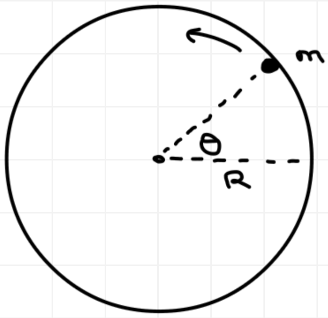
P1



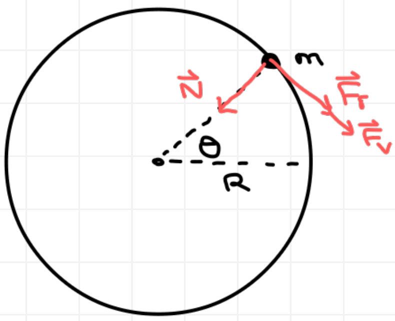
- Roce viscoso cuadrático coef. η
- Roce cinético coef. μ
- $\eta = m\mu / R$

A)
- Partícula lanzada en $t=0$ desde $\theta = 0$, con vel. angular ω_0
- Calcular W de $\vec{F}_{neta}$ en fn. de t.

Se define el trabajo W de una fuerza $\vec{F}$ como la integral de línea sobre la trayectoria de la partícula:

$$W = \int_C \vec{F} \cdot d\vec{r}$$

<u>dcl</u>:



<u>fuerzas</u>: contacto: $\vec{N}$ y $\vec{F}_r$

roce visc.: $\vec{F}_v$

en polares:

$$\vec{N} = -N\hat{\rho} \qquad \vec{F}_r = -\mu N \hat{\theta}$$

$$\vec{v} = \cancelto{0}{\dot{\rho}\hat{\rho}} + \rho\dot{\theta}\hat{\theta} = R\dot{\theta}\hat{\theta} \longrightarrow v^2 = R^2\dot{\theta}^2$$

$$\vec{F}_v = -\eta v^2 \hat{\theta} = -\eta R^2 \dot{\theta}^2 \hat{\theta}$$

$$\vec{F} = \vec{N} + \vec{F}_r + \vec{F}_v$$

Como queremos $W(t)$, tomamos $d\vec{r} = \vec{v}\, dt$:

$$W = \int_0^t \vec{F} \cdot \vec{v}\, dt$$

$$= \int \cancelto{0}{\vec{N} \cdot \vec{v}}\, dt + \int \vec{F}_r \cdot \vec{v}\, dt + \int \vec{F}_v \cdot \vec{v}\, dt$$

$$= \int -\mu N \cdot R\dot{\theta}\, dt + \int -\eta R^2 \dot{\theta}^2 R\dot{\theta}\, dt$$

$$= -\mu R \int N\dot{\theta}\, dt - \eta R^3 \int \dot{\theta}^3\, dt$$

Necesitamos $N(t)$ y $\dot{\theta}(t)$ para poder integrar. Podemos sacar info. de las ec. de mov:

$$\vec{a} = (\cancel{\ddot{\rho}} - \rho\dot{\theta}^2)\hat{\rho} + (\cancel{2\dot{\rho}\dot{\theta}} + \rho\ddot{\theta})\hat{\theta}$$

$$\vec{a} = -R\dot{\theta}^2\hat{\rho} + R\ddot{\theta}\hat{\theta}$$

Newton:

$$-mR\dot{\theta}^2\hat{\rho} + mR\ddot{\theta}\hat{\theta} = -N\hat{\rho} - \mu N\hat{\theta} - \eta R^2\dot{\theta}^2\hat{\theta}$$

• $\hat{\rho}$: $\quad -mR\dot{\theta}^2 = -N \longrightarrow \underline{N = mR\dot{\theta}^2}$

• $\hat{\theta}$: $\quad mR\ddot{\theta} = -\mu N - \eta R^2\dot{\theta}^2$

$$\cancel{m}\cancel{R}\ddot{\theta} = -\mu\cancel{m}\cancel{R}\dot{\theta}^2 - \frac{\cancel{m}\mu}{\cancel{R}}\cancel{R^2}\dot{\theta}^2$$

(*) $$\ddot{\theta} = -2\mu\dot{\theta}^2 \qquad \Big/ \ \ddot{\theta} = \frac{d\dot{\theta}}{dt}$$

$$\frac{d\dot{\theta}}{dt} = -2\mu\dot{\theta}^2$$

$$\frac{1}{\dot{\theta}^2}\,d\dot{\theta} = -2\mu\,dt \qquad \Big/ \int_{t=0}^{t} dt$$

$$\int_{\omega_0}^{\dot{\theta}} \frac{1}{\dot{\theta}^2}\,d\dot{\theta} = -2\mu t$$

$$-\frac{1}{\dot{\theta}} + \frac{1}{\omega_0} = -2\mu t$$

$$\frac{1}{\dot{\theta}} = \frac{1}{\omega_0} + 2\mu t = \frac{1 + 2\omega_0\mu t}{\omega_0}$$

$$\underline{\dot{\theta}(t) = \frac{\omega_0}{1 + 2\omega_0\mu t}} \longrightarrow N(\dot{\theta}(t)) = mR\dot{\theta}^2(t)$$

$$W(t) = -\mu R\int N\dot{\theta}\,dt - \eta R^3\int\dot{\theta}^3\,dt$$

$$= -\mu R\int mR\dot{\theta}^3\,dt - \frac{m\mu}{R}R^3\int\dot{\theta}^3\,dt$$

$$= -2\mu m R^2\int_0^t \dot{\theta}^3(t)\,dt$$

$$\int_0^t \dot{\theta}^3\, dt = \int_0^t \left(\frac{\omega_0}{1+2\omega_0\mu t}\right)^3 dt \qquad \tau = 2\omega_0\mu t \longrightarrow d\tau = 2\omega_0\mu\, dt$$

$$= \omega_0^3 \int_0^{2\omega_0\mu t} \frac{1}{(1+\tau)^3}\,\frac{d\tau}{2\omega_0\mu}$$

$$= \frac{\omega_0^2}{2\mu}\int \frac{d\tau}{(1+\tau)^3}$$

$$= \frac{\omega_0^2}{2\mu}\left[-\frac{1}{2(\tau+1)^2}\right]_0^{2\omega_0\mu t}$$

$$= \frac{\omega_0^2}{2\mu}\cdot\left(\frac{1}{2} - \frac{1}{2(1+2\omega_0\mu t)^2}\right)$$

$$= \frac{\omega_0^2}{4\mu}\left(\frac{(1+2\omega_0\mu t)^2 - 1}{(1+2\omega_0\mu t)^2}\right)$$

$$= \frac{\omega_0^2}{\cancel{4}\cancel{\mu}}\cdot\frac{\cancel{4}\omega_0\cancel{\mu}t + \cancel{4}\omega_0^2\mu^{\cancel{2}}t^2}{(1+2\omega_0\mu t)^2}$$

$$= \omega_0^3 t\cdot\frac{1+\mu\omega_0 t}{(1+2\omega_0\mu t)^2}$$

$$\boxed{W(t) = -2\mu m R^2\omega_0^3 t\cdot\frac{1+\mu\omega_0 t}{(1+2\omega_0\mu t)^2}}$$

B) Trabajo al recorrer media circ. $\longrightarrow$ trayectoria $\theta = 0$ a $\theta = \pi$

$$W = -2\mu m R^2\int_0^t \dot{\theta}^3(t)\, dt$$

$$W(\theta) = -2\mu m R^2\int_0^\theta \dot{\theta}^2\,\dot{\theta}\,\frac{dt}{d\theta}$$

Necesitamos $\dot{\theta}(\theta)$. De $(*)$:

$$\ddot{\theta} = -2\mu\dot{\theta}^2 \qquad \ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\cdot\dot{\theta}$$

$$\cancel{\dot{\theta}}\, d\dot{\theta} = -2\mu\dot{\theta}^{\cancel{2}}\, d\theta$$

$$\frac{1}{\dot\theta}\, d\dot\theta = -2\mu\theta \qquad / \int_0^\theta$$

$$\ln \dot\theta \,/\, \omega_0 = -2\mu\theta$$

$$\dot\theta = \omega_0 e^{-2\mu\theta}$$

$$W(\theta) = -2\mu m R^2 \int_0^\theta \dot\theta^2 \, d\theta$$

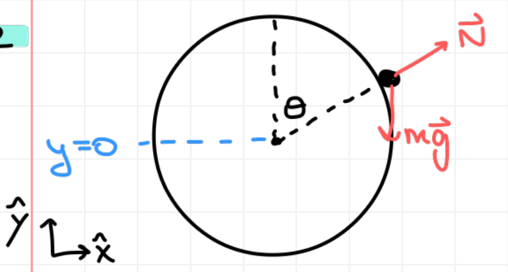
$$= -2\mu m R^2 \int \omega_0^2 e^{-4\mu\theta} \, d\theta$$

$$= -2\omega_0^2 \cancel{\mu} m R^2 \cdot -\frac{1}{4\cancel{\mu}} \left(e^{-4\mu\theta} - 1\right)$$

$$= \frac{1}{2}\omega_0^2 m R^2 \left(e^{-4\mu\theta} - 1\right) \qquad / \theta = \pi$$

$$\boxed{W(\pi) = \frac{1}{2}\omega_0^2 m R^2 \big(\underbrace{e^{-4\mu\pi} - 1}_{<0}\big) < 0}$$ pérdida de E

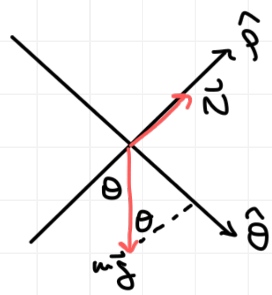
P2



$\vec{N} = N\hat{\rho}$

$m\vec{g} = mg(-\cos\theta\,\hat{\rho} + \sin\theta\,\hat{\theta})$

$\vec{a} = (\cancel{\ddot{\rho}} - \rho\dot{\theta}^2)\hat{\rho} + (\cancel{2\dot{\rho}\dot{\theta}} + \rho\ddot{\theta})\hat{\theta}$



$m\vec{a} = \vec{F}$

$\cdot\hat{\rho}: \quad -mR\dot{\theta}^2 = \cancel{N} - mg\cos\theta$ — condición en $\dot{\theta}$ y θ para despegue

$$\underline{R\dot{\theta}^2 = g\cos\theta} \qquad (1)$$

conservación de energía:

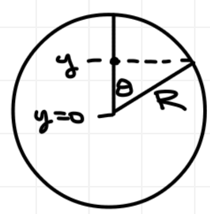
en el punto inicial, $\theta = 0$

$K_i = \frac{1}{2}mv_0^2 \approx 0$ ($v_0 > 0$ muy pequeño)

$U_i = mgR$

en el punto de despegue $v^2 = R^2\dot{\theta}^2$ $(\vec{v} = \cancel{\dot{R}\hat{\rho}}^{\,0} + R\dot{\theta}\hat{\theta})$

$K_d = \frac{1}{2}mR^2\dot{\theta}^2$



$\cos\theta = \frac{y}{R}$

$U_d = mgR\cos\theta$

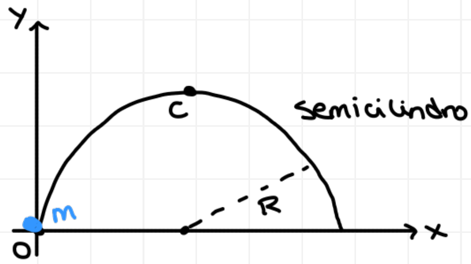
$\Delta E = 0 \longrightarrow 0 + \cancel{m}g\cancel{R} = \frac{1}{2}\cancel{m}R^{\cancel{2}}\dot{\theta}^2 + \cancel{m}g\cancel{R}\cos\theta$

$$\underline{g(1-\cos\theta) = R\dot{\theta}^2} \qquad (2)$$

(1) = (2) $\searrow$ $\frac{2g}{R}(1-\cos\theta) = \frac{g}{R}\cos\theta$

$\cos\theta = \frac{2}{3} \rightarrow \theta = \frac{\pi}{3}$

P3



- $\vec{F}_1 = -c(xy^2\hat{x} + x^2y\hat{y})$
- $\vec{F}_2$ permite rapidez cte en la superficie

A) $\vec{F}$ es conservativa si y solo si

$$\oint \vec{F}\cdot d\vec{r} = 0 \qquad (1)$$

o $\quad \vec{F} = -\nabla U \quad (2)$

o $\quad \nabla \times \vec{F} = 0 \longrightarrow \begin{cases} \partial_y F_z = \partial_z F_y \\ \partial_z F_x = \partial_x F_z \\ \partial_x F_y = \partial_y F_x \end{cases} \qquad (3)$

Método (3):

$$\vec{F}_1 = -c(xy^2\hat{x} + x^2y\hat{y})$$

$$\partial_y F_{1z} = 0 = \partial_z F_{1y}$$

$$\partial_z F_{1x} = 0 = \partial_x F_{1z}$$

$$\partial_x F_{1y} = -2cxy \quad = \quad \partial_y F_{1x} = -2cxy \longrightarrow \vec{F}_1 \text{ es cons.}$$

Método (2):

Supongamos que existe U tal que $-\nabla U = \vec{F}_1$. Si lo encontramos, entonces tamos bn :)

$$\vec{F}_1 \overset{!}{=} -\nabla U \longrightarrow \begin{array}{l} \partial_x U = cxy^2 \longrightarrow U = \frac{c}{2}x^2y^2 + A(y,z) \\ \partial_y U = cx^2y \longrightarrow U = \frac{c}{2}x^2y^2 + A(z) \\ \partial_z U = 0 \longrightarrow U \text{ cte en } z \end{array}$$

$U = \frac{1}{2}cx^2y^2 + A \rightarrow \vec{F}_1 = -\nabla U$ y es conservativa, asociada al pot. U.