Chapter 6.2
MINE FEASIBILITY STUDIES*

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Feasibility studies are the heart of the mine evaluation process. A *feasibility study* of a mining project represents an engineering/economic appraisal of the commercial viability of that project. As such, it is the result of a relatively formal procedure for assessing the various relationships that exist among the myriad of factors that directly or indirectly affect the project in question. In essence, the objective of a feasibility study is to clarify the basic factors that govern the chances for project success. Once all the factors relative to the project have been defined and studied, an attempt is made to quantify as many variables as possible in order to arrive at a potential value or worth of the property.

As a mining project progresses from raw exploration through to the time when a management decision is made to develop and mine the property, a number of analyses will be conducted on the property, each of which will be based on increasing amounts of data, will require increasing amounts of time (and therefore expense) to prepare, and will have increasing degrees of accuracy. For example, as exploration occurs on a mining property, the intersection of mineralization by a few drillholes typically triggers the need for some type of initial analysis to assist with necessary decision making. These types of studies are identified by various names (Gocht et al., 1988; Taylor, 1977), but in each case they are designed to answer questions pertaining to (1) what magnitude of deposit might exist rather than what is known to exist, (2) should further expenditures be incurred to look for what might exist, (3) should the project be abandoned, or (4) what additional effort and/or expense is necessary before making any of these decisions.

Assuming a favorable decision for continuation of the project, the next sequence of decisions must be predicated on studies utilizing much more detailed information. These so-called prefeasibility studies or intermediate economic studies are based on increasing amounts of data pertaining to geologic information, preliminary engineering designs and plans for mining and processing facilities, and initial estimates of project revenues and costs. They are constructed to support a continuum of decisions relating to the next major spending requirement. Intermediate economic studies of this type typically contain the following information and analysis (Gocht et al., 1988):

1. *Project Description:* geographic area, existing access routes, topography, climate, project history, concessionary terms, schedule for development of mine and any processing facilities.

2. Geology: regional geology, detailed description of the project area, preliminary reserve calculations, plans for detailed target evaluation.

3. *Mining:* geometry of the ore body, proposed mining plan (and alternatives), required plant and equipment.

4. *Processing:* technical descriptions of the ore and concentrate, processing facilities.

5. *Other Operating Needs:* availability of energy, water, spare parts, and equipment (diesel oil, explosives, replacement parts, etc.).

6. *Transportation:* description of the additional, necessary transportation facilities (roads, air strips, bridges, harbors, rail lines).

7. *Towns and Related Facilities:* housing for workers, schools for children of workers, medical facilities, company offices.

8. *Labor Requirements:* estimates of work force broken down according to qualifications (skills) and local availability.

9. *Environmental Protection:* plans to reduce or minimize environmental damage, description of relevant environmental legislation.

10. *Legal Considerations:* review of mining laws, taxation, foreign-investment regulations, political risk.

11. *Economic Analysis:* cost estimates for plant and equipment, infrastructure, materials, labor, other factors; market analysis, including production, consumption, and price formation for the relevant minerals; revenue forecasts based on expected production and mineral prices; cash flow and net present value analysis; sensitivity analysis.

Assuming the project continues to appear favorable throughout the intermediate economic studies, as these studies progressively focus more on engineering and economic aspects and less on geologic parameters, the project must be formally assessed through a comprehensive feasibility study. The feasibility study represents a detailed analysis of all the parameters contained in the intermediate economic studies, along with other pertinent factors relating to political and legal aspects affecting project viability. Specific data requirements for incorporation into feasibility studies are contained in the following segment of this chapter; however, in general, the study contains analyses of the project's geology and deposit characteristics, mineralogy, mineral processing characteristics, designs and plans for mining and processing equipment requirements, construction schedules, investment requirements and timing, estimates of revenues and costs, marketing plans, cash flow calculations, sources and methods of financing, and risk and sensitivity analyses of important project variables. As stated previously, the purpose of the feasibility study is to assess the technical and economic viability of the project and to assist the organization in making the "go/no-go" decision regarding project development.

Although there is no prescribed format for reporting the results of a project feasibility study, the final report must fulfill the following essential functions (Taylor, 1977):

1. Provide a comprehensive framework of established and detailed facts concerning the mineral project.

2. Present an appropriate scheme of exploitation complete with plans, designs, equipment lists, etc., in sufficient detail for accurate cost estimation and associated economic results.

3. Indicate the most likely profitability on investment in the project, assuming the project is equipped and operated as specified in the report.

4. Provide an assessment of pertinent legal factors, financing alternatives, fiscal regimes, environmental regulations, and risk and sensitivity analyses on important technical, economic, political, and financial variables affecting the project.

5. Present all information in a manner intelligible to the owner and suitable for presentation to prospective partners or to sources of finance. The document must be "bankable."

* This chapter draws heavily from various chapters in *Mine Investment Analysis* by D.W. Gentry and T. J. O'Neil (1984).

6.2.1 DATA REQUIREMENTS

Nothing improves the results of a project feasibility study more than good input data. Unfortunately, those preparing feasibility studies for mining projects never possess all the information they would like. In addition to inadequacy or unavailability of some needed data, care must be taken not to overlook any variable that may influence project viability. In this regard, it is often helpful to compile an outline of factors to be considered when preparing feasibility studies on mining properties.

6.2.1.1 Factors for Consideration

Table 6.2.1. is an outline of some of the pertinent variables that must be studied, considered, and analyzed when evaluating mining properties. The significance of each variable will be a function of the specific property being investigated and the mineral commodity (metallic, nonmetallic, fuel) involved. Nonetheless, all these variables should be assessed during preparation of the final feasibility study.

A review of Table 6.2.1. suggests there are some fundamental issues that are applicable to all mining property feasibility studies—regardless of the commodity involved. For instance, one of the first tasks associated with any mining property is estimating the magnitude and quality of the ore reserves (Chapter 5.6). Ore is, of course, an economic term and is a function of commodity prices, production costs, mining method, recoveries, dilution, and a number of other variables. Because ore reserves are determined by ever-changing economic conditions, the exact amount of ore contained in a deposit cannot be precisely determined until production ceases. This uncertainty with respect to ore reserves has a significant impact on the evaluation of mineral properties where long-term contracts at stipulated selling prices are not available.

Production technology is another key area of concern when performing mining feasibility studies. Technological advancements in equipment, mineral processing, and other areas can significantly impact projected operating and capital costs. A good example of the impact of technology changes on mining costs is the comparison between direct mining costs per ton (tonne) of rock for underground and surface operations. While underground operating costs have been increasing at significant rates in recent years (mainly because of the lack of major technological advancements and the labor intensity that exists), unit operating costs at surface operations have changed far less. The technological advancements in mining equipment, yielding productivity increases, have prevented direct operating costs in surface operations from escalating at the rates experienced by underground producers. The relative stability in surface operating costs has not been entirely free, however. The technological advancements in mining equipment that contributed to this stability in operating costs carried with them significant increases in capital costs.

Operating and capital cost requirements must be determined separately when formulating the feasibility study. However, in the limiting case, it is the combination of the magnitude and timing of both these costs that ultimately influence the analysis. Any changes in future production technologies must be carefully analyzed and the impact assessed on overall operating and capital cost requirements. The estimation of capital and operating costs for mining projects that have not progressed to the detailed planning and layout stage is discussed in Chapter 6.3.

Another area of fundamental interest in feasibility studies of mining properties that requires considerable data generation and analysis is the estimation of project revenues. The timing and magnitude of mining revenues depends upon factors such as ore reserves, production rates, commodity prices, markets, and metallurgical recoveries. These variables are often extremely difficult to estimate or predict—particularly for commodities traded in international markets. This topic is discussed in more detail in the following portion of this chapter.

The overall operating environment is another area of major concern. In recent years, the national and, to a lesser but growing extent, the international operating environment of mining properties has been impacted significantly by environmental and other regulatory requirements. These constraints have invariably increased operating and capital cost requirements for the industry and have reduced or delayed the production of mineral commodities. The operating environment of mining operations is also affected by direct economic variables, such as royalties and taxes mandated by federal, state, and local taxing authorities. All these costs, whether direct or indirect, impact profit margins, ore reserves, mineral conservation, and ultimately project viability.

6.2.1.2 Variable Quantification

As stated previously, the objective of any feasibility study is to assess, clarify, and ultimately quantify the basic factors that govern the chances for project success. Once all the geologic, engineering, and other technology-related factors relative to the property have been defined and studied, an attempt is made to quantify as many of the variables as possible to arrive at a potential value or worth of the property. In this regard, two general categories of quantification are extremely important yet quite troublesome to compute: revenue estimation and cost estimation.

Revenue Estimation: Annual mine revenue is calculated by multiplying the number of units produced and sold throughout the year by the sales price received per unit. While the arithmetic associated with calculating annual mine revenue is trivial, determining the best value to use for each of these two critical variables is much more difficult.

There are a number of important considerations in estimating the number of units produced and sold annually. For example, estimates must be made of the tonnage of ore produced, the grade of ore mined (including dilution), the percentage recovery of the valuable mineral in the ore, and, finally, the number of payable units available for sale.

The second major component of the mine revenue calculation is the unit sales price. Estimating future mineral prices—particularly prices far enough into the future to be of use in mine investment analysis—is much more difficult than estimating the production-related variables and is an exercise in which a high error of estimation invariably exists. As pointed out in Section 2.0, mineral prices, like those of any other product, are ultimately determined by supply and demand. However, there are major complications on both sides of the supply-demand equation that seriously impair the value of quantitative econometric modeling for estimating mineral prices.

Clearly, the analysis of supply and demand for most mineral commodities is complicated. In fact, few analysts are willing to suggest that reliable forecasts of prices useful in mine investment analysis are possible. The current popular approach to this problem is for analysts to occupy safer ground and issue price *projections*—that is, prices that are likely if certain assumed events actually occur.

Regardless of the difficulties associated with forecasting or projecting mineral prices into the future, estimates of mining project annual revenue must be established. Generally, fungible

Table 6.2.1. Salient Factors Requiring Consideration in a Mining Project Feasibility Study

I. Information on Deposit
- A. Geology
  1. Mineralization: type, grade, uniformity
  2. Geologic structure
  3. Rock types: physical properties
  4. Extent of leached or oxidized zones
  5. Possible genesis
- B. Geometry
  1. Size, shape, and attitude
  2. Continuity
  3. Depth
- C. Geography
  1. Location: proximity to population centers, supply depots, services
  2. Topography
  3. Access
  4. Climatic conditions
  5. Surface conditions: vegetation, stream diversion
  6. Political boundaries
- D. Exploration
  1. Historical: district, property
  2. Current program
  3. Reserves
     - a. Tonnage-grade curve for deposit, distribution classification; computation of complete mineral inventory (geological and mining reserves) segregated by ore body, ore type, elevation and grade categories
     - b. Derivation of dilution and mining recovery estimates for mining reserves.
  4. Sampling: types, procedures, spacing
  5. Assaying: procedures, check assaying
  6. Proposed program

II. Information on General Project Economics
- A. Markets
  1. Marketable form of product: concentrates, direct shipping ore, specifications, regulations, restrictions
  2. Market location and alternatives: likely purchasers, direct purchase vs. toll treatment
  3. Expected price levels and trends: supply-demand, competitive cost levels, new source of product substitutions, tariffs
  4. Sales characteristics: further treatment, sales terms, letters of intent, contract duration, provisions for amendments and cost escalations, procedures/requirements for sampling, assaying, and umpiring.
- B. Transportation
  1. Property access
  2. Product transportation: methods, distance, costs
- C. Utilities
  1. Electric power: availability, location, ownership right-of-way, costs
  2. Natural gas: availability, location, costs
  3. Alternatives: on-site generation
- D. Land, Water, and Mineral Rights
  1. Ownership: surface, mineral, water, acquisition or securement by option or otherwise, costs
  2. Acreage requirements: concentrator site, waste dump location, tailings pond location, shops, offices, changehouses, laboratories, sundry buildings, etc.
- E. Water
  1. Potable and process: sources, quantity, quality, availability, costs
  2. Mine water: quantity, quality, depth and service, drainage method, treatment
- F. Labor
  1. Availability and type: skilled/unskilled in mining
  2. Rates and trends
  3. Degree of organization: structure and strength
  4. Local/district labor history
  5. Housing and transport of employees
- G. Government Considerations
  1. Taxation: federal, state, local
     - a. Organization of the enterprise
     - b. Tax authorities and regimes
     - c. Special concessions, negotiating procedures, duration
     - d. Division of distributable profits
  2. Reclamation and operating requirements and trends: pollution, construction, operating and related permits, reporting requirements
  3. Zoning
  4. Proposed and pending mining legislation
  5. Legal issues: employment laws, licenses and permits, currency exchange, expatriation of profits, agreements among partners, type of operating entity for tax and other purposes.
- H. Financing
  1. Alternatives: sources, magnitudes, issues of ownership
  2. Obligations: repayment of debt, interest
  3. Type of operating entity: organizational structure
  4. Division of profits: legal considerations

III. Mining Method Selection
- A. Physical Controls
  1. Strength: ore, waste, relative
  2. Uniformity: mineralization, blending requirements
  3. Continuity: mineralization
  4. Geology: structure
  5. Surface disturbance: subsidence
  6. Geometry
- B. Selectivity
  1. Dilution, ore recovery estimates
  2. Waste mining and disposal
- C. Preproduction Requirements
  1. Preproduction development or mining requirements: quantity, methods, time
  2. Layout and plans: schedule
  3. Capital requirements
- D. Production Requirements
  1. Relative production
  2. Continuing development: methods, quantity, time requirements
  3. Labor and equipment requirements
  4. Capital requirements vs. availability

IV. Processing Methods
- A. Mineralogy
  1. Properties of ore: metallurgical, chemical, physical
  2. Ore hardness
- B. Alternative Processes
  1. Type and stages of extraction process
  2. Degree of processing: nature and quality of products
  3. Establish flowsheet: calculation of quantities flowing, specification of recovery and product grade
  4. Production schedule
- C. Production Quality vs. Specifications
- D. Recoveries and Product Quality
  1. Estimate effects of variations in ore type or head grade
- E. Plant Layout
  1. Capital requirements
  2. Space requirements
  3. Proximity to deposit

V. Capital and Operating Cost Estimates
- A. Capital Costs
  1. Exploration
  2. Preproduction development (may also be considered operating costs)
     - a. Site preparation
     - b. Development of deposit for extraction
  3. Working capital
     - a. Spares and supplies (inventory)
     - b. Initial operations
     - c. Financing costs (when appropriate)

4. Mining
   a. Site preparation
   b. Mine buildings
   c. Mine equipment: freight, taxes and erection costs, replacement schedule
   d. Engineering and contingency fees
5. Mill
   a. Site preparation
   b. Mill buildings
   c. Mill equipment: freight, taxes and erection costs, replacement schedules
   d. Tailings pond
   e. Engineering and contingency fees

B. Operating Costs
1. Mining
   a. Labor: pay rates plus fringes
   b. Maintenance and supplies: quantities, unit
   c. Development
2. Milling
   a. Labor: pay rates plus fringes
   b. Maintenance and supplies: quantities, unit costs
3. Administrative and supervisory
   a. Overhead charges
   b. Irrecoverable social costs

Source: Gentry and Hrebar, 1978; Taylor, 1977.

commodities—such as most metals traded on exchanges—suffer from the greatest future price uncertainty. Most metal markets are notoriously cyclical, and the amplitude and the period of the cycles defy accurate prediction. The recommended approach to mineral price forecasting is not limited to the application of any one or two specific analytical techniques, and it definitely is not a mechanical process. Rather, it is a painstaking blend of economic theory, industry analysis, market analysis, and competitor analysis combined with sound, experienced judgment.

Cost Estimation: The economic evaluation portion of a feasibility study ultimately must be based on information that provides an answer to the question, "what is it going to cost?" Unfortunately, the answer to this question is not simple, primarily because of the significant misunderstandings associated with cost data. Therefore, the components of so-called "total production cost" or "total operating cost," for example, must be carefully identified and defined.

When preparing a mine feasibility study, it is essential also to distinguish between operating costs, expenses, and capital costs. *Operating costs* are considered to be all expenses incurred at the plant site, whereas *general expenses* are off-site management or corporate-level expenditures. This latter classification of expenses may be directly related to mine or plant size, or it may contain indirect items incurred by headquarters and allocated across all production divisions in accordance with some corporate allocation scenario.

Direct costs, or variable costs, relate to items such as labor, materials, energy, and supplies that are consumed directly in the production process and are used roughly in direct proportion to the level of production. On the other hand, *indirect costs,* or fixed costs, are expenditures that are independent of the level of production—at least over certain ranges. It is obvious that, in the limit, few costs are absolutely fixed.

Capital costs (or first cost, or capital investment) are those expenditures made to acquire or develop capital assets, the benefits of which will be derived over several years. The largest portion of capital costs is incurred in the initial stages of project start-up, but some capital expenditures are incurred annually throughout the life of the mine.

In general, capital costs fall into one of three classes, depending on the treatment of the cost for tax purposes. These are depreciable investment, expensed or amortizable investment, and nondeductible investment. Because of differing tax treatments, the *type* of capital expenditure involved can be a very important factor in the evaluation of a new project, in addition to its magnitude.

The estimation of operating and capital costs for mining projects is extremely difficult and must be performed with great care. Chapter 6.3 illustrates procedures for estimating initial capital and operating costs for mining projects being analyzed via intermediate economic studies. However, the final feasibility must contain operating and capital cost estimates based on actual design and layout drawings, manning tables, flow charts, and equipment lists, specifications, and manufacturer quotations. These estimates should be predicated on data relative to unit operations, job functions, job requirements, timetables, and so on. This requires considerable time and effort; there are few, if any, acceptable shortcuts.

6.2.2 CASH FLOW ANALYSIS

The importance of the *pro forma* income statement in establishing the value of a mining project via the income (earnings) approach was alluded to in Chapter 6.1. Inasmuch as there are generally major differences between accounting profits and actual net cash benefits derived from an investment, investors are using almost exclusively the concept of project cash flows as the primary measure of real benefits produced by a capital project. This is predicated on the knowledge that cash flow analyses and accounting concepts depict investments differently, as a result of the timing of costs, and on the belief that the proper method for evaluating a capital investment is to compare the present investment outlay with the expected positive net cash flows that will accrue from the project in the future. In making this comparison, it is essential that the timing of the various cash flows be recognized by the use of an appropriate interest (discount) rate. This aspect of cash flow analysis is discussed briefly in the following portion of this chapter.

As indicated, cash flow analyses relate the expenditures associated with investment to the subsequent revenues or benefits generated from such investment. Cash flows are routinely calculated on an annual basis for evaluation purposes and are determined by subtracting the annual cash outflows from the annual cash inflows that result from the investment. Consequently, a cash flow analysis may be performed for any investment with which income and expenditure are associated. Also the annual cash flows resulting from an investment may be either positive or negative. Typically, the net annual cash flows for a new mining property will be negative during the preproduction years due to large capital expenditures. After production commences, the cash flows will usually be positive, and an inflow of cash results from investment in the project.

In the US income tax law, net annual cash flow is treated basically as a combination of two components: the return *on* the investment and the recoupment *of* the investment. In the minerals industry, net cash flow is generally defined as net profit after taxes plus depreciation and depletion minus capital expenditures and working capital. Within this definition, net profit after taxes represents the return on the investment, whereas depreciation and depletion represent the recoupment of the investment.

In a cash flow analysis, each investment receives credit for income taxes saved. Since the accounting allowances for depreci-

Table 6.2.2. Components and Basic Calculation Procedure for Developing Cash Flows

Calculation	Component
	Revenue
Less	Royalties
Equal	Gross income from mining
Less	Operating costs
Equal	Net operating income
Less	Depreciation and amortization allowance
Equal	Net income after depreciation and amortization
Less	Depletion allowance
Equal	Net taxable income
Less	State income tax
Equal	Net federal taxable income
Less	Federal income tax
Equal	Net profit after taxes
Add	Depreciation and amortization allowances
Add	Depletion allowance
Equal	Operating cash flow
Less	Working capital
Equal	Net annual cash flow

Source: Gentry and O'Neil, 1984.

Table 6.2.3. Parameters for Consideration in Cash Flow Analysis of a Mining Property

Preproduction Period	
Exploration expenses	Land and mineral rights
Water rights	Environmental costs
Mine and plant capital requirements	Development costs
Sunk costs	Financial structure
Working capital	Administration
Production Period	
Price	Capital investment—replacement of expansions
Processing costs	Royalty
Recovery	Mining cost
Postconcentrate cost	Development cost
Revenues and percent removable	Exploration cost
Grade	General and administration
Investment tax credit	Insurance
State taxes	Production rate in tons per year
Federal taxes	Financial year production begins
Depletion rate	Percent production not sent to processing plant
Depreciation	Operating days per year
Postproduction	
Salvage value	Contractual and reclamation expenditures

Source: Laing, 1977.

ation and depletion reduce the amount of taxable income (and therefore reduce the amount of taxes paid), they have the effect of saving the organization money. Consequently, they constitute a credit in the cash flow calculation and are added to net income after taxes. It must be recognized that depreciation and depletion are noncash items and do not actually "flow" anywhere.

Table 6.2.2. illustrates the components and basic calculation procedure for determining annual cash flows for a mining project. Table 6.2.3. lists some of the more important parameters relating to preproduction, production, and postproduction mining activities that require consideration in the preparation of cash flow analyses. The appropriate use and manipulation of these input variables are an extremely important facet of the cash flow analysis. The concept of a cash flow analysis is a particularly useful technique for the evaluation of mineral-related projects because of the important impact of the depletion allowance in the United States.

In view of the foregoing, it is worthwhile to reiterate the fact that in a cash flow analysis, each investment receives credit for income taxes saved. Therefore, for profitable organizations, it is advantageous to maximize pretax deductions and thereby reduce the amount of taxable income and, consequently, income taxes paid. In order to take advantage of these tax savings as soon as possible, the firm would opt to expense all possible expenditures in the year incurred as opposed to capitalizing them followed by subsequent write-offs over the amortization period. Although the total amount of the pretax deduction would be the same in either case, by expensing as soon as possible, the firm realizes an earlier return of the resulting tax savings. This early return of tax savings enables the firm to utilize these dollars sooner than would otherwise be possible.

6.2.3 TIME VALUE OF MONEY

If it were not for the existence of interest, the analysis of investment opportunities would be greatly simplified. In the absence of interest, investors would be indifferent as to when cash outlays are made or cash benefits received. It would, in fact, be irrelevant whether the outlays preceded or followed inflows, as long as both amounts are known with certainty.

Of course, it does make a considerable difference whether, for example, a firm receives $1 million now or five years from now. The reason is that money does have a value, which is a function of time. Interest is how this time value is measured.

Interest is generally defined as money paid for the use of borrowed money. Interest may be likened to a rental charge for using an asset over some specific time period. The rate of interest is the ratio of the interest chargeable at the end of a specific period of time to the money owed, or borrowed, at the beginning of that period. Interest exists to compensate for a number of concerns experienced by lenders; these are related primarily to risk, inflation, transaction costs, opportunity costs, and postponement of pleasures. The level of interest is, like the price of other assets, determined by supply and demand.

The history, philosophy, and theoretical underpinnings of interest are covered exhaustively in a large number of texts and reference books (e.g., Newman, 1980; Smith, 1973). This material need not be repeated here.

It is sufficient simply to recognize that money has earning power. That is, the timing of when payments are made and earnings are received in a capital project is very important.

6.2.3.1 Interest Formulas

Cash flows at different points in time are related by a series of six basic interest formulas. These formulas in turn are based on five variables, as follows:

> F is a future sum of money, P is a present sum of money, A is a payment in a series of n equal payments, made at the end of each period of interest, i is effective interest rate per period, and n is number of interest periods.

Every interest problem is composed of four of these variables: three are given, and the fourth must be determined. The standard notation for describing the particular type of problem involved lists all four variables of concern, in the following manner:

For example: *(F/P,i,n)*, means, "Find F, given P,i, and n"

Similarly, *(A/F,i,n)*, means, "Find A, given F,i, and n"

The notation is often shortened to simply *F/P* or *A/F*, etc., where it is understood that i and n are given. This notation is widely accepted in the field of engineering economy.

The six basic interest equations are developed and described.

1. Single payment compound amount, *(F/P,i,n)*.

$$F = P(1 + i)^n \tag{6.2.1}$$

Example 6.2.1. Find the amount that will accrue at the end of 7 years if \$1250 is invested now at 8%, compounded annually. Given: P = \$1250, n = 7 years, i = 8%.

Solution.

$$F = P(1 + i) = \$1250\ (1 + 0.08)^7 = \$2142.28$$

The quantity $(1 + i)^n$ = *(F/P,i,n)* has, like other interest factors, been tabulated for various values of i and n in so-called interest tables. Such tables are provided in the appendix of this volume for select interest rates and discrete compounding.

However, many hand-held calculators can now solve such problems directly so that interest tables are becoming less necessary.

2. Single payment, present worth *(P/F,i,n)*.

This is Eq. 6.2.1 solved for P.

$$P = F/(1 + i)^n \tag{6.2.2}$$

Example 6.2.2. If \$6500 will be needed in 5 years, how much should be invested now at an interest rate of 7.5%, compounded annually?

Solution. Given: F = \$6500, n = 5 years, i = 7.5%

$$P = F(1 + i)^n$$
$$P = \$6500/(1 + 0.075)^5 = \$4527.63$$

Using interest tables to solve this problem becomes somewhat more difficult because one must interpolate between 7 and 8%. For most interest problems, a linear interpolation is satisfactory. The widespread use of programmed calculators and computers for solving interest problems facilitates exact solutions so that interpolation is often unnecessary. However, the intellectual problem of calculating exact solutions from highly inexact data is of major concern in all investment studies.

3. Uniform series, compound amount *(F/A,i,n)*.

Here the concern is to determine the terminal amount when equal annual payments are made to an interest-bearing account for a specified number of years. It is important at this point to recall that A is defined as occurring at the *end* of the interest period. Therefore, for

$$n = 1,\ F = A$$
$$n = 2,\ F = A(1 + i) + A \tag{6.2.3}$$
$$n = 3,\ F = A\ (1 + i)^2 + A\ (1 + i) + A$$

or in general,

$$F = A\ (1 + i)^{n-1} + A\ (1 + i)^{n-2} + \dots + A\ (1 + i) + A$$
$$= A\ [(1 + i)^{n-1} + (1 + i)^{n-2} + \dots + (1 + i) + 1]$$

Multiplying Eq. 6.2.3 by $(1 + i)$ yields

$$F(1 + i) = A[(1 + i)^n + (1 + i)^{n-1} + \dots + (1 + i)^2 + (1 + i)] \tag{6.2.4}$$

Now subtracting Eq. 6.2.3 from 6.2.4,

$$iF = A[(1 + i)^n - 1]$$

or (6.2.5)

$$F = \frac{A[(1 + i)^n - 1]}{i}$$

Example 6.2.3. If payments of \$725 are made at the end of each year for 12 years to an account which pays interest at the rate of 9% per year, what will be the terminal amount?

Solution .

$$F = \frac{A\ [(1 + i)^n - 1]}{i}$$
$$= \frac{725[(1 + 0.09)^{12} - 1]}{0.09} = \$14{,}602.02$$

It obviously makes a considerable difference if the annual payments are made at the beginning of the year (an *annuity due)* rather than at the end of the year (an *immediate annuity)*. However, the end-of-year convention is more common, and nearly all interest tables and computer programs are constructed on this basis. To verify easily that any particular table or computer program adheres to this convention, note that for n = 1, $F = A$ regardless of the interest rate.

4. Uniform series, sinking fund, *(A/F,i,n)*.

Solving Eq. 6.2.5 for A will enable the analyst to determine what annual payments must be made to accumulate a specified amount by some future date at an interest rate i:

$$A = iF/[(1 + i)^n - 1] \tag{6.2.6}$$

Example 6.2.4. With interest at 6%, how much must be deposited at the end of each year to yield a final amount of \$2825 in 7 years?

Solution .

$$A = iF/[(1 + i)^n - 1]$$
$$A = 0.06\ (\$2825)/[(1 + 0.06)^7 - 1]$$
$$= \$336.58$$

The concept of a sinking fund is well known, so that the *A/F* interest factor is often called the *sinking fund factor*.

5. Uniform series, present worth *(P/A,i,n)*.

This type of problem arises when the current value of a future series of cash flows is desired. This is often the case with investments in securities where an expenditure now will provide equal interest or dividend payments for several future periods.

For Eq. 6.2.5,

$$F = \frac{A[(1+i)^n - 1]}{i}$$

Substitute Eq. 6.2.1,

$$F = P(1+i)^n$$

$$P(1+i)^n = \frac{A[(1+i)^n - 1]}{i}$$

and

$$P = \frac{A[(1+i)^n - 1]}{i(1+i)^n} \qquad (6.2.7)$$

Example 6.2.5. An investment will yield $610 at the end of each year for 15 years. If interest is 10%, what is the maximum purchase price (i.e., present value) for this investment?

Solution.

$$P = \frac{A[(1+i)^n - 1]}{i(1+i)^n}$$

$$= \frac{610[(1+0.1)^{15} - 1]}{0.1(1+0.1)^{15}}$$

$$= \$4639.71$$

6. Uniform series, capital recovery, *(A/P,i,n)*

This is the reverse of the previous problem, and the equation is derived simply by solving Eq. 6.2.7 for *A:*

$$A = \frac{iP(1+i)^n}{[(1+i)^n - 1]}$$

Example 6.2.6. If an investment opportunity is offered now for $3500, how much must it yield at the end of every year for 6 years to justify the investment if interest is 12%?

Solution.

$$A = \frac{iP(1+i)^n}{[(1+i)^n - 1]}$$

$$= \frac{0.12\ (\$3500)\ (1+0.12)^6}{[(1+0.12)^6 - 1]}$$

$$= \$851.67$$

The preceding formulas pertain to annual compounding and, where *A* enters the problem, to annual cash flows. In practice, these constraints are routinely violated as neither payments/receipts nor compounding need be on an annual basis. In fact, there is a continuous spectrum of possibilities with discrete annual compounding and discrete cash flows at one end of the continuum, and continuous compounding and continuous flow of funds at the other end. The reader interested in learning more about the procedures for handling specific cases within this spectrum is referred to Chapter 3 in Gentry and O'Neil (1984).

6.2.3.2 Interest Factor Relationships

The relationships between the interest factors identified in the preceding formulas are often not as clear as they might be—particularly when values are extracted from interest tables. Following are some relationships based on given values of *i* and *n*.

1. The single payment compound amount factor *(F/P,i,n)* and the single payment present value factor *(P/F,i,n)* are reciprocals:

$$\frac{1}{(P/F,i,n)} = (F/P,i,n)$$

2. The sinking fund factor *(A/F,i,n)* and the compound amount factor for an annuity *(F/A,i,n)* are reciprocals:

$$\frac{1}{(A/F,i,n)} = (F/A,i,n)$$

3. The capital recovery factor *(A/P,i,n)* and the present value factor for an annuity *(P/A ,i,n)* are reciprocals:

$$\frac{1}{(A/P,i,n)} = (P/A,i,n)$$

4. The capital recovery factor *(A/P,i,n)* is equal to the sinking fund factor *(A/F,i,n)* plus interest rate:

$$(A/P,i,n) = (A/F,i,n) + i$$

5. The present value factor for an annuity *(P/A,i,n)* is equal to the sum of the first n terms of single payment value factors *(P/F,i,n):*

$$(P/A,i,n) = (P/F,i,1) + (P/F,i,2) + \ldots + (P/F,i,n)$$

6. The compound amount factor for an annuity *(F/A,i,n)* is equal to 1.00 plus the sum of the first (n-l) terms of the single-payment compound amount factor *(F/P,i,n):*

$$(F/A,i,n) = 1.00 + (F/P,i,1) + (F/P,i,2) + \ldots + (F/P,i,n\text{-}1)$$

6.2.4. SELECTING A DISCOUNT RATE

As indicated in the previous section of this chapter, future project receipts and expenditures must be discounted to permit valid comparisons with current cash flows. Although the concept of discounting is widely accepted, selection of an appropriate discount rate has been the source of considerable debate and much disagreement.

The fact that interest exists suggests that all money has a cost associated with its use. The cost of this money may be the result of either explicit or implicit charges or some combination of the two. Indeed, any time an individual or a firm consumes less than the total earnings generated in a specified time period, that individual or corporation has either consciously or unconsciously decided to "invest" these excess funds in some type of activity—even if the funds are maintained in a cash account.

Intuitively, it seems reasonable to accept the fact that no money is free. Certainly, there is a cost associated with raising investment capital. For instance, there is a cost associated with going to the debt markets and borrowing money for investment purposes. This explicit cost of borrowing results from the fact that interest exists. Similarly, it is important to recognize that an investor who purchases the firm's common stock (equity) has the same expectations of making a return on his investment as a banker who loans money. Additionally, it is important to

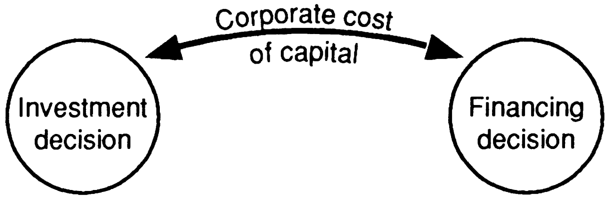

Fig. 6.2.1. Relationship between investment decision and financing decision.

recognize that a firm may allocate investment capital to alternative investment opportunities that may be available and that promise specific rates of return on the investment capital. This concept of alternative investment opportunities suggests that there also are *opportunity costs* (implicit costs) associated with investment capital. Clearly, then, there are explicit and implicit costs associated with procuring and utilizing investment capital, and, at least to some extent, these costs are influenced by inflationary trends and expectations.

Remembering that the firm's primary objective should be the maximization of shareholder wealth, it then intuitively seems reasonable to suggest that the firm should not invest in any project where the anticipated return does not exceed the cost of funds (capital) committed to the project. Indeed, if the firm always invests in projects having returns in excess of the cost of capital committed to them, then the wealth of the firm (as measured by the price of common stock) should be increased to the stockholders. This cost of funds, known as the *cost of capital,* is the direct linkage between investment policy and the firm's objective.

Further, the cost of capital also relates the financing decision to the investment decision. Indeed, the cost of capital is the *only* link between these two decisions. The relationship between the financing decision and the investment decision in corporate finance via the cost of capital is illustrated in Fig. 6.2.1.

An alternative way to view the cost of capital is to recognize that the *minimum acceptable rate of return* (MARR) on an investment may be defined as the minimum rate of return a firm must earn on its investments in order to leave unchanged the value of existing shares of its common stock. Investing at the cost of capital will achieve this objective, and therefore the MARR is quantitatively equal to the cost of capital. The logical conclusion from this observation is that this cost of capital represents the "hurdle rate" or the appropriate discount rate to be used in conjunction with discounted cash flow analyses of investment opportunities.

6.2.4.1 Components of Discount Rate

Following are four major components to the discount rate. Some of these components stem from sound theoretical and practical foundations, while others are judgmental. Rarely is the combined discount rate calculated by summing these separate components, as the capital markets do a good job of determining the overall cost of funds to various uses. Nonetheless, the combined rate must be sufficient to cover these four elements.

Base Opportunity Cost: There is always some base opportunity cost associated with procuring and utilizing investment capital. This cost is the return foregone by diverting funds from the next most attractive project and exists whether the funds are obtained externally or internally. In either case, funds appropriated for a particular use will carry a cost related to the return those funds could have achieved elsewhere. The other components of the discount rate are typically added to this base opportunity cost for investment capital.

Transaction Cost: Whether corporate investment capital is procured from the debt market or the equity market, the firm will experience transaction costs regardless of whether or not a new security issue is involved. These costs include broker and investment banker fees, costs of prospectuses and various filings, sales discounts, and other flotation costs. Although the aggregate transaction costs are generally much less than the base opportunity cost, they are significant and should be incorporated into the determination of the appropriate discount rate.

Increment for Risk: The cost of procuring funds, either debt or equity, includes a component for the investor's perception of risk. Consequently, some organizations add some *additional* percentage increment for risk to the cost of raising investment capital before applying this rate to discounted cash flow analyses. The rationale here is that high-risk projects should be discounted at some higher rate to compensate for their relative riskiness.

There are two fundamental problems associated with this approach to accounting for project risk. First, the increments for risk assigned to the discount rate are subjective by nature and cannot be quantitatively equated to project risk in any systematic manner. The second problem with this procedure is the manner in which project risk (business risk) is addressed. Clearly, the discount rate only adjusts for the present value of money; it does not by itself adjust for project business risk, as measured by the variability in annual project cash flows. Thus the discount rate provides no information on how annual project cash flows might vary, nor does it provide any insight into why these cash flows might fluctuate from year to year. Consequently, the discount rate does not provide a measure of project risk and, in general, should not be used to do so. Superior approaches to accounting for project risk are discussed in Chapter 6.5.

Increment for Inflation: It seems reasonable to assume that investment capital raised in the marketplace includes some cost premium for investor expectations of future inflation. Therefore, presumably a market-determined cost of capital includes a component that is based on investors' perceptions of future inflation. As a result, this market-determined cost of capital should be used as the appropriate discount rate when analyzing inflation-adjusted project cash flows. The amount of downward adjustment in the market-derived cost of capital for use with constant-dollar analyses can be shown to be roughly equal to the general rate of inflation.

6.2.4.2 Cost of Specific Types of Capital

The manner in which a firm chooses to finance itself will largely determine the cost of capital to the organization, since each source of financing has a specific cost. Thus the *capital structure* of the firm, defined as the mix of long-term sources of funds used to finance assets, becomes important in that the resulting cost of capital to the firm directly influences the investment decision. Since the firm's investment decisions influence common stock prices and therefore the value of the firm, the capital structure of the firm and the resulting cost of capital become key parameters in corporate finance.

Many corporations choose to develop a capital structure by utilizing a mixture of debt, preferred stock, and equity financing. Each of the various financial instruments available within these general categories has its own specific cost to the organization. The cost of each specific type of financing follows; however, the cost is essentially that rate which equates the funds received with all discounted future outlays relevant to that source. This may be expressed as

$$I_o = \frac{C_1}{(1 + k)} + \frac{C_2}{(1 + k)^2} + \ldots\ldots + \frac{C_i}{(1 + k)^i} \quad (6.2.9)$$

where I_o is net proceeds received at time zero, C_i is cash outflow in the ith period, and k is cost of this specific source of financing. The C_i's are fairly easily determined in the case of debt and preferred stock financing. Equity financing costs, however, have generated a great deal of debate, and, while Eq. 6.2.9 applies in theory, one of several proposed simplifying approximations must be adopted to determine the costs of equity financing in practice.

Cost of Debt: In keeping with the preceding discussion, the cost of debt is that discount rate that equates net proceeds from the debt issue to the future cash outlays. Further, these future payments are net of taxes. This is very significant with debt financing since interest payments, unlike dividends, are tax deductible. If k_d is the cost of debt and k is the yield to maturity of the debt issue determined by the application of Eq. 6.2.9, then

$$k_d = (1 - t)^k \quad (6.2.10)$$

where t is the marginal corporate tax rate.

Cost of Preferred Stock: In theory, because preferred stock dividends do not represent a legal obligation on the part of the firm, preferred stocks offer a distinct advantage over debt financing. In practice, however, the omission of a preferred stock dividend is viewed by the financial community as an indication of serious financial difficulty in the firm. Thus firms usually view preferred stock dividend payments as de facto obligations.

The cost of preferred stock is usually computed in a manner similar to debt perpetuities since both instruments have fixed annual costs and no maturity. There is one important difference, however. Preferred stock dividend payments are not tax deductible to the firm.

If the annual dividend paid is D dollars, and I_o represents the net proceeds per share for a new stock issue, the cost of preferred stock, k_P, is

$$k_p = \frac{D}{I_o} \quad (6.2.11)$$

From an investment viewpoint, preferred stocks offer one advantage over bond investments to a corporation. Eighty-five percent of the dividend income to the corporation is nontaxable, while interest income from bonds is taxable at the full applicable rate.

Cost of Equity Capital: Equity financing comes from two sources—new common stock issues and retained earnings. Clearly, there is a cost of funds obtained through the issuance of new common stock. Proceeds from such a transaction must be invested to return sufficient earnings to maintain the firm's earnings per share. While all analysts agree that there are real costs involved in equity financing through the issuance of common stock, no such agreement exists on calculating these costs in practice.

The so-called *dividend valuation model* is one of the popular methods for calculating the cost of equity financing. The premise for this model is that common stocks only have a value because of an expected future stream of dividends. In theory, this model is consistent with the generalized cost of capital model previously discussed and may be represented as

$$P_o = \frac{D_1}{(1 + k_e)} + \frac{D_2}{(1 + k_e)^2} + \ldots\ldots + \frac{D_i}{(1 + k_e)^i} = \sum_{i=1}^{\infty} \frac{D_i}{(1 + k_e)} \quad (6.2.12)$$

where P_O is value of stock at time O, D_i is dividend payment in the ith year, and k_e is cost of equity applicable to firms in this risk class.

Obviously, dividend payments in future periods are unknown at the present time. Investors, however, have subjective estimates of what these payments will be, and these estimates are generally based on the current dividend and the historical long-run growth of dividends. An intuitive expression of these relationships might be:

$$k_e = \frac{D_o}{P_o} + g \quad (6.2.13)$$

where D_o is current dividend and g is annual growth rate of dividends.

Another popular method for computing the cost of common stock financing is the *earnings:price ratio* (E/P ratio). This approach is based on the assumption that investors really purchase earnings when buying common stock and that anticipated future earnings per share determine the value of a share of common stock.

The E/P model obviously is more suitable than the dividend valuation model for firms that pay low dividends or none at all. For firms that have strong earnings growth records, a growth term is sometimes appropriate in the E/P model to reflect shareholder expectations that such growth will continue. The mathematical expression for this model is

$$k_e = E/P_o + g \quad (6.2.14)$$

where E represents current earnings per share, P_O is value of stock at present, and g is growth term for earnings per share.

It should be noted that both of these models incorporate market prices for common stock. However, if a new common issue is planned, net proceeds per share, P_f should be used rather than the current market price, P_O. P_f will be lower than P_O by the amount of discount required to sell the new issue, plus the underwriting costs.

Although other models exist for estimating the cost of equity, they are more complex to use in practice and are not covered here (see Gentry and O'Neil, 1984, for more complete coverage).

That there is also a cost of *retained earnings* (net profits minus dividends) is less obvious than for common stock. However, if the firm retains, rather than distributes, earnings, and if it is dedicated to the maximization of shareholder wealth, it clearly must invest these funds above some minimum rate of return. Furthermore, the minimum rate in general is the cost of common stock financing, for this is the rate the investor implicitly agreed to by purchasing stock in the first place. If no investment opportunities are available to the firm above this minimum rate, all earnings should be returned to the owners (shareholders) to enable them to reinvest the funds themselves. Thus the cost of retained earnings is equal to the cost of common stock financing adjusted, where applicable, for flotation costs.

Readers interested in the application of the foregoing equations to calculating a firm's cost of capital are referred to Chapter 11 in Gentry and O'Neil (1984).

6.2.4.3 Weighted Average Cost of Capital

Given that each of the various types of financing available to a corporation has a specific cost, it seems reasonable to conclude that it should be possible to combine these various forms of financing into a corporate capital structure that would result in a minimum weighted average cost of capital to the firm. In other words, the implication is that there should be an optimum capital structure that minimizes the firm's weighted average cost of capital. Thus if a firm finances itself in this optimum manner, it should generate the lowest possible weighted average cost of capital. If this cost of capital is then used as the appropriate discount rate when evaluating new investment opportunities, the firm should find that more investments become acceptable. This should subsequently result in an optimum investment program and generate more wealth to shareholders.

Gentry and O'Neil (1984) provide more extensive discussions on the issues of optimum capital structures, marginal weighted average cost of capital calculations, and the practical problems associated with these calculations.

6.2.4.4 Common Errors in Evaluation

One of the most common errors made in discounted cash flow financial analysis involves using the cost of specific financing in evaluating projects rather than the average cost of all capital used by the firm. When a capital project involves an expenditure of, say, \$20 million through a bank loan carrying an interest rate of 12%, it often is difficult to understand that (1) the interest charges from the loan should *not* be levied against the project for evaluation purposes, and (2) the appropriate discount rate to use is *not* 12% (or its approximate after-tax equivalent), but rather the marginal weighted average cost of *all* capital used by the firm.

Another common error in financial analyses is the inappropriate use of a market-derived cost of capital as the discount rate with constant dollar project cash flow estimates. These errors are sufficiently common to deserve further discussion.

Error No. 1: Charging Specific Capital Costs to the Project. The cost of capital, being the link between the investment decision and the financing decision, occupies a unique place in the evaluation procedure. All other project costs (i.e., operating costs, plant overhead, depreciation, etc.) are handled explicitly in the pro forma income statement. The cost of capital, however, *is* the discount rate and does not receive a separate line in these statements. While such a procedure may be applicable in other types of economic studies or analyses of financing alternatives, it is incorrect to do so in discounted cash flow analyses.

Unfortunately, this procedure has led many practitioners to believe that the cost of funds has been omitted in the analysis and should be listed separately in a manner similar to the other cost items. This misconception occurs most frequently when debt capital is raised and the accompanying interest payments are not charged directly to the project in the discounted cash flow return on investment (DCFROI) calculations. However, it is important to recognize that to do so would clearly double-count the cost of capital—once through the discount rate and a second time through the explicit interest charges. This point is illustrated in the following.

Example 6.2.7. (Gentry and O'Neil, 1984)

Suppose Hikki Mining Co. contemplates an investment opportunity having the following anticipated revenues and costs.

Investment = \$100
Net annual cash flows (year 1-5) = \$27.74
Cost of capital = 12%
NPV = \$27.74 $(P/A, 12, 5) - 100 = 0$

In other words, the project should just break even after returning the investment plus the 12% cost of capital. This is verified in the following.

Solution .

	Year 1	2	3	4	5	Total	
Cost of capital @ 12%	12.00	10.11	8.00	5.63	2.97	38.71	
Total money owed before year-end payment	112.00	94.37	74.63	52.49	27.72		
General payment	27.74	27.74	27.74	27.74	27.74	138.70	Total payments
Money owed after year-end payment	84.26	66.63	46.86	24.75	—		
						100.00	Principal

Thus the discounting procedure assures that all project costs are covered—including return *of* and return *on* capital. To have levied financing charges explicitly against the project prior to discounting would clearly have forced the project to pay these costs twice.

Nonetheless, it continues to bother many that interest payments resulting from additional debt obligations are not charged directly to the project, even though these obligations must be met from the earning power of the firm's investments. This has caused some firms to incorporate a procedure whereby projects are evaluated on (1) a full equity basis and (2) various degrees of debt leveraging. The following example serves to illustrate the types of calculations resulting from this *improper* procedure.

Example 6.2.8. (Gentry and O'Neil, 1984)

An underground conveyor system is proposed to reduce haulage costs at a zinc mine. Benefits to be derived from this project are estimated to be \$200,000 annual savings in operating costs. Depreciation is straight-line, and the income tax rate is 50%. Depletion is constrained by 22% of gross income from mining and can, therefore, be ignored as being constant in all alternatives. If the total investment is estimated to be \$1,146,000, and the service life is 12 years with no salvage value, calculate the rate of return on equity capital for (1) all equity case, (2) 50/50, debt/equity financing, and (3) 95/5, debt/equity financing.

Assume the before-tax cost of debt is 8% compounded annually, and for simplicity assume that the entire principal amount will be repaid at the termination of the project.

Solution .

1. Equity investment
   A. \$1,146,000
   B. \$ 573,000
   C. \$ 57,000
2. Debt investment to be repaid at the end of the year 12
   A. 0
   B. \$ 573,000
   C. \$1,088,700
3. Annual cash flows

	Case A	**Case B**	**Case C**
Net operating savings	\$200,000	\$200,000	\$200,000
Less: depreciation	− 95,500	− 95,500	− 95,500
: interest	0	− 45,840	− 87,096
Pretax net income	104,500	58,660	17,404
Less: income tax	52,250	29,330	8,702
Incremental net profit	52,250	29,330	8,702
Add: depreciation	95,500	95,500	95,500
Incremental net cash flow	\$147,750	\$124,830	\$104,202

4. DCF return on equity

Case A: \$1,146,000 = 147,750 $(P/A, i, 12)$
$i = 7.5\%$

Case B: $573,000 = 124,830 (*P/A*, *i*, 12)–573,000)(*P/F*, *i*, 12)
$i = 14.8\%$

Case C: $57,300 = 104,202 (*P/A*, *i*, 12)–1,088,700 (*P/F*, *i*, 12)
$i = 182.5\%$

The foregoing analysis seems to suggest that the use of debt financing can turn a very marginal project into one yielding a phenomenal rate of return. However, it must be recognized that the DCFROI on the equity portion of the investment is always increased with leveraged financing if the after-tax cost of debt is less than the DCFROI of the project figured for the all-equity case. Thus the most marginal of investments can often be made to appear positively superb when a high percentage of debt capital is used!

A basic principle of investment analysis is that a project must stand on its own merits and must compete for funds on an equal basis with other investment opportunities. Each such opportunity must share fully in the benefits of debt leveraging if the same weighted marginal average cost of capital is used for all projects.

In summary, the following should always be observed in project investment analysis:

1. To avoid counting capital costs twice, do not charge costs of specific capital sources against the project. These charges are embedded in the discount rate.

2. Calculate returns on investments on total capital invested (i.e., all-equity case) to insure that all projects are compared on an equal basis.

Error No. 2: Using Specific Capital Costs as the Discount Rate. Debt financing is the source of another common error in capital project evaluations. There is a tendency on the part of some analysts to use the cost of a specific source of financing—rather than a weighted average of all capital sources—as the discount rate in measuring the attractiveness of a project. The following example illustrates this flaw.

Example 6.2.9. (after Quirin, 1967)

Global Mineral Ventures, Inc. was presented with a similar set of investment opportunities in three successive years, as shown:

	Year 1		Year 2		Year 3	
Project	Amount of investment $	Rate of return $	Amount of investment $	Rate of return $	Amount of investment $	Rate of return $
A	100,000	20	100,000	20	100,000	20
B	200,000	15	200,000	15	200,000	15
C	200,000	11	200,000	11	200,000	11
D	200,000	8	200,000	8	200,000	8
E	200,000	6	200,000	6	200,000	6

Solution.

In year one, Global had no long-term debt, and the corporate treasurer found that the full $900,000 could be raised by selling debentures bearing an annual interest rate of 5½%. He convinced Global's board that, since every project returned more than 5½%, they should all be accepted.

In year two, the treasurer was able to borrow a further $700,000 at 7%, and using the same logic as the previous year, accepted all projects except E, which offered a return below the 7% marginal cost of debt.

In the third year, however, Global's treasurer found only a limited amount of additional debt available to him—$100,000 at 18% from a finance company. Since this was still below the estimated 19% for a new equity issue, the treasurer used the debt to accept only project A.

Thus over the three-year period by using the cost of a specific source of debt as his investment criterion, the treasurer had invested $1.7 million at a weighted average return of 12%. It is clear, however, that the treasurer's eagerness to accept projects in year 1 precluded the acceptance of better projects in year 3. In fact, the same $1.7 million could have been invested to yield an average rate of 13.3% by accepting projects in the following sequence:

Year 1 A, B, C, D
Year 2 A, B, C
Year 3 A, B, C

This example illustrates that debt financing is only possible if an adequate equity base exists. Clearly, the marginal cost of capital is not simply the cost of debt but also the cost of equity that will be required in the future to support the added debt. Thus every investment must carry its proportionate share of the necessary—but higher cost—equity funds whenever debt is added to the capital structure of the firm. In the foregoing example, using the marginal weighted average cost of *all* capital sources would have resulted in rejecting the lower-value projects in year one, thereby permitting the acceptance of higher-value projects in year three.

In summary, it is important to recognize that it is incorrect to use the cost of a specific source of capital as the cash flow discount rate, or equivalently, as the minimum acceptable rate of return for capital projects.

Error No. 3: Using the Appropriate Discount Rate with Inflated and Constant Dollar Cash Flows. Some organizations insist on performing cash flow analyses in terms of constant dollars, while others prefer to perform analyses in terms of inflated cash flows. Although the preparation of constant dollar pro forma cash flows is highly recommended to promote better technical understanding of the project, discounting should always be performed on inflated dollars to avoid miscalculation of income taxes. Although there are specific virtues associated with both approaches, the important point here is to recognize that a market-determined cost of capital includes a component that is based on investors' perceptions of future inflation. As a result, this market-derived cost of capital should be used as the appropriate discount rate when analyzing inflation-adjusted project cash flows. If this rate is applied to project cash flows that are not adjusted for inflation, the project will be seriously *undervalued*. The amount of downward adjustment in the market-derived cost of capital for use with constant-dollar analysis is roughly equal to the general rate of inflation.

A more complete discussion of the impact and treatment of inflation in mining investment decision analysis is provided in Chapter 10 of Gentry and O'Neil (1984).

6.2.5 AN ITERATIVE PROCESS

This chapter has focused on the engineering and economic parameters associated with project feasibility studies. Estimates necessary for the quantification of many of these parameters in the pro forma income statement are provided in Chapter 6.3. The calculation of project net annual cash flows can be performed after completion of the pro forma income statement using the appropriate cost of capital or discount rate. Given these estimates of relative benefits, costs, and annual cash flows for a project, it then becomes necessary to convert these estimates into measures of relative desirability or attractiveness. The criteria and techniques typically utilized to determine project acceptability or desirability are the topics of discussion in Chapter 6.5.

It must be remembered, however, that completion of a feasibility study or an intermediate economic study incorporating one set of project parameters represents only one alternative in an iterative process. As noted in the introduction to this section, the process of evaluating mine investment opportunities is iterative nature. Each time a project variable or parameter changes, it is necessary to assess the impact of this change on all other project variables and on the subsequent financial results.

This iterative process must be repeated until the most economic design is achieved for the project being analyzed. This may require changes in cutoff grade, mining reserves, mine size or mining rate, or alternative extraction scenarios having differing capital and operating cost characteristics. Whatever the case, the most efficient combination of project parameters must be combined to meet the firm's primary objective: wealth maximization to its shareholders.

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