Pauta Auxiliar 5

$\boxed{P1}$ $¿P(C_1 \cap C_2) = P(C_1)\cdot P(C_2)?$ R: Si $\Rightarrow$ Independientes

R: No $\Rightarrow$ No lo son

$P(C_1) = ?$ (No sabemos que moneda se utilizó

$$\Rightarrow P(C_1) \overset{\text{Totales}}{=} P(C_1|M_1)P(M_1) + P(C_1|M_2)P(M_2)$$

$$= \frac{2}{3}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}\left(\frac{4}{6}+\frac{3}{6}\right) = \boxed{\frac{7}{12}}$$

$$\boxed{P(C_2) \overset{\text{Análogo}}{=} \frac{7}{12}}$$

$$P(C_1\cap C_2) \overset{\text{Totales}}{=} P(C_1\cap C_2|M_1)P(M_1) + P(C_1\cap C_2|M_2)P(M_2)$$

$$= \left(\frac{2}{3}\right)^2\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2\cdot\frac{1}{2} = \frac{1}{2}\left(\frac{4}{9}+\frac{1}{4}\right) = \frac{25}{72} = \boxed{\frac{50}{144}}$$

$$\frac{50}{144} \neq \frac{7}{12}\cdot\frac{7}{12} = \frac{49}{144} \Rightarrow \text{No son independientes}$$

$\boxed{P2}$ $\underset{\text{Mentirosa}}{P(M)} = \frac{4}{10} = \frac{2}{5} \Rightarrow P(M^c) = \frac{3}{5}$

$P(\text{Saber}) = \frac{3}{4} = P(\text{Saber}|M) = P(\text{Saber}|M^c) \Rightarrow P(\text{Saber}\cap M) = P(\text{Saber})\cdot P(M)$

$P(\text{Saber}\cap M^c) = P(\text{Saber})\cdot P(M^c)$

$P(C|\text{Saber}^c) = \frac{1}{2}$

$P(C|\text{Saber}\cap M^c) = 1 \quad \Rightarrow P(C|\text{Saber}\cap M) = 0$

Preguntarle a Amiga

$$P(C) \underset{\text{Fobs}}{=} P(C \mid Saber \cap M)P(Saber \cap M) + P(C \mid Saber \cap M^c)P(Saber \cap M^c) +$$
$$P(C \mid Saber^c \cap M)P(Saber^c \cap M) + P(C \mid Saber^c \cap M^c)P(Saber^c \cap M^c)$$

$$= 0 + 1 \cdot P(Saber \cap M^c) + \underbrace{P(C \cap Saber^c \cap M) + P(C \cap Saber^c \cap M^c)}_{\text{Disjuntas}}$$

$$= 0 + P(Saber \cap M^c) + P(C \cap Saber^c)$$

$$= P(Saber) \cdot P(M^c) + P(C \mid Saber^c) \cdot P(Saber^c)$$

$$= \frac{3}{4} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{1}{4}$$

$$= \frac{9}{20} + \frac{1}{8} = \frac{18+5}{40} = \boxed{\frac{23}{40}}$$

Solo $\quad P(Saber) = \frac{1}{2}, \quad P(C \mid Saber^c) = \frac{1}{5}$

$$P(C) = P(C \mid Sabe)P(Saber) + P(C \mid Saber^c) \cdot P(Saber^c)$$

$$= \cancel{\frac{1}{4} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{1}{4} = \frac{15}{20} + \frac{1}{20} = \frac{16}{20}}$$

$$= 1 \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{1}{2} = \frac{1}{2}\left(\frac{6}{5}\right) = \frac{3}{5} = \boxed{\frac{24}{40}}$$

Conviene solo

P3

$P(A) = \frac{2}{3}$

$P(B) = \frac{1}{3}$

$P(\text{Estar a favor}) = p$

$P(Si \mid \text{Estar a favor} \cap A) = 1$

$P(Si \mid \text{No estar a favor} \cap A) = 0$

$P(Si \mid \text{Estar a favor} \cap B) = 0$

$P(Si \mid \text{No Estar a favor} \cap B) = 1$

$$P(\text{Estar a favor} \mid Si) = \frac{P(\text{Estar a favor} \cap Si)}{P(Si)}$$

$$\overset{\text{Disjuntos}}{=} \frac{P(Si \cap \text{Estar a favor} \cap A) + P(Si \cap \text{Estar a favor} \cap B)}{P(Si)}$$

$$= \Big[P(Si \mid \text{Estar a Favor} \cap A) \cdot P(\text{Estar a Favor} \cap A) + \underbrace{P(Si \mid \text{Estar a favor} \cap B)}_{0} \cdot P(\text{Estar a Favor} \cap B)\Big] \cdot P(Si)^{-1}$$

Estar a favor y A independientes

$$= \frac{P(Si \mid \text{Estar a favor} \cap A) \cdot P(\text{Estar a Favor}) \cdot P(A)}{P(Si)}$$

$$\overset{\text{p. Totales}}{=} \frac{1 \cdot p \cdot \frac{2}{3}}{P(NEF) \cdot P(B) \cdot P(Si \mid NEF \cap B) + P(Si \mid EF \cap A) \cdot P(EF) P(A) + P(Si \mid EF \cap B) P(EF \cap B) + P(Si \mid NEF \cap A) P(NEF) \cdot P(A)}$$

$$= \frac{\frac{2p}{3}}{1\, p \cdot \frac{2}{3} + 1 \cdot (1-p) \cdot \frac{1}{3}} = \frac{2p}{2p + (1-p)} = \boxed{\frac{2p}{1+p}}$$

b) $P(S_i) = q$

Cuanto vale p

de a) $$P(S_i) = \frac{2p + (1-p)}{3} \Rightarrow q = \frac{2p + (1-p)}{3}$$

$$\Rightarrow 3q = 1 + p \Rightarrow \boxed{p = 3q - 1}$$

P4 $$P(E_N) = \left(\frac{5}{6}\right)^{N-1} \cdot \frac{1}{6}$$

Con E_N el primer uno aparece en el N-ésimo Tiro

$$\Rightarrow P(P4) = P\left(\bigcup_{N>3} E_N\right)$$

Son disjuntos

$$= \sum_{N>3} P(E_N) = \sum_{N=4}^{\infty} \left(\frac{5}{6}\right)^{N-1} \cdot \frac{1}{6}$$

$$= \sum_{N=0}^{\infty} \left(\frac{5}{6}\right)^{N+3} \frac{1}{6} = \frac{1}{6} \cdot \left(\frac{5}{6}\right)^3 \cdot \sum_{N=0}^{\infty} \left(\frac{5}{6}\right)^N$$

$$= \cancel{\frac{1}{1 - \frac{5}{6}}} \cdot \cancel{\frac{1}{6}} \left(\frac{5}{6}\right)^3 = \boxed{\left(\frac{5}{6}\right)^3}$$