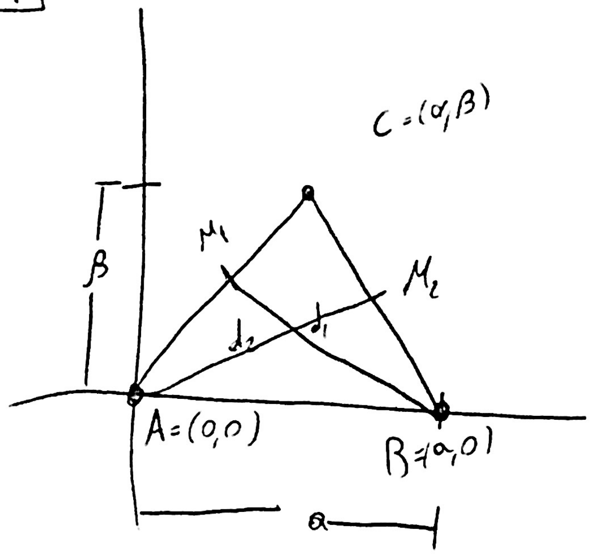
P1



$$\Delta = \frac{\text{Base} \cdot \text{Altura}}{2} = \frac{a \cdot \beta}{2}$$

$$M_1 = \left(\frac{\alpha}{2}, \frac{\beta}{2}\right), \quad M_2 = \left(\frac{a+\alpha}{2}, \frac{\beta}{2}\right)$$

$$d_1^2 = \left(a - \frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2, \quad d_2^2 = \left(\frac{a+\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2$$

Imponiendo $d_1^2 + d_2^2 = 5\Delta \iff \boxed{\left(a - \frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 + \left(\frac{a+\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 = \frac{5}{2} a \cdot \beta}$

Trabajar la ecuación y obtener relación entre (α, β)

$$\iff a^2 - a\alpha + \frac{\alpha^2}{4} + \frac{\beta^2}{4} + \frac{a^2}{4} + \frac{a\alpha}{2} + \frac{\alpha^2}{4} + \frac{\beta^2}{4} = \frac{5}{2} a\beta$$

$$\iff \frac{\alpha^2}{2} + \frac{a\alpha}{2} + \frac{\beta^2}{2} - \frac{5}{2} a\beta = -\frac{5a^2}{4} \qquad / \cdot 2$$

$$\iff \alpha^2 + 2 \cdot \alpha \cdot \left(\frac{a}{2}\right) + \beta^2 - 2 \cdot \beta \cdot \left(\frac{5}{2}a\right) = -\frac{5a^2}{2}$$

$$\iff \alpha^2 + 2\alpha\left(\frac{a}{2}\right) + \frac{a^2}{4} + \beta^2 - 2 \cdot \beta\left(\frac{5}{2}a\right) + \frac{25}{4}a^2 = \frac{26}{4}a^2 - \frac{5a^2}{2}$$

$$\iff \left(\alpha + \frac{a}{2}\right)^2 + \left(\beta - \frac{5}{2}a\right)^2 = (2a)^2$$

$\iff (\alpha, \beta)$ es una circunferencia de centro $\left(-\frac{a}{2}, \frac{5}{2}a\right)$ y

Radio $2a$

P2) a) $\frac{x^2}{25}+\frac{y^2}{9}=1 \iff \frac{(x-0)^2}{5^2}+\frac{(y-0)^2}{3^2}=1 \iff$ elipse $a=5$, $b=3$ y centro $(0,0)$

$\Rightarrow$ Horizontal

$a\cdot e=\sqrt{a^2-b^2}=4 \Rightarrow \boxed{\text{Focos}=(\pm 4,0)}$

Como la parabola pasa por $(\pm 4,0)$

$\Rightarrow \quad 4p(0-y_0)=(4-x_0)^2 \iff -4py_0=16-8x_0+x_0^2 \quad (1)$

$4p(0-y_0)=(-4-x_0)^2 \iff -4py_0=16+8x_0+x_0^2 \quad (2)$

Restando (1) y (2) $\Rightarrow 0=16x_0 \Rightarrow \boxed{x_0=0}$

Remplazando $\overset{(1)}{\Rightarrow} -4py_0=16 \iff \boxed{-py_0=4}$

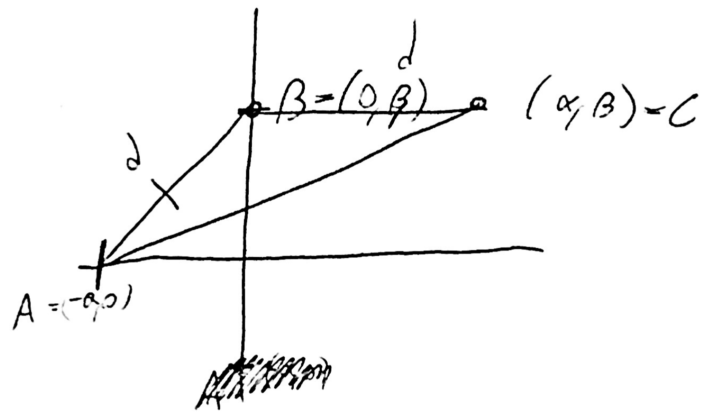
b) Directriz $y=-5 \Rightarrow y_0-p=-5$, como $p\neq 0$

$\Rightarrow py_0-p^2=-5p \Rightarrow 0=p^2-5p+4 \Rightarrow P=4$ o $P=1$

Usando que $P\geq 2 \Rightarrow \boxed{P=4} \rightarrow \boxed{y_0=-1}$

P3 | Asumir

$a > 0$



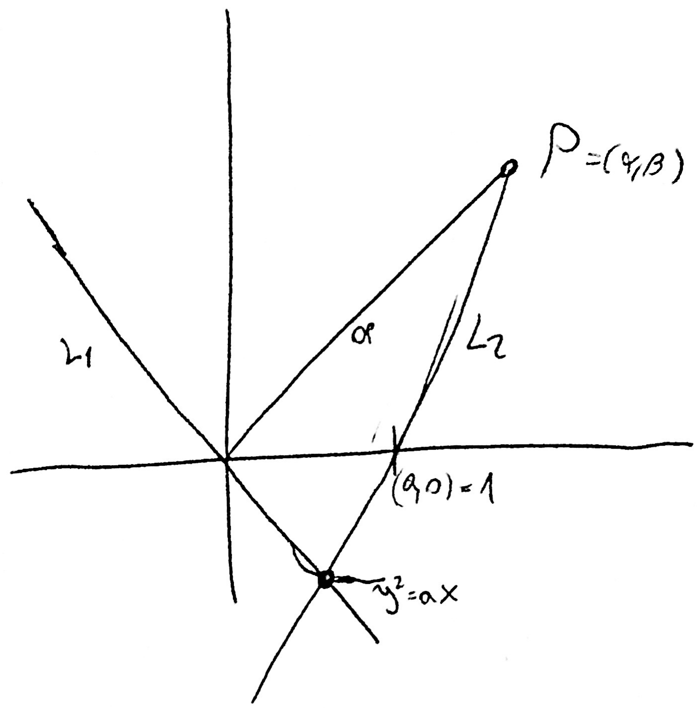
Buscar relación (α, β)

$$\left.\begin{array}{l} d^2 = a^2 + \beta^2 \\ d^2 = \alpha^2 \end{array}\right\} \Rightarrow \alpha^2 = a^2 + \beta^2$$

$$\Rightarrow \alpha^2 - \beta^2 = a^2 \Rightarrow \frac{\alpha^2}{a^2} - \frac{\beta^2}{a^2} = 1$$

Hiperbola centrada en $(0,0)$ con $a = b$

P4



$m_{op}=\frac{\beta}{\alpha} \Rightarrow Op: y=\frac{\beta}{\alpha}x$, $m_{L_2}=\frac{\beta}{\alpha-a} \Rightarrow L_2\ y=\frac{\beta}{\alpha-a}(x-a)$

$m_{L_1}=-\frac{\alpha}{\beta} \Rightarrow L_1: y=-\frac{\alpha}{\beta}x$

$L_1 \cap L_2 \doteq \frac{\beta}{\alpha-a}(\tilde{x}-a) = \frac{-\alpha}{\beta}\tilde{x} \Rightarrow \tilde{x}\left(\frac{\beta}{\alpha-a}+\frac{\alpha}{\beta}\right) = \frac{\beta a}{\alpha-a}$

$\frac{\beta^2+\alpha^2-a\alpha}{(\alpha-a)\cdot\beta}$

$\Rightarrow \tilde{x}=\frac{\beta^2 a}{\beta^2+\alpha^2-a\alpha} \Rightarrow \tilde{y}=\frac{-\alpha\beta a}{\beta^2+\alpha^2-a\alpha}$

Como cumple $\tilde{y}^2=a\tilde{x} \Rightarrow$

$\frac{\alpha^2\beta^2a^2}{(\beta^2+\alpha^2-a\alpha)^2} = \frac{a^2\beta^2}{(\beta^2+\alpha^2-a\alpha)}$

$\Rightarrow \cancel{\alpha^2} = \beta^2+\cancel{\alpha^2}-a\alpha$

$\Rightarrow \beta^2=a\alpha \Rightarrow$ $\boxed{\text{La parabola } y^2=ax}$

Esta su vertice en $(0,0)$, Foco $(\frac{a}{4}, 0)$, Directriz : $X = -\frac{a}{4}$

$$X - 0 = \frac{1}{4(\frac{a}{4})}(y-0)^2$$