Pauta Auxiliar 6

P1 $f: \mathbb{R}^2 \to \mathbb{R}$

e)
$$f(x,y) = \begin{cases} \dfrac{x^3 y \sin(xy^2)}{x^6 + y^6} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

a)
$$\frac{\partial f}{\partial x} = \frac{\left(3x^2 y \sin(xy^2) + x^3 y \cos(xy^2)\, y^2\right)(x^6 + y^6) - 6x^5 \cdot x^3 y \sin(xy^2)}{(x^6 + y^6)^2}$$

$$\frac{\partial f}{\partial y} = \frac{\left(x^3 \sin(xy^2) + x^3 y \cos(xy^2)\, 2xy\right)(x^6 + y^6) - 6y^5 x^3 y \sin(xy^2)}{x^6 + y^6}$$

$$Df(x,y) = \left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}\right]$$

b)
$$\frac{\partial f}{\partial x}(0,0) = \lim_{t \to 0} \frac{f((0,0) + t(1,0)) - f(0,0)}{t} = \frac{0 - 0}{t} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{t \to 0} \frac{f((0,0) + t(0,1)) - f(0,0)}{t} = \frac{0 - 0}{t} = 0$$

$$Df((0,0) + t(e_1,e_2)) = \lim_{t \to 0} \frac{f((0,0) + t(e_1,e_2)) - f(0,0)}{t}$$

$$= \lim_{t \to 0} \frac{t^4 e_1^3 e_2 \sin(t^3 e_1 e_2^2)}{\cancel{t^7}\, t^3 (e_1^6 + e_2^6)} = \frac{e_1^4 e_2^3}{(e_1^6 + e_2^6)}$$

c) Suponiendo que es Diferenciable

$\Rightarrow Df(0,0) = [0 \quad 0]$

$\Rightarrow Df((0,0))\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = Df((0,0); (e_1, e_2))$

$\neq \Rightarrow \cancel{\ }$

$\Rightarrow$ f que suponí no era f Diferenciable

P2) Sea $(x_n, y_n) \in A \Rightarrow x_n \to \bar{x}$ e $y_n \to \bar{y}$, como

$(x_n, y_n) \in A \Rightarrow y_n = e^{x_n}$

Como $A \Rightarrow y_n = e^{x_n} \xrightarrow[\text{continuidad}]{\text{por}} e^{\bar{x}} = \bar{y}$ por unicidad

$\Rightarrow (\bar{x}, \bar{y}) = (\bar{x}, e^{\bar{x}}) \in A \Rightarrow$ Cerrado

Supongamos que $\exists M > 0$ tq que $\forall \vec{x} \in A \quad \|\vec{x}\| \leq M$

Sea $\vec{x} \in A$ ~~$y \|\vec{x}\| \leq \varepsilon$~~

Sea $x = M \Rightarrow e^M > M \Rightarrow \|\vec{x}\| > e^M > M$

Con $\vec{x} = (M, e^M)$

Es fácil ver que $\text{int}(A) = \emptyset$, $\text{Adh}(A) = A \Rightarrow \text{Fr}(A) = A$

es una línea

$B = \{ (x,y) \in \mathbb{R}^2 : 0 < -x^2 - \operatorname{Sen}(y) + \ln(1+|x|+|y|) \}$

$= \{ (x,y) \in \mathbb{R}^2 : 0 < f(x,y) \}$

$= \{ (x,y) \in \mathbb{R}^2 : f(x,y) \in (0,\infty) \}$

$=$ [illegible] $f^{-1}((0,\infty))$

Si f continua, estamos listos, por hy estamos

$\Rightarrow B$ abierto $\Rightarrow$ No cerrado $\Rightarrow$ No compacto

$B = \operatorname{int}(B)$ $\Rightarrow$ $\operatorname{Adh}(B) = f^{-1}([0,\infty)) \Rightarrow \operatorname{Fr}(B)$ $f^{-1}(\{0\})$

P3) $f(0,2) = (1,0)$

$\frac{\partial f_1}{\partial x} = 4y^2 \qquad \frac{\partial f_1}{\partial y} = 8yx$

$\frac{\partial f_2}{\partial x} = \operatorname{Cos}(3x+y-2) \cdot 3 \qquad \frac{\partial f_2}{\partial y} = \operatorname{Cos}(3x+y-2)$ } Son Contin

$$Df(0,2) = \begin{bmatrix} 16 & 0 \\ 3 & 1 \end{bmatrix}$$

$$T(x,y) = f(0,2) + Df(0,2) \begin{pmatrix} x-0 \\ y-2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{bmatrix} 16 & 0 \\ 3 & 1 \end{bmatrix} \begin{pmatrix} x \\ y-2 \end{pmatrix} = \begin{pmatrix} 1+16x \\ 3x+y-2 \end{pmatrix}$$

$$f(0{,}1,\ 2{,}2) \approx \begin{pmatrix} 2{,}6 \\ 0.3+0.2-2 \end{pmatrix} = \boxed{\begin{pmatrix} 2{,}6 \\ -1{,}5 \end{pmatrix}}$$

P4 $g(x,y) = f_1(x,y) + f_2(x,y) f_3(x,y)$

$$\nabla g(x,y) = \begin{pmatrix} \frac{\partial g(x,y)}{\partial x} \\ \frac{\partial g(x,y)}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1(x,y)}{\partial x} + \frac{\partial f_2(x,y)}{\partial x} \cdot f_3(x,y) + \frac{\partial f_3}{\partial x} \cdot f_2(x,y) \\ \frac{\partial f_1}{\partial y} + \frac{\partial f_2}{\partial y} f_3 + \frac{\partial f_3}{\partial y} f_2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 0 \cdot 0 + 1 \cdot 4 \\ 0 + 4 \cdot 0 + 1 \cdot 4 \end{pmatrix} = \boxed{\begin{pmatrix} 5 \\ 4 \end{pmatrix}}$$

P5 No Va

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