P1 a) $G(s) = \frac{1}{s^3} + \frac{2}{s^2-(\sqrt{7})^2} + \frac{4s}{s^2+(\sqrt{2})^2} + \frac{e^{-2s}}{s}$.

$= \mathcal{L}(f_1(t))(s) + \mathcal{L}(f_2(t))(s) + \mathcal{L}(f_3(t))(s) + \mathcal{L}(f_4(t))(s)$

$\mathcal{L}(f_1(t)) = \frac{1}{s^3} = \mathcal{L}\left(\frac{t^2}{2}\right)(s) \Rightarrow \boxed{f_1(t) = \frac{t^2}{2}}$, $\mathcal{L}(t^k e^{\alpha t}) = \frac{k!}{(s-\alpha)^{k+1}}$

$\mathcal{L}(f_2(t)) = \frac{2}{s^2-(\sqrt{7})^2} = \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{s^2-(\sqrt{7})^2} = \frac{2}{\sqrt{7}} \cdot \mathcal{L}(\sinh(\sqrt{7}t))(s)$

$= \mathcal{L}\left(\frac{2}{\sqrt{7}} \sinh(\sqrt{7}\,t)\right)(s) \Rightarrow \boxed{f_2(t) = \frac{2}{\sqrt{7}} \sinh(\sqrt{7}\,t)}$

$\mathcal{L}(f_3(t)) = \frac{4s}{s^2+(\sqrt{2})^2} = 4 \cdot \frac{s}{s^2+(\sqrt{2})^2} = 4\mathcal{L}(\cos(\sqrt{2}t)) = \mathcal{L}(4\cos(\sqrt{2}t))$

$\Rightarrow \boxed{f_3(t) = 4\cos(\sqrt{2}\,t)}$

$\mathcal{L}(f_4(t)) = \frac{e^{-2s}}{s} = \mathcal{L}(H_2(t)) \Rightarrow \boxed{f_4(t) = H_2(t)}$

$\Rightarrow \boxed{G(s) = \mathcal{L}\left(\frac{t^2}{2} + \frac{2}{\sqrt{7}} \sinh(\sqrt{7}\,t) + 4\cos(\sqrt{2}t) + H_2(t)\right)(s)}$

$$b)\ G(s) = e^{-2s} \cdot \frac{1}{s^2+2s-1} = e^{-2s} \cdot \frac{1}{s^2+2s+1-2} = e^{-2s} \cdot \mathcal{L}(f)(s)$$

Subproblema

$$\frac{1}{s^2+2s-1} = \frac{1}{s^2+2s+1-2} = \frac{1}{(s+1)^2-(\sqrt{2})^2} = \mathcal{L}(f(t))(s) = \mathcal{L}(g(t))(s+1)$$

Sub-subproblema

$$\frac{1}{s^2-(\sqrt{2})^2} = \mathcal{L}(g)(s) = \frac{\sqrt{2}}{s^2-(\sqrt{2})^2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \mathcal{L}(\sinh(\sqrt{2}t))(s)$$

$$= \mathcal{L}\left(\frac{\sinh(\sqrt{2}t)}{\sqrt{2}}\right)(s)$$

$$\mathcal{L}(f(t))(s) = \mathcal{L}\left(\frac{\sinh(\sqrt{2}t)}{\sqrt{2}}\right)(s-(-1)) = \mathcal{L}\left(e^{-t}\frac{\sinh(\sqrt{2}t)}{\sqrt{2}}\right)(s)$$

$$\Rightarrow f(t) = e^{t}\frac{\sinh(\sqrt{2}t)}{\sqrt{2}}$$

$$G(s) = e^{-2s}\mathcal{L}\left(e^{-t}\frac{\sinh(\sqrt{2}t)}{\sqrt{2}}\right)(s) = \mathcal{L}\left(H_2(t)\, e^{-(t-2)}\frac{\sinh(\sqrt{2}(t-2))}{\sqrt{2}}\right)$$

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c) $G(s) = \ln\left(\frac{s-3}{s+1}\right) = \ln(s-3) - \ln(s+1) = \mathcal{L}(f_1(t))(s) - \mathcal{L}(f_2(t))(s)$

$\ln(s-3) = \mathcal{L}(f_1(t))(s)$

$\Rightarrow \frac{1}{s-3} = \frac{d}{ds}\mathcal{L}(f_1(t))(s) \Rightarrow -\left(\frac{1}{s-3}\right) = -\frac{d}{ds}\mathcal{L}(f_1(t))(s)$

$\Rightarrow \mathcal{L}(-e^{3t}) = -\left(\frac{1}{s-3}\right) = -\frac{d}{ds}\mathcal{L}(f_1(t))(s) = \mathcal{L}(t f_1(t))(s)$

$\Rightarrow \boxed{-\frac{e^{3t}}{t} = f_1(t)}$

$\ln(s+1) = \mathcal{L}(f_2(t))(s) \Rightarrow \frac{1}{s+1} = \frac{d}{ds}\mathcal{L}(f_2(t))(s)$

$\Rightarrow \mathcal{L}(-e^{-t})(s) = -\left(\frac{1}{s+1}\right) = -\frac{d}{ds}\mathcal{L}(f_2(t))(s) = \mathcal{L}(t f_2(t))(s)$

$\Rightarrow \boxed{-\frac{e^{-t}}{t} = f_2(t)}$

$\Rightarrow \boxed{G(s) = \frac{-e^{3t} + e^{-t}}{t}}$

d) $G(s) = \dfrac{1}{(s^2+1)^2} = \mathcal{L}(f(t))(s) = \dfrac{(-1)}{2} \cdot \dfrac{1}{s} \cdot \dfrac{(-2s)}{(s^2+1)^2}$

$\Rightarrow \left(-\dfrac{1}{2}\right) \cdot \dfrac{1}{s} \cdot \dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right) = \mathcal{L}(f(t))(s)$

$\dfrac{1}{s^2+1} = \mathcal{L}(\sin(t))(s) \Rightarrow \dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right) = \dfrac{d}{ds}\mathcal{L}(\sin(t))(s) = \mathcal{L}(-t\sin(t))(s)$

$\Rightarrow \left(-\dfrac{1}{2}\right)\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right) = \mathcal{L}\left(\dfrac{t\sin(t)}{2}\right)(s)$

$\boxed{\begin{array}{l} \text{Usando } \frac{1}{s} H(s) = \frac{1}{s}\mathcal{L}(g(t))(s) \\ \Rightarrow \frac{1}{s}\mathcal{L}(g(t))(s) = \mathcal{L}\left(\int_0^t g(t)\,dt\right)(s) \end{array}}$

$\Rightarrow \dfrac{1}{s}\left[\left(-\dfrac{1}{2}\right)\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right)\right] = \mathcal{L}\left(\displaystyle\int_0^t \dfrac{t\sin(t)}{2}\,dt\right)(s)$

$= \mathcal{L}\left(\dfrac{\sin(t) - t\cos(t)}{2}\right)(s)$

$\Rightarrow \boxed{G(s) = \dfrac{1}{(s^2+1)^2} = \left(\dfrac{1}{s}\right)\left[\left(-\dfrac{1}{2}\right)\dfrac{d}{ds}\left(\dfrac{1}{s^2+1}\right)\right] = \mathcal{L}\left(\dfrac{\sin(t) - t\cos(t)}{2}\right)(s)}$

$=$

P2 $\quad y'' - 4y = \begin{cases} 1 & 5 \le t < 20 \\ 0 & \sim \end{cases} \quad , \quad y(0) = y'(0) = 0$

$$\Rightarrow y'' - 4y = H_5(t) - H_{20}(t) = g(t)$$

$$\overset{\mathcal{L}()(s)}{\Rightarrow} \mathcal{L}(y''(t))(s) - 4\mathcal{L}(y(t))(s) = \mathcal{L}(H_5(t) - H_{20}(t))(s)$$

$$\Rightarrow s^2 \mathcal{L}(y(t))(s) - s\cdot 0 - 1\cdot 0 - 4\mathcal{L}(y(t))(s) = \frac{e^{-5s} - e^{-20s}}{s}$$

$$\Rightarrow (s^2 - 4)\mathcal{L}(y(t))(s) = \frac{e^{-5s} - e^{-20s}}{s}$$

$$\Rightarrow \mathcal{L}(y(t))(s) = \frac{1}{s(s^2-4)} \cdot \left(e^{-5s} - e^{-20s}\right)$$

$$= \frac{1}{s} \cdot \frac{1}{s^2-4} \cdot \left(e^{-5s} - e^{-20s}\right)$$

$$= \frac{1}{s} \cdot \mathcal{L}\left(\frac{\sin(2t)}{2}\right)(s) \cdot \left(e^{-5s} - e^{-20s}\right)$$

$$= \mathcal{L}\left(\int_0^t \frac{\sin(2t)}{2}\,dt\right)(s) \cdot \left(e^{-5s} - e^{-20s}\right)$$

$$= \mathcal{L}\left(\frac{\cos(2t) - 1}{4}\right)(s) \cdot e^{-5s} - \mathcal{L}\left(\frac{\cos(2t) - 1}{4}\right)(s)\, e^{-20s}$$

$$= \mathcal{L}\left(H_5(t) \cdot \left(\frac{\cos(2(t-5)) - 1}{4}\right) - H_{20}(t)\left(\frac{\cos(2(t-20)) - 1}{4}\right)\right)$$

AntiTransformada

$$\rightarrow \boxed{y(t) = H_5(t) \cdot \left(\frac{\cos(2(t-5)) - 1}{4} \right) - H_{20}(t) \left(\frac{\cos(2(t-20)) - 1}{4} \right)}$$

$\boxed{P3}$ $y' + 2y + \int^t y(\tau)\,d\tau \quad \begin{cases} t & 0 \le t < 1 \end{cases}$

P3 $\quad y' + 2y + \int_0^t y(\tau)\,d\tau = t - tH_1(t) + (2-t)H_1(t) - (2-t)H_2(t)$ $\quad$ Con $y(0)=1$

$$= t + (2-2t)H_1(t) - (2-t)H_2(t)$$

Aplicando Transformada

$$\mathcal{L}(y')(s) + 2\mathcal{L}(y)(s) + \mathcal{L}\left(\int_0^t y(\tau)d\tau\right)(s) = \frac{1}{s^2} + 2\mathcal{L}(H_1(t)(t-1))(s) + \mathcal{L}(H_2(t)(t-2))(s)$$

Se usa que $\mathcal{L}(H_a(t)f(t-a)) = \mathcal{L}(f)e^{-as}$

$$\Rightarrow s\mathcal{L}(y)(s) - y(0) + 2\mathcal{L}(y)(s) + \frac{1}{s}\mathcal{L}(y)(s) = \frac{1}{s^2} - 2\frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$$

Se usó $\mathcal{L}(t)(s) = \frac{1}{s^2}$

$\cdot s$

$$\Rightarrow \underbrace{(s^2 + 2s + 1)}_{(s+1)^2}\mathcal{L}(y)(s) = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s} + s$$

$\Rightarrow$

$$\mathcal{L}(y)(s) = \frac{1}{s}\cdot\frac{1}{(s+1)^2}\cdot\underbrace{\left[1-2e^{-s}+e^{-2s}\right]}_{\text{Lo colocaremos al final}} + \frac{s}{(s+1)^2}$$

$$\frac{1}{(s+1)^2} = \mathcal{L}(te^{-t})(s) \Rightarrow \frac{1}{s}\cdot\frac{1}{(s+1)^2} = \frac{1}{s}\mathcal{L}(te^{-t})(s)$$

$$= \mathcal{L}\left(\int_0^t u e^{-u}\,du\right)(s)$$

$\Rightarrow$

$$\Rightarrow \frac{1}{s}\cdot\frac{1}{(s+1)^2} = \mathcal{L}\left(\left(\left(-e^{-u}u - e^{-u}\right)\Big|_0^t\right)\right)(s)$$

$$= \mathcal{L}\left(1 - e^{-t}(t+1)\right)(s)$$

$$\Rightarrow \frac{1}{s}\cdot\frac{1}{(s+1)^2}\cdot e^{-s} = \mathcal{L}\left(1 - e^{-t}(t+1)\right)(s)\cdot e^{-s}$$

$$= \mathcal{L}\left(\left[1 - e^{-(t-1)}\cdot t\right]H_1(t)\right)(s)$$

$$\Rightarrow \frac{1}{s}\cdot\frac{1}{(s+1)^2}\cdot e^{-2s} = \mathcal{L}\left(\left[1 - e^{-(t-2)}(t-1)\right]H_2(t)\right)(s)$$

$$\frac{s}{(s+1)^2} = \frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2} = \frac{1}{s+1} - \frac{1}{(s+1)^2} = \mathcal{L}(e^{-t})(s) - \mathcal{L}(te^{-t})(s)$$

$$= \mathcal{L}\left(e^{-t}[1-t]\right)(s)$$

AntiTrasformando

$$\Rightarrow \boxed{y(t) = \left[1 - e^{-t}(t+1)\right] - 2\left[1 - e^{-(t-1)}t\right]H_1(t) + \left[1 - e^{-(t-2)}(t-1)\right]H_2(t) + e^{-t}[1-t]}$$