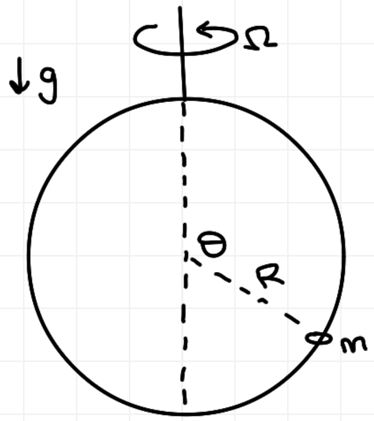
P1



$$\vec{a} = \underbrace{\vec{a}_{o'}}_{\text{trasl.}} + \vec{a}' + \underbrace{\dot{\vec{\Omega}} \times \vec{r}'}_{\text{transv.}} + \underbrace{2\vec{\Omega} \times \vec{v}'}_{\text{cor.}} + \underbrace{\vec{\Omega} \times (\vec{\Omega} \times \vec{r}')}_{\text{cent.}}$$

$$m\vec{a} = \vec{F}_{reales} \qquad m\vec{a}' = \vec{F}_{reales} + \vec{F}_{ficticias}$$

$$\vec{\Omega} = \Omega\hat{z} = \Omega\hat{z}'$$

sistema S: vemos el aro girando

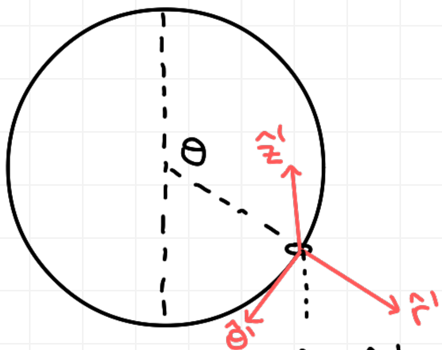
" S': vemos siempre la misma cara del aro, observando fuerzas ficticias sobre m

$\Big\}\ \vec{a}_{o'} = 0$

Trabajemos en S' usando esféricas (en S' !!!)

$\dot{\phi} = 0$ porque en S' no vemos mov. azimutal

$$\vec{r}' = R\hat{r}'$$
$$\vec{v}' = R\dot{\theta}\hat{\theta}'$$
$$\vec{a}' = -R\dot{\theta}^2\hat{r}' + R\ddot{\theta}\hat{\theta}'$$



$$\hat{z}' = (\hat{z}' \cdot \hat{r}')\hat{r}' + (\hat{z}' \cdot \hat{\theta}')\hat{\theta}'$$
$$= \cos\theta\,\hat{r}' + \cos(\pi/2 - \theta)\hat{\theta}'$$
$$= \cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}'$$

Las fzas reales son

$$\vec{N} = N_r\hat{r}' + N_\phi\hat{\phi}$$
$$m\vec{g} = -mg\hat{z}'$$
$$= -mg(\cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}')$$

y las ficticias:

$$\dot{\vec{\Omega}} = 0$$

$$\vec{\Omega} = \Omega\hat{z}' = \Omega(\cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}')$$

$$\vec{\Omega} \times \vec{v}' = \Omega(\cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}') \times R\dot{\theta}\hat{\theta}'$$
$$= \Omega R\dot{\theta}\cos\theta\,\hat{\phi}'$$

$$\vec{\Omega} \times \vec{r}' = \Omega(\cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}') \times R\hat{r}'$$
$$= \Omega R\sin\theta\,\hat{\phi}'$$

$$\vec{\Omega} \times (\vec{\Omega} \times \vec{r}') = \Omega(\cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}') \times \Omega R\sin\theta\,\hat{\phi}'$$
$$= -\Omega^2 R(\cos\theta\sin\theta\,\hat{\theta}' + \sin^2\theta\,\hat{r}')$$

En la ec. de mov.

$$m\left[-R\dot{\theta}^2\hat{r}' + R\ddot{\theta}\hat{\theta}' + 2\Omega R\dot{\theta}\cos\theta\,\hat{\phi}' - \Omega^2 R\left(\cos\theta\sin\theta\,\hat{\theta}' + \sin^2\theta\,\hat{r}'\right)\right]$$
$$= N_r\hat{r}' + N_\phi\hat{\phi}' - mg\left(\cos\theta\,\hat{r}' - \sin\theta\,\hat{\theta}'\right)$$

- $\hat{r}'$: $\quad mR\dot{\theta}^2 + m\Omega^2 R\sin^2\theta = mg\cos\theta - N_r$
- $\hat{\theta}'$: $\quad mR\ddot{\theta} - m\Omega^2 R\cos\theta\sin\theta = mg\sin\theta$
- $\hat{\phi}'$: $\quad 2m\Omega R\dot{\theta}\cos\theta = N_\phi$

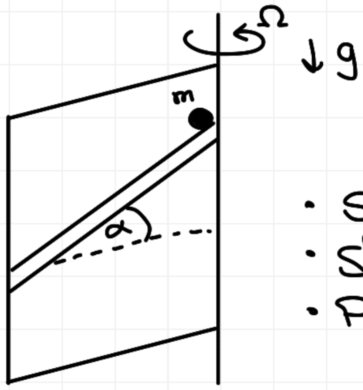
Buscamos puntos de equilibrio $(\ddot{\theta} = 0)$, así que de la ec. en $\hat{\theta}'$:

$$-m\Omega^2 R\cos\theta\sin\theta = mg\sin\theta$$

$$\sin\theta = 0 \longrightarrow \theta_{eq} = 0, \pi$$

$$\cos\theta = -\frac{g}{\Omega^2 R} \leq 1 \quad \longrightarrow \theta_{eq} = \arccos\left(-\frac{g}{\Omega^2 R}\right)$$

P2



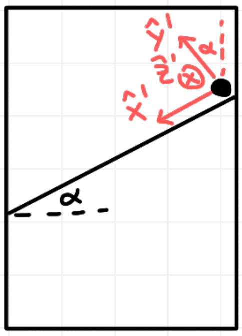
$$\vec{\Omega} = \Omega \hat{z}$$

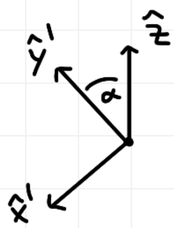
- S: vemos la rotación de la puerta con $\vec{\Omega}$
- S': vemos la puerta siempre de frente sin rotar

$\Big\}\ \vec{a}_{0'} = 0$

- Partícula soltada desde el punto más alto

A)





$$\begin{aligned}\hat{z} &= (\hat{z}\cdot\hat{x}')\hat{x}' + (\hat{z}\cdot\hat{y}')\hat{y}' \\ &= \cos(\pi/2+\alpha)\hat{x}' + \cos\alpha\,\hat{y}' \\ &= -\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}'\end{aligned}$$

$$\vec{\Omega} = \Omega(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')$$

Como trabajamos en S':

$$\begin{aligned}\vec{r}' &= x\hat{x}' \\ \vec{v}' &= \dot{x}\hat{x}' \\ \vec{a}' &= \ddot{x}\hat{x}'\end{aligned}$$

Las fuerzas reales: 2 normales (puerta y vara) y peso

$$\begin{aligned}\vec{N} &= N_v\hat{y}' - N_p\hat{z}' \\ m\vec{g} &= -mg\hat{z} \\ &= -mg(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')\end{aligned}$$

y las ficticias

$$\vec{a}_{0'} = 0 \qquad \dot{\vec{\Omega}} = 0$$

$$\begin{aligned}\vec{\Omega}\times\vec{v}' &= \Omega(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')\times\dot{x}\hat{x}' \\ &= -\Omega\dot{x}\cos\alpha\,\hat{z}'\end{aligned}$$

$$\begin{aligned}\vec{\Omega}\times\vec{r}' &= \Omega(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')\times x\hat{x}' \\ &= -\Omega x\cos\alpha\,\hat{z}'\end{aligned}$$

$$\begin{aligned}\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') &= \Omega(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')\times -\Omega x\cos\alpha\,\hat{z}' \\ &= -\Omega^2 x\sin\alpha\cos\alpha\,\hat{y}' - \Omega^2 x\cos^2\alpha\,\hat{x}'\end{aligned}$$

B) $m\vec{a} = \vec{F}_{reales}$

$$m(\ddot{x}\hat{x}' - 2\Omega\dot{x}\cos\alpha\,\hat{z}' - \Omega^2 x\sin\alpha\cos\alpha\,\hat{y}' - \Omega^2 x\cos^2\alpha\,\hat{x}')$$

$$= N_v\hat{y}' - N_p\hat{z}' - mg(-\sin\alpha\,\hat{x}' + \cos\alpha\,\hat{y}')$$

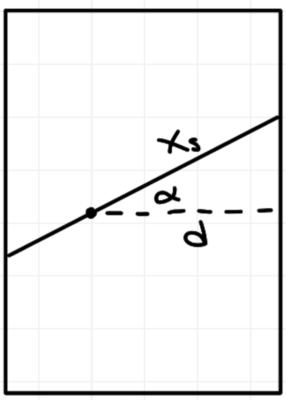
- $\hat{x}'$: $\quad m\ddot{x} - m\Omega^2 x\cos^2\alpha = mg\sin\alpha \qquad (1)$
- $\hat{y}'$: $\quad -m\Omega^2 x\sin\alpha\cos\alpha = N_v - mg\cos\alpha \qquad (2)$
- $\hat{z}'$: $\quad -2m\Omega\dot{x}\cos\alpha = -N_p \qquad (3)$

C) Si se despega de la vara $N_v = 0$:

$$(2) \longrightarrow +\cancel{m}\Omega^2 x\sin\alpha\cancel{\cos\alpha} = 0 + \cancel{m}g\cancel{\cos\alpha}$$

$$x = \frac{g}{\Omega^2\sin\alpha} = x_s$$

y por geometría, la distancia d al eje de rotación es:



$$\cos\alpha = \frac{d}{x_s}$$

$$d = \frac{g}{\Omega^2}\cot\alpha$$

D) Buscamos $N_p(x_s)$.

$$(3) \longrightarrow N_p = 2m\Omega\dot{x}\cos\alpha$$

así que necesitamos $\dot{x}(x)$:

$$(1) \longrightarrow m\ddot{x} - m\Omega^2 x\cos^2\alpha = mg\sin\alpha$$

$$\ddot{x} = \Omega^2\cos^2\alpha\, x + g\sin\alpha \qquad / \ \ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{dx}\cdot\frac{dx}{dt} = \dot{x}\frac{d\dot{x}}{dx}$$

$$\dot{x}\frac{d\dot{x}}{dx} = \Omega^2\cos^2\alpha\, x + g\sin\alpha$$

$$\int\dot{x}\,d\dot{x} = \Omega^2\cos^2\alpha\int x\,dx + g\sin\alpha\int dx$$

$$\frac{1}{2}(\dot{x}^2 - \cancel{\dot{x}_0^2}) = \frac{1}{2}\Omega^2\cos^2\alpha(x^2 - \cancel{x_0^2}) + g\sin\alpha(x - \cancel{x_0})$$

$\dot{x}_0 \to 0$ (reposo) $\quad x_0 \to 0$ pequeño $\quad x_0 \to 0$

$$\dot{x}^2 = \Omega^2\cos^2\alpha\, x^2 + 2g\sin\alpha\, x$$

$$\dot{x} = \sqrt{\Omega^2\cos^2\alpha\, x^2 + 2g\sin\alpha\, x}$$

y en el punto de separación

$$\dot{x}_s = \sqrt{\Omega^2\cos^2\alpha\left(\frac{g}{\Omega^2\sin\alpha}\right)^2 + 2g\cancel{\sin\alpha}\frac{g}{\Omega^2\cancel{\sin\alpha}}}$$

$$\dot{x}_s = \frac{g}{\Omega}\sqrt{\cot^2\alpha + 2}$$

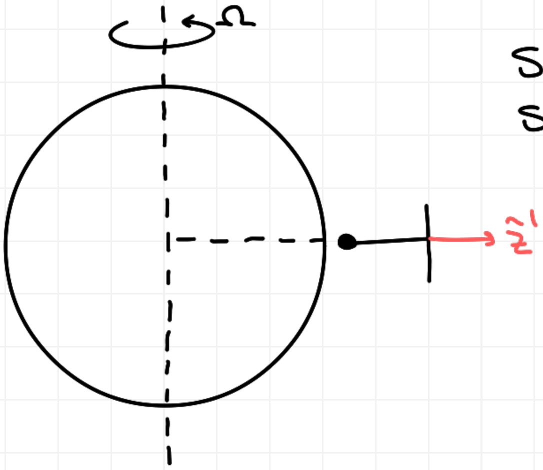
Finalmente,

$$N_p(x_s) = 2m\Omega\dot{x}_s\cos\alpha$$

$$= 2m\Omega\frac{g}{\Omega}\sqrt{\cot^2\alpha + 2}\,\cos\alpha$$

$$N_p = 2mg\cos\alpha\sqrt{\cot^2\alpha + 2}$$

P3



S: vemos la Tierra rotar

S': estamos parados en la superficie, viendo la masa colgar

En el sistema S' las fuerzas son

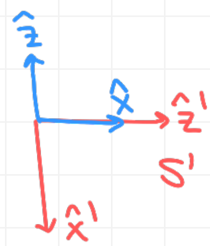
$$m\vec{g} = -mg\hat{z}' \qquad \vec{T} = T\hat{z}'$$

y las fuerzas ficticias

$$\vec{a}_{o'} = 0 \qquad \dot{\vec{\Omega}} \approx 0$$

$$\vec{v}' = 0 \qquad \vec{a}' = 0 \quad \text{(masa en reposo)}$$

$$\vec{r}' = (R+h)\hat{z}' \qquad \vec{\Omega} = \Omega\hat{z}$$
$$= (R+h)\hat{x}$$



$$\vec{\Omega}\times\vec{r}' = \Omega\hat{z}\times(R+h)\hat{x}$$
$$= \Omega(R+h)\hat{y}$$

$$\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = \Omega\hat{z}\times\Omega(R+h)\hat{y}$$
$$= -\Omega^2(R+h)\hat{x}$$

y la ec. de mov. es

$$m\vec{a} = \vec{F}_{\text{reales}} = -mg\hat{x} + T\hat{x}$$

$$m(-\Omega^2(R+h)\hat{x}) = -mg\hat{x} + T\hat{x}$$

$$T = mg - m\Omega^2(R+h)$$