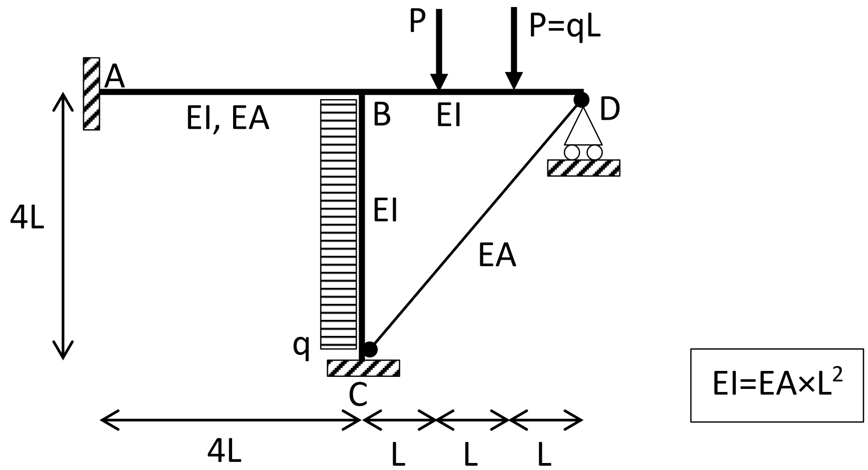
Examen

CI4202 - Análisis Estructural 2020-1
Profesores: J. Felipe Beltrán – Ricardo Herrera
Auxiliares: Benjamín Arellano – Maximiliano Zelada

Pregunta 1 (50%)

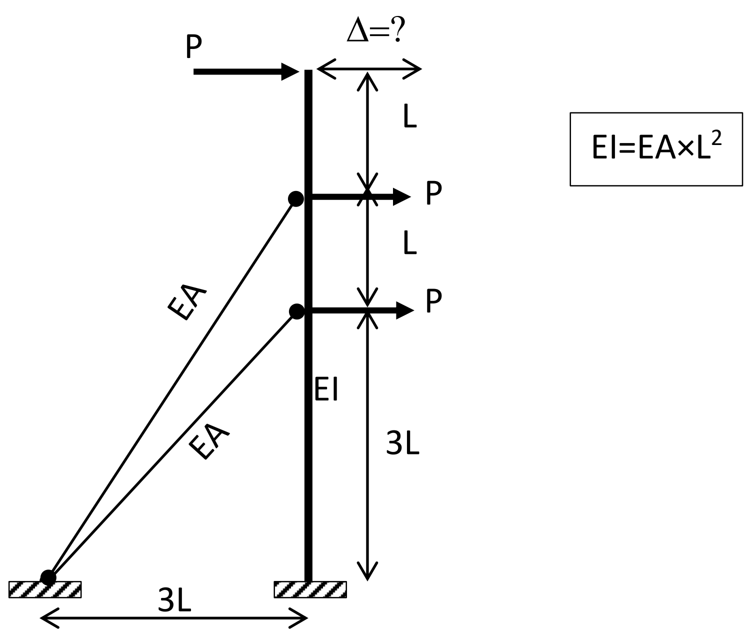
Encuentre diagramas de esfuerzo y reacciones del modelo de la figura utilizando el método de flexibilidad o el de rigidez. ¿Cuánto vale el desplazamiento horizontal del punto D?



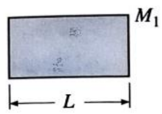
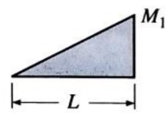
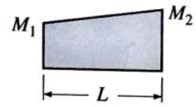
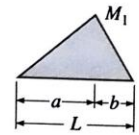
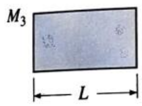
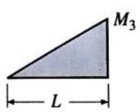
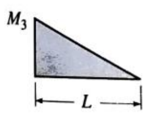
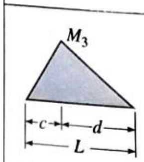
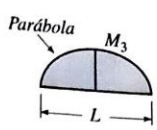
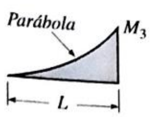
¿Cómo cambia el problema si la barra BD es axialmente deformable?

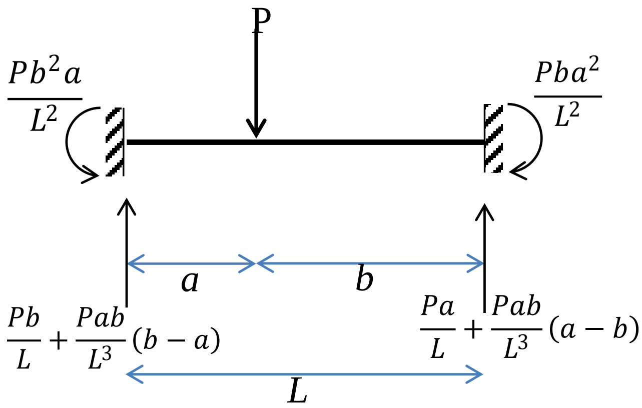
Pregunta 2 (50%)

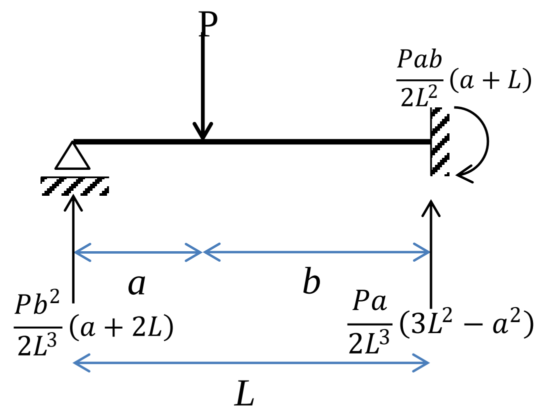
El modelo de la figura representa una torre de transmisión atirantada por cables en dos puntos en su altura. Las tres cargas P representan el efecto del viento sobre las antenas instaladas en la torre. Encuentre diagramas de esfuerzo y reacciones utilizando el método de flexibilidad o rigidez, pero sin repetir el método usado en la P1. Calcule el desplazamiento en la punta de la viga.

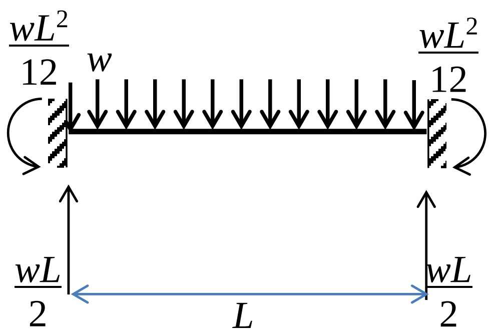


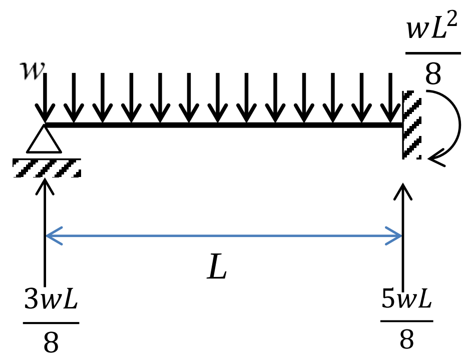
Valores de las integrales de productos $\int_{x=0}^{x=L} M_Q M_P \, dx$

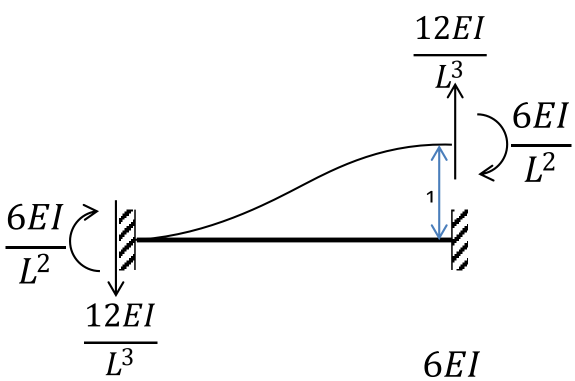
M_Q / M_P				
	M_1M_3L	$\frac{1}{2}M_1M_3L$	$\frac{1}{2}(M_1 + M_2)M_3L$	$\frac{1}{2}M_1M_3L$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{6}(M_1 + 2M_2)M_3L$	$\frac{1}{6}M_1M_3(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3L$	$\frac{1}{6}(2M_1 + 2M_2)M_3L$	$\frac{1}{6}M_1M_3(L + b)$
	$\frac{1}{2}M_1(M_3 + M_4)L$	$\frac{1}{6}M_1(M_3 + 2M_4)L$	$\frac{1}{6}M_1(2M_3 + M_4)L$ $+\frac{1}{6}M_2(M_3 + 2M_4)L$	$\frac{1}{6}M_1M_3(L + b)$ $+\frac{1}{6}M_1M_4(L + a)$
	$\frac{1}{2}M_1M_3L$	$\frac{1}{6}M_1M_3(L + c)$	$\frac{1}{6}M_1M_3(L + d)$ $+\frac{1}{6}M_2M_3(L + c)$	para $c \le a$: $\left(\frac{1}{3} - \frac{(a-c)^2}{6ad}\right)M_1M_3L$
	$\frac{2}{3}M_1M_3L$	$\frac{1}{3}M_1M_3L$	$\frac{1}{3}(M_1 + M_2)M_3L$	$\frac{1}{3}M_1M_3\left(L + \frac{ab}{L}\right)$
	$\frac{1}{3}M_1M_3L$	$\frac{1}{4}M_1M_3L$	$\frac{1}{12}(M_1 + 3M_2)M_3L$	$\frac{1}{12}M_1M_3\left(3a + \frac{a^2}{L}\right)$

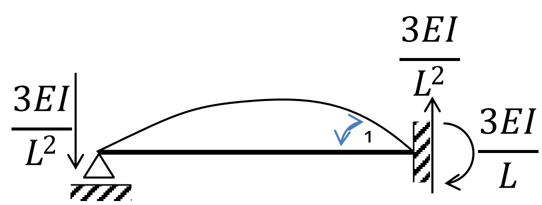

P
$\frac{Pb^2a}{L^2}$
$\frac{Pba^2}{L^2}$
a
b
$\frac{Pb}{L}+\frac{Pab}{L^3}(b-a)$
$\frac{Pa}{L}+\frac{Pab}{L^3}(a-b)$
L

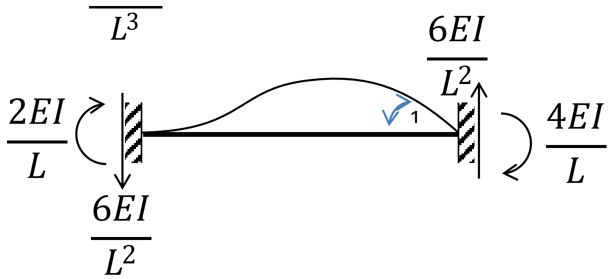

P
$\frac{Pab}{2L^2}(a+L)$
a
b
$\frac{Pb^2}{2L^3}(a+2L)$
$\frac{Pa}{2L^3}(3L^2-a^2)$
L

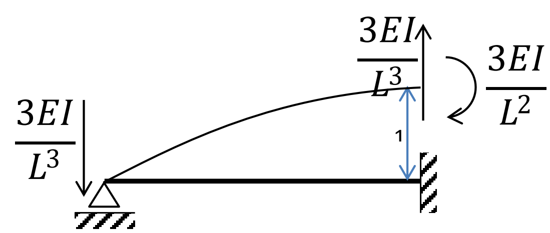

$\frac{wL^2}{12}$
w
$\frac{wL^2}{12}$
$\frac{wL}{2}$
L
$\frac{wL}{2}$


$\frac{wL^2}{8}$
w
L
$\frac{3wL}{8}$
$\frac{5wL}{8}$


$\frac{12EI}{L^3}$
$\frac{6EI}{L^2}$
1
$\frac{6EI}{L^2}$
$\frac{12EI}{L^3}$

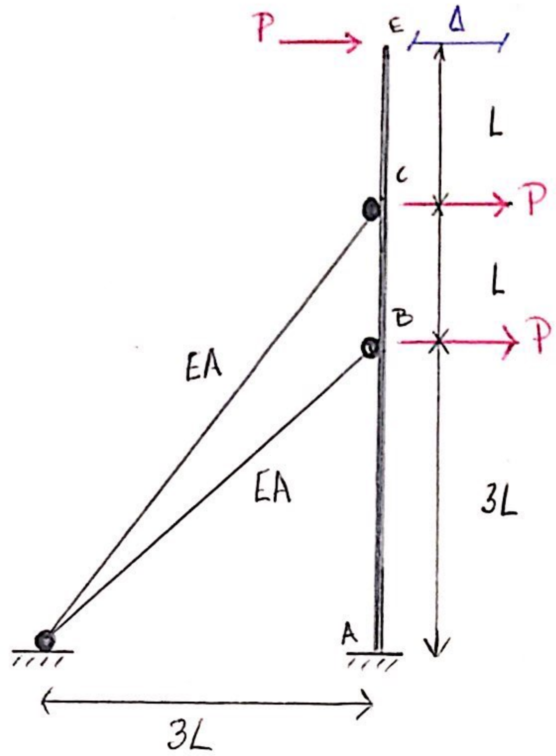

$\frac{3EI}{L^2}$
$\frac{3EI}{L^2}$
1
$\frac{3EI}{L}$


$\frac{6EI}{L^2}$
$\frac{2EI}{L}$
1
$\frac{4EI}{L}$
$\frac{6EI}{L^2}$


$\frac{3EI}{L^3}$
$\frac{3EI}{L^2}$
1
$\frac{3EI}{L^3}$

Pauta Examen N°1 - P2

Luis Carcamo



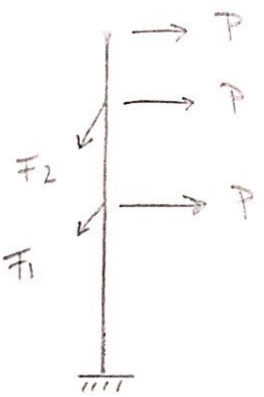
$EI = EA \times L^2$

- Determinar diagramas de esfuerzos
- Reacciones
- Desplazamiento en la punta de la viga

1ro: GIE

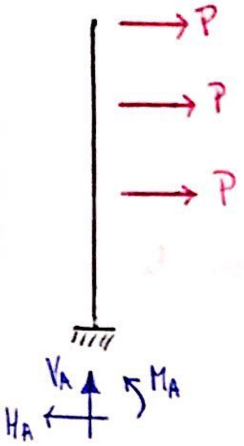
Voladizo + 2 bielas $\longrightarrow$ $\underline{GIE = 2}$

2do: Definir EIF



Escaneado con CamScanner

3ro: Cargas Reales $(F_1 = F_2 = 0)$

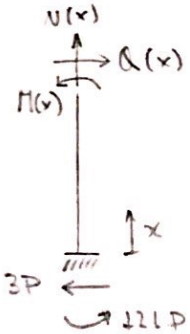


$\sum F_x = 0 \rightarrow H_A = 3P$

$\sum F_y = 0 \rightarrow V_A = 0$

$\sum M = 0 \rightarrow M_A = 12LP$

Tramo AB:

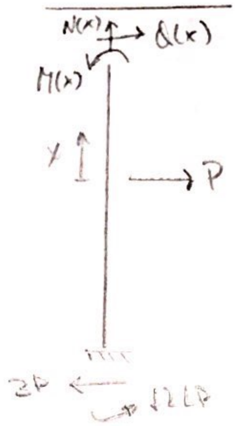


$N(x) = 0$

$Q(x) = 3P$

$M(x) = 3Px - 12LP$

Tramo BC:



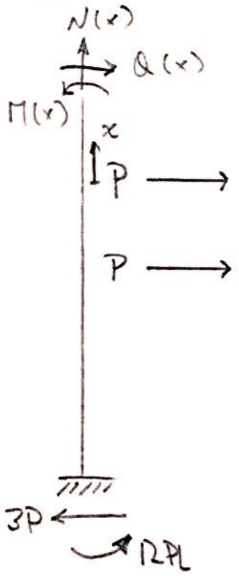
$N(x) = 0$

$Q(x) = 2P$

$M(x) = 3P(x+3L) - 12PL - Px$

$\hookrightarrow M(x) = P(2x - 3L)$

Tramo CD:



$N(x) = 0$

$Q(x) = P$

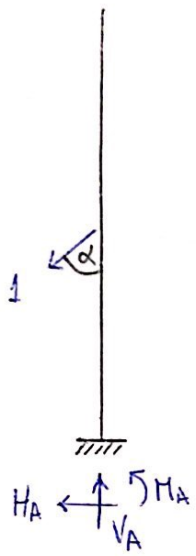
$M(x) = -12PL + 3P(x + 4L) - P(x+L) - Px$

$\hookrightarrow M(x) = P(x + L)$

Escaneado con CamScanner

4to: Cargas Virtuales

4.1: $F_1 = 1 \wedge F_2 = 0 \quad (\alpha = 45°)$

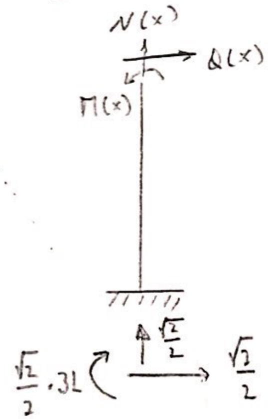


$$\sum F_x = 0 \rightarrow H_A = -\operatorname{sen}\alpha$$

$$\sum F_y = 0 \rightarrow V_A = \cos\alpha$$

$$\sum M = 0 \rightarrow M_A = -3L\operatorname{sen}\alpha$$

Tramo AB



$$N(x) = -\frac{\sqrt{2}}{2}$$

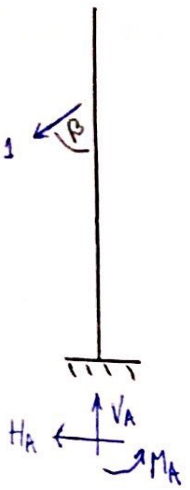
$$Q(x) = -\frac{\sqrt{2}}{2}$$

$$M(x) = \frac{3L\sqrt{2}}{2} - \frac{\sqrt{2}}{2}x$$

$$\hookrightarrow M(x) = \frac{\sqrt{2}}{2}(3L - x)$$

En los tramos BC y CD no hay fuerzas y por tanto tampoco esfuerzos.

4.2 $F_1 = 0 \wedge F_2 = 1$

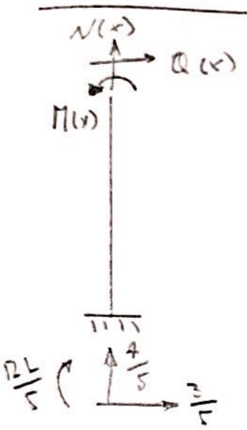


$\sum F_x = 0 \longrightarrow H_A = -\operatorname{sen}\beta$

$\sum F_y = 0 \longrightarrow V_A = \cos\beta$

$\sum M = 0 \longrightarrow M_A = -4L\operatorname{sen}\beta$

Tramo AC:



$N(x) = -\frac{4}{5}$

$Q(x) = -\frac{3}{5}$

$M(x) = \frac{12L}{5} - \frac{3}{5}x$

Tramo CD

↳ Esfuerzos nulos

5to: Determinación matriz de flexibilidad

$$f_{11} = \frac{1}{EI}\int_0^{3L}\left(-\frac{\sqrt{2}}{2}(3L-x)\right)^2 dx + \frac{1^2}{EA}\cdot 3L\sqrt{2} \longrightarrow \boxed{f_{11} = \frac{8{,}74L^3}{EI}}$$

$$f_{22} = \frac{1}{EI}\int_0^{4L}\left(\frac{12L}{5} - \frac{3}{5}x\right)^2 dx + \frac{1^2}{EA}\cdot 5L \longrightarrow \boxed{f_{22} = \frac{12{,}68L^3}{EI}}$$

$$f_{12} = f_{21} = \frac{1}{EI}\left[\int_0^{3L}\left(\frac{\sqrt{2}}{2}(3L-x)\cdot\left(\frac{12L}{5} - \frac{3}{5}x\right)\right)dx\right] \rightarrow \boxed{f_{12} = \frac{5{,}728L^3}{EI}}$$

6to Determinacion del vector de cargas

$$\Delta Q_1 = \frac{1}{EI}\left[\int_0^{3L}\left(\frac{\sqrt{2}}{2}(3L-x)\cdot(3Px-12PL)\,dx\right)\right] \longrightarrow \boxed{\Delta Q_1 = -\frac{28{,}64L^3P}{EI}}$$

$$\Delta Q_2 = \frac{1}{EI}\left[\int_0^{3L}\left(\frac{12L}{5}-\frac{3}{5}x\right)\cdot(3Px-12PL)\,dx + \int_0^{L}\left(\left(12\frac{L}{5}-\frac{3}{5}(3L+x)\right)\cdot P(2x-3L)\right)dx\right]$$

Ojo TRAMO BC

$$\longrightarrow \boxed{\Delta Q_2 = -\frac{38{,}5L^3P}{EI}}$$

7mo Método de Flexibilidad

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} = \begin{pmatrix} 1{,}828P \\ 2{,}210P \end{pmatrix} \longrightarrow \boxed{\begin{array}{c} F_1 = 1{,}828\,P \\ \hline F_2 = 2{,}210\,P \end{array}}$$

8vo: Reacciones

$$H_A = 3P - \frac{\sqrt{2}}{2}\cdot F_1 - \frac{3}{5}\cdot F_2 \longrightarrow \boxed{H_A = 0{,}38P}$$

$$Y_A = 0$$

$$M_A = 12PL - \frac{\sqrt{2}}{2}\cdot 3L\cdot F_1 - \frac{12}{5}L\cdot F_2 \longrightarrow \boxed{M_A = 2{,}82PL}$$

9no: Diagramas de esfuerzos internos

Tramo AB:

$$N_{AB} = -3{,}06P$$

$$Q_{AB} = 0{,}38P$$

$$M_{AB} = 0{,}3815(x-7{,}386L)$$

Escaneado con CamScanner

Tramo BC:

$N_{BC} = -1,77P$

$Q_{BC} = 0,67P$

$M_{BC} = 0,6734P(x - 2,485L)$

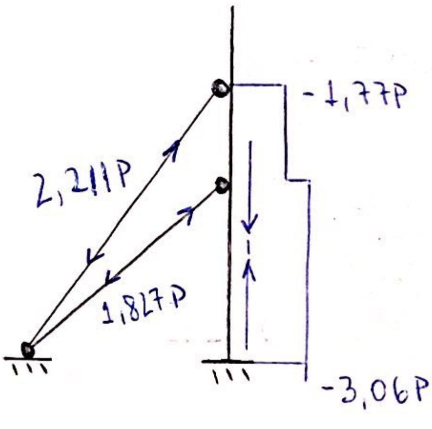
Tramo CD:

$N_{CD} = 0$

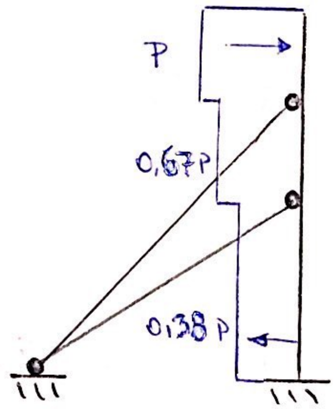
$Q_{CD} = P$

$M_{CD} = P(x - L)$

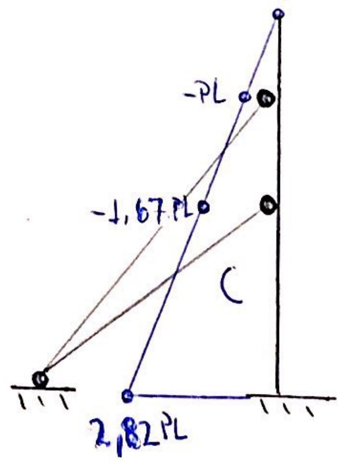
N(x):



Q(x):

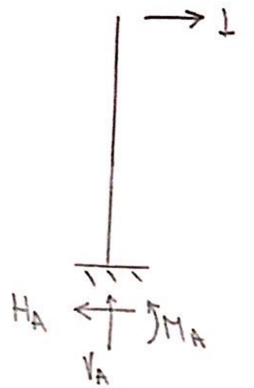


M(x):



10mo Determinación del desplazamiento Δ.

Usamos TTV.



$H_A = 1$

$V_A = 0$

$M_A = 5L$

Momentos

$M_{AB}(x) = -5L + x$

$M_{BC}(x) = -5L + (3L + x) = -2L + x$

$M_{CD}(x) = -5L + (4L + x) = -L + x$

$$\Delta = \frac{1}{EI}\left[\int_0^{3L}(-5L+x)(3Px-12PL)\,dx + \int_0^{L}(-2L+x)\cdot P(2x-3L)\,dx + \int_0^{L}(-L+x)P(x-L)\,dx\right]$$

$$\rightarrow \Delta = \frac{26{,}83\,PL^3}{EI}$$