Oscilaciones:
- mov. periódico en torno a algún equilibrio
- fuera del equilibrio existen fuerzas que nos devuelven
- puede haber pérdida de energía → amortiguamiento
- " " inyección " → forzamiento

- 3 tipos: libres, amort., forzadas

- partícula m sujeta a fuerza neta $\vec{F}(\vec{r},\vec{v},t)$
- equilibrio en $\vec{r}_{eq} \rightarrow \vec{F}(\vec{r}_{eq}) = 0$
- aprox. lineal: $\vec{F} = -k(\vec{r} - \vec{r}_{eq})$
  $\hookrightarrow \vec{r}_{eq} = \vec{0} \rightarrow \vec{F} = -k\vec{r}$

Osc. libre: $m\vec{a} = -k\vec{r}$

$$m\ddot{x} = -kx$$

$$\ddot{x} + \omega^2 x = 0, \qquad \omega \equiv \frac{k}{m}$$

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t}$$
$$= C\cos\omega t + D\sin\omega t$$

Osc. amortiguada $F \propto v^n$

$n=1 \longrightarrow \quad m\ddot{x} = -c\dot{x} - kx$

$$\ddot{x} + 2\gamma\dot{x} + \omega^2 x = 0 \qquad \gamma \equiv \frac{c}{2m}$$

$\gamma = \omega \rightarrow$ amor. crítico

$$x(t) = (A + Bt)e^{-\gamma t}$$

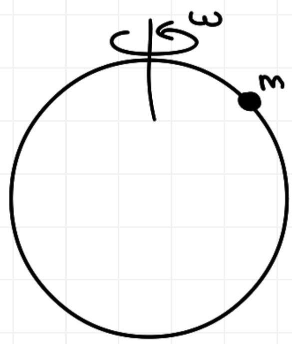
$\gamma < \omega \longrightarrow$ amor. subcrítico

$$x(t) = e^{-\gamma t}\left(Ae^{i\Omega t} + Be^{-i\Omega t}\right) \qquad \Omega \equiv \omega\sqrt{1-(\gamma/\omega)^2}$$

$\gamma > \omega \longrightarrow$ amort. supercrítico

$$x(t) = e^{-\gamma t}\left(Ae^{\Gamma t} + Be^{-\Gamma t}\right) \qquad \Gamma \equiv \gamma\sqrt{1-(\omega/\gamma)^2}$$

P1



$$\vec{a} = (\ddot{r} - r\dot{\phi}^2\sin^2\theta - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta + 2\dot{r}\dot{\theta})\hat{\theta} + (2\dot{r}\dot{\phi}\sin\theta + r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\hat{\phi}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\,\hat{\phi}$$

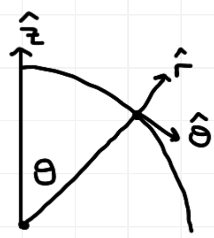
$$\vec{v} = R(\dot{\theta}\hat{\theta} + \dot{\phi}\sin\theta\,\hat{\phi}) \qquad \dot{\phi} = \omega$$

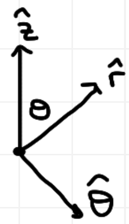
$$\vec{a} = -R(\dot{\phi}^2\sin^2\theta + \dot{\theta}^2)\hat{r} + R(\ddot{\theta} - \dot{\phi}^2\sin\theta\cos\theta)\hat{\theta} + 2R\dot{\theta}\dot{\phi}\cos\theta\,\hat{\phi}$$

$$\vec{a} = -R(\omega^2\sin^2\theta + \dot{\theta}^2)\hat{r} + R(\ddot{\theta} - \omega^2\sin\theta\cos\theta)\hat{\theta} + 2R\omega\dot{\theta}\cos\theta\,\hat{\phi}$$

fuerzas: peso, normal $\qquad \vec{N} = N_\phi\hat{\phi} + N_r\hat{r}$

$$m\vec{g} = -mg\hat{z}$$





$$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$

$$mR(\ddot{\theta} - \omega^2\sin\theta\cos\theta) = mg\sin\theta$$

$$\ddot{\theta} - \omega^2\sin\theta\cos\theta = \frac{g}{R}\sin\theta$$

$\omega = 0$:

$\hookrightarrow \quad \ddot{\theta} = \frac{g}{R}\sin\theta \longrightarrow$ oscilaciones tipo péndulo simple

$$\theta \ll 1 \rightarrow \quad \ddot{\theta} = \frac{g}{R}\theta$$

$\underline{\omega \neq 0}$:

$$\ddot{\theta} - \omega^2 \sin\theta \cos\theta = \frac{g}{R}\sin\theta$$

$$\ddot{\theta} = 0 \longrightarrow \quad 0 = \sin\theta\left(\omega^2\cos\theta + \frac{g}{R}\sin\theta\right)$$

$$\sin\theta = 0 \longrightarrow \theta = 0, \pi$$

$$\cos\theta = -\frac{g}{R\omega^2} \longrightarrow \theta = \arccos\left(-\frac{g}{R\omega^2}\right)$$

supong. pequeñas osc. en torno a alguno de los equilibrios:

$$\theta(t) = \theta_0 + \eta(t) \qquad \eta \ll 1$$

$0, \pi, \arccos \ldots$

$$\longrightarrow \ddot{\theta} = \frac{d^2}{dt^2}(\theta_0 + \eta) = \ddot{\eta}$$

$$\sin\theta = \sin(\theta_0 + \eta) = \sin\theta_0 \cos\eta + \cos\theta_0 \sin\eta$$

$$\sin\theta \approx \sin\theta_0 + \eta\cos\theta_0 + \cancelto{0}{O(\eta^2)}$$

$$\cos\theta = \cos(\theta_0 + \eta) = \cos\theta_0 \cos\eta - \sin\theta_0 \sin\eta$$

$$\cos\theta \approx \cos\theta_0 - \eta\sin\theta_0 + \cancelto{0}{O(\eta^2)}$$

$$\sin\theta\cos\theta \approx (\sin\theta_0 + \eta\cos\theta_0)(\cos\theta_0 - \eta\sin\theta_0)$$

$$= \sin\theta_0\cos\theta_0 - \eta\sin^2\theta_0 + \eta\cos^2\theta_0 - \cancelto{0}{\eta^2}\sin\theta_0\cos\theta_0$$

$$\approx \sin\theta_0\cos\theta_0 + \eta(\cos^2\theta_0 - \sin^2\theta_0)$$

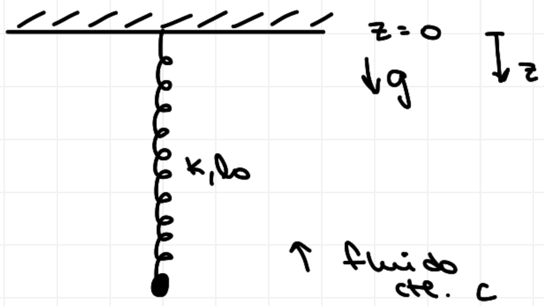
$$\hookrightarrow \quad \ddot{\eta} - \omega^2\left[\sin\theta_0\cos\theta_0 + \eta(\cos^2\theta_0 - \sin^2\theta_0)\right] = \frac{g}{R}(\sin\theta_0 + \eta\cos\theta_0)$$

$\theta_0 = 0$

$$\ddot{\eta} - \omega^2\eta = \frac{g}{R}\eta \longrightarrow \ddot{\eta} - \overbrace{\left(\omega^2 + \frac{g}{R}\right)}^{\Omega^2}\eta$$

$$\ddot{\eta} - \Omega^2\eta = 0 \longrightarrow \text{oscilación en } \hat{\theta}$$

P2



a) Encontrar ec. de mov. c/r pos. de equilibrio

El movimiento solo es vertical → $\vec{a} = \ddot{z}\hat{z}$

fuerzas: peso, resorte, fluido

$$m\vec{g} = mg\hat{z}$$

$$\vec{F}_e = -k(z - l_0)\hat{z}$$

$$\vec{F}_v = -c\vec{v} = -c\dot{z}\hat{z}$$

Newton:

$$m\ddot{z} = mg - k(z - l_0) - c\dot{z} \qquad \Big/ \cdot \frac{1}{m}$$

$$\ddot{z} = g - \frac{k}{m}(z - l_0) - \frac{c}{m}\dot{z} \qquad \Big/ \; \omega^2 \equiv \frac{k}{m} \qquad b \equiv \frac{c}{m}$$

$$\ddot{z} = g - \omega^2(z - l_0) - b\dot{z}$$

$$\ddot{z} = -\omega^2\left(z - l_0 - \frac{g}{\omega^2}\right) - b\dot{z}$$

$$\ddot{z} + b\dot{z} + \omega^2\Big[z - \underbrace{\left(l_0 + \frac{g}{\omega^2}\right)}_{z_{eq}}\Big]$$

¿pq? para encontrar z_{eq} se impone $\dot{z} = \ddot{z} = 0$, y coincide con esta cantidad → es efectivamente equilibrio

Definimos y según el equilibrio:

$$y \equiv z - \left(l_0 + \frac{g}{\omega^2}\right) \longrightarrow \dot{y} = \dot{z} \longrightarrow \ddot{y} = \ddot{z}$$

$$\ddot{y} + bz + \omega^2 y = 0$$

$$\boxed{y(t) = e^{-\gamma t}\left(Ae^{\Gamma t} + Be^{-\Gamma t}\right)} \qquad \left(\gamma \equiv \frac{b}{2},\ \Gamma = \gamma\sqrt{1-\left(\frac{\omega}{\gamma}\right)^2}\right)$$

solución sobreamortiguada

b) $y(0) = H,\ \dot{y}(0) = v_0$

$$y(0) = H$$

$$H = A + B \longrightarrow \boxed{B = H - A}$$

$$y(t) = e^{-\gamma t}\left[Ae^{\Gamma t} + (H-A)e^{-\Gamma t}\right]$$

$$\dot{y}(t) = e^{-\gamma t}\left[A\Gamma e^{\Gamma t} - (H-A)\Gamma e^{-\Gamma t} - A\gamma e^{\Gamma t} - (H-A)\gamma e^{-\Gamma t}\right]$$

$$\dot{y}(t) = e^{-\gamma t}\left[Ae^{\Gamma t}(\Gamma-\gamma) - (H-A)e^{-\Gamma t}(\Gamma+\gamma)\right]$$

$$\dot{y}(0) = v_0$$

$$v_0 = A(\Gamma-\gamma) - (H-A)(\Gamma+\gamma)$$

$$v_0 = A(\Gamma-\gamma) - H(\Gamma+\gamma) + A(\Gamma+\gamma)$$

$$v_0 = 2A\Gamma - H(\Gamma+\gamma)$$

$$\boxed{\frac{v_0 + H(\Gamma+\gamma)}{2\Gamma} = A}$$

$$B = H - A = \frac{2\Gamma H - v_0 - H(\Gamma+\gamma)}{2\Gamma}$$

$$\boxed{B = \frac{-v_0 - H(\gamma-\Gamma)}{2\Gamma}}$$

c) Condición sobre v_0 para que cruce $y=0$ en instante posterior al inicial.

$$y(T) = 0$$

$$\cancel{e^{-\gamma T}}\left(Ae^{\Gamma T} + Be^{-\Gamma T}\right) = 0 \qquad / \cdot e^{\Gamma T}$$

$$Ae^{2\Gamma T} + B = 0$$

$$e^{2\Gamma T} = -\frac{B}{A}$$

$$2\Gamma T = \ln\left(-\frac{B}{A}\right)$$

como $T>0$ (y $\Gamma>0$), $\ln(-B/A)>0$

$$-\frac{B}{A} > 1$$

caso 1: $A>0 \longrightarrow -B>A$

$$\frac{\cancel{v_0} + \cancel{H}(\gamma - \Gamma)}{\cancel{2\Gamma}} > \frac{\cancel{v_0} + \cancel{H}(\Gamma+\gamma)}{\cancel{2\Gamma}}$$

$$\cancel{\gamma} - \Gamma > \Gamma + \cancel{\gamma}$$

$$-\Gamma > \Gamma \longrightarrow \text{no es posible}$$

caso 2: $A<0 \longrightarrow -B<A$

$$\frac{\cancel{v_0} + H(\cancel{\gamma} - \Gamma)}{\cancel{2\Gamma}} < \frac{\cancel{v_0} + H(\Gamma+\cancel{\gamma})}{\cancel{2\Gamma}}$$

$$-\Gamma < \Gamma \longrightarrow \text{sí es posible}$$

Sigue que $A<0$:

$$\frac{v_0 + H(\Gamma+\gamma)}{2\Gamma} < 0$$

$$\boxed{v_0 < -H(\Gamma+\gamma)}$$