \cdot \lim_{x\to 1}(x+1)$$

$u = x^2-1$
$u \to 0$

$$= \lim_{u\to 0} \frac{\sin(u)}{u} \cdot \lim_{x\to 1}(x+1)$$

$$= 1\cdot(2) = \boxed{2}$$

b) $\lim\limits_{x\to 1} \dfrac{1-\sqrt{x}}{1-x} = \lim\limits_{x\to 1} \dfrac{\cancel{1-\sqrt{x}}}{\cancel{1-\sqrt{x}}}\cdot\dfrac{1}{1+\sqrt{x}} = \lim\limits_{x\to 1}\dfrac{1}{1+\sqrt{x}} = \boxed{\dfrac{1}{2}}$

c) $\lim\limits_{x\to 1} \dfrac{\ln(3-x)}{e^{2(x-2)}-1} = \boxed{\dfrac{\ln(2)}{e^{-2}-1}}$

d) $$\lim_{x\to 0} x \cdot \underbrace{e^{\sin\left(\frac{\pi}{x}\right)}}_{\text{Acotada}}$$

$$\sin\left(\frac{\pi}{x}\right) \in [-1,1] \Rightarrow e^{\sin\left(\frac{\pi}{x}\right)} \in [e^{-1}, e]$$

$$\Rightarrow e^{\sin\left(\frac{\pi}{x}\right)} \text{ es Acotada}$$

$$\rightarrow \lim_{x\to 0} \underbrace{x}_{\text{Nula}} \cdot \underbrace{e^{\sin\left(\frac{\pi}{x}\right)}}_{\text{Acotada}} = 0$$

e) $$\lim_{x\to 0} \left(1+\tan(x)\right)^{\frac{3}{x}} = L$$

$$\lim_{x\to 0} \ln\left(\left(1+\tan(x)\right)^{\frac{3}{x}}\right) = \ln(L)$$

$$\lim_{x\to 0} 3\,\frac{\ln(1+\tan(x))}{x} = \lim_{x\to 0} 3\,\frac{\ln(1+\tan(x))}{\tan(x)} \cdot \frac{\sin(x)}{x} \cdot \frac{1}{\cos(x)}$$

$$= \lim_{u\to 0} \frac{\ln(1+u)}{u} \cdot \lim_{x\to 0} \frac{\sin(x)}{x} \cdot \lim_{x\to 0} \frac{3}{\cos(x)} = 1 \cdot 1 \cdot 3 = \boxed{3}$$

Creado con Tiny Scanner

f) $\lim_{x \to 1} \frac{1+\cos(\pi x)}{x-1} = \lim_{x \to 1} \frac{1+\cos(\pi(x-1)+\pi)}{x-1}$

$\cos(\pi(x-1)) \cdot \overset{-1}{\cos(\pi)} - 0$

$= \lim_{x \to 1} \frac{1+\cos(\pi(x-1)+\pi)}{x-1}$

$= \lim_{x \to 1} \frac{1-\cos(\pi(x-1))}{x-1}$

$= \lim_{u \to 0} \frac{1-\cos(u)}{u} \cdot \pi = \boxed{0}$

g) $\lim_{x \to 0} \frac{e^{|x|}-1}{x} \nexists \quad \lim_{x \to 0^+} \frac{e^{x}-1}{x} = \boxed{1}$

$\lim_{x \to 0^-} \frac{e^{-x}-1}{x} = \lim_{x \to 0^-} \frac{e^{-x}-1}{-x} \cdot -1$

$= \lim_{u \to 0} \frac{e^{u}-1}{u} \cdot (-1) = \boxed{-1}$

$\Rightarrow$ Que el límite no existe

h) $\lim_{x\to 1} \left\lfloor \frac{2}{x^2+1} \right\rfloor \Rightarrow$ Estudiar los límites laterales

$$\lim_{x\to 1^-} \quad x^2+1 < 2 \Rightarrow 2 > \frac{2}{x^2+1} > 1 \Rightarrow \left\lfloor \frac{2}{x^2+1} \right\rfloor = 1$$

$$\lim_{x\to 1^+} \quad x^2+1 > 2 \Rightarrow 0 < \frac{2}{x^2+1} < 1 \Rightarrow \left\lfloor \frac{2}{x^2+1} \right\rfloor = 0$$

$\Rightarrow$ No existe el límite

P2 $\lim_{x\to 0} g(x)$

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \pi \frac{e^x-1}{x^2-x} = \lim_{x\to 0^+} \pi \cdot \cancel{\frac{e^x-1}{x}}^{\,1} \cdot \cancel{\frac{1}{x-1}}^{\,-1} = \boxed{-\pi}$$

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{\sin((1+x)\pi)}{x} = \lim_{x\to 0^-} -\frac{\sin(x\pi)}{x\cdot\pi}\cdot\pi$$

$$= \lim_{u\to 0} -\frac{\sin(u)}{u}\cdot\pi = \boxed{-\pi}$$

$$\Rightarrow \lim_{x\to 0} g(x) = -\pi = g(0) \checkmark$$