P1 $I = \int_0^{\frac{\pi}{2}} \int_0^{x} (x^3 y + \cos(x))\, dy\, dx$

$$I = \int_0^{\frac{\pi}{2}} \frac{x^5}{2} + \cos(x) \cdot x \, dx = \frac{x^6}{12} + x\sin(x) + \cos(x) \Big|_0^{\frac{\pi}{2}} = \boxed{\frac{\left(\frac{\pi}{2}\right)^6}{12} + \frac{\pi}{2} - 1}$$

P2 $\boxed{V(D) = \iiint_D 1 \; dx\,dy\,dz}$

d enunciado era $\boxed{x, y, z \geq 0}$

$z \in [0, 1]$

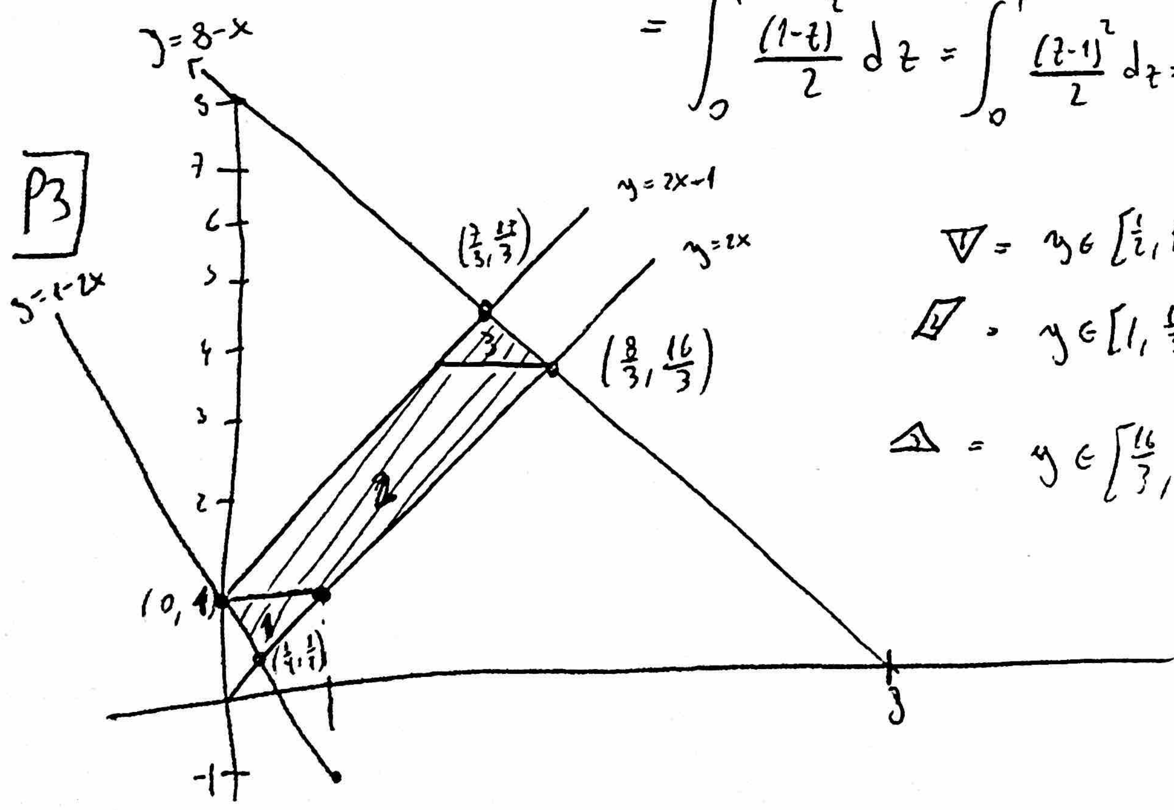
$x \in [0, 1-z]$

$y \in [0, 1-z-x]$

$$\Rightarrow \int_0^1 \int_0^{1-z} \int_0^{1-z-x} 1 \cdot dy\, dx\, dz = \int_0^1 \int_0^{1-z} (1-z-x)\, dx\, dz = \int_0^1 (1-z)^2 - \frac{(1-z)^2}{2}\, dz$$

$$= \int_0^1 \frac{(1-z)^2}{2}\, dz = \int_0^1 \frac{(z-1)^2}{2}\, dz = \frac{(z-1)^3}{6} \Big|_0^1 = \boxed{\frac{1}{6}}$$

P3



∇ = $y \in \left[\frac{1}{2}, 1\right]$, $x \in \left[\frac{1}{2} - \frac{y}{2}, \frac{y}{2}\right]$

▱ = $y \in \left[1, \frac{16}{3}\right]$, $x \in \left[\frac{y-1}{2}, \frac{y}{2}\right]$

Δ = $y \in \left[\frac{16}{3}, \frac{17}{3}\right]$, $x \in \left[\frac{y-1}{2}, 8-y\right]$

Se puede despejar x, lo hicimos en clases

$$\iint_D (y-2x)\,ds = \iint_{\Delta} (y-2x) + \iint_{\square} (y-2x) + \iint_{\nabla} (y-2x)$$

$$\iint_{\Delta} (y-2x)\,dx\,dy = \int_{\frac12}^{1} \int_{\frac12-\frac{y}{2}}^{\frac{y}{2}} (y-2x)\,dx\,dy = \int_{\frac12}^{1} y\left(\frac{y}{2}-\frac12+\frac{y}{2}\right) - x^2 \Big|_{\frac12-\frac{y}{2}}^{\frac{y}{2}} dy$$

$$= \int_{\frac12}^{1} y^2 - \frac{y}{2} - \frac{y^2}{4} + \left(\frac{y-1}{2}\right)^2 dy = \int_{\frac12} y^2 - y + \frac14\,dy$$

$$= \int_{\frac12}^{1} \left(y-\frac12\right)^2 dy = \frac{\left(y-\frac12\right)^3}{3} \Bigg|_{\frac12}^{1} = \boxed{\frac{1}{24}}$$

$$\iint_{\square} (y-2x)\,dx\,dy = \int_1^{\frac{16}{3}} \int_{\frac{y-1}{2}}^{\frac{y}{2}} (y-2x)\,dx\,dy = \int_1^{\frac{16}{3}} y\left(\frac{y}{2}-\left(\frac{y-1}{2}\right)\right) - x^2 \Big|_{\frac{y-1}{2}}^{\frac{y}{2}} dy$$

$$= \int_1^{\frac{16}{3}} \frac{y}{2} - \frac{y^2}{4} + \left(\frac{y-1}{2}\right)^2 dy = \int_1^{\frac{16}{3}} \cancel{\frac{y}{2}} - \cancel{\frac{y^2}{4}} + \cancel{\frac{y^2}{4}} - \cancel{\frac{y}{2}} + \frac14\,dy$$

$$= \frac14\left(\frac{16}{3}-\frac33\right) = \boxed{\frac{13}{12}}$$

$$\iint_{\nabla} (y-2x)\,dx\,dy = \int_{\frac{16}{3}}^{\frac{17}{3}} \int_{\frac{y-1}{2}}^{8-y} (y-2x)\,dx\,dy = \int_{\frac{16}{3}}^{\frac{17}{3}} y\left[8-y-\left(\frac{y-1}{2}\right)\right] - x^2 \Big|_{\frac{y-1}{2}}^{8-y} dy$$

$$= \int_{\frac{16}{3}}^{\frac{17}{3}} \frac{17y}{2} - \frac{3y^2}{2} - (8-y)^2 + \frac{(y-1)^2}{4}\,dy = \int_{\frac{16}{3}} \frac{-9}{4}y^2 \quad \frac{48y}{2} \quad \frac{255}{4}\Big)\,dy = \boxed{\frac{1}{18}}$$

$$\iint_D (y-2x)\,ds = \frac{1}{24} + \frac{13}{12} + \frac{1}{18} = \frac{3+78+4}{72} = \boxed{\frac{85}{72}}$$

P4 $V(D)=\iiint_D 1\,dz\,dy\,dx$, $D=\{(x,y,z): x\in[a,b]\ \ y^2+z^2\le f(x)^2\}$

$D \Leftrightarrow \{x\in[a,b],\ z\in[-f(x),f(x)],\ y\in[-\sqrt{f(x)^2-z^2},\sqrt{f(x)^2-z^2}]$

$$V(D)=\int_a^b\int_{-f(x)}^{f(x)}\int_{-\sqrt{f(x)^2-z^2}}^{\sqrt{f(x)^2-z^2}} 1\,dy\,dz\,dx = \int_a^b\int_{-f(x)}^{f(x)} 2\sqrt{f(x)^2-z^2}\,dz\,dx$$

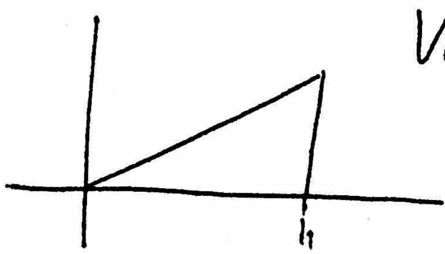
$$=4\int_a^b\int_0^{f(x)}\sqrt{f(x)^2-z^2}\,dz\,dx$$

$z=f(x)\sin(u)$ / $dz=f(x)\cos(u)$

$$=4\int_a^b\int_0^{\frac{\pi}{2}} f(x)^2\cos(u)^2\,du\,dx = 4\int_a^b\int_0^{\frac{\pi}{2}} f(x)^2\left(\frac{1+\cos(2u)}{2}\right)du\,dx$$

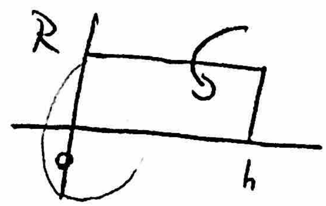
$$=2\int_a^b f(x)^2\left[u+\frac{\sin(2u)}{2}\right]\Big|_0^{\frac{\pi}{2}}dx = \boxed{\pi\int_a^b f(x)^2\,dx}$$

Cono $f(x)=\frac{R}{h}x$ $a=0$, $b=h$



$$V(D)=\pi\int_0^h \frac{R^2x^2}{h^2}dx = \frac{\pi R^2 x^3}{3h^2}\Big|_0^h = \boxed{\frac{\pi R^2 h}{3}}$$

Cilindro $f(x)=R$ $a=0$, $b=h$



$$\Rightarrow V(D)=\pi\int_0^h R^2dx = \boxed{\pi R^2 h}$$