P1

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$P(X=1)=\frac{1}{36}$ $P(X=8)=\frac{2}{36}$ $P(X=18)=\frac{2}{36}$

$P(X=2)=\frac{2}{36}$ $P(X=9)=\frac{1}{36}$ $P(X=20)=\frac{2}{36}$

$P(X=3)=\frac{2}{36}$ $P(X=10)=\frac{2}{36}$ $P(X=24)=\frac{2}{36}$

$P(X=4)=\frac{3}{36}$ $P(X=12)=\frac{4}{36}$ $P(X=25)=\frac{1}{36}$

$P(X=5)=\frac{2}{36}$ $P(X=15)=\frac{2}{36}$ $P(X=30)=\frac{2}{36}$

$P(X=6)=\frac{4}{36}$ $P(X=16)=\frac{1}{36}$ $P(X=36)=\frac{1}{36}$

$\forall k \neq \quad P(X=k)=0$

$$\boxed{F_X(K)=\sum_{i=0}^{K} P(X=i)}$$

P2 a) $X \sim \text{Geom}(p) \Rightarrow P(X=k)=(1-p)^{k-1}p$

$$P(X\geq N)=\sum_{i=N}^{\infty} P(X=i)=\sum_{i=N}^{\infty}(1-p)^{i-1}p=\sum_{i=0}^{\infty}(1-p)^{i+N-1}p$$

$$=p(1-p)^{N-1}\sum_{i=0}^{\infty}(1-p)^{i}=p(1-p)^{N-1}\cdot\frac{1}{\underbrace{1-(1-p)}_{p}}=\boxed{(1-p)^{N-1}}$$

$\boxed{\Leftarrow}$ Si $P(X \geq N) = (1-p)^{N-1}$

$$\Rightarrow \quad P(X=N) = P(\{X \geq N\} \cap \{X \geq N+1\}^c)$$

$$= P(\{X \geq N\}) - P(X \geq N+1)$$

$$= (1-p)^{N-1} - (1-p)^N = (1-p)^{N-1}[(1-(1-p)]$$

$$\boxed{P(X=N) = p\,(1-p)^{N-1}} \qquad X \sim \text{Geométrica de } (p)$$

b) $$P(Y = X+1) \overset{\text{Totales}}{=} \sum_{k=1}^{\infty} P(Y = X+1 \mid X=k) \cdot P(X=k)$$

$Y \perp X$

$$= \sum_{k=1}^{\infty} P(Y = k+1)\, P(X=k)$$

$$= \sum_{k=1}^{\infty} (1-q)^k\, p\, (1-p)^{k-1}$$

$$= \left[\sum_{k=1}^{\infty} (1-p-q+pq)^{k-1}\right] p(1-q)$$

$$= \sum_{k=0}^{\infty} (1-p-q-pq)^k \quad p(1-q)$$

$$= \frac{1}{1-(1-p-q-pq)} \cdot p(1-q) = \boxed{\frac{p(1-q)}{p+q+pq}}$$

c) Usando a) $P(Z \geq N) = P(\text{Min}(X,Y) \geq N)$

$$= P(X \geq N, Y \geq N) \quad \text{de } \text{[illegible]}$$

$$= P(X \geq N) \cdot P(Y \geq N) \quad \text{Son independientes}$$

$$= [1-p-q+pq]^{N-1} \;\; = [1-p]^{N-1}[1-q]^{N-1}$$

$$= \Big[1 - (\underbrace{p+q-pq}_{\in(0,1)})\Big]^{N-1}$$

$\Longrightarrow \quad Z \sim \text{Geom}(p+q-pq)$ Usando la a) Al reves

Propuesta 1

$$P(A \text{ invertible}) = P(\det(A) \neq 0)$$

$$= P\Big(Y \cdot \prod_{i=1}^{N}(Y-i) + \cancel{aY} - \cancel{aY} \neq 0\Big)$$

$$= P\Big(\prod_{i=0}^{N}(Y-i) \neq 0\Big)$$

Como es una multiplicación

$$= P(Y \neq 0, Y-1 \neq 0, \dots, Y-N \neq 0)$$

$$= P(Y > N) \underset{\text{discreta}}{=} P(Y \geq N+1) \underset{\text{por P2) a)}}{=} \boxed{(1-p)^N}$$