Pauta Auxiliar 2 - P1

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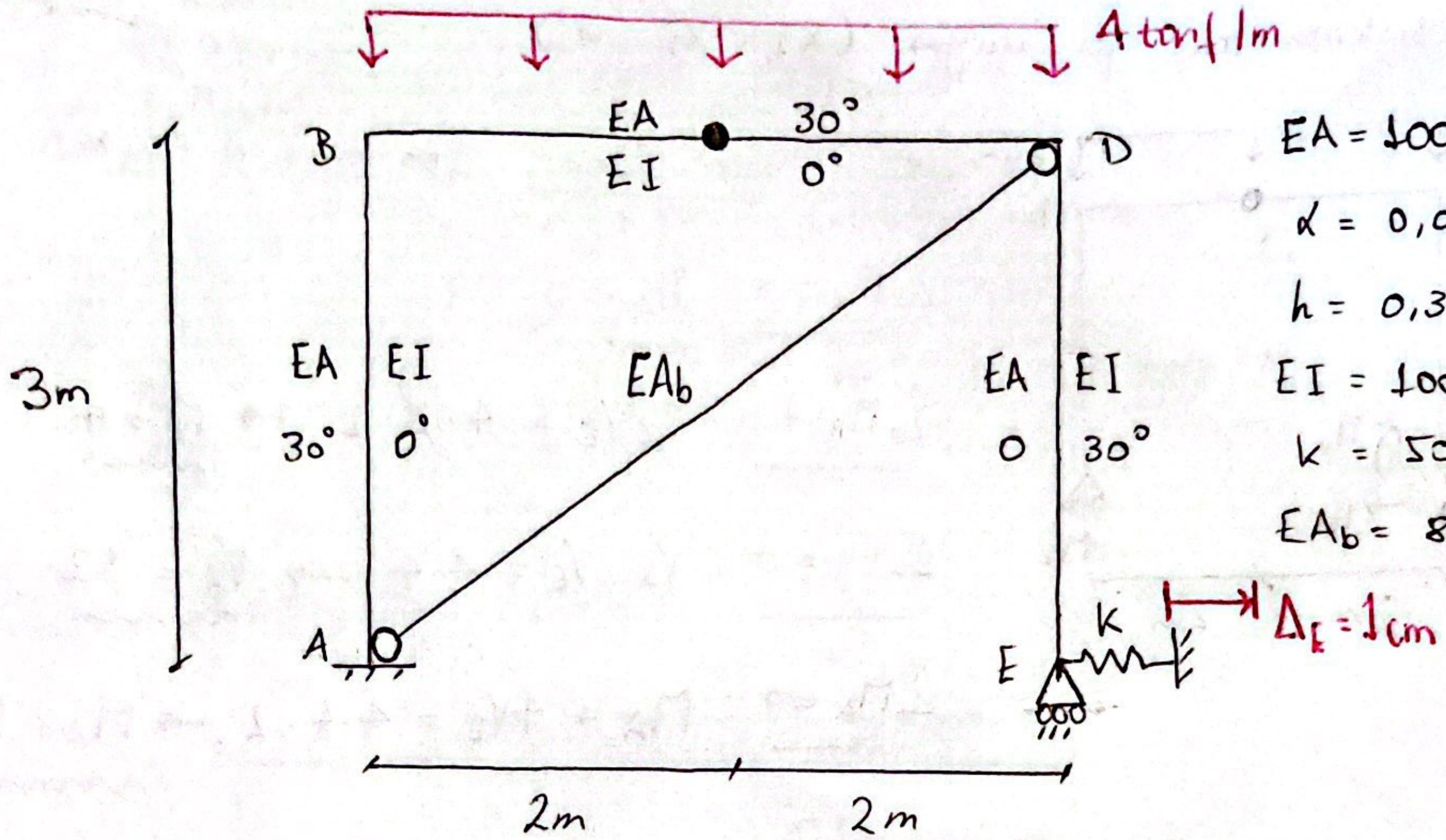


$EA = 10000$ [tonf]

$\alpha = 0{,}0001$ [$1/C°$]

$h = 0{,}3$ [m]

$EI = 1000$ [$tonf \cdot m^2$]

$k = 50$ [$tonf/m^2$]

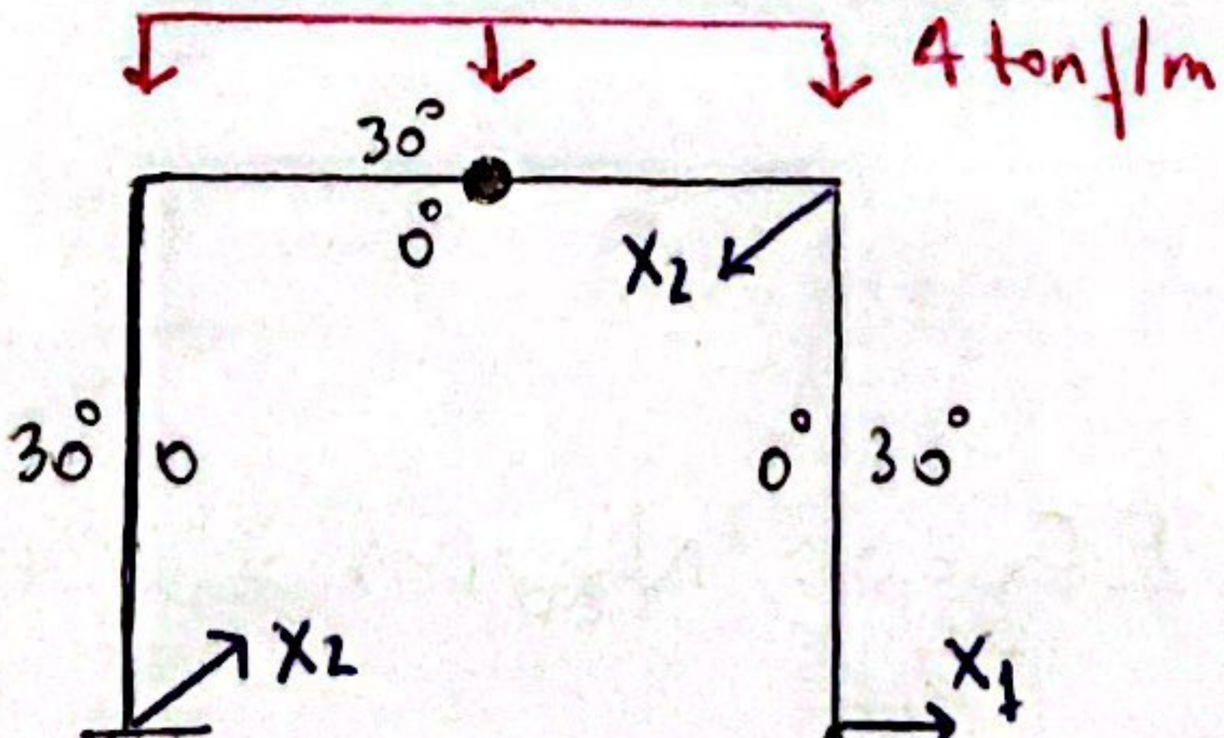
$EA_b = 800$ [tonf]

1ro : Determinar GIE. (# Incognitas - # Ecuaciones)

$$GIE = 3 + 2 + 1 - \underbrace{(3+1)}_{\substack{3 \text{ de equilibrio} \\ 1 \text{ de rótula}}} = 2$$

2do : Definir Estructura Isostatica Fundamental (EIF)

⊛ Se recomienda liberar bielas y resortes.

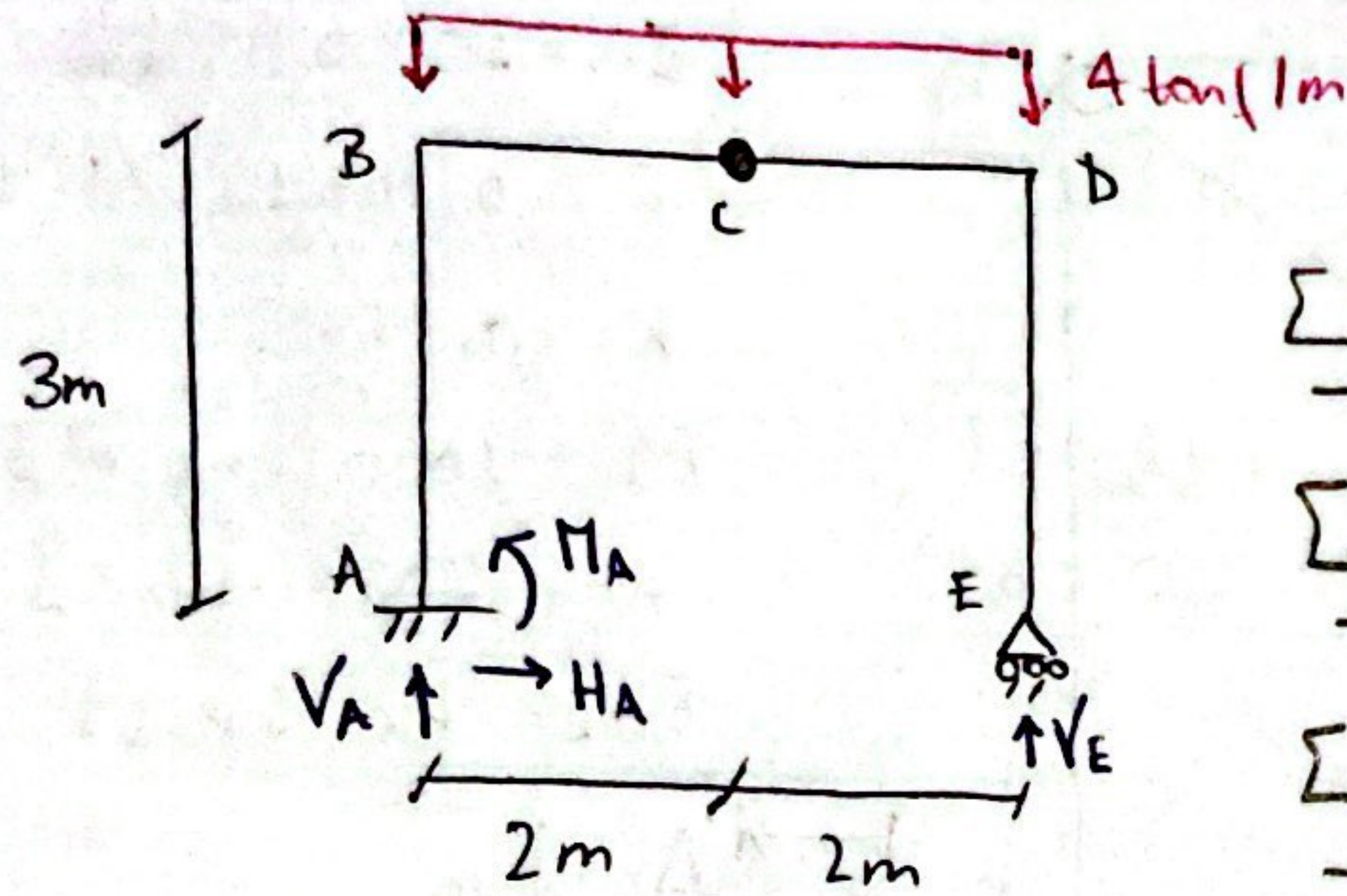


⊛ Recordando que el método de flexibilidad es:

$$\underbrace{\{\Delta\}_H}_{\text{(HIPERESTATICO)}} + \underbrace{\{\Delta\}_A}_{\text{(ASENTAMIENTO)}} = \underbrace{\{\Delta\}_Q}_{\text{(CARGA)}} + \underbrace{\{\Delta\}_T}_{\text{(TEMPERATURA)}} + \underbrace{\{\Delta\}_c}_{\text{(ERROR DE FABRI)}} + \underbrace{[F]}_{\text{[MATRIZ DE FLEX]}} \cdot \overbrace{\{R\}}^{\text{(REDUNDANTES)}}$$

3ro: Obtención de diagramas de esfuerzos internos (EIF, X_1, X_2)

3.1: EIF, sistema real de cargas ($X_1 = X_2 = 0$)



Reacciones: (en ton ó ton m)

$\Sigma F_x = 0: \; H_A = 0$

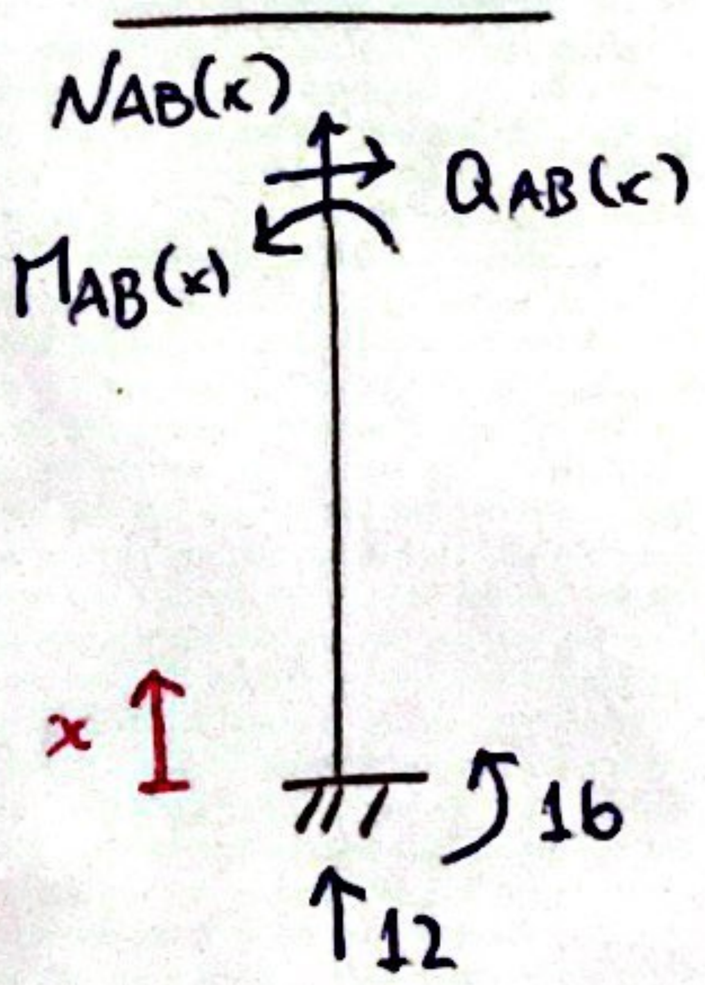
$\Sigma \vec{M}_R = 0: \; \cancel{2} V_E = 4 \cdot \cancel{2} \cdot 1 \rightarrow V_E = 4$

$\Sigma F_y = 0: \; V_A + V_E = 4 \cdot 4 \rightarrow V_A = 12$

$\Sigma M_A = 0: \; M_A + 4V_E = 4 \cdot 4 \cdot 2 \rightarrow M_A = 16$

D.E.I:

- Tramo AB: $x \in [0,3]$

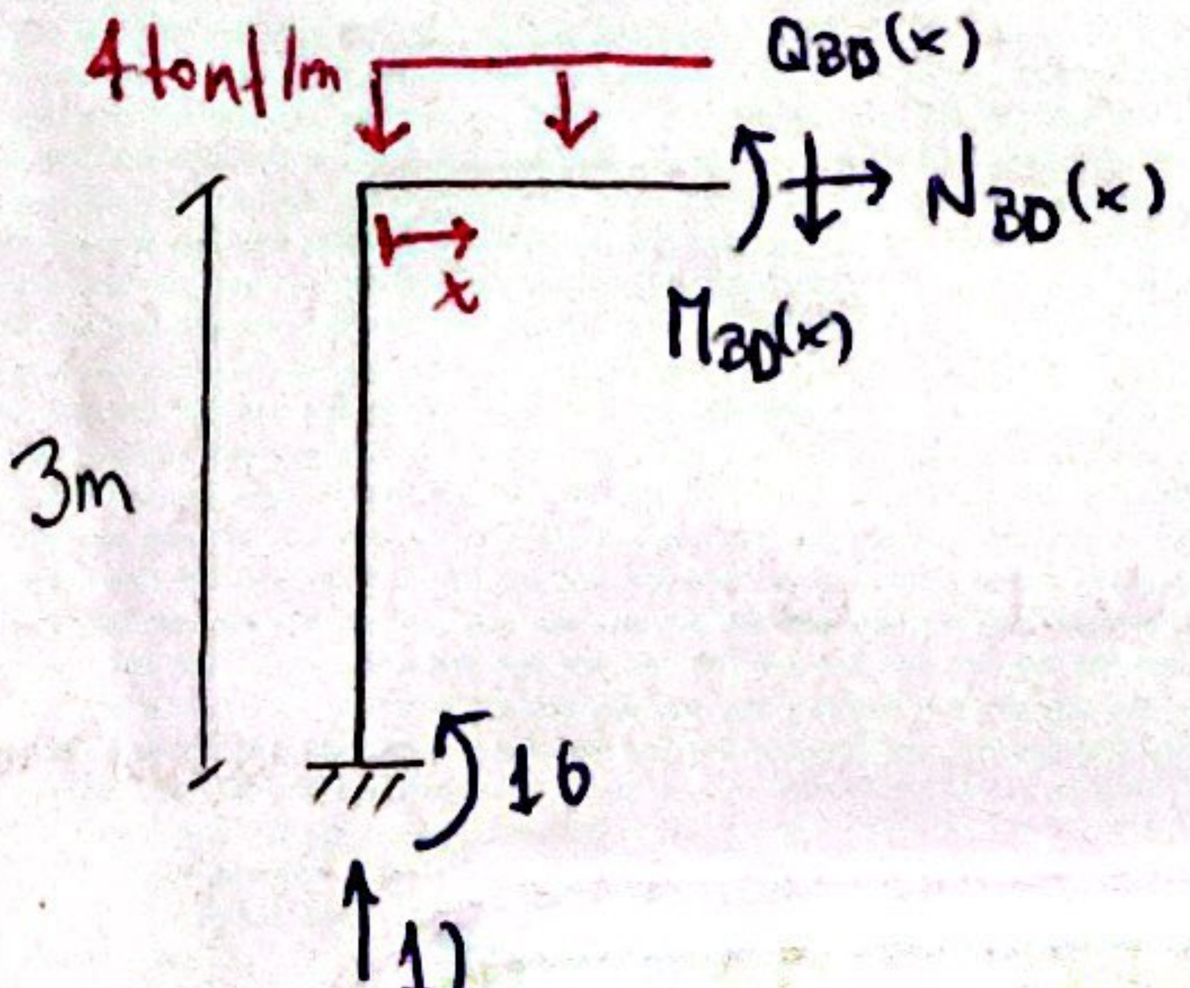


$\Sigma F_y = 0: \; N_{AB}(x) = -12$ ton

$\Sigma F_x = 0: \; Q_{AB}(x) = 0$ ton

$\Sigma M_x = 0: \; M_{AB}(x) = -16$ ton-m

- Tramo BD: $x \in [0,4]$



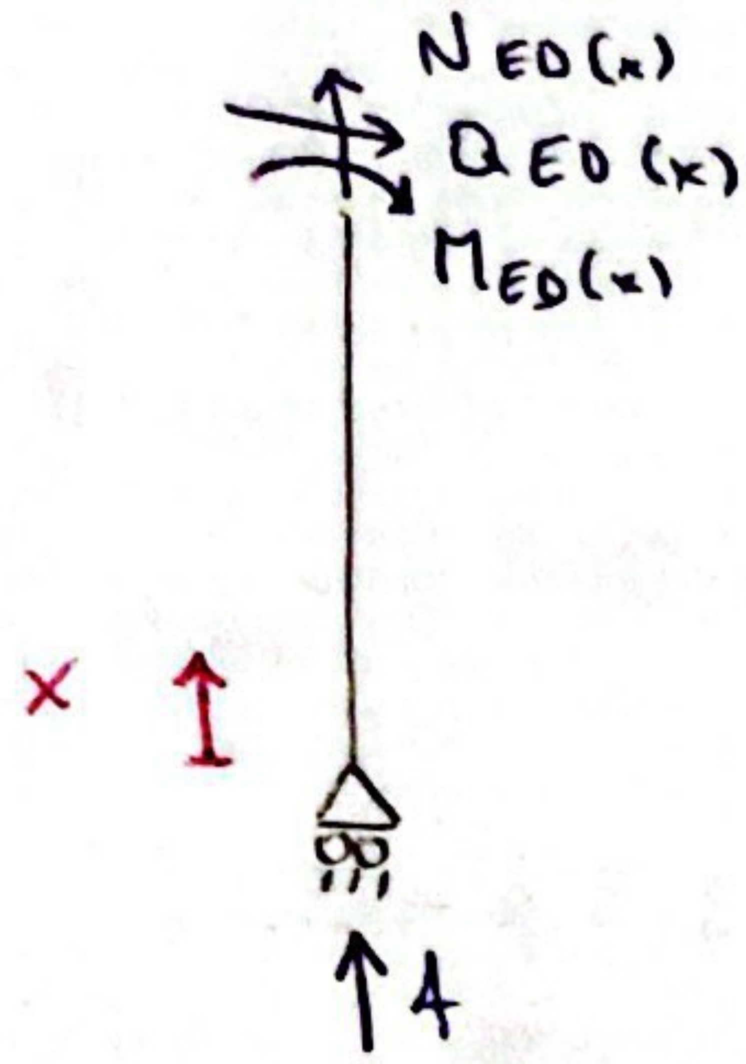
$\Sigma F_x = 0: \; N_{BD}(x) = 0$ ton

$\Sigma F_y = 0: \; Q_{BD}(x) = 12 - 4x$

$\Sigma M_x = 0: \; M_{BD}(x) = -16 + 12x - 2x^2$

(Evaluar en $x = 0$, $x = 2$ y $x = 4$)

- Tramo ED : $x \in [0,3]$

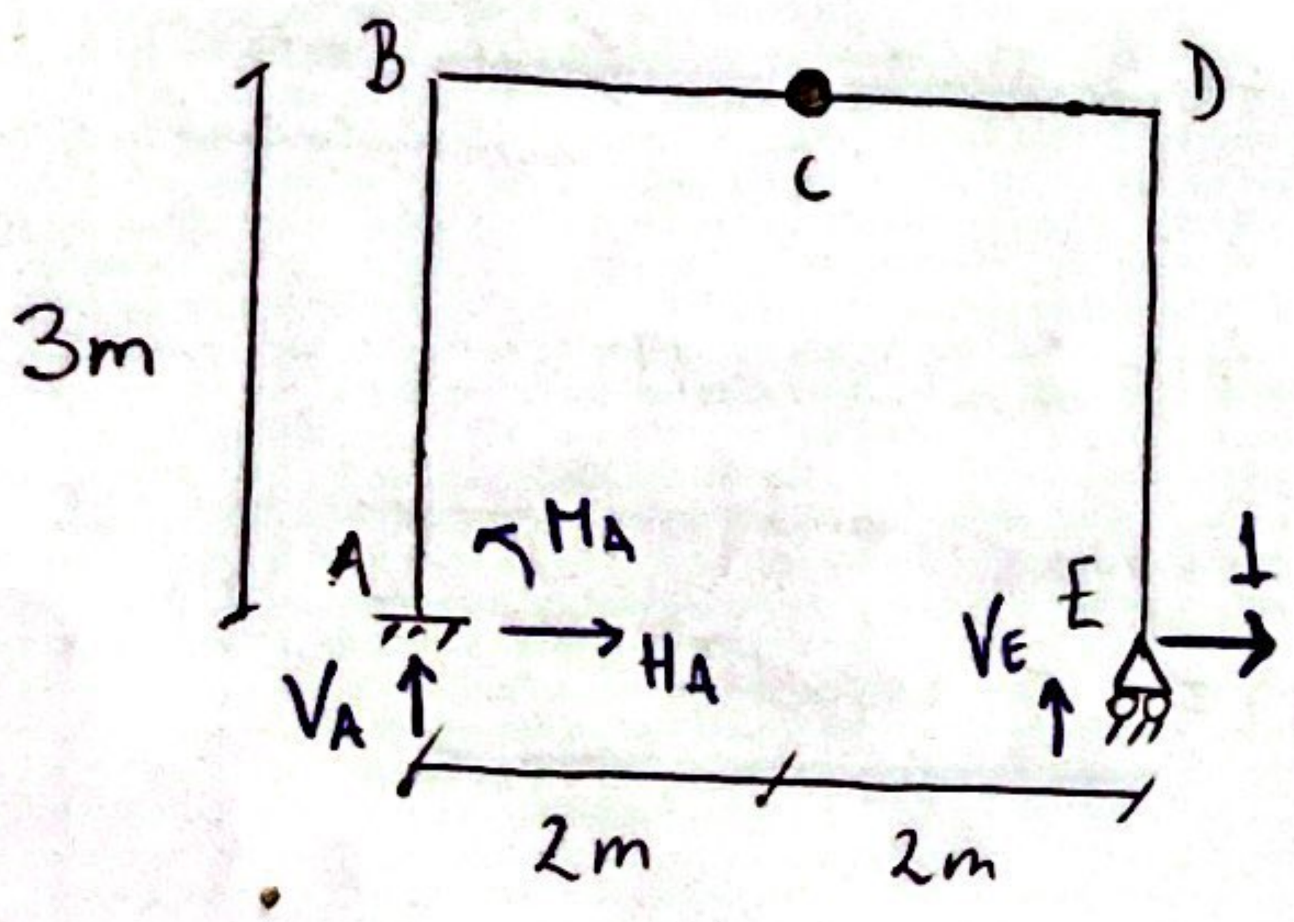


$\underline{\Sigma F_y = 0}$: $N_{ED}(x) = -4$ tonf

$\underline{\Sigma F_x = 0}$: $Q_{ED}(x) = 0$ tonf

$\underline{\Sigma M_x = 0}$: $M_{ED}(x) = 0$ tonf - m

3.2 : $(x_1 = 1 \wedge x_2 = 0)$



Reacciones :

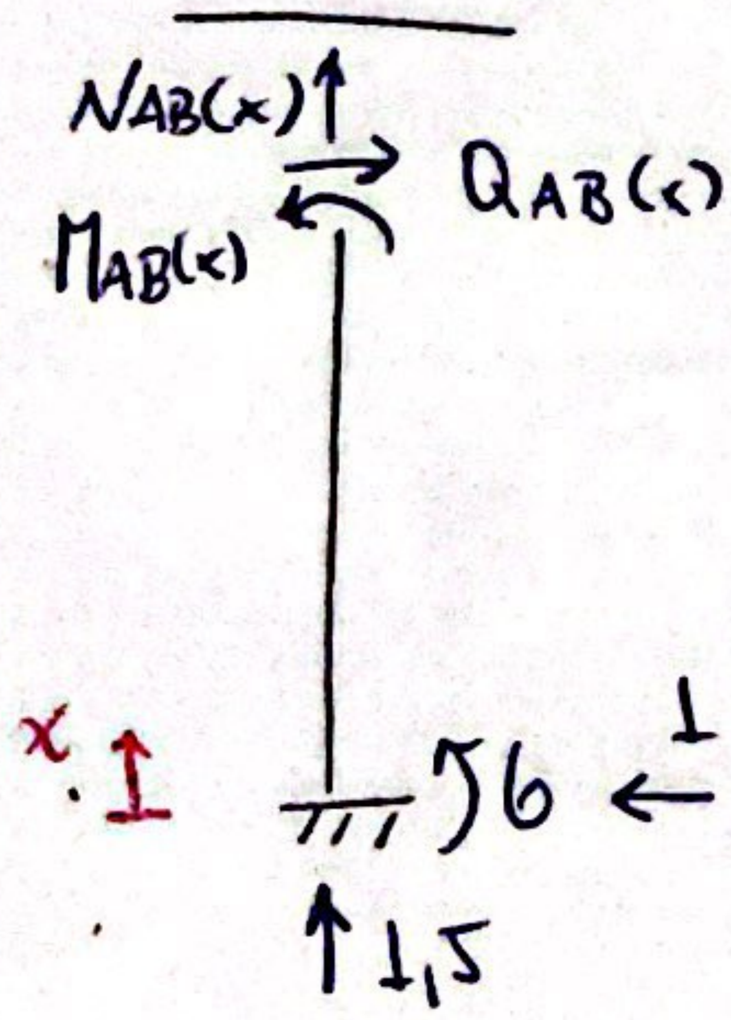
$\underline{\Sigma F_x = 0}$: $H_A = -1$

$\underline{\Sigma \vec{M}_R = 0}$: $2V_E = -1 \cdot 3 \rightarrow V_E = -1,5$

$\underline{\Sigma F_y = 0}$: $V_A + V_E = 0 \rightarrow V_A = 1,5$

$\underline{\Sigma M_A = 0}$: $M_A + 4V_E \rightarrow M_A = 6$

- Tramo AB: $x \in [0,3]$

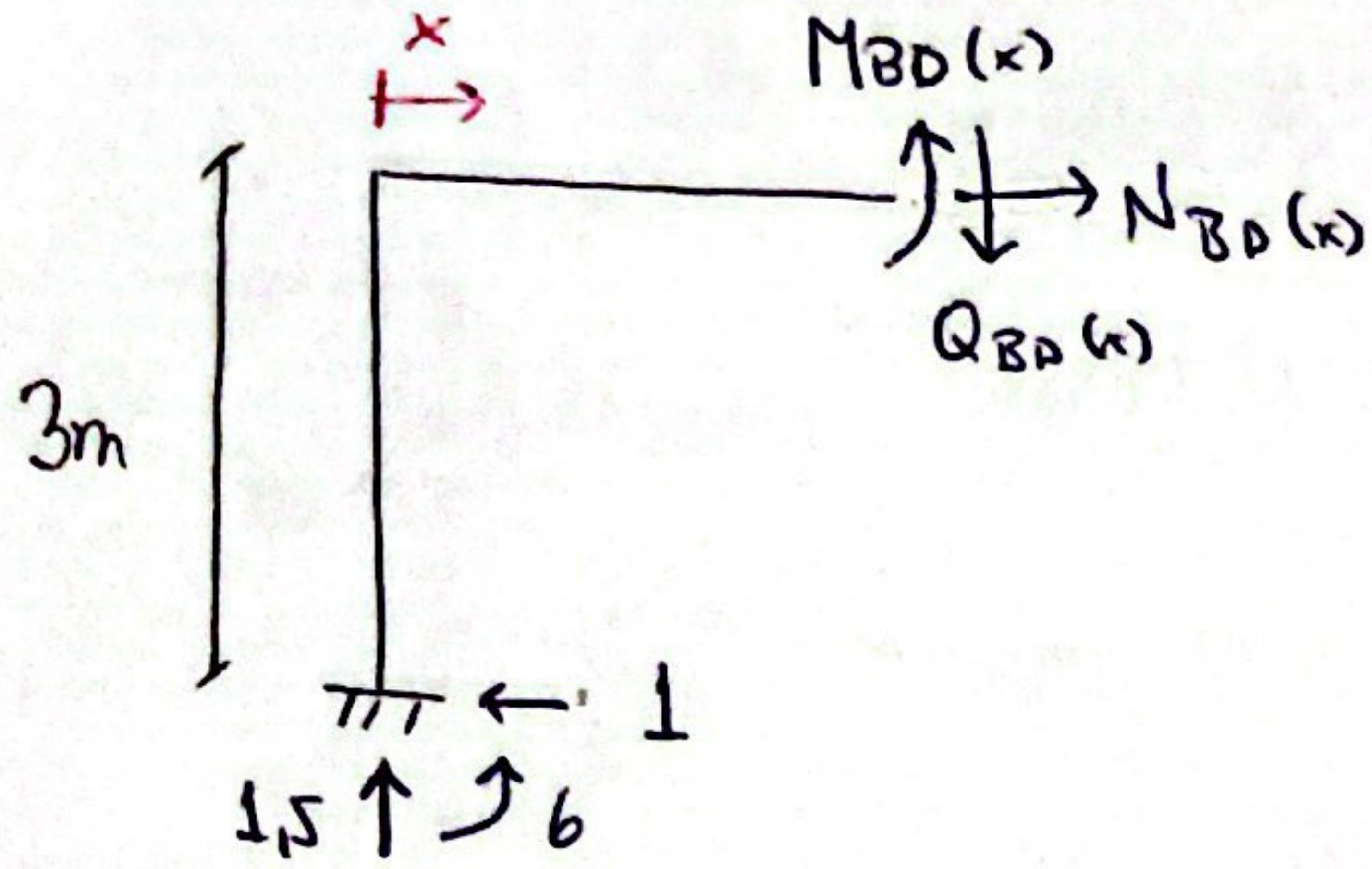


$\underline{\Sigma F_y = 0}$: $N_{AB}(x) = -1,5$

$\underline{\Sigma F_x = 0}$: $Q_{AB}(x) = 1$

$\underline{\Sigma M_x = 0}$: $M_{AB}(x) = x - 6$

- Tramo BD : $x \in [0,4]$

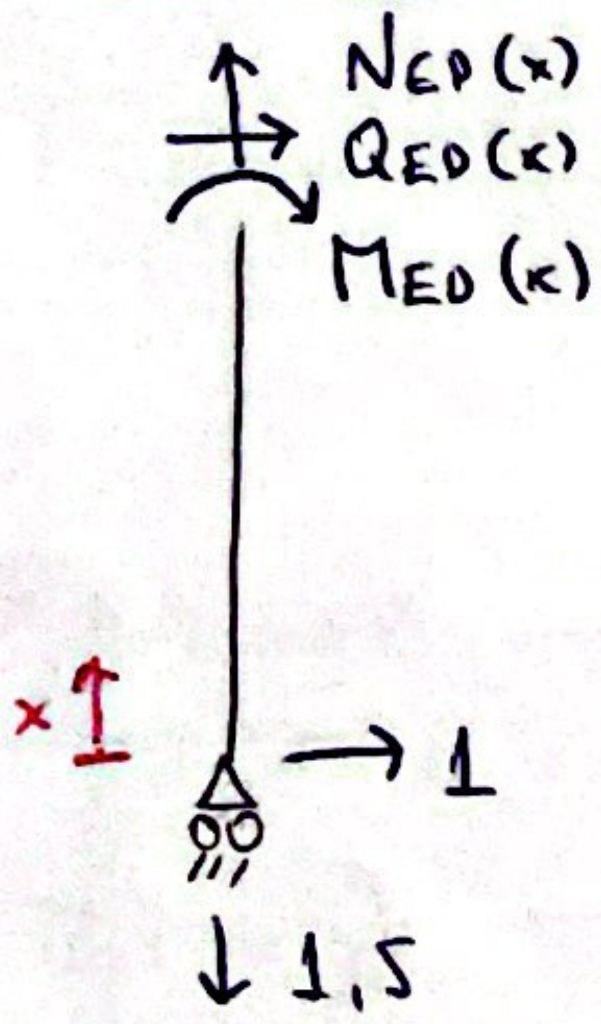


$\sum F_x = 0$: $N_{BD}(x) = 1$

$\sum F_y = 0$: $Q_{BD}(x) = 1,5$

$\sum M_x = 0$: $M_{BD}(x) = 1,5x - 3$

- Tramo ED : $x \in [0,2]$

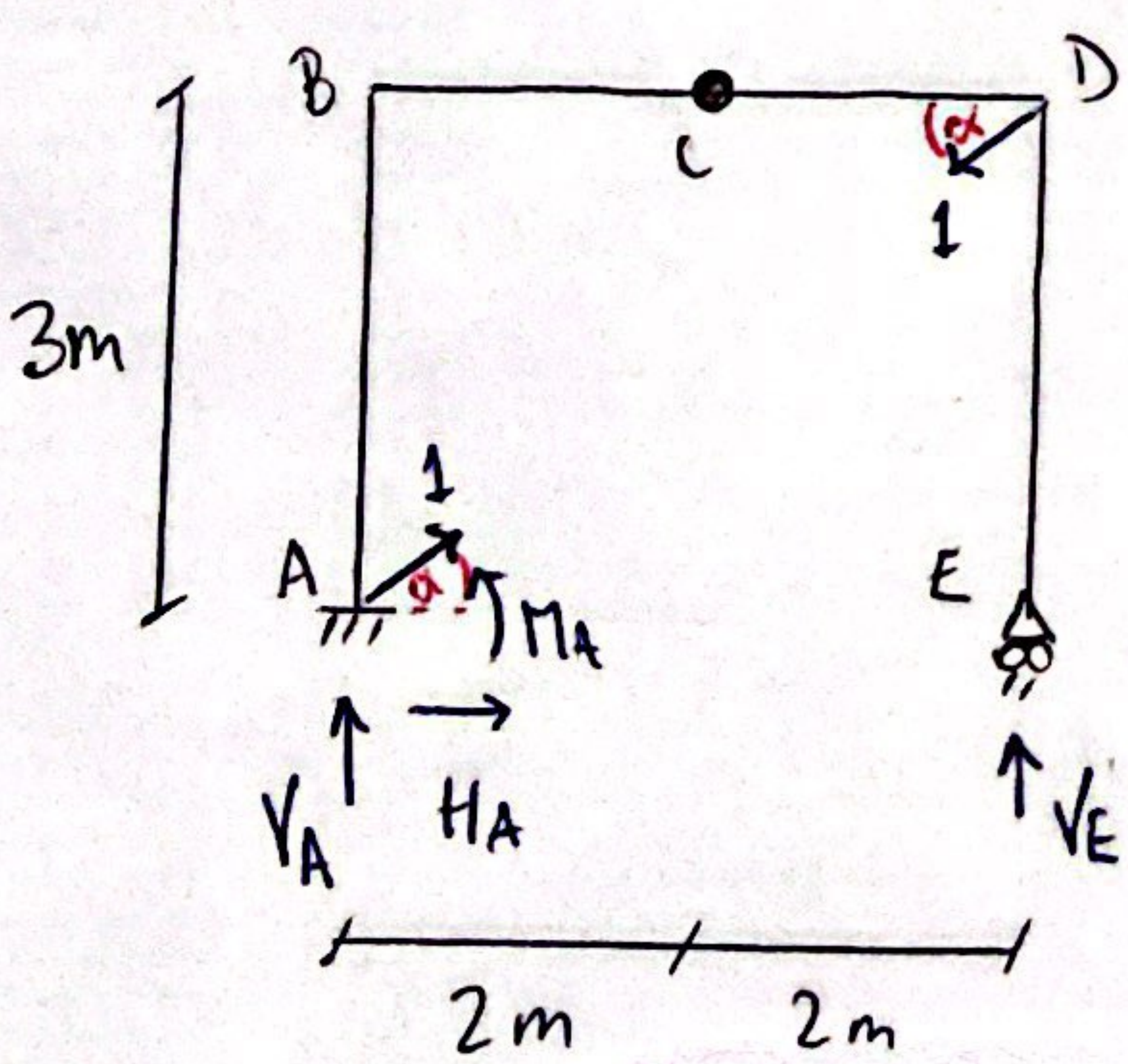


$\sum F_y = 0$: $N_{ED}(x) = 1,5$

$\sum F_x = 0$: $Q_{ED}(x) = -1$

$\sum M_x = 0$: $M_{ED}(x) = x$

3.3 : $(x_1 = 0 \wedge x_2 = 1)$



Reacciones :

$\sum F_x = 0$: $H_A = 0$

$\sum \overrightarrow{M_R} = 0$: $\cancel{2} V_E = \frac{3}{5} \cdot \cancel{2} \rightarrow V_E = 0,6$

$\sum F_y = 0$: $V_A + V_E = 0 \rightarrow V_A = -0,6$

$\sum M_A = 0$: $M_A + 4V_E + \cancel{\frac{4}{5} \cdot 3} - \cancel{\frac{3}{5} \cdot 4} = 0$

$\rightarrow M_A = -2,4$

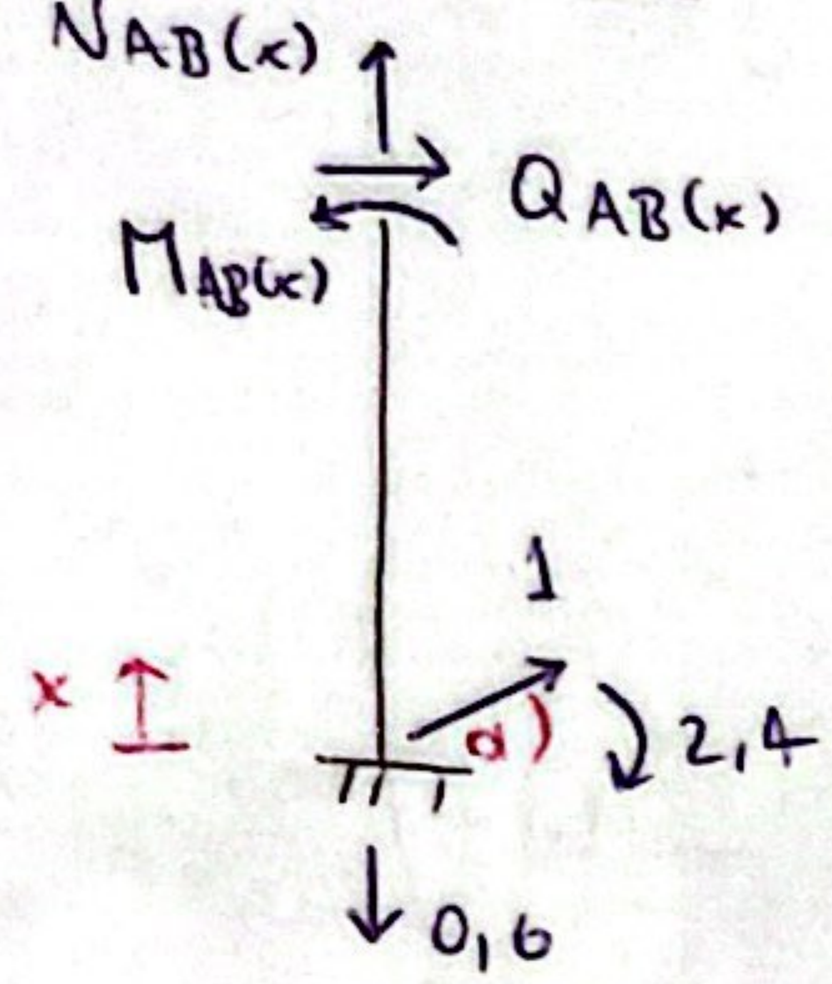
① Algunas cosas :

$L_B = 5m$.

$\sin \alpha = \frac{3}{5}$; $\cos \alpha = \frac{4}{5}$

D.E.I:

- Tramo AB: $x \in [0,3]$

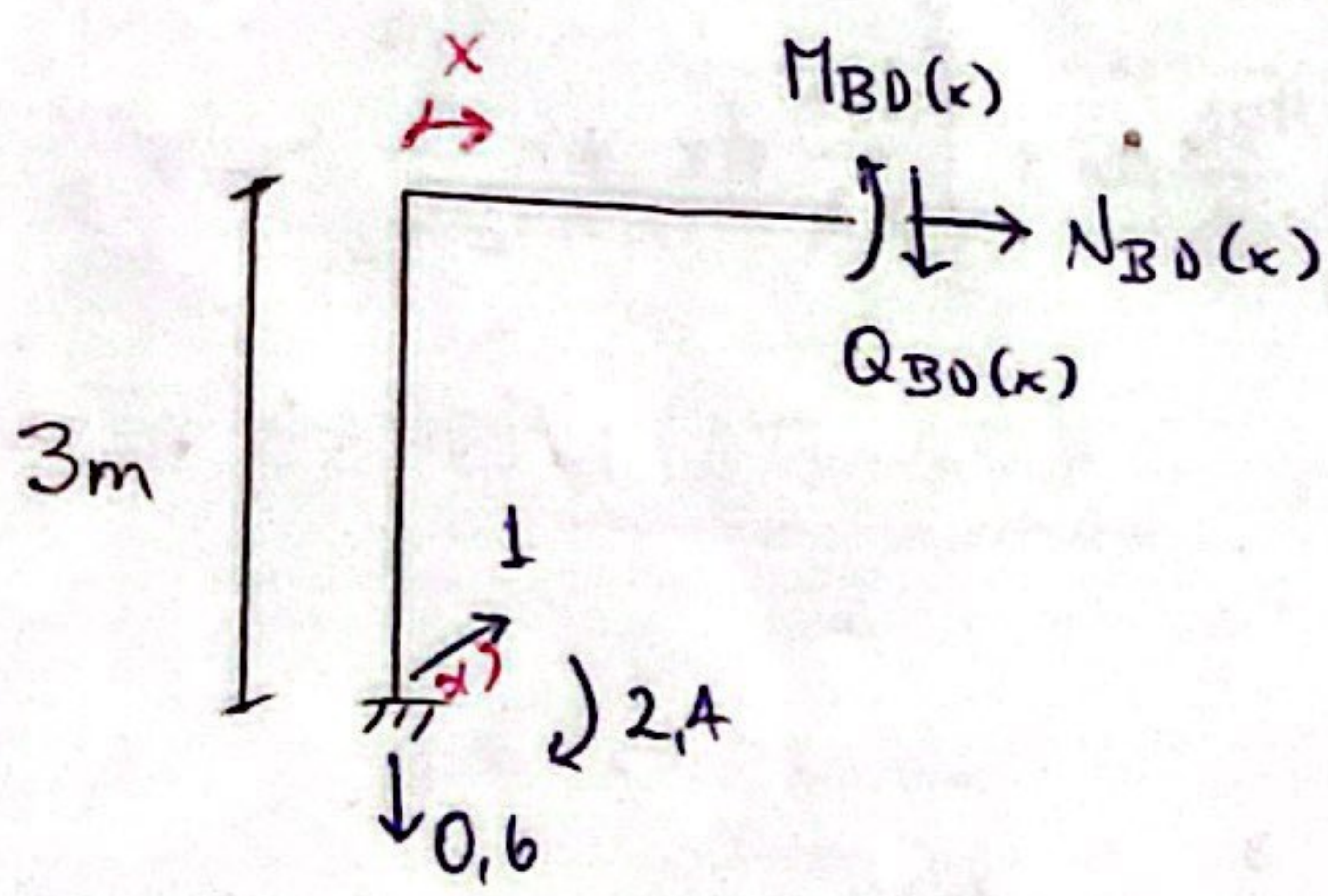


$\sum F_y = 0$: $N_{AB}(x) = 0$

$\sum F_x = 0$: $Q_{AB}(x) = -0{,}8$

$\sum M_x = 0$: $M_{AB}(x) = 2{,}4 - 0{,}8x$

- Tramo BD: $x \in [0,4]$

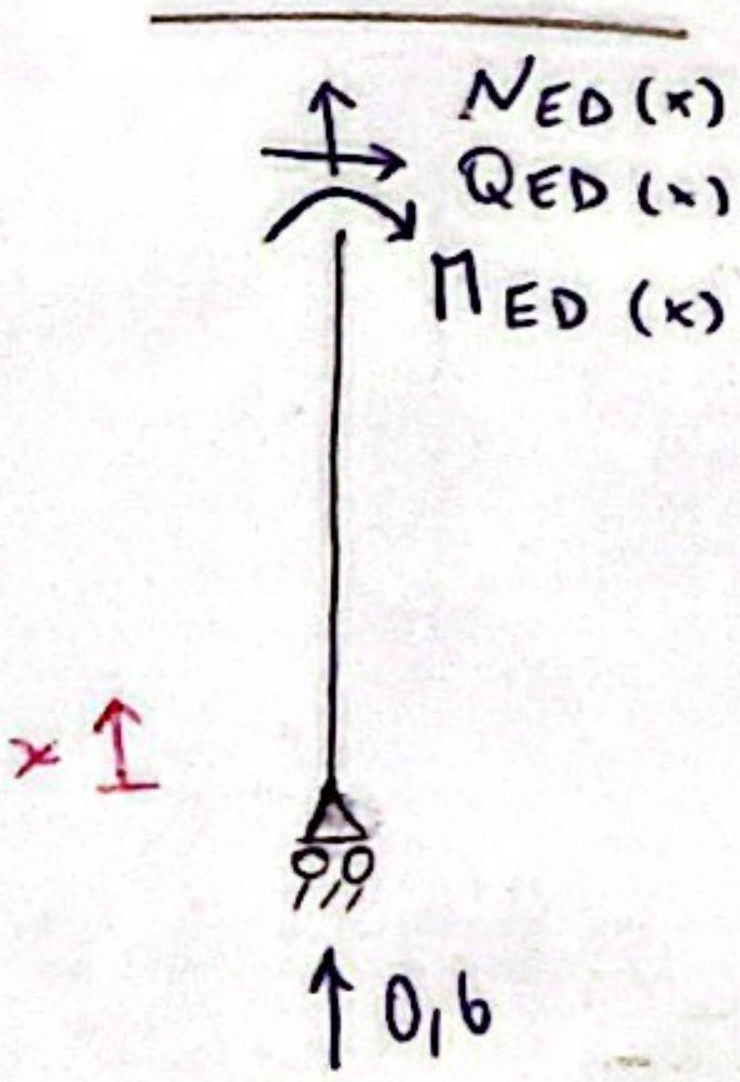


$\sum F_x = 0$: $N_{BD}(x) = -0{,}8$

$\sum F_y = 0$: $Q_{BD}(x) = 0$

$\sum M_x = 0$: $M_{BD}(x) = 0$

- Tramo ED: $x \in [0,3]$



$\sum F_y = 0$: $N_{ED}(x) = -0{,}6$

$\sum F_x = 0$: $Q_{ED}(x) = 0$

$\sum M_x = 0$: $M_{ED}(x) = 0$

$\underline{\underline{4^{to}}}$: Determinacion de coeficientes matriz de flexibilidad [F].

$$f_{ij} = \int_0^x \frac{M_i M_j}{EI} dx + \underbrace{\int_0^x \frac{N_i N_j}{EA} dx}_{} + \int_0^x \frac{Q_i Q_j}{GA_c} dx + \left(\sum_{i=1}^{k} \frac{F_i F_j}{k} \right) \quad \text{(RESORTE)}$$

!! ; No olvidar Bielas.

Calculamos:

$$f_{11} = \int_0^3 \frac{M_{AB}^2}{EI} dx + \int_0^4 \frac{M_{BD}^2}{EI} dx + \int_0^3 \frac{M_{DE}^2}{EI} dx + \int_0^3 \frac{N_{AB}^2}{EA} dx + \int_0^4 \frac{N_{BD}^2}{EA} dx + \int_0^3 \frac{N_{ED}^2}{EA} + \left(\frac{1^2}{k} \right)$$

RESORTE!!

$$\longrightarrow \boxed{f_{11} = 0,1058 \ m/tonf}$$

$$f_{22} = \int_0^3 \frac{M_{AB}^2}{EI} dx + \cancel{\int_0^4 \frac{M_{BD}^2}{EI} dx}^{\,0} + \cancel{\int_0^3 \frac{M_{ED}^2}{EI} dx}^{\,0} + \cancel{\int_0^3 \frac{N_{AB}^2}{EA} dx}^{\,0} + \int_0^4 \frac{N_{BD}^2}{EA} dx + \int_0^3 \frac{N_{ED}^2}{EA} dx + \left(\frac{1^2}{EA_b} \cdot L_B \right)$$

FUERZA BIELA!!

$$\longrightarrow \boxed{f_{22} = 0,0124 \ m/tonf}$$

$$f_{12} = f_{21} = \int_0^3 \frac{M_{AB}^{(1)} M_{AB}^{(2)}}{EI} dx + \int_0^4 \frac{N_{BD}^{(1)} N_{BD}^{(2)}}{EA} dx + \int_0^3 \frac{N_{ED}^{(1)} N_{ED}^{(2)}}{EA} dx$$

$$\longrightarrow \boxed{f_{12} = f_{21} = -0,0186 \ m/tonf}$$

$\underline{\underline{5^{to}}}$: Determinación del vector de carga $\{\Delta\}_Q$

$$\Delta_{Qi} = \int_0^x \frac{M_Q M_i}{EI} dx + \underbrace{\int_0^x \frac{N_Q N_i}{EA} dx}_{} + \int_0^x \frac{Q_Q Q_i}{GA_c} dx + \underbrace{\sum_{i=1}^{k} \frac{F_Q F_i}{k}}_{}$$

¡¡APLICAN BIELAS Y RESORTES!!

Calculamos: (Los terminos no considerados son xq' algún diag. = 0)

$$\Delta Q_1 = \int_0^3 \frac{M_{AB}^{EIF} M_{AB}^{(1)}}{EI} dx + \int_0^4 \frac{M_{BD}^{EIF} M_{BD}^{(1)}}{EI} + \int_0^3 \frac{N_{AB}^{EIF} N_{AB}^{(1)}}{EA} dx + \int_0^3 \frac{N_{ED}^{EIF} N_{ED}^{(1)}}{EA} dx$$

$$\longrightarrow \boxed{\Delta_{Q1} = 0{,}2516 \text{ m}}$$

$$\Delta Q_2 = \int_0^3 \frac{M_{AB}^{EIF} M_{AB}^{(2)}}{EI} dx + \int_0^3 \frac{N_{ED}^{EIF} N_{ED}^{(2)}}{EA} dx$$

$$\longrightarrow \boxed{\Delta Q_2 = -0{,}0569 \text{ m}}$$

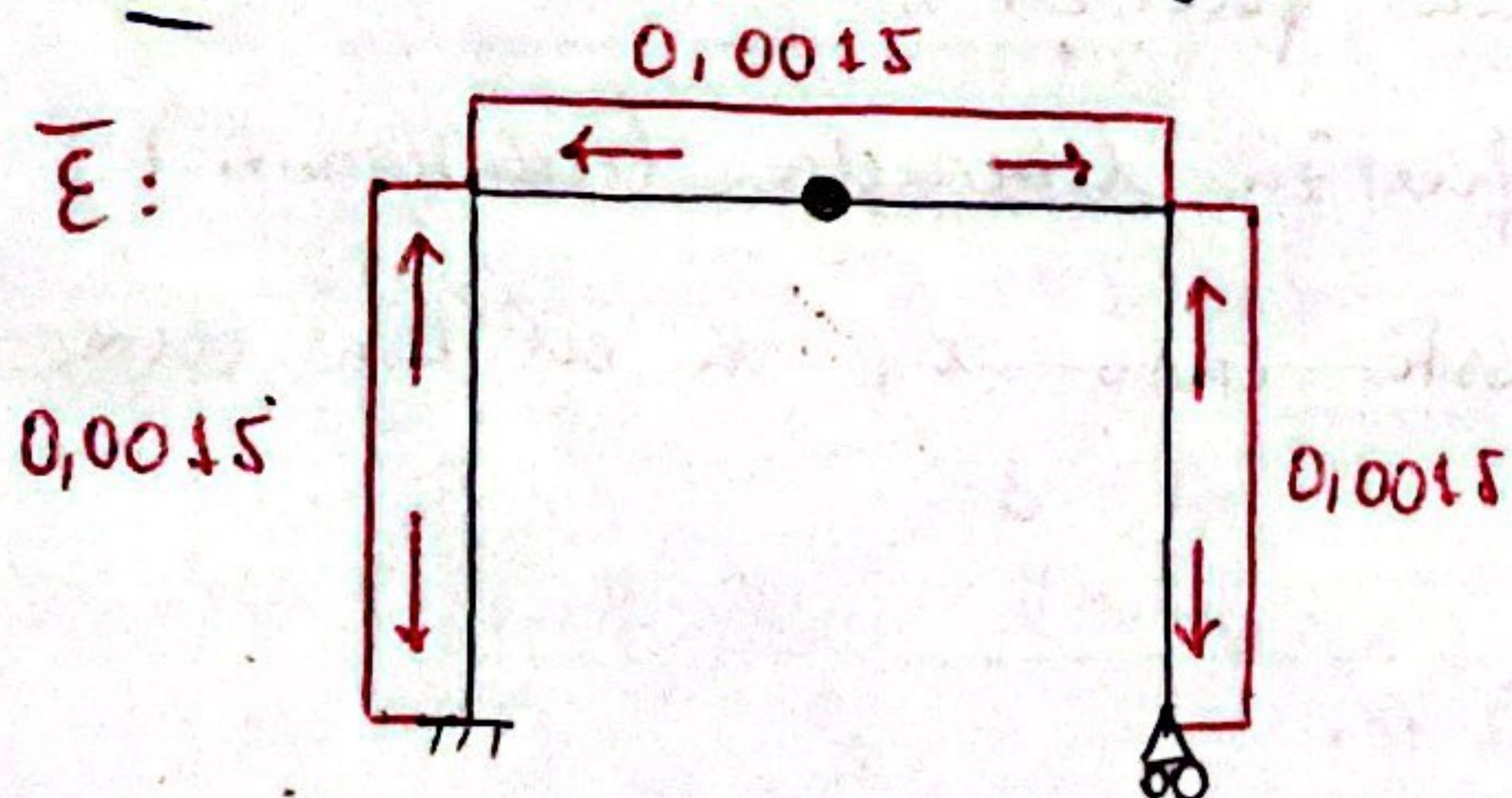
6to: Determinación del vector de temperatura $\{\Delta\}_T$

$$\bar{\varepsilon} = \frac{\alpha(\Delta T_1 + \Delta T_2)}{2} \quad \wedge \quad \bar{\phi} = \frac{\alpha(\Delta T_2 - \Delta T_1)}{h}$$

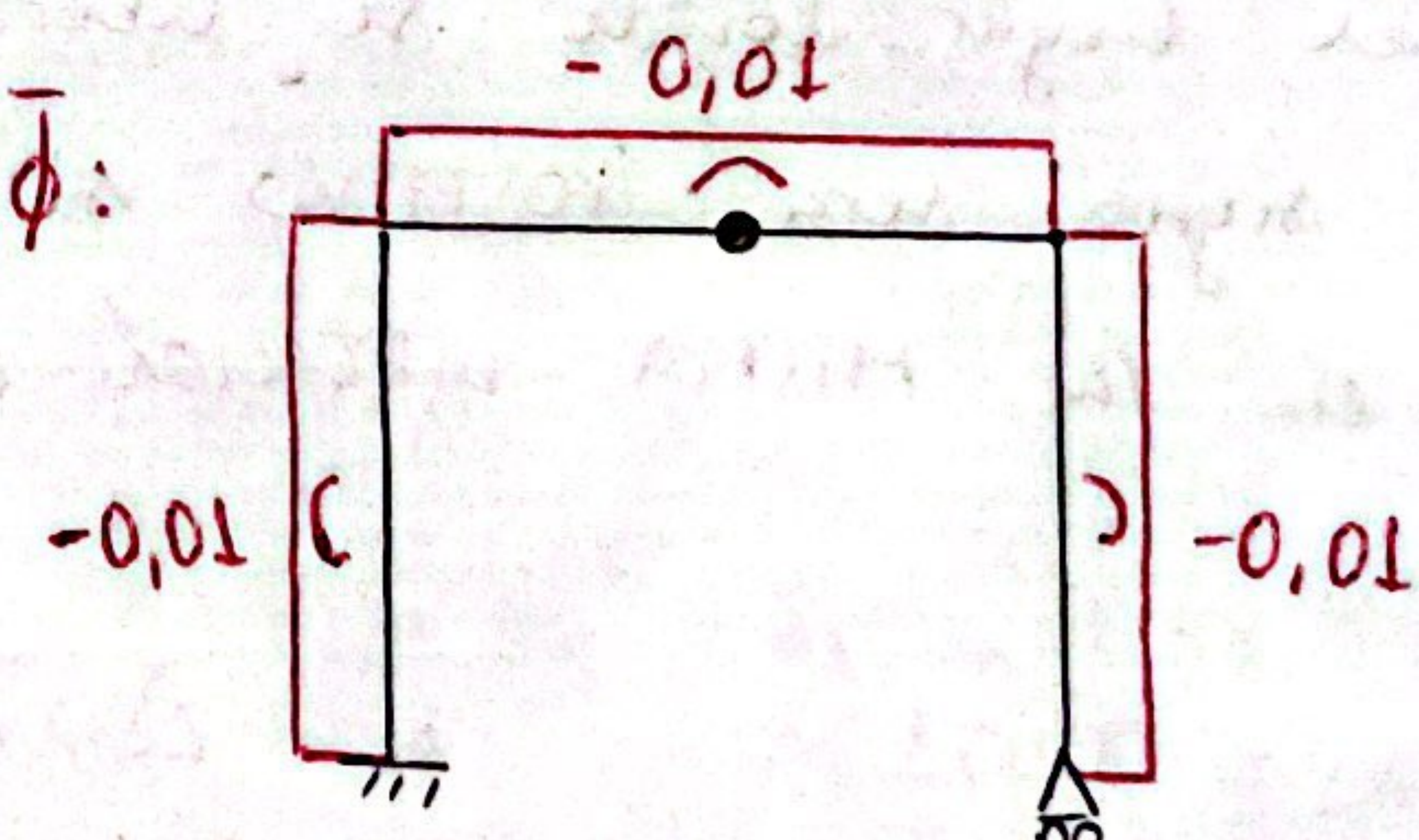
(Elongación) (curvatura)

6.1: Calculo de $\bar{\varepsilon}$ y $\bar{\phi}$:

$\bar{\varepsilon}$:



$\bar{\phi}$:



(el signo lo da la curvatura !!)

6.2: Cálculo $\{\Delta\}_T$:

$$\Delta_{Ti} = \sum_{barra} A_{M_i(x)} \cdot \bar{\phi} + \sum_{barra} A_{N_i(x)} \cdot \bar{\varepsilon}$$

$$\Delta_{T_1} = -0{,}01 \left[\int_0^3 M_{AB}^{(1)}\,dx + \int_0^4 M_{BD}^{(1)}\,dx + \int_0^3 M_{ED}^{(1)}\,dx \right]$$

$$+\, 0{,}0015 \left[\int_0^3 N_{AB}^{(1)}\,dx + \int_0^4 N_{BD}^{(1)}\,dx + \int_0^3 N_{ED}^{(1)}\,dx \right]$$

$$\longrightarrow \boxed{\Delta_{T_1} = 0{,}096\,m}$$

$$\Delta_{T_2} = -0{,}01 \left[\int_0^3 M_{AB}^{(2)}\,dx + \int_0^4 M_{BD}^{(2)}\,dx + \int_0^3 M_{ED}^{(2)}\,dx \right]$$

$$+\, 0{,}0015 \left[\int_0^3 N_{AB}^{(2)}\,dx + \int_0^4 N_{BD}^{(2)}\,dx + \int_0^3 N_{ED}^{(2)}\,dx \right]$$

$$\longrightarrow \boxed{\Delta_{T_2} = -0{,}0435\,m}$$

7mo: Determinación del vector de asentamientos $\{\Delta\}_A$

Este vector es nulo, a menos que haya un asentamiento en el lugar donde se liberó una fuerza.
El signo será POSITIVO si la fuerza liberada (redundante), va en la MISMA DIRECCIÓN (en este caso X_1 va en esa direc.).

$$\underbrace{\Delta_{A1} = 0{,}01\,m} \quad \wedge \quad \Delta_{A2} = 0\,m$$

¡¡ojo las unidades!!

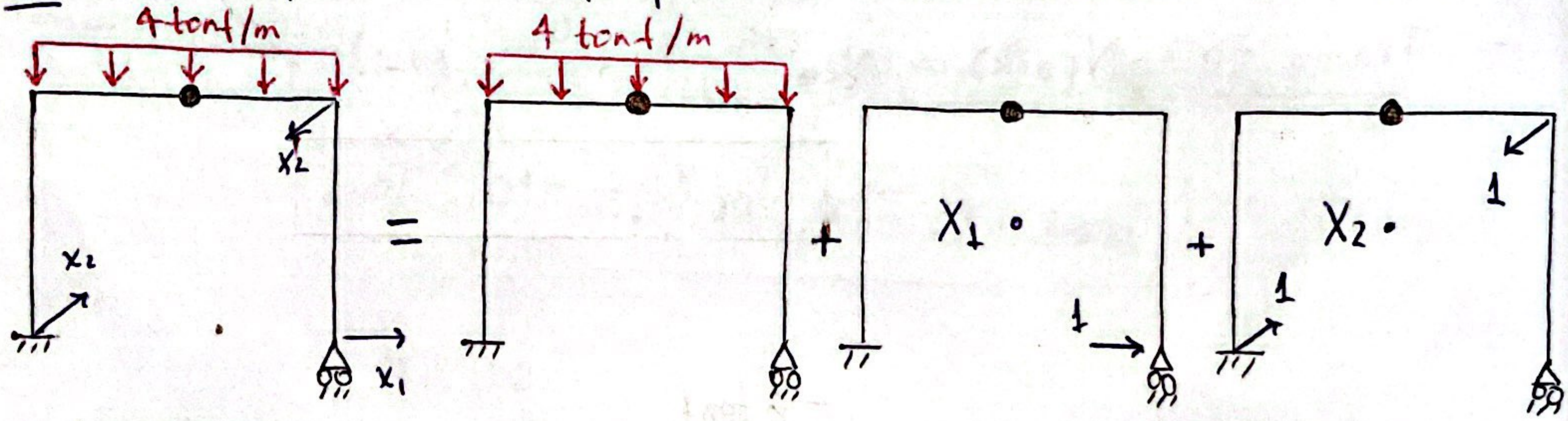
8vo: Resolver el problema de flexibilidad ᵕ̈.

$$\{\Delta\}_A = \{\Delta\}_Q + \{\Delta\}_T + [F]\{R\}$$

$$\rightarrow \begin{Bmatrix} 0,01 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0,2156 \\ -0,0569 \end{Bmatrix} + \begin{Bmatrix} 0,096 \\ -0,0435 \end{Bmatrix} + \begin{bmatrix} 0,1058 & -0,0186 \\ -0,0186 & 0,0124 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

$$\therefore \boxed{\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} -2,4 \\ 4,5 \end{Bmatrix} [\text{tonf}]}$$

9no: Principio de superposición → Reacciones + D.E.I



$$\boxed{\text{Estructura Real} = \text{EIF} + X_1 \cdot \text{Virtual 1} + X_2 \cdot \text{Virtual 2}}$$

9.1: Reacciones + FBIELA

1) $F_b = 4,5$ tonf (Tracción)

2) $H_E = -2,4$ tonf

3) $V_A = V_A^{EIF} + X_1 \cdot V_A^{(1)} + X_2 \cdot V_A^{(2)}$

$\rightarrow V_A = 5,7$ tonf

4) $H_A = H_A^{EIF} + X_1 \cdot H_A^{(1)} + X_2 \cdot H_A^{(2)}$

$\rightarrow H_A = 2,4$ tonf

5) $M_A = M_A^{EIF} + X_1 M_A^{(1)} + X_2 M_A^{(2)}$

$\rightarrow M_A = -9,2$ tonf-m

6) $V_E = V_E^{EIF} + X_1 V_E^{(1)} + X_2 V_E^{(2)}$

$\rightarrow V_E = 4,9$ tonf

9.2: Diagramas

$N(x)$

- **Tramo AB**: $N_{AB}(x) = N_{AB}^{EIF} + X_1 \cdot N_{AB}^{(1)} + X_2 \cdot N_{AB}^{(2)}$

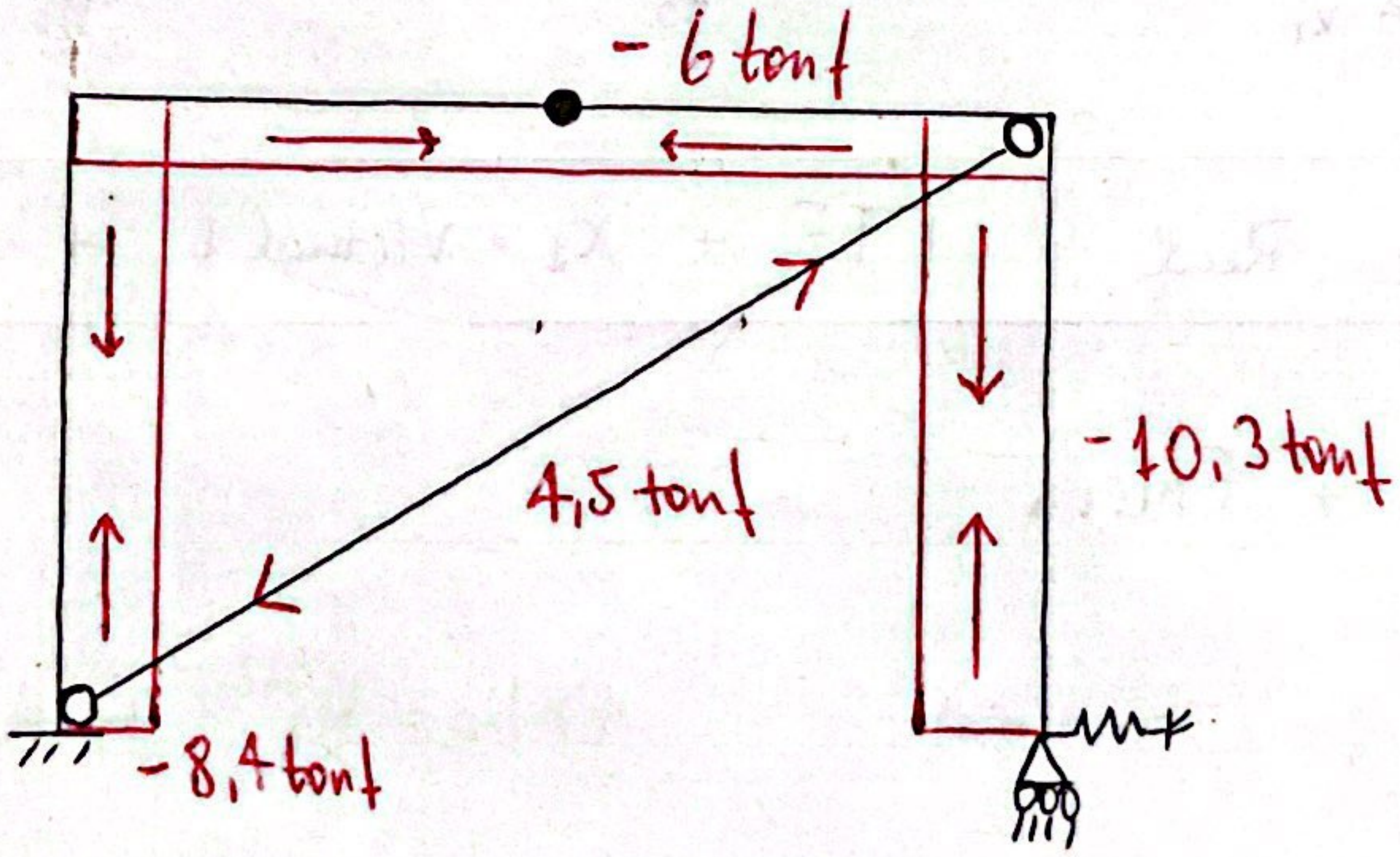
$$\longrightarrow \boxed{N_{AB}(x) = -8{,}4 \text{ tonf}}$$

- **Tramo BD**: $N_{BD}(x) = N_{BD}^{EIF} + X_1 \cdot N_{BD}^{(1)} + X_2 \cdot N_{BD}^{(2)}$

$$\longrightarrow \boxed{N_{BD}(x) = -6 \text{ tonf}}$$

- **Tramo ED**: $N_{ED}(x) = N_{ED}^{EIF} + X_1 \cdot N_{ED}^{(1)} + X_2 \cdot N_{ED}^{(2)}$

$$\longrightarrow \boxed{N_{ED}(x) = -10{,}3 \text{ tonf}}$$



$Q(x)$

- **Tramo AB:** $Q_{AB}(x) = Q_{AB}^{EIF} + X_1 \cdot Q_{AB}^{(1)} + X_2 \cdot Q_{AB}^{(2)}$

$$\longrightarrow \boxed{Q_{AB}(x) = -6\,\text{tonf}}$$

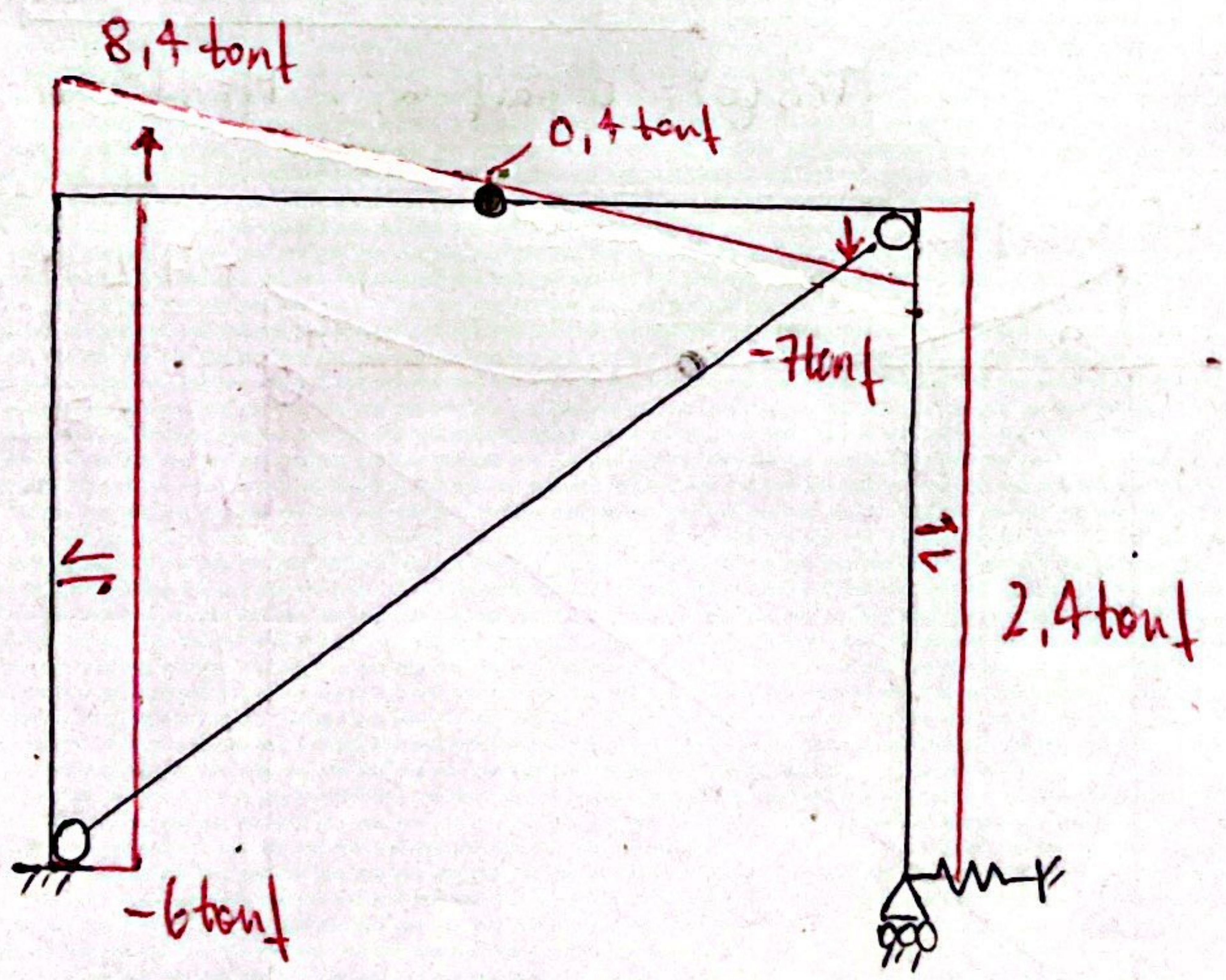
- **Tramo BD:** $Q_{BD}(x) = Q_{BD}^{EIF} + X_1 \cdot Q_{BD}^{(1)} + X_2 \cdot Q_{BD}^{(2)}$

$$\longrightarrow \boxed{Q_{BD}(x) = 8{,}4 - 4x}$$

$Q_{BD}(0) = 8{,}4$ tonf ; $Q_{BD}(2) = 0{,}4$ tonf ; $Q_{BD}(4) = -7{,}6$ tonf

- **Tramo ED:** $Q_{ED}(x) = Q_{ED}^{EIF} + X_1 \cdot Q_{ED}^{(1)} + X_2 \cdot Q_{ED}^{(2)}$

$$\longrightarrow \boxed{Q_{ED}(x) = 2{,}4\,\text{tonf}}$$



$M(x)$

- **Tramo AB:** $M_{AB}(x) = M_{AB}^{EIF} + X_1 \cdot M_{AB}^{(1)} + X_2 \cdot M_{AB}^{(2)}$

$$\longrightarrow \boxed{M_{AB}(x) = 9.2 - 6x}$$

$M_{AB}(0) = 9,2$ tonf-m ; $M_{AB}(3) = -8,8$ tonf-m

- **Tramo BD:** $M_{BD}(x) = M_{BD}^{EIF} + X_1 \cdot M_{BD}^{(1)} + X_2 \cdot M_{BD}^{(2)}$

$$\longrightarrow \boxed{M_{BD}(x) = -2x^2 + 8,4x - 8,8}$$

$M_{BD}(0) = -8,8$ tonf-m ; $M_{BD}(2) = 0$ tonf-m ; $M_{BD}(4) = -7,2$ tonf-m

- **Tramo ED:** $M_{ED}(x) = M_{ED}^{EIF} + X_1 \cdot M_{ED}^{(1)} + X_2 \cdot M_{ED}^{(2)}$

$$\longrightarrow \boxed{M_{ED}(x) = -2,4x}$$

$M_{ED}(0) = 0$ tonf-m ; $M_{ED}(3) = -7,2$ tonf-m