P1 Sea $y \in [0,1] \Rightarrow x \in [y,1] = D_y$

$$f_Y(y) = \int_{D_y} f_{X,Y}(x,y)\,dx = \int_y^1 8xy\,dx = 4y(1-y^2)$$

$$\Rightarrow f_{X|Y}(x|y) = \frac{8xy}{4y(1-y^2)} \mathbb{1}\{y<x<1\}$$

$$= \frac{2x}{(1-y^2)} \mathbb{1}\{y<x<1\}$$

$$\Rightarrow P(X<\tfrac{1}{2} \mid Y=y) = \int_{-\infty}^{1/2} \frac{2x}{(1-y^2)} \mathbb{1}\{y<x<1\}\,dx$$

Si $Y > \frac{1}{2}$	Si $Y \leq \frac{1}{2}$
$\Rightarrow \int_{-\infty}^{1/2} \mathbb{1}_{\{y<x<1\}}\,dx = \int_{-\infty}^{1/2} 0\,dx = 0$	$\int_{-\infty}^{1/2} \frac{2x}{(1-y^2)} \mathbb{1}\{y<x<1\}\,dx = \int_y^{1/2} \frac{2x}{(1-y^2)}\,dx$
$\Rightarrow P(X<\frac{1}{2} \mid Y=y) = 0$	$P(X<\frac{1}{2} \mid Y=y) = \frac{(1/2)^2 - y^2}{1^2 - y^2}$

$$E(X|Y) = \int_{-\infty}^{\infty} x \, f_{X|Y}(x|y)\,dx = \int_{-\infty}^{\infty} \frac{2x^2}{(1-y^2)} \mathbb{1}_{\{y<x<1\}}\,dx$$

$$= \int_{y}^{1} \frac{2x^2}{(1-y^2)}\,dx = \boxed{\frac{2}{3}\frac{(1-y^3)}{(1-y^2)}}$$

depende de y

$\Rightarrow$ No son ind

Para $E(Y|X)$ necesitamos $f_{Y|X}(y|x)$

$x \in [0,1] \Rightarrow y \in [0,x] = D_x$

$$f_X(x) = \int_{D_x} 8xy\,dy = \int_0^x 8xy\,dy = 4x^3$$

$$\Rightarrow f_{Y|X}(y|x) = \frac{2y}{x^2} \mathbb{1}_{\{0<y<x\}}$$

$$\Rightarrow E(Y|X) = \int_{-\infty}^{\infty} \frac{2y^2}{x^2} \mathbb{1}_{\{0<y<x\}}\,dy$$

$$= \int_0^x \frac{2y^2}{x^2}\,dy = \boxed{\frac{2}{3}X}$$

P2 $P(X=i)$ directo dificil

Pero si en vez de Y lanzamientos

$Y \sim$ Poisson $\Rightarrow P(Y=k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

fueran $\underline{N \text{ lanzamientos}}$

$$\Rightarrow P(X=i \mid Y=N) = \binom{N}{i} p^i (1-p)^{N-i} \quad i \leq N$$

$$= 0 \quad i > N$$

Usando Totales

$$P(X=i) = \sum_{N=0}^{\infty} P(X=i \mid Y=N)\, P(Y=N)$$

$$= \sum_{N=i}^{\infty} P(X=i \mid Y=N)\, P(Y=N)$$

$$= \sum_{N=i}^{\infty} \binom{N}{i} p^i (1-p)^{N-i} \frac{e^{-\lambda}\lambda^N}{N!} = \sum_{N=i}^{\infty} \frac{\cancel{N!}}{(N-i)!\, i!} p^i (1-p)^{N-i} \frac{e^{-\lambda}\lambda^N}{\cancel{N!}}$$

$$= \frac{(\lambda p)^i}{i!} e^{-\lambda} \sum_{N=i}^{\infty} \frac{(1-p)^{N-i}\lambda^{N-i}}{(N-i)!} = \frac{(\lambda p)^i}{i!} \sum_{N=0}^{\infty} \frac{(1-p)^N \lambda^N}{N!}$$

$$P(X=i) = \frac{(\lambda p)^i e^{-\lambda}}{i!} e^{\lambda(1-p)} = \frac{e^{-\lambda p} (\lambda p)^i}{i!} \sim Poisson(\lambda p)$$

$\boxed{P3}$ $E(X_i) = 100$, $Var(X_i) = 900 \Rightarrow \sigma = 30$

$X_i \sim iid$

$$P\left(\sum_{i=1}^{N} X_i \geq 20 \cdot 365\right) \geq 0.99$$

$$P\left(\sum_{i=1}^{N} X_i - \mu N \geq 100(73-N)\right) \geq 0.99$$

$$P\left(\frac{\sum_{i=1}^{N} X_i - \mu N}{\sigma\sqrt{N}} \geq \frac{10(73-N)}{3\sqrt{N}}\right) \geq 0.99$$

$$\Leftrightarrow P\left(\frac{\sum_{i=1}^{N} X_i - \mu N}{\sigma\sqrt{N}} < \frac{10}{3}\frac{(73-N)}{\sqrt{N}}\right) \leq 0.01$$

Como $P(Z \leq -2.33) = 0.01$

$$\Longleftrightarrow \frac{10}{3}\frac{(73-N)}{\sqrt{N}} < -2.33$$

$$\Longleftrightarrow N > 79.22 \Rightarrow \boxed{N^* \geq 80}$$