Cut-off grade determination for the maximum value of a small Wits-type gold mining operation

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Synopsis

Work undertaken in the field of cut-off grade optimization has not advanced much beyond the work undertaken by Lane in 1988. His definitive work is based on the calculus of the Net Present Value criterion, which is the most widely understood, consistent, and appropriate method by which sequential cash flows arising from the extraction of mineral reserves from an exhaustible resource can be represented. Although the mathematics is not complex Lane's method is not a widely appreciated or applied approach to maximizing the value of a mining operation through selection and balancing of operational cut-off grades. Three stages in a mining operation, namely mining, processing and marketing were defined by Lane and the economics of each stage are identified and isolated to provide an optimum cut-off for each stage. Points of intersection along the present value curves for each stage of a mining operation are used to identify balancing cut-off grades at points where the capacity of the mining, processing and marketing stages is fully utilized and the Net Present Value of the operation is optimized. Data that simulate a small, deep-level Witwatersrand-type gold mine with an average grade of 6 g/t Au and a logrithmic variance of 1.2 are applied to the model to illustrate the benefits of representing cut-off grades in terms of present value curves for different segments of a mining operation.

Keywords: cut-off grade, Net Present Value, grade-tonnage curve, balancing cut-off grade, mining, processing, marketing.

Introduction

The purpose of this paper is to re-examine the work undertaken by Lane (1988, 1997) with regard to the optimization of cut-off grades in mining operations. The method has been couched in mathematics that detracts from its value and as a consequence it has not been as widely applied as might otherwise be the case. This is evidenced by the fact that many mining engineers and geologists keep a copy of Lane's original work on their shelves but the books generally tend to be in pristine condition. The Net Present Value (NPV) criterion was used by Lane as the basis for deriving a set of equations that allow one to identify the constraining factors in a mining operation and thereby determine the maximum present value of a mining operation. The NPV criterion is consistently quoted as the principal determinant of economic value in mining operations and the relationship between cut-off grade and NPV provides a means by which cut-off grades can be optimized.

For the purposes of a discounted cash flow analysis Lane defined value as a function of two factors, namely the size of the remaining ore reserve (S) and the rate of extraction (q). Intuitively these two factors will inform decisions about the capacity of the mine, for the larger the rate of extraction the shorter will be the life of a mine for any given size of mineral reserve and vice versa. These two factors (S) and (q) also define the life of the mine (T). The present value (PV) for any mining operation is the sum of all future cash flows discounted by an appropriate rate of interest, which should at least be the cost of capital. Both Lane (1997) and Hartwick and Olewiler (1997) arrived at the same conclusion using slightly different techniques, but Lane used the following expression, where Cq is the cash flow in the initial period associated with q tons of extracted ore. The value V of the mining operation at any time is a function of the life of the mine, the size of the remaining reserve and the rate of extraction $V(T + t, S - q)$, so we write the wellknown Net Present Value notation as follows:

$$PV(T,S) = V(T+1, S-q_0) + \frac{V(T+t_1, S-q_1)}{(1+r)} + \frac{V(T+t_2, S-q_2)}{(1+r)^2} + ..$$

Differentiating the present value, PV in this equation with respect to S the remaining ore reserve and with respect to time, the life of the mine, produces the following relationships. So we have:

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$$\frac{dV}{dS} = C - \frac{t}{q}\left(rV + \frac{dV}{dT}\right) \qquad [1]$$

The last term in Equation [1] is negative and is made up of an interest term that reflects the depreciation in value of the operation (rV) as a consequence of extraction, and the first derivative of the value of the mine with respect to time, $^{dV}/_{dT}$ which together are referred to as the opportunity cost. Opportunity cost is a loss incurred by investors for having tied up their capital in the present mining operation. This means they have to forego the benefits that would have accrued to them as a result of investing their capital in the next best mining investment opportunity and hence they incurs a cost

$$F = \left(rV + {}^{dV}\!/_{dT}\right)$$

that will be effective for the life of the operation. So we have terms F and t where

$$\tau = {}^{t}\!/_{q}$$

This equation gives the time taken to work through one unit of mineralized material and the term (τ) is simply the tonnage milled, processed or marketed in one time period, typically a year. Thus the term reduces to a rand per ton term (exactly the same units as the rest of the terms in the equation) and is simply an additional cost attributable to each ton of ore in the same way as any other costs. So we have the result that the *maximum present value of a mining operation* based on a finite resource S, is:

$$\frac{dV}{dS} = C - F\tau$$

This is the elegant solution that Lane derived as long ago as 1988. Details of the derivation are provided in Appendix 1. In fact, this is a remarkable result for it tells us how the value of a mining operation declines as the primary asset, the mineral resource, is progressively depleted. This equation takes account of both the present value maximization and of the opportunity cost and can be rearranged to show that the change in value that accompanies the depletion of the ore reserves is simply the cash flow associated with the mining of those reserves minus the opportunity cost. On a per unit basis the change in value (dV) with each unit q that is mined (dS) is given by the equation:

$$^{dV}\!/_{dS} = p(q_t) - c(q_t) - rV_t - {}^{dV}\!/_{dT} \qquad [2]$$

This is exactly the equality used by the United Nations (2000) in their calculation of royalty, which is defined as the change in value associated with the extraction of one unit of reserves. Furthermore, we can write an equation for the change of value of the mining operation with time:

$$\frac{dV}{dT} = \frac{q}{t} \times \left(\frac{dV}{dS} - C\right) + rV$$

This equation says that the change in value of the mining operation with time is equal to the production rate multiplied by the change in value of the stock minus the cash flow, plus the capital appreciation.

Model parameters

For this particular exercise the average values of key data from the Chamber of Mines Annual Report (2000), for a relatively small, deep-level Witwatersrand-type gold mine in South Africa, are presented in Table I.

Mine, mill and market capacity

Three main components of a mining operation, namely mining, processing and marketing, were identified by Lane (1988, 1997) in his approach to the analysis of cut-off grades, and are shown with symbols for capacity and costs in Table II. Mining capacity, M in Mt/a, is primarily a function of mine design, access, labour, infrastructure, and available face length, whereas mill capacity, H in Mt/a, is a function of the number of crushers, ball mills and absorption tanks in the mill. Marketing capacity may be constrained through selling constraints or long-term contracts, but is probably only related to the smelting and refining capacity of the refinery.

Table I

Key data for an industry average, small South African gold mine

Rock mined	Shaft capacity	Mill capacity	Ore milled	Average grade	Gold produced	Capital expenditure	Total cash costs
tons 1 200 000	Mt pa ~1.404	Mt pa ~1.21	tons 700 000	g/t 6.00	kg 4 200	R (Million) 24.50	R (Million) 38.0

Modified after: Key data for small gold mining operations in South Africa. Source: Chamber of Mines (2000)

Table II

Notation for mine analysis (after Lane, 1997)

Component	Material	Quantity	Variable cost	Capacity Mt/a
Mining	Ore and waste	1.0	m	M
Processing	Ore	Proportion above cut-off (x)	h	H
Marketing	Metal	$xy\bar{g}$	k	K

Output in some mining operations may be limited by capacity in one of the three stages, mining, processing or marketing and could affect the economic cut-off grade. For the bulk of South African Wits-type deposits no such constraints exist. The average large South African gold mine hoists in the order of 2.25 Mt of rock annually comprising about 1.8 Mt of ore and approximately 0.45 Mt (20 per cent) waste. Key data from Anglogold (2001) suggest an average shaft overcapacity of 17% in respect of rock hoisted and an average mill overcapacity of 42%, in respect of ore hoisted for the larger mining operations in South Africa. Thus gold production at these mining operations is constrained by the rate of extraction rather than infrastructure capacity.

The results for the industry average, small gold mine shown in Table I are applied in Table III by way of an example in which the present value of a mining operation can be optimized through tactical application of appropriate cut-off grades.

Application of the concept

The cash flow shown in Equation [1] yields the following expression:

$$\frac{dV}{dS} = (p-k)xy\bar{g} - xh - m - f\tau - \left(rV - \frac{dV}{dT}\right)\tau$$

$$= (p-k)xy\bar{g} - xh - m - (f+F)\tau$$

By setting the differentiated equation above equal to zero it is possible to find a maximized solution for any of the variables and to maximize the present value (*PV* in *R/t*) arising from the extraction, processing and marketing of one unit of mineral reserve given by:

$$PV = (p-k)xy\bar{g} - xh - m - (f+F)\tau \qquad [3]$$

where:

x = the proportion of mineral resource above cut-off, i.e. the ratio of mineral reserve to mineral resource (payability), and

$\bar{g}$ is the average grade of the ore above cut-off.

In the limiting cases the following formulae apply for mining:

$$\bar{g}_m = \frac{h}{(p-k) \times y}$$

for processing

$$\bar{g}_h = \frac{[h + (f+F)/H]}{(p-k) \times y}$$

and for marketing

$$\bar{g}_k = \frac{h}{[(p-k) \times y - (f+F) \times y]/K}$$

Note the last two terms for the processing and marketing equations are identical in form and in each case the cut-off grade will fall as *F* declines because the remaining life of the mine is reduced. None of the formula makes direct reference to the grades present in the mineralized rock. The cut-off is calculated with reference to costs, prices and capacities regardless of the way the grades actually vary within the mineralized body. Making the appropriate substitutions of the data for a small Wits-type gold mine in Table III produce the following results for limiting economic cut-off grades for mining, processing and marketing. These are shown in Figure 1 as gm, gh, and gk respectively.

Limiting economic cut-off grades

Mine limiting cut-off grade

The mine limiting capacity *M* is the shaft capacity shown in Table I given in mining units (SMUs) per year, but in every case these shafts have significant overcapacity in terms of their ability to hoist broken rock. One unit of mineral reserve that is mined gives rise to one unit that is sent for processing. On average the time to handle one unit (1/M) is very small for Archean gold deposits (1/160 000) and even smaller for large Wits-type gold deposits (1/1 200 000). So Equation [3] becomes

$$\frac{dV}{dS} = (p-k)xy\bar{g} - xh - m - (f+F)/M$$

Table III

Model parameters for an industry average, small South African gold mine

Operation component	Symbol	Utilization
Mining capacity (rock mined)	M	1.2 Mt/a
Ore		0.7 Mt/a
#Waste*		0.5 Mt/a
Processing capacity (rock processed)	H	0.7 Mt/a
Marketing capacity (metal recovered)	K	4.20 t/a
	Model parameters	
Mining variable costs	(m) R/t	180
Processing variable costs	(h) R/t	200
Marketing variable costs	(k) R/t	300 000
Total fixed costs	(f) R/a	38.0
Opportunity cost	(F) R/a	24.50
Average grade above cut-off	$\bar{g}$ g/t	Values from 0 to 45 g/t
Proportion above cut-off	x	Values from 0.0 to 1.0
Yield or recovery	(y)	0.82 (82%)
Price	(p) R/g	101

*Average values calculated from data for Tau Lekoa and Kopanang mines indicate about 20 per cent waste (Anglogold, 2001)

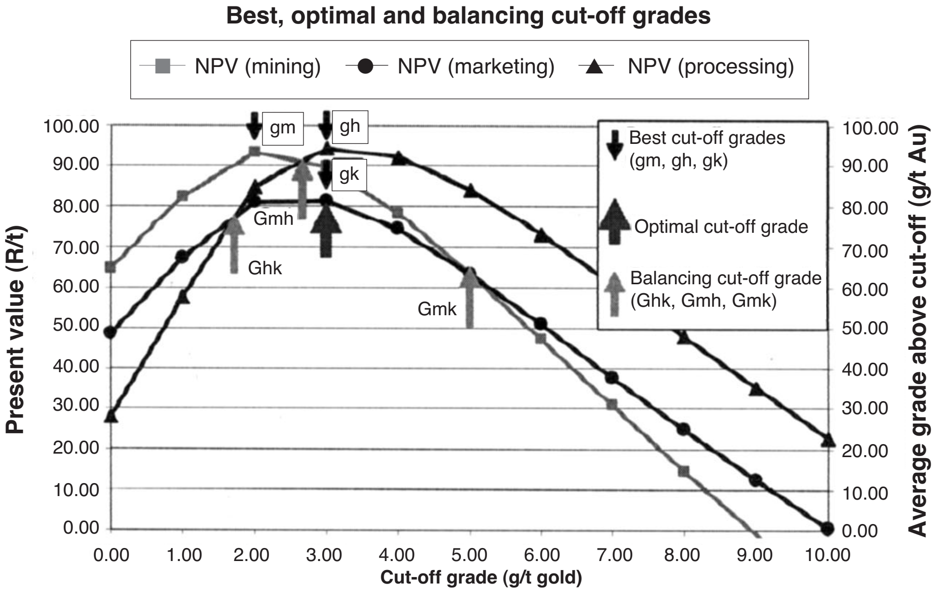

Figure 1—Effective optimum cut-off grades for the mining, processing and marketing stages of a mining operation. The thick line at 2.44 g/t defines the optimum operational cut-off grade

and since the terms –m and –(f + F)/M do not vary with changes in $\bar{g}$ we can write:

$$\frac{dV}{dS} = (p - k)xy\bar{g} - xh = 0$$

Setting the equation equal to zero and solving for $\bar{g}_m$ we have the formula for the mine limiting cut off grade:

$$\bar{g}_m = \frac{h}{(p-k) \times y}$$

This equation tells us that one unit of mineralized rock is part of mineral reserves if the value of the unit is greater than the cost of further processing $((p - k)yg > h)$.

The following points are worth noticing:

- That after allowing for marketing cost the value of mineral reserve need only cover the variable cost of treatment for it to make a contribution to mining operation. This is the clearest definition of marginal ore that is available.
- Neither time cost nor mining/development costs are relevant.
- There is no reference to present value; hence a mine that is limited by mining capacity should be operated on a tactical rather than a strategic basis. This means no matter what your current cut-off grade policy is, there is no way to make gains now that you trade off against losses in the future. Where a decision has been made to continue operating, there is no limit to treatment capacity—you should increase output as price rises.

Substituting the parameters from Table III into the mine limiting equation we have:

$$\bar{g}_m = \frac{h}{(p-k)y} = \frac{200}{(101 - 0.3) \times 0.82} = 2.42 \text{ g/t } gold$$

This cut-off arises because if the extraction process is the limiting constraint then plant and market are starved of ore; so everything above 2.42 g/t gold is classified as ore as shown for the mining curve in Figure 1.

Process limiting cut-off grade

For the average large South African Wits-type gold mine the processing plant is never a constraint on the rate of gold production. The most common constraint in mining operations is in the processing stage, i.e. tramming, hoisting, crushing, concentrating or processing facility. The process limiting capacity is *H* units per year. One unit of mineralized material gives rise to x units of ore. The time τ to handle x units of ore is $\tau = x/H$. So Equation [3] becomes

$$\frac{dV}{dS} = (p - k)xy\bar{g} - xh - m - (f + F) \times \frac{x}{H} = 0$$

Again the term –m does not vary with the grade $\bar{g}$ and $\tau = x/H$, so solving for $\bar{g}_h$ we have the following equation for the process limiting cut-off grade:

$$\bar{g}_h = \frac{(h + (f+F)/H)}{(p-k)y}$$

From this we see that the opportunity cost $F = rV - dV/dT$ appears simply as an additional time cost factor divided by a tonnage (*H*), and this makes the cut-off determination significantly different from conventional methods. The cut-off declines as the mine ages, because the older the mine the smaller will *F* be. Substituting the model parameters from Table III into the process limiting equation we get:

$$\bar{g}_h = \frac{(h + (f+F)/H)}{(p-k)y} = \frac{(200 + (38.00 + 24.5)/0.7)}{(101 - 0.3) * 0.82} = 3.50 \text{ g/t } gold$$

This is a higher cut-off grade than if the mine capacity is the constraining factor as shown for the processing curve in Figure 1.

Market limiting cut-off grade

For the average gold mine the market may be limited in the short-term by an exclusive sales contract, or by imposing constraining capacity of a refinery or smelter. This does not present any significant restriction on the sale of the metal in the medium-to long-term, but the concept is carried forward by Lane (1997) and is applied here because of potential application in other commodity markets. The market limiting capacity is K units per year, meaning one unit of mineralized material gives rise to $xy\bar{g}$ units of concentrate or metal and the time to handle (process or sell) the $xy\bar{g}$ units of material is $xy\bar{g}/K$, where $\tau = xy\bar{g}$.

So Equation [3] becomes

$$\frac{dV}{dS} = (p-k)xy\bar{g} - xh - m - (f+F) \times \frac{xy\bar{g}}{K} = 0$$

Setting it equal to zero and solving for $\bar{g}k$ gives the market limiting cut-off grade and through substitution of the model parameters in Table III this equation gives:

$$\begin{aligned}\bar{g}_k &= \frac{h}{(p-k)y - y(f+F)/K} \\ &= \frac{200}{(101-0.3)*0.82 - 0.82(38.0+24.50)/4.20} \\ &= 2.84\ g/t\ gold\end{aligned}$$

The cut-off grade is low as shown in Figure 1 and a constraint in the market means that cash flow patterns cannot be influenced very much by cut-off grade policy.

Effective optimum cut-off grades

The best way to examine the relationships between cut-offs at different stages of the mining operation is to calculate and compare the present value V for each of the constraints. With present value as a function of the remaining reserves the following equation was derived:

$$dV/dS = PV = (p-k)xy\bar{g} - xh - m - (f+F)\tau$$

The three forms of limiting cut-offs were all related to the variable τ in the following way:

- Mine limiting $\tau = 1/M$
- Process limiting $\tau = x/H$
- Market limiting $\tau = xy\bar{g}/K$

Present value for each constraint takes the form:

- $V_m = (p - k)\ xy\bar{g} - xh - m - (f+F)/M$
- $V_h = (p - k)\ xy\bar{g} - xh - m - (f+F)x/H$
- $V_k = (p - k)\ xy\bar{g} - xh - m - (f+F)\ xy\bar{g}/K$

Graphs of these three representations of present value (in R/t) as a function of the cut-off grade are all convex upwards with a single maximum that is the limiting cut-off grade for the constraint concerned. The grade-tonnage curve (Figure 2) for a deposit with an average grade of 6 g/t and a logrithmic variance of 1.2 is shown together with the present value data (Table AI) in Appendix 2. Curves for mining, processing and marketing in a small Wits-type gold mine are shown in Figure 1.

Balancing cut-off grades

Optimal cut-off grades can be determined at each stage of the mining operation when capacity related factors are incorporated in the calculation. Some mining operations may be constrained by the capacity of mining (M), processing (H) and marketing (K) operations, but data from Table I indicate that gold production on the average large scale mine is not constrained by any of these factors. The only major constraint on the rate of gold production is the thin tabular aspect of the orebody itself, the mining method, the extreme depths at which the reef occurs, the high rock temperatures

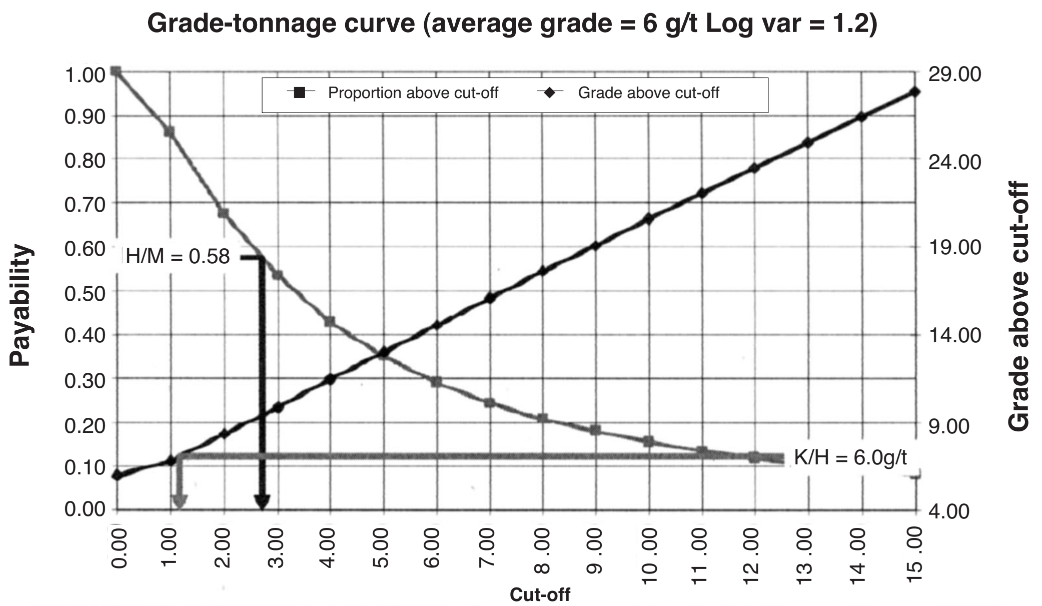

Figure 2—The grade-tonnage curve for a gold deposit (lognormal distribution) with an average grade of 6 g/t gold and a logrithmic variance of 1.2. The balancing cut-off grades (H/M = 2.84 g/t) and (K/H = 1.2 g/t)) are indicated

and the distances that have to be travelled underground. Relative utilization of a Wits-type orebody is determined by the grade distribution of the orebody, the applied cut-off grade and the variability of the ore. At lower cut-offs orebody utilization is high, the payability is high, the average grade is low, rates of development are low and selectivity is low, but the opposite is true at high cut-off grades. The grade-tonnage curve graphically describes this relationship as shown in Figure 2.

At full utilization there is a cut-off grade at which the different components of the mining operation are balanced. Payability is the ratio of ore milled to total rock mined, so the following relationship is true for the average Wits gold mine:

$$\text{Payability }(x) = \frac{\text{Ore sent to the mill}}{\text{Rock that is mined}} =$$
$$\frac{H}{M} = \frac{0.7\text{Mt/a}}{1.2\text{Mt/a}} = 0.583$$

Mining capacity *M* and processing capacity *H* are balanced when the payability is in the ratio of existing capacities and from this we derive the mining/processing balancing cut-off grade, Gmh. In this particular case the cut-off grade is about 2.8 g/t according to Figure 2 and operating at this cut-off keeps the two parts of the mining system at full capacity.

In a similar way the average grade above cut-off is the ratio of total metal recovered (marketing capacity *K*) to total tonnes processed (processing capacity *H*) and can be represented by the following equation: (These two factors are balanced when the average grade above cut-off is in the ratio of the existing capacities.)

$$\text{Average grade above cut}-\text{off} =$$
$$\frac{\text{Total metal recovered}}{\text{Total tons processed}} = \frac{K}{H}$$
$$= \frac{4\ 200\ 000\text{g}}{700\ 000t} = 6.0\text{g/t}$$

Operating at the cut-off grade where average grade above cut-off = K/H means we keep the two parts of the mining system at full capacity. A recovery factor of 0.82 accounts for the difference between calculated and actual metal recovered and gives an average grade above cut-off of about 5 g/t gold and a cut-off grade of about 1.2 g/t gold (Figure 2).

We now have three limiting cut-off grades and two balancing cut-off grades (Figure 1), but only one is feasible and we are looking for a logical procedure for identifying it. The solution for the maximum cut-off grades at the intersection points of the various curves marked Vm, Vh, and Vm in Figure 1 are known as the balancing cut-off grades. At these points the full capacity of all parts of the mining operation is utilized. Lane (1997) referred to these points shown on Figure 1 as follows:

- Ghk = gk if ghk < gk
- Ghk = gh if ghk > gh
- Otherwise Ghk = ghk.

The overall effective optimum cut-off grade is now one of the two, either Gmh or Ghk. The largest PV is limited by the least of Vm, Vh or Vk and in this particular example it is the process limiting grade of 2.84 g/t gold (Figure 1).

Conclusions

The method suggested by Lane (1988, 1997) is an elegant and simple way of optimizing cut-off grades in mining operations where grades are low and selective mining on a strategic basis can be applied. In most South African operations there is little in the way of constraint on mining infrastructure and in this particular model it can be shown that the optimal cut-off grade at which to run the operation is the marketing limiting cut-off of 2.84 g/t. This provides the miner with a tactical tool for maximizing the cash flow from the operation on a local scale and a year-by-year basis.

Appendix 1

Derivation of the relevant equations used in this paper

The value of the mining operation *V*, in the first period, is the cash flow (*Cq*) associated with mining *q* units of stock and the present value (*PV*) of any facility is given by:

$$V(T,S) = Cq + \frac{1}{(1+r)^t}\left[V(T+t,S-q)\right] = Cq + \left[V(T+t,S-q)\right]/(1+r)^t \qquad [1]$$

Let's focus on the second part of the equation, namely

$\left[V(T+t,S-q)\right]/(1+r)^t$.

Using the Binomial Series expansion,

$1+n)^n \approx (1+nn)$ if $|n| << 1$

we rewrite the equation as follows:

$$V(T+t,S-q)/(1+r)^t$$
$$= V(T+t,S-q) \times (1+rt)$$

Now using the Taylor Series expansion for two variables for this part of the equation we get:

$$V(T+t) = V(T) + t\frac{dV}{dT} + \frac{t^2}{2}\frac{d^2V}{dT^2} + \ldots\text{and} \qquad [2]$$

$$V(S-q) = V(S) - q\frac{dV}{dS} - \frac{q^2}{2}\frac{d^2V}{dS^2} + \ldots \qquad [3]$$

Combining Equations [2] and [3] we get:

$$[4] \quad V(T+t,S-q) = \left[V(T,S) + t\frac{dV}{dT} - q\frac{dV}{dS}\right] \times (1+rt)$$

And multiply Equation [4] by $-rt(t\frac{dV}{dT})\approx 0$ and because $-rt$ is very small, it means that $-rt^2$ and $-rtq$ are very, very small so we ignore these terms and from Equation [4] we get:

$$V(T,S) - rtV + t\frac{dV}{dT} - q\frac{dV}{dS} \qquad [5]$$

Return to Equation [1] and substitute Equation [5] into it to get:

$$V(T,S) = Cq + V(T,S) - rtV + t\frac{dV}{dT} - q\frac{dV}{dS} \qquad [6]$$

Having performed the differentiation and cancelling common terms on either side of the equation we can now set Equation [6] equal to zero:

Appendix 2

Present Values for mining (Vm), processing (Vh), and marketing (Vk) at different cut-off grades of the mining operation are represented in the table below.

Table A1

Data used in the simulation of an industry average, small Wits-type gold mine

INDUSTRY AVERAGE SIMULATION								
Parameters	**Factor**		**Proportion (x)**	**Average grade**	**Cut-off grade**	**Vm**	**Vh**	**Vk**
Mining (t/a)	M	1 200 000	1.00	6.00	0.00	63.36	26.16	47.11
Treating (t/a)	H	700 000	0.86	6.86	1.00	81.07	56.22	65.81
Marketing (t/a)	K	4.5	0.68	8.35	2.00	91.80	83.57	79.68
Mining (R/t rock)	m	200.00000	0.53	9.90	3.00	88.35	92.77	80.23
Processing (R/t ore)	h	180.00000	0.43	11.46	4.00	77.19	90.92	73.19
Marketing (R/g metal)	k	0.30000	0.35	13.01	5.00	62.39	83.09	62.37
Fixed costs total (Rm/a)	f	38 000 000	0.29	14.55	6.00	46.17	72.19	49.87
Opportunity cost (Rm/a)	F	24 500 000	0.25	16.08	7.00	29.71	59.87	36.83
Price (R/g metal)	p	101.0	0.21	17.59	8.00	13.64	47.07	23.89
Recovery (per cent)	100y	0.82	0.18	19.09	9.00	-1.73	34.34	11.37
			0.16	20.57	10.00	-16.24	21.98	-0.54

$$Cq - rtV + t\frac{dV}{dT} - q\frac{dV}{dS} = 0 \qquad [7]$$

Solve Equation [7] for the variables of interest. So we have

$$q\frac{dV}{dS} = Cq - t\left(rV + \frac{dV}{dT}\right)$$

$$\frac{dV}{dS} = C - \frac{t}{q}\left(rV + \frac{dV}{dT}\right)$$

and if

$$F = \left(rV + \frac{dV}{dT}\right) \text{ and } \tau = \frac{t}{q}$$

we can make the appropriate substitutions and we have the result that the *maximum PV of a mining operation* based on a finite resource S, is

$$\frac{dV}{dS} = C - F\tau \qquad [8]$$

which was the elegant solution that Lane derived. Furthermore, we can write an equation for the change of value of the mining operation with time:

$$\frac{dV}{dT} = \frac{q}{t}\times\left(\frac{dV}{dS} - C\right) + rV$$

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