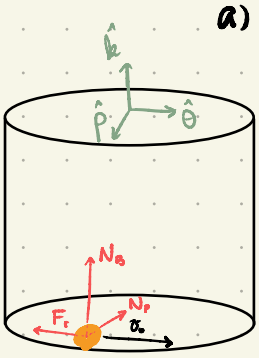
P1 Auxiliar 8



a) Usamos coord cilíndricas. Las fuerzas involucradas son: el peso en $-\hat{k}$, la normal de la base en $\hat{k}$, la normal de la pared en $-\hat{\rho}$ y la fuerza de roce cinético en $-\hat{\theta}$

$$\Rightarrow m\left((\ddot{\rho}-\rho\dot{\theta}^2)\hat{\rho}+(2\dot{\rho}\dot{\theta}+\rho\ddot{\theta})\hat{\theta}+\ddot{z}\hat{k}\right)=-mg\hat{k}+N_\theta\hat{k}-N_\rho\hat{\rho}-N_\rho\mu_c\hat{\theta}$$

donde $\ddot{\rho}=\dot{\rho}=\ddot{z}=0 \wedge \rho=R$, por lo que las ecs escalares quedan como:

$$\hat{\rho})\ -mR\dot{\theta}^2=-N_\rho \quad (1)$$
$$\hat{\theta})\ mR\ddot{\theta}=-N_\rho\mu_c \quad (2)$$
$$\hat{k})\ 0=-mg+N_\theta \quad (3)$$

Reemplazando (1) en (2), se tiene:

$$mR\ddot{\theta}=-mR\dot{\theta}^2\mu_c$$
$$\Leftrightarrow \dot{\theta}\frac{d\dot{\theta}}{d\theta}=-\dot{\theta}^2\mu_c \quad / \int d\theta$$
$$\Rightarrow \int_{\dot{\theta}_0}^{\dot{\theta}}\frac{d\dot{\theta}}{\dot{\theta}}=-\mu_c\int_0^\theta d\theta$$
$$\Leftrightarrow \ln\left(\frac{\dot{\theta}}{\dot{\theta}_0}\right)=-\mu_c\theta$$

donde se tiene $v_0=R\dot{\theta}_0$

$$\Rightarrow \ln\left(\frac{R\dot{\theta}}{v_0}\right)=-\mu_c\theta$$
$$\Leftrightarrow \dot{\theta}(\theta)=\frac{v_0}{R}e^{-\mu_c\theta} \quad (4)$$
$$\Leftrightarrow \frac{d\theta}{dt}=\frac{v_0}{R}e^{-\mu_c\theta} \quad / \int dt$$
$$\Rightarrow \int_0^\theta e^{\mu_c\theta}d\theta=\frac{v_0}{R}\int_0^t dt$$
$$\Leftrightarrow \frac{e^{\mu_c\theta}}{\mu_c}\Big|_0^\theta=\frac{v_0 t}{R}$$
$$\Leftrightarrow e^{\mu_c\theta}-1=\frac{\mu_c v_0 t}{R}$$
$$\Rightarrow \theta(t)=\frac{1}{\mu_c}\ln\left(\frac{\mu_c v_0 t}{R}+1\right) \quad (5)$$

La partícula se detiene si $\dot{\theta}=0$, de (4) vemos que esto ocurre cuando $\theta\to\infty$ y de (5) sabemos que esto se tiene para $t\to\infty$

b) La partícula da la vuelta cuando $\theta=2\pi$, reemplazando en (4) tenemos

$$\dot{\theta}(\theta=2\pi)=\frac{v_0}{R}e^{-2\pi\mu_c} \Rightarrow v(\theta=2\pi)=v_0e^{-2\pi\mu_c}$$

c) Imponemos $\theta=2\pi$ en (5)

$$\Rightarrow 2\pi=\frac{1}{\mu_c}\ln\left(\frac{\mu_c v_0 t_1}{R}+1\right) \Rightarrow t_1=\frac{R}{\mu_c v_0}\left(e^{2\pi\mu_c}-1\right)$$

d) Ahora (2) pasaría a ser $mR\ddot{\theta} = -N_0\mu_c = -mg\mu_c$

$$\Leftrightarrow \dot{\theta}\frac{d\dot{\theta}}{d\theta} = -\frac{g}{R}\mu_c \quad / \int d\theta$$

$$\Rightarrow \int_{\omega_0/R}^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = -\frac{g}{R}\mu_c \int_0^{\theta} d\theta$$

$$\Leftrightarrow \dot{\theta}^2 - \frac{v_0^2}{R^2} = -\frac{2g}{R}\mu_c\,\theta$$

$$\Rightarrow \dot{\theta}(\theta) = \sqrt{-\frac{2g\mu_c}{R}\theta + \frac{v_0^2}{R^2}} \quad (6)$$

$$\Leftrightarrow \left(-\frac{2g\mu_c}{R}\theta + \frac{v_0^2}{R^2}\right)^{-1/2} d\theta = dt \quad / \int$$

$$\Rightarrow \int_0^{\theta}\left(-\frac{2g\mu_c}{R}\theta + \frac{v_0^2}{R^2}\right)^{-1/2} d\theta = \int_0^t dt$$

$$\Leftrightarrow -\frac{R}{g\mu_c}\sqrt{-\frac{2g\mu_c}{R}\theta + \frac{v_0^2}{R^2}}\Bigg|_0^{\theta} = t$$

$$\Leftrightarrow \sqrt{-\frac{2g\mu_c}{R}\theta + \frac{v_0^2}{R^2}} - \frac{v_0}{R} = -\frac{g\mu_c}{R}t$$

$$\Leftrightarrow \theta(t) = -\frac{R}{2g\mu_c}\left[\left(-\frac{g\mu_c}{R}t + \frac{v_0}{R}\right)^2 - \frac{v_0^2}{R^2}\right] = -\frac{R}{2g\mu_c}\left[\left(\frac{g\mu_c}{R}\right)^2 t^2 - \frac{2g\mu_c v_0}{R^2}t\right]$$

$$= -\frac{g\mu_c}{2R}t^2 + \frac{v_0}{R}t \quad (7)$$

También podemos integrar la ec de mov. c/r al tiempo

$$\frac{d\dot{\theta}}{dt} = -\frac{g}{R}\mu_c \quad / \int dt$$

$$\Rightarrow \int_{\omega_0/R}^{\dot{\theta}} d\dot{\theta} = -\frac{g}{R}\mu_c \int_0^t dt$$

$$\Leftrightarrow \dot{\theta}(t) = -\frac{g}{R}\mu_c t + \frac{v_0}{R}$$

Imponiendo que pare en $t_2 \Rightarrow 0 \overset{!}{=} -\frac{g\mu_c}{R}t_2 + \frac{v_0}{R} \Leftrightarrow t_2 = \frac{v_0}{g\mu_c}$

e) Usamos (6) con $\theta = 2\pi \Rightarrow \dot{\theta}(\theta = 2\pi) = \sqrt{\frac{v_0^2}{R^2} - \frac{4\pi g\mu_c}{R}}$

f) Usamos (7) con $\theta = 2\pi \Rightarrow 2\pi \overset{!}{=} -\frac{g\mu_c}{2R}t_3^2 + \frac{v_0}{R}t_3$

$$\Leftrightarrow at_3^2 + bt_3 + c = 0, \text{ con } a = \frac{g\mu_c}{4\pi R},\ b = -\frac{v_0}{2\pi R} \text{ y } c = 1$$

$$\Rightarrow t_{3,1,2} = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{v_0 \pm \sqrt{v_0^2 - 4\pi g\mu_c R}}{g\mu_c}$$

donde consideramos la primera solución ($\sqrt{v_0^2 - 4\pi g\mu_c R} < v_0$)

$$\Rightarrow t_3 = \frac{v_0 - \sqrt{v_0^2 - 4\pi g\mu_c R}}{g\mu_c}$$