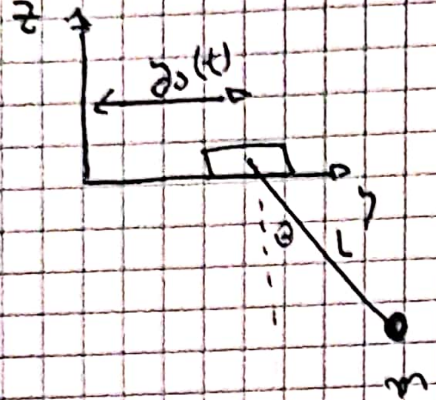
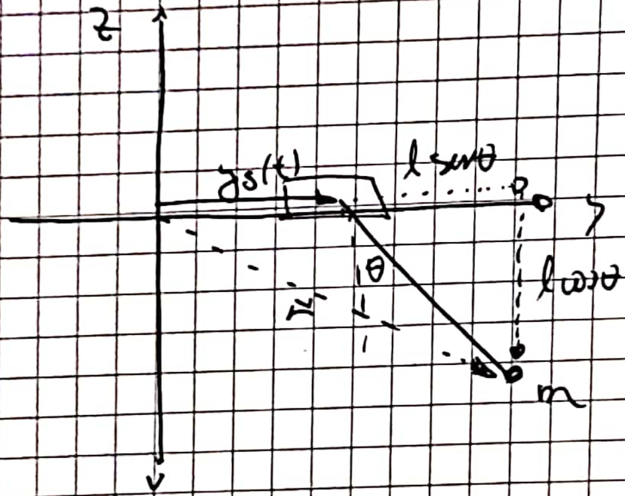


Posición masa:



Posición = $\vec{r} = \underbrace{\left(y_s(t) + l\,\text{sen}\,\theta\right)}_{y(t)}\hat{y}\ \underbrace{-\ l\cos\theta}_{z(t)}\hat{z}$

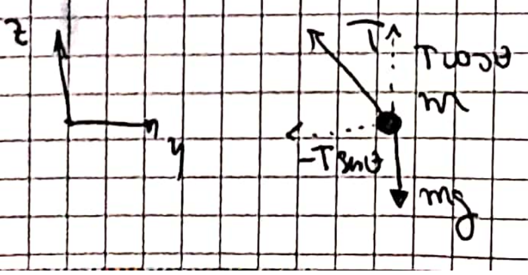
Ahora debemos encontrar ecuación de mov para θ, es decir

$$\ddot{\theta} = \text{algo}$$

→ Hacemos DCL y luego 2da ley de Newton

$(\ddot{y} = a_y)$ aceleración

$(\ddot{z} = a_z)$



$$\boxed{\begin{aligned} m\ddot{y} &= -T\,\text{sen}\,\theta \\ m\ddot{z} &= T\cos\theta - mg \end{aligned}}$$

→ Tenemos

$$m\ddot{y} = -T\sin\theta$$

$$m\ddot{z} = T\cos\theta - mg$$

y además $\vec{r} = \underbrace{(y_s(t) + l\sin\theta)}_{\text{es } y}\,\hat{y} + \underbrace{-l\cos\theta}_{\text{es } z}\,\hat{z}$

→ $\dot{y} = \dfrac{d}{dt}(y_s(t) + l\sin\theta)$ $\qquad \dot{z} = \dfrac{d}{dt}(-l\cos\theta)$

$\dot{y} = \dot{y}_s(t) + l\cos\theta\,\dot{\theta}$ $\qquad \dot{z} = l\sin\theta\,\dot{\theta}$

→ $\ddot{y} = \ddot{y}_s(t) + (-l\sin\theta\,\dot{\theta}^2) + l\cos\theta\,\ddot{\theta}$ $\qquad \ddot{z} = l\cos\theta\,\dot{\theta}^2 + l\sin\theta\,\ddot{\theta}$

Con esto las ecuaciones quedan:

$$m(\ddot{y}_s(t) + l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}^2) = -T\sin\theta \qquad (1)$$

$$m(l\cos\theta\,\dot{\theta}^2 + l\sin\theta\,\ddot{\theta}) = T\cos\theta - mg \qquad (2)$$

queremos $\ddot{\theta}$ = algo , entonces nos esto molestaría el $\dot{\theta}^2$

→ despejamos de ② $\dot{\theta}^2$

$$\dot{\theta}^2 = \frac{-g + \frac{T}{m}\cos\theta - l\,\text{sen}\theta\,\ddot{\theta}}{l\cos\theta}$$

y usando esto en ①

$$m\left(\ddot{y}_s + l\cos\theta\,\ddot{\theta} - l\,\text{sen}\theta\left(\frac{-g + \frac{T}{m}\cos\theta - l\,\text{sen}\theta\,\ddot{\theta}}{l\cos\theta}\right)\right) = -T\,\text{sen}\theta$$

$$m\left(\ddot{y}_s + l\cos\theta\,\ddot{\theta} + g\frac{\text{sen}\theta}{\cos\theta} - \cancel{\frac{T}{m}\text{sen}\theta} + l\frac{\text{sen}^2\theta\,\ddot{\theta}}{\cos\theta}\right) = \cancel{-T\,\text{sen}\theta}$$

$$m\left(\ddot{y}_s + l\left(\cos\theta + \frac{\text{sen}^2\theta}{\cos\theta}\right)\ddot{\theta} + g\frac{\text{sen}\theta}{\cos\theta}\right) = 0 \quad /\cdot\cos\theta$$

$$\ddot{y}_s\cos\theta + l\,\ddot{\theta} + g\,\text{sen}\theta = 0$$

$$\ddot{\theta} + \frac{g}{l}\text{sen}\theta = -\ddot{y}_s\frac{\cos\theta}{l}$$

Aproximamos a pequeñas oscilaciones: $\operatorname{sen}\theta = \theta$

$\cos\theta = 1$

$$\longrightarrow \boxed{\ddot{\theta} = -\frac{g}{l}\theta + \frac{\ddot{y}_s}{l}}$$

Por otra parte $y_s = y_0 \cos(\omega t)$

$$\longrightarrow \dot{y}_s = -y_0\omega \operatorname{sen}(\omega t)$$

$$\ddot{y}_s = -y_0\omega^2 \cos(\omega t)$$

$$\longrightarrow \ddot{\theta} = \underbrace{-g/l}_{-\omega_0^2}\,\theta + \frac{y_0\omega^2}{l}\cos(\omega t)$$

$$\boxed{\ddot{\theta} = -\omega_0^2\theta + \frac{y_0\omega^2}{l}\cos(\omega t)} \quad \star$$

La solución para este tipo de problemas es

$$\boxed{\theta(t) = A\cos(\omega_0 t + \phi) + A'\cos(\omega t + \phi')}$$

$$\longrightarrow \boxed{\ddot{\theta} = -A\omega_0^2\cos(\omega_0 t + \phi) - A'\omega^2\cos(\omega t + \phi')}$$

Usando esto en $\star$

$$-A\omega_0^2\cos(\omega_0 t+\phi)-A'\omega^2\cos(\omega t+\phi')=-\omega_0^2\left(A\cos(\omega_0 t+\phi)+A'\cos(\omega t+\phi')\right)+\frac{y_0\omega^2}{l}\cos(\omega t)$$

$$\cancel{-A\omega_0^2\cos(\omega_0 t+\phi)}-A'\omega^2\cos(\omega t+\phi')=\cancel{-A\omega_0^2\cos(\omega_0 t+\phi)}-A'\omega_0^2\cos(\omega_0 t+\phi')+\frac{y_0\omega^2}{l}\cos(\omega t)$$

$$A'(\omega_0^2-\omega^2)\cos(\omega t+\phi')=\frac{y_0\omega^2}{l}\cos(\omega t)$$

$$\boxed{A'\cos(\omega t+\phi')=\frac{\frac{y_0\omega^2}{l}}{\omega_0^2-\omega^2}\cos(\omega t)}$$

$$\boxed{\phi'=0,\ \pi}$$

$$\boxed{A'=\frac{y_0\omega^2/l}{\omega_0^2-\omega^2}}$$

Con esto la solución es finalmente:

hay resonancia cuando $\omega^2\sim g/l$

$$\theta(t)=A\cos(\omega_0 t+\phi)+A'\cos(\omega t+\phi')$$

$$\boxed{\theta(t)=A\cos(\omega_0 t+\phi)+\frac{\frac{y_0\omega^2}{l}}{\omega_0^2-\omega^2}\cos(\omega t)}$$

Luego en un tiempo largo, solo tomamos la 2da solución

$t\to\infty$ $\boxed{\theta(t)=A'\cos(\omega t)}$