P1 Notar que la posición está dada por

$$X = D - I$$

Movimientos a la derecha / Movimientos a la izquierda

Como hay N movimientos $\Rightarrow D + I = N$

$\Rightarrow I = N - D$

$$\Rightarrow X = 2D - N$$

Notemos que $D \sim Binomial(p, N)$ (Éxitos son pasos a la derecha y con prob de éxito p)

$$\Rightarrow P(D = k) = \binom{N}{k} p^k (1-p)^{N-k}, \text{ con } k \in \{0, \dots, N\}$$

Ahora para $P(X = k) = P(2D - N = k)$

$\Rightarrow P\left(D = \frac{k+N}{2}\right)$, Si $\frac{k+N}{2} \in \{0, \dots, N\}$ Tiene fórmula

Si $\frac{k+N}{2} \notin \{0, \dots, N\} \Rightarrow P\left(D = \frac{k+N}{2}\right) = 0$

$$\Rightarrow \boxed{\binom{N}{\frac{N+k}{2}} \cdot p^{\frac{k+N}{2}} (1-p)^{\frac{N-k}{2}} = P(X = k)}$$

$\boxed{k \in \{-N, -N+2, \dots, N-2, N\}}$

2] a) $X \sim$ Binomial (p, N)

$$P(X=k) = \binom{N}{k} p^k (1-p)^{N-k}$$

) $$P(Y=l) = \sum_{k=0}^{N} P(Y=l \mid X=k) P(X=k)$$

Prob Totales

Condicionando Tenemos que $P(Y=l \mid X=k) = \binom{k}{l} q^l (1-q)^{k-l}$

esto es solo valido para $l \leq k$

pues si $l > k$ es imposible

$$\Rightarrow P(Y=l) = \sum_{k=l}^{N} P(Y=l \mid X=k) P(X=k)$$

$$= \sum_{k=l}^{N} \binom{k}{l} q^l (1-q)^{k-l} \binom{N}{k} p^k (1-p)^{N-k}$$

$$= \sum_{k=l}^{N} \frac{\cancel{k!}}{l!(k-l)!} \cdot \frac{N!}{\cancel{k!}(N-k)!} q^l \cdot (1-q)^{k-l} p^k (1-p)^{N-k}$$

$$= \binom{N}{l} q^l \sum_{k=l}^{N} \frac{(N-l)!}{(k-l)!(N-k)!} (1-q)^{k-l} p^k (1-p)^{N-k}$$

$$= \binom{N}{l} q^l \sum_{K=0}^{N-l} (1-q)^K p^{K+l} (1-p)^{N-K-l} \cdot \frac{(N-l)!}{K!\,(N-K-l)!}$$

$$= \binom{N}{l} (pq)^l \sum_{K=0}^{N-l} \binom{N-l}{K} \left[p(1-q)\right]^K \left[1-p\right]^{N-l-K}$$

$$= \binom{N}{l} (pq)^l \left[1-p+p-pq\right]^{N-l}$$

$$= \binom{N}{l} (pq)^l \left[1-pq\right]^{N-l} \Rightarrow \boxed{Y \sim Bin(N, pq)}$$

$\boxed{P3}$ $P(H_1 \cap H_2) = P(H_1)P(H_2) = \frac{1}{4}$ /

H_1 = Hombre Hijo 1
H_2 = Hombre Hijo 2
M_1 = Mujer Hij@ 1
M_2 = Mujer Hij@ 2

$P(H_1 \cap M_2) = \frac{1}{4}$

$P(H_2 \cap M_1) = \frac{1}{4}$

$P(M_1 \cap M_2) = \frac{1}{4}$

La función de Ganancia es

$P(G=1000) = \underbrace{\frac{1}{4}+\frac{1}{4}}_{\text{Uno hijo}} = \frac{1}{2}$

$P(G=0) = \frac{1}{4}$ { Dos hijos no juegan

$P(G=-1000) = \underbrace{\frac{1}{4}}_{\text{Dos Hijas pierde.}}$

Ganancia esperada = $E(G) = \sum K P(X=K) = 1000 \cdot \frac{1}{2} + (-1000) \cdot \frac{1}{4} = \boxed{250}$

$Var(G) = E(G^2) - E(G)^2$

$E(G^2) = \sum K^2 P(X=K) = (1000)^2 \cdot \frac{1}{2} + (-1000)^2 \cdot \frac{1}{4} + 0^2 \cdot \frac{1}{4} =$

$= 1000^2 \cdot \frac{3}{4} = 750.000$

$\Rightarrow Var(G) = 750.000 - 62500 = \boxed{687.500}$

$\boxed{P4}$ a) Usando que $\int_{-\infty}^{\infty} f_X(t)dt = 1$

Nos queda $\int_{-1}^{1} c\,t^2\,dt = 1$

Si $f(t)$ es par $\Rightarrow \int_{-a}^{a} f(t)dt = 2\int_0^a f(t)dt$
$f(t) = f(-t)$

$\Rightarrow 2c\int_0^1 t^2 dt = 1 \Rightarrow 2c \cdot \frac{t^3}{3}\Big|_0^1 = 1$

$\Rightarrow \frac{2c}{3} = 1 \Rightarrow c = \frac{3}{2} \Rightarrow f_X(t) = \begin{cases} \frac{3}{2}t^2 & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$

b) $P(X \ge \frac{1}{2}) = \int_{\frac{1}{2}}^{\infty} f_X(t)dt = \int_{\frac{1}{2}}^{1} \frac{3}{2}t^2 dt = \frac{t^3}{2}\Big|_{\frac{1}{2}}^{1} = \frac{1}{2} - \frac{1}{16} = \boxed{\frac{7}{16}}$

c) $P(X = \frac{1}{2}) = 0$, las variables continuas nunca son =

d) $E(X) = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_{-1}^{1} x \cdot x^2 \cdot \frac{3}{2}\,dx = \frac{3}{2}\int_{-1}^{1} \underbrace{x^3}_{\text{es impar}}\,dx = \boxed{0}$

$f(x) = -f(-x)$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_{-1}^{1} \underbrace{\frac{3}{2} x^4}_{\text{Es par}}\,dx = 3\int_{0}^{1} x^4\,dx = \frac{3}{5} x^5 \Big|_0^1 = \boxed{\frac{3}{5}}$

$Var(X) = E(X^2) - [E(X)]^2 = \boxed{\frac{3}{5}}$

$\boxed{P5}$

$Y \sim Exp(\lambda)$ si $f_Y(t) = \begin{cases} \lambda e^{-\lambda t} & \text{si } t \geq 0 \\ 0 & \text{si } t < 0 \end{cases}$

$F_{|X|}(t) = \mathbb{P}(|X| \leq t) \overset{\text{si } t>0}{=} \mathbb{P}(X \in [-t, t])$

$= \int_{-t}^{t} \frac{1}{2}\lambda e^{-\lambda|t|}\,dt = \int_0^t \lambda e^{-\lambda|t|}\,dt = \int_0^t \lambda e^{-\lambda t}\,dt$

$\Rightarrow \quad F_{|X|}(t) = \begin{cases} \int_0^t \lambda e^{-\lambda u}\,du & \text{si } t \geq 0 \\ 0 & \text{si } t < 0 \quad \text{(Pues } |X| \leq \text{Negativo es Imposible)} \end{cases}$

Derivando

$f_{|X|}(t) = \begin{cases} \lambda e^{-\lambda t} & \text{si } t \geq 0 \\ 0 & \text{si } t < 0 \end{cases} = f_Y(t)$

Por lo Tanto $|X| \sim \exp(\lambda)$

b) $$\text{Signo}(X) = \begin{cases} 1 & \text{si } X \geq 0 \\ -1 & \text{si } X < 0 \end{cases}$$

$Y \sim \exp(\lambda)$

$\int_{-\infty}^{\infty} f_Y(t)\,dt = 1$

$$P(\text{Signo}(X)=1) = P(X \geq 0) = \int_0^{\infty} \frac{1}{2}\lambda e^{-\lambda|t|}\,dt \overset{(t\geq 0)}{=} \frac{1}{2}\left[\int_0^{\infty} \lambda e^{-\lambda t}\,dt\right]$$

$$= \boxed{\frac{1}{2}}$$, Se uso un Truco, como $\int_{-\infty}^{\infty} f_Y(t) = 1$ para cualquier v.a, se encontro un caso

$$P(\text{Signo}(X)=-1) = P(X<0) = \int_{-\infty}^{0} \frac{1}{2}\lambda e^{-\lambda|t|}\,dt$$

$$\overset{u=-t}{=} \int_{\infty}^{0} \frac{1}{2}\lambda e^{-\lambda|-u|}(-du) = \int_0^{\infty} \frac{1}{2}\lambda e^{-\lambda|-u|}\,du$$

$$= \int_0^{\infty} \frac{1}{2}\lambda e^{-\lambda u}\,du = \boxed{\frac{1}{2}}$$

$$\Rightarrow \boxed{\text{Signo}(X) = \begin{cases} 1 & \text{con } p=\frac{1}{2} \\ -1 & \text{con } 1-p=\frac{1}{2} \end{cases}} \sim \text{Bernulli}\left(\tfrac{1}{2}\right)$$

Propuesta 1 a) $P(\text{Regalon}) = \frac{1}{5}$, $P(\text{Acierto} \mid \text{Regalon}) = \frac{3}{4}$

$P(\text{Normal}) = \frac{4}{5}$, $P(\text{Acierto} \mid \text{Normal}) = \frac{1}{3}$

$P(\text{Tener 8 o mas ciertos en 10 Tiros})$

$$\overset{\text{Totales}}{=} P(\text{Tener 8 en 10} \mid \text{Regalon}) \cdot P(\text{Regalon}) + P(\text{Tener 8 de 10} \mid \text{Normal})\, P(\text{Normal}))$$

* No Tenemos que esto es una binomial de parametro p y $\boxed{n=10}$ después de condicionar al cacho

$$= \left[\sum_{k=8}^{10} \binom{10}{k} \left(\frac{3}{4}\right)^k \left(\frac{1}{4}\right)^{10-k}\right]\left[\frac{1}{5}\right] + \left[\sum_{k=8}^{10} \binom{10}{k} \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{10-k}\right]\left[\frac{4}{5}\right]$$

b) $$P(\text{Regalon} \mid \text{9 de 10 Aciertos}) \overset{\text{bayes}}{=} \frac{P(\text{9 de 10 Aciertos} \mid \text{Regalon}) \cdot P(\text{Regalon})}{P(\text{9 de 10 Aciertos})}$$

$$\overset{\text{Totales}}{=} \frac{P(\text{9 de 10} \mid \text{Regalon})\, P(\text{Regalon})}{P(\text{9 de 10} \mid \text{Regalon})\, P(\text{Regalon}) + P(\text{9 de 10} \mid \text{Normal})\, P(\text{Normal})}$$

$$\boxed{P(\text{Regalon} \mid \text{9 de 10}) = \frac{\cancel{\binom{10}{9}} \left(\frac{3}{4}\right)^9 \left(\frac{1}{4}\right) \cdot \frac{1}{\cancel{5}}}{\cancel{\binom{10}{9}} \left[\left(\frac{3}{4}\right)^9 \left(\frac{1}{4}\right) \cdot \frac{1}{\cancel{5}} + \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right) \cdot \frac{4}{\cancel{5}}\right]}}$$

Propuesta 2

a) Por ser A.C $\Rightarrow f_x(t)$ es continua

$\Rightarrow \frac{A}{2} = \frac{B}{2} \Rightarrow \boxed{A = B}$, pues en $x = \frac{1}{2}$ dan esos

$$\Rightarrow f_x(t) = \begin{cases} At & \text{si } 0 \le t < \frac{1}{2} \\ A(1-t) & \text{si } \frac{1}{2} < t \le 1 \\ 0 & \sim \end{cases}$$

$$1 = \int_{-\infty}^{\infty} f_x(t)\,dt = \int_0^{\frac{1}{2}} At\,dt + \int_{\frac{1}{2}}^{1} A(1-t)\,dt$$

$$= \left.\frac{At^2}{2}\right|_0^{\frac{1}{2}} + \left. A\frac{(1-t)^2}{2}(-1)\right|_{\frac{1}{2}}^{1} = \frac{A}{8} + 0 - 0 + \frac{A}{8} = \frac{A}{4}$$

$$\Rightarrow A = 4$$

$$F_x(t) = \begin{cases} \int_{-\infty}^{t} f_x(x)\,dx = 0, & \text{si } t < 0 \\ = \int_0^t 4x\,dx = \left.2x^2\right|_0^t = 2t^2 & \text{si } t \in [0, \frac{1}{2}[\\ = \int_0^{\frac{1}{2}} 4x\,dx + \int_{\frac{1}{2}}^{t} 4(1-x)\,dx = \frac{1}{2} + \left.2(1-x)^2(-1)\right|_{\frac{1}{2}}^{t} = 1 - 2(1-t)^2 & \text{si } t \in [\frac{1}{2}, 1] \end{cases}$$

3) $F_X(t) = 1$ si $t > 1$

$\Rightarrow$ $F_X(t) = \begin{cases} 0 & \text{si } t < 0 \\ 2t^2 & \text{si } t \in [0, \frac{1}{2}[\\ 1 - 2(1-t)^2 & \text{si } t \in [\frac{1}{2}, 1] \\ 1 & \text{si } t > 1 \end{cases}$

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c) $E(X) = \int_{-\infty}^{\infty} t f_X(t) dt = \int_0^{\frac{1}{2}} 4t^2 dt + \int_{\frac{1}{2}}^{1} 4t(1-t) dt$

$= \frac{4}{3} t^3 \Big|_0^{\frac{1}{2}} + 2t^2 \Big|_{\frac{1}{2}}^{1} - \frac{4}{3} t^3 \Big|_{\frac{1}{2}}^{1}$

$= \cancel{\frac{1}{6}} + \cancel{2} - \frac{1}{2} - \cancel{\frac{4}{3}} + \cancel{\frac{1}{6}} = \boxed{\frac{1}{2}}$

$E(X^2) = \int_{-\infty}^{\infty} t^2 f_X(t) dt = \int_0^{\frac{1}{2}} 4t^3 dt + \int_{\frac{1}{2}}^{1} 4t^2(1-t) dt$

$= t^4 \Big|_0^{\frac{1}{2}} + \frac{4}{3} t^3 \Big|_{\frac{1}{2}}^{1} - t^4 \Big|_{\frac{1}{2}}^{1} = \frac{1}{16} + \frac{4}{3} - \frac{1}{6} - 1 + \frac{1}{16}$

$= \frac{1}{8} + \frac{1}{3} - \frac{1}{6} = \frac{1}{8} + \frac{1}{6} = \boxed{\frac{7}{24}}$

$\Rightarrow Var(X) = \frac{7}{24} - \frac{1}{4} = \boxed{\frac{1}{24}}$