P1 $f(x,y) = e^x y^2$

a) $z = f(x,y)$ en $(1,2) \Rightarrow z = 4e$

$$S = \{ (x,y,z) \in \mathbb{R}^3 \mid \underbrace{z - e^x y^2}_{g(x,y,z)} = 0 \}$$

Usando la formula d plano Tangente en (x_0, y_0, z_0) es

$$\boxed{T_{(x_0,y_0,z_0)} = \{ (x,y,z) \in \mathbb{R}^3 \mid \nabla g(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0 \}}$$

$$\nabla g(x,y,z) = (-e^x y^2, -2e^x y, 1)$$

$$\nabla g(1,2,4e) = (-4e, -4e, 1)$$

$$\boxed{\begin{array}{l} T_{(1,2,4e)} = \{ (x,y,z) \in \mathbb{R}^3 \mid 4e((x-1)+(y-2)) = z - 4e \} \\ \qquad\qquad\qquad\qquad 4e(x+y-2) = z \end{array}}$$

b) Taylor de orden 2 es $f(x+h_1, y+h_2) \approx f(x,y) + \nabla f(x,y) \cdot (h_1, h_2)$
en Torno A (x,y)

$$+ \frac{(h_1,h_2)\, H_f(x,y)\, (h_1,h_2)}{2}$$

$\boxed{f(1,2) = 4e}$, $\nabla f(x,y) = (e^x y^2, 2e^x y) \Rightarrow \boxed{\nabla f(1,2) = (4e, 4e)} \Rightarrow \nabla f(1,2)(h_1,h_2) = 4e(h_1+h_2)$

$$Hf(x,y) = \begin{bmatrix} e^x y^2 & 2e^x y \\ 2e^x y & 2e^x \end{bmatrix} \Rightarrow Hf(1,2) = \begin{bmatrix} 4e & 4e \\ 4e & 2e \end{bmatrix} \Rightarrow (h_1,h_2) H_f(1,2) (h_1,h_2) = (h_1,h_2) \begin{bmatrix} 4e & 4e \\ 4e & 2e \end{bmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

$$\Rightarrow (h_1,h_2) H_f(1,2) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = (h_1,h_2) \begin{pmatrix} 4e(h_1+h_2) \\ 2e(2h_1+h_2) \end{pmatrix} = 4e(h_1^2 + h_1h_2) + 2e(2h_1h_2 + h_2^2)$$

$$= 2e(2h_1^2 + 4h_1h_2 + h_2^2)$$

$$\boxed{T f_{(1,2)}(h_1,h_2) = 4e + 4e(h_1+h_2) + e\left(2h_1^2 - 4h_1h_2 + h_2^2\right)}$$

c)

$$f(0{,}9,\, 2{,}05) \approx T f_{(1,2)}(-0{,}1,\, 0.05) = e\left[4 + (-0{,}4) + (0{,}2) + 2(0{,}1)^2 + 4(0{,}1)(0{,}05) + (0.05)^2\right]$$

$$= e\left[3{,}8 + \cancel{0{,}02} - \cancel{0{,}02} + 0.0025\right]$$

$$= e\left[3{,}8025\right] \approx 2{,}71 \cdot 3.8025$$

$$
\begin{array}{r}
271 \cdot 38025 \\
1355 \\
542* \\
0** \\
2168*** \\
813**** \\
\hline
10{,}204775
\end{array}
$$

$$\boxed{\approx 10{,}204}$$

P2 $f(x,y) = 3x^3 + y^2 - 9x - 6y + 1$

$\nabla f(x,y) = (9x^2 - 9, 2(y-3)) \overset{\substack{\text{Impongo} \\ \downarrow}}{=} (0,0)$

$\Rightarrow \quad x = \pm 1 \quad , \quad y = 3$

Hay 2 puntos críticos $(1,3)$ y $(-1,3)$

$Hf(x,y) = \begin{bmatrix} 18x & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow Hf(1,3) = \begin{bmatrix} 18 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow$ Definida Positiva $\Rightarrow$ $(1,3)$ es mínimo local

$Hf(-1,3) = \begin{bmatrix} -18 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow$ Punto silla (No es definida)

No hay más puntos críticos

Si $(x,y) \to (\infty, 0) \Rightarrow f(x,y) \to \infty$ } No hay máximo global

$(x,y) \to (-\infty, 0) \Rightarrow f(x,y) \to -\infty$ } No hay mínimo global

P3 a) $\nabla f(x) = (A + A^t)x + c \overset{\substack{\text{Imponiendo} \\ \downarrow}}{=} 0 \Rightarrow \boxed{\bar{x} = -(A+A^t)^{-1} c}$

¿Existe $(A+A^t)^{-1}$?

A definida positiva $\Rightarrow x^t A x > 0$ (Trasponiendo)

$\Rightarrow x^t A^t x > 0$

$\Rightarrow x^t (A + A^t) x > 0$

$\Rightarrow \det(A + A^t) > 0 \Rightarrow$ Es invertible

$Hf(x) = A + A^t$ (definida positiva) $\forall x \Rightarrow$ Hay mínimo global

$\Rightarrow \boxed{x = -(A+A^t)^{-1} c}$ es un mínimo global

b) Para $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ y $c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$\Rightarrow \quad x = -\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -1/2 \\ -1/2 \end{pmatrix}}$$

$$\nabla g(x,y) = (2x+1,\ 2y+1) = (0,0) \Rightarrow \boxed{x = -\tfrac{1}{2},\ y = -1/2}$$

$H_g(x,y) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ definido positivo, se verifica

P4

$$g(x,y) = x^3 - 2y^3 - \cos(x) + y^2$$

$$\nabla g(x,y) = (3x^2 + \sin(x),\ 2y(-3y+1)) = (0,0)$$

$$\Rightarrow \quad 3x^2 + \sin(x) = 0$$

$$y = 0 \quad \vee \quad y = \tfrac{1}{3}$$

¿Cuándo $3x^2 + \sin(x) = 0$? R: en $x = 0$ ✓ ¿hay otro?

~~$h(x) = 3x^2 + \sin(x)$~~

~~$h(x) = 6x + \cos(x)$~~

~~Sea $x \in$~~ $(0,$

$$h(x) = 3x^2 + \sin(x)$$

$$h'(x) = 6x + \cos(x)$$

$$h''(x) = 6 - \sin(x) \neq 0$$

Notemos que $h(-0.1) < 0$, $h(-5) > 0 \Rightarrow$ Hay otro 0 $(\tilde{x}_1)$

$h'(0) = 1 > 0$ y $h'(-5) =$ Negativo $\Rightarrow$ $h'(x)$ Tiene un 0

Supongamos que hay un Tercer 0 de $h(x)$ $\Rightarrow$

$h'(x)$ Tiene al menos 2 ceros por TVM $\Rightarrow$ $h''(x)$ Tiene al menos un cero ✗

$$\frac{\partial^2 f(x)}{\partial x_j \partial x_i} = \frac{\partial}{\partial x_j}\left(e^{u} \cdot \frac{\partial g}{\partial x_i}\right)$$

$$= \frac{\partial e^{u}}{\partial x_j} \cdot \frac{\partial g}{\partial x_i} + e^{u} \cdot \frac{\partial^2 g}{\partial x_j \partial x_i}$$

$$= \frac{\partial e^{u}}{\partial u} \cdot \frac{\partial u}{\partial x_j} \cdot \frac{\partial g}{\partial x_i} + e^{u} \frac{\partial^2 g}{\partial x_j \partial x_i}$$

$$= e^{u}\left(\frac{\partial g}{\partial x_j} \cdot \frac{\partial g}{\partial x_i} + \frac{\partial^2 g}{\partial x_j \partial x_i}\right)$$

$$\Rightarrow \left[H_f(x)\right]_{ij} = e^{u} \cdot \left(\frac{\partial g}{\partial x_j} \cdot \frac{\partial g}{\partial x_i} + \frac{\partial^2 g}{\partial x_j \partial x_i}\right)$$

$$h^t \cdot H_f(x) \cdot h = \sum_{i=1}^{N}\sum_{j=1}^{N} h_i \left[H_f(x)\right]_{ij} h_j$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} h_i \left[\frac{\partial^2 g}{\partial x_j \partial x_i}\right] h_j + \left(\sum_{i=1}^{N} \frac{\partial g}{\partial x_i} \cdot h_i\right)^2$$

$$= \underbrace{h^t H_g(x,u)\, h}_{\text{Positivo}} + \underbrace{\left(\sum_{i=1}^{N} \frac{\partial g}{\partial x_i} h_i\right)^2}_{\text{Positivo}}$$

$$\geq 0$$

Se concluye que es semidefinido positivo