

## VECTOR DERIVATIVES

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**Cartesian.**  $d\mathbf{l} = dx \hat{\mathbf{x}} + dy \hat{\mathbf{y}} + dz \hat{\mathbf{z}}; \quad d\tau = dx dy dz$

*Gradient:*  $\nabla t = \frac{\partial t}{\partial x} \hat{\mathbf{x}} + \frac{\partial t}{\partial y} \hat{\mathbf{y}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$

*Divergence:*  $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

*Curl:*  $\nabla \times \mathbf{v} = \left( \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{\mathbf{x}} + \left( \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{\mathbf{y}} + \left( \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{\mathbf{z}}$

*Laplacian:*  $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

**Spherical.**  $d\mathbf{l} = dr \hat{\mathbf{r}} + r d\theta \hat{\boldsymbol{\theta}} + r \sin \theta d\phi \hat{\boldsymbol{\phi}}; \quad d\tau = r^2 \sin \theta dr d\theta d\phi$

*Gradient:*  $\nabla t = \frac{\partial t}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\boldsymbol{\phi}}$

*Divergence:*  $\nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$

*Curl:*  $\nabla \times \mathbf{v} = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{\mathbf{r}}$   
 $+ \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\boldsymbol{\theta}} + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\boldsymbol{\phi}}$

*Laplacian:*  $\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$

**Cylindrical.**  $d\mathbf{l} = ds \hat{\mathbf{s}} + s d\phi \hat{\boldsymbol{\phi}} + dz \hat{\mathbf{z}}; \quad d\tau = s ds d\phi dz$

*Gradient:*  $\nabla t = \frac{\partial t}{\partial s} \hat{\mathbf{s}} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z} \hat{\mathbf{z}}$

*Divergence:*  $\nabla \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s} (s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$

*Curl:*  $\nabla \times \mathbf{v} = \left[ \frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{\mathbf{s}} + \left[ \frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\boldsymbol{\phi}} + \frac{1}{s} \left[ \frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{\mathbf{z}}$

*Laplacian:*  $\nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$

### Triple Products

$$(1) \quad \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$(2) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

### Product Rules

$$(3) \quad \nabla(fg) = f(\nabla g) + g(\nabla f)$$

$$(4) \quad \nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$(5) \quad \nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$$

$$(6) \quad \nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$(7) \quad \nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$$

$$(8) \quad \nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$$

### Second Derivatives

$$(9) \quad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$(10) \quad \nabla \times (\nabla f) = 0$$

$$(11) \quad \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

## FUNDAMENTAL THEOREMS

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**Gradient Theorem:**  $\int_a^b (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

**Divergence Theorem:**  $\int (\nabla \cdot \mathbf{A}) d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

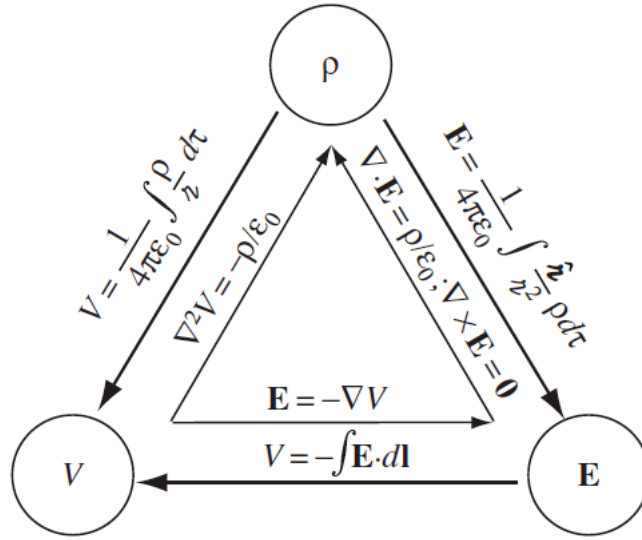
**Curl Theorem:**  $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

$$W = \frac{1}{2} \int \rho V d\tau.$$

$$W = \frac{\epsilon_0}{2} \int E^2 d\tau$$

$$C \equiv \frac{Q}{V}.$$

$$W = \frac{1}{2} CV^2,$$



$$V_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}.$$

$$\mathbf{p} \equiv \int \mathbf{r}' \rho(\mathbf{r}') d\tau'.$$

$$V = \frac{1}{4\pi\epsilon_0} \oint_S \frac{1}{r} \mathbf{P} \cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} (\nabla' \cdot \mathbf{P}) d\tau'.$$

$$\mathbf{D} \equiv \epsilon_0 \mathbf{E} + \mathbf{P},$$

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}.$$

$$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$

$$\mathbf{F} = Q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})].$$

$$\mathbf{F}_{\text{mag}} = \int I(d\mathbf{l} \times \mathbf{B}).$$

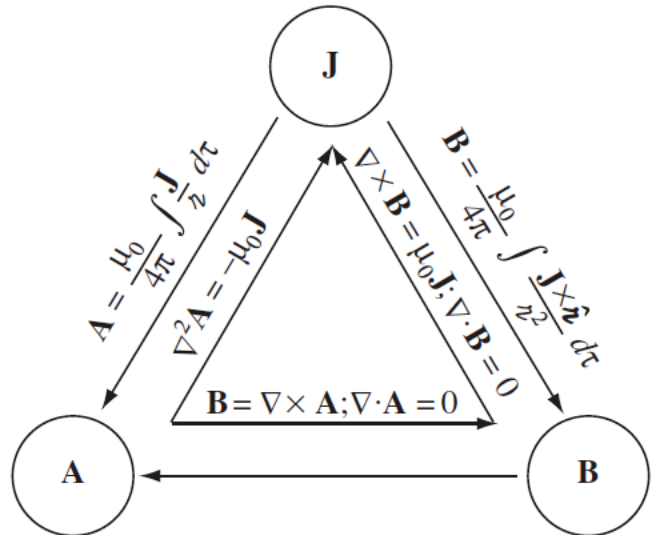
$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}.$$

$$E_{\text{above}}^\perp - E_{\text{below}}^\perp = \frac{1}{\epsilon_0} \sigma,$$

$$\mathbf{E}_{\text{above}}^\parallel - \mathbf{E}_{\text{below}}^\parallel = \mathbf{0}.$$

$$D_{\text{above}}^\perp - D_{\text{below}}^\perp = \sigma_f,$$

$$\mathbf{D}_{\text{above}}^\parallel - \mathbf{D}_{\text{below}}^\parallel = \mathbf{P}_{\text{above}}^\parallel - \mathbf{P}_{\text{below}}^\parallel.$$



$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times \hat{\mathbf{r}}}{r^2} dl' = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\mathbf{r}}}{r^2}. \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}')}{r} d\tau'.$$

$$\mathbf{B}_{\text{above}} - \mathbf{B}_{\text{below}} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}}), \quad \mathbf{A}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2}, \quad \mathbf{m} \equiv I \int d\mathbf{a} = I\mathbf{a}.$$

$$\mathbf{B}_{\text{dip}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} [3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}], \quad \mathbf{J}_b = \nabla \times \mathbf{M}, \quad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}},$$

$$\mathbf{H} \equiv \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}, \quad \nabla \times \mathbf{H} = \mathbf{J}_f,$$

$$\mathbf{H}_{\text{above}}^{\parallel} - \mathbf{H}_{\text{below}}^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}}, \quad \mathbf{M} = \chi_m \mathbf{H}, \quad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi_m)\mathbf{H},$$

$$P = VI = I^2 R, \quad \mathcal{E} = -\frac{d\Phi}{dt}, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

$$\Phi_2 = M_{21}I_1, \quad M_{21} = M_{12}, \quad \Phi = LI, \quad \mathcal{E} = -L \frac{dI}{dt}.$$

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M \frac{dI_1}{dt}, \quad W = \frac{1}{2} LI^2, \quad W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2 d\tau.$$

(i)	$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$	(Gauss's law),	$u = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right).$	$\mathbf{S} \equiv \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}).$
(ii)	$\nabla \cdot \mathbf{B} = 0$	(no name),		
(iii)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(Faraday's law),		
(iv)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	(Ampère's law with Maxwell's correction).	$\frac{dW}{dt} = -\frac{d}{dt} \int_V \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau - \frac{1}{\mu_0} \oint_S (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{a},$	

$$\mathbf{g} = \mu_0 \epsilon_0 \mathbf{S} = \epsilon_0 (\mathbf{E} \times \mathbf{B}), \quad \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}, \quad \nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.$$