P2 a. $\varepsilon = \frac{p_z^2}{2m} + \frac{\hbar e B}{mc}\left(n + \frac{1}{2}\right)$

No interactuantes y fermiones

$$\Rightarrow \ln(\Xi) = \ln\left(\prod_{\varepsilon}\left(1 + e^{-\beta\varepsilon}e^{\beta\mu}\right)^g\right)$$

$$= \sum_{\varepsilon} g \ln\left(1 + e^{-\beta\varepsilon}e^{\beta\mu}\right) \quad ; \quad z = e^{\beta\mu}$$

$$= g \sum_{p_z, n} \ln\left(1 + z e^{-\beta\left(\frac{p_z^2}{2m} + \frac{\hbar e B}{mc}\left(n + \frac{1}{2}\right)\right)}\right)$$

b. $\sum_{p_z} \rightarrow \frac{1}{(2\pi/L)}\int_{-\infty}^{+\infty} dp_z$

$$\Rightarrow \ln(\Xi) = \frac{gL}{2\pi}\int_{-\infty}^{+\infty} dp_z \sum_{n} \ln\left(1 + z e^{-\beta\left(\frac{p_z^2}{2m} + \frac{\hbar e B}{mc}(n + 1/2)\right)}\right)$$

T suficientemente alto $\Rightarrow \beta(\varepsilon - \mu) \ll 1 \wedge z \ll 1$

$$\Rightarrow \ln\left(1 + z e^{-\beta\varepsilon}\right) \approx z e^{-\beta\varepsilon}$$

$$\Rightarrow \ln(\Xi) = \frac{gzL}{2\pi}\int_{-\infty}^{+\infty} dp_z \sum_{n} e^{-\beta\left(\frac{p_z^2}{2m} + \frac{\hbar e B}{mc}\left(n + \frac{1}{2}\right)\right)}$$

Forma 1 (Incorrecta, pero evaluada igual en el control)

$$\sum_n \to \int dn$$

$$\Rightarrow \ln(\Xi) = \frac{gzL}{2\pi} \underbrace{\int_{-\infty}^{+\infty} dp_z \, e^{-\frac{\beta p_z^2}{2m}}}_{1/\lambda_T} \int_0^{+\infty} dn \, e^{-\frac{\beta \hbar e B}{mc}(n+\frac{1}{2})}$$

Con $x = \frac{\mu_B B}{k_B T}$ $\wedge$ $\mu_B = \frac{e\hbar}{2mc}$

$$\Rightarrow \int_0^{+\infty} dn \, e^{-\frac{\beta \hbar e B}{mc}(n+\frac{1}{2})} = \int_0^{+\infty} dn \, e^{-x(2n+1)}$$

$$= e^{-x}\left(0 - (-xe^0)\right) = xe^{-x}$$

$$\Rightarrow \ln(\Xi) = \frac{gzL}{2\pi\lambda_T} \cdot xe^{-x}$$

Forma 2 (Correcta):

$$\ln(\Xi) = \frac{gzL}{2\pi} \underbrace{\int_{-\infty}^{+\infty} dp_z \, e^{-\frac{\beta p_z^2}{2m}}}_{1/\lambda_T} \sum_n e^{-x(2n+1)}$$

$$\sum_n e^{-x(2n+1)} = \frac{xe^{-x}}{1-e^{-2x}} = \frac{x}{e^x - e^{-x}} = \frac{x}{2\,\mathrm{senh}(x)}$$

$$\Rightarrow \ln(\Xi) = \frac{gzxL}{4\pi\,\mathrm{senh}(x)\,\lambda_T}$$

PROARTE

c. Siguiendo Forma 2 (Forma 1 da un resultado $\neq$)

$$N = z\frac{\partial(\ln(\Xi))}{\partial z} = z \cdot \frac{g x L}{4\pi \lambda_t \operatorname{senh}(x)}$$

$$\Rightarrow N = \ln(\Xi) \Rightarrow J = -k_B t N$$

d. $$M = -\frac{\partial J}{\partial B} = -\frac{\partial J}{\partial x}\frac{\partial x}{\partial B}$$

$$= \frac{\cancel{k_B t}\, g L}{4\pi \lambda_t} \cdot \frac{\mu_B}{\cancel{k_B t}} \cdot \frac{\partial(x/\operatorname{senh}(x))}{\partial x}$$

$$= \frac{g L \mu_B}{4\pi \lambda_t}\frac{(1 - x\operatorname{cotanh}(x))}{\operatorname{senh}(x)} \quad ; \quad \mu_B = \frac{k_B t x}{B}$$

$$\Rightarrow M = \frac{g L k_B t}{4\pi \lambda_t B} \cdot \frac{x(1 - x\operatorname{cotanh}(x))}{\operatorname{senh}(x)}$$

Si $x \ll 1 \Rightarrow M \approx 0$

En el límite clásico $\hbar \to 0 \Rightarrow x \to 0$

$\Rightarrow$ ¡No existe magnetismo clásicamente!