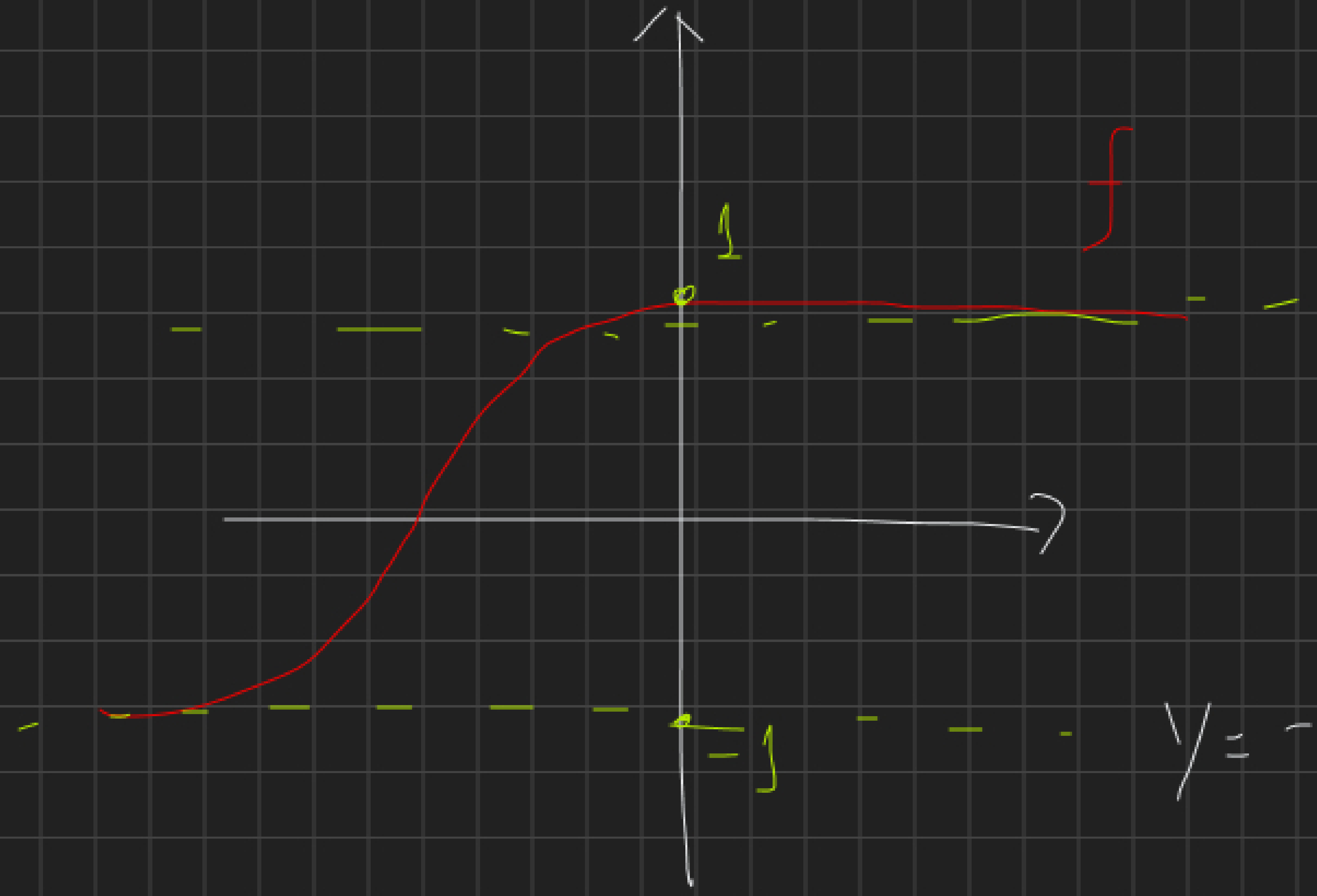
Repaso Asíntotas

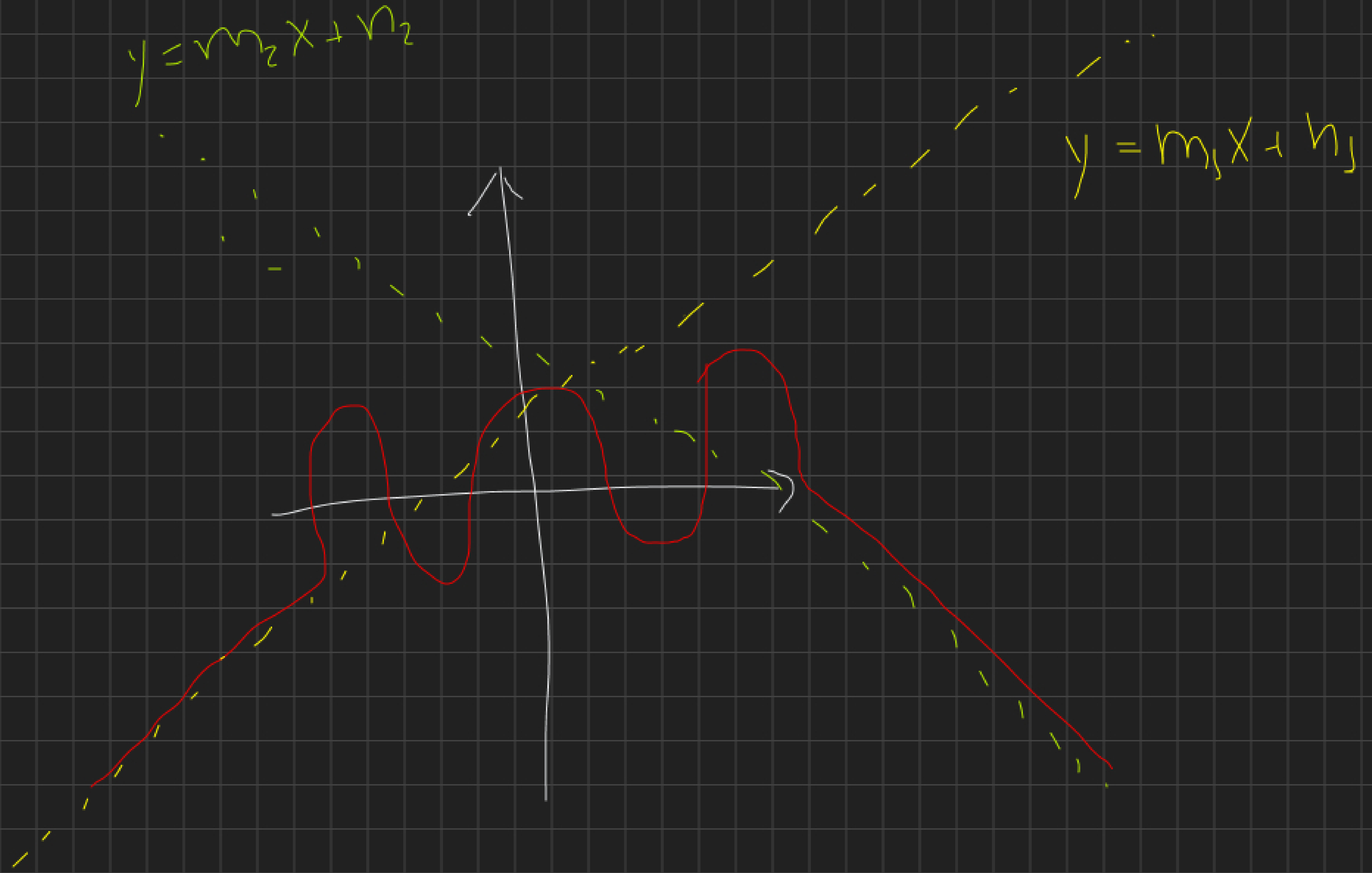
Horizontales:



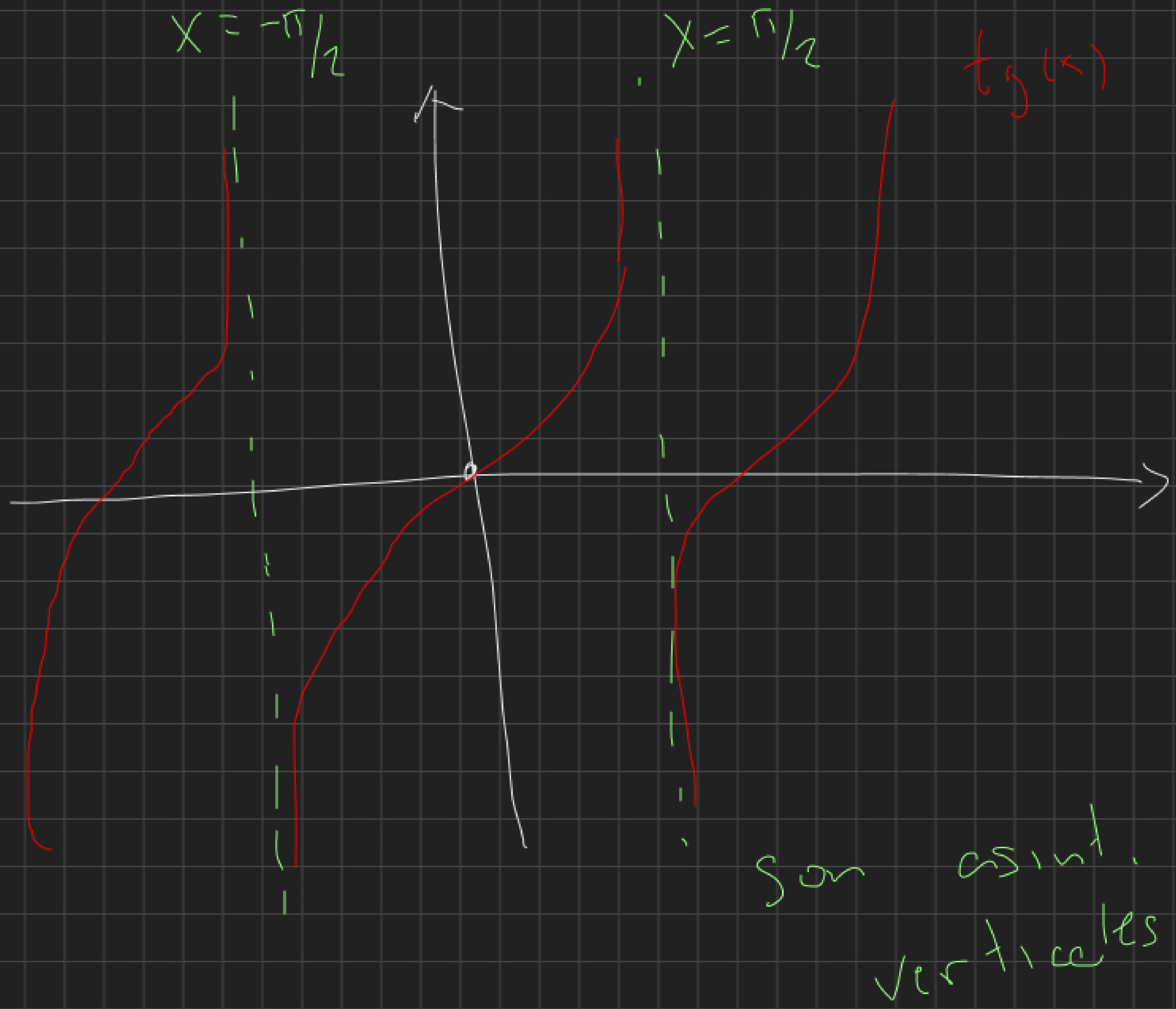
$y = 1$ es asint.

$y = -1$ es asint.

Oblicuas:



Verticales:



P1) • Veamos asíntotas horizontales y/o oblicuas:

1) $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \operatorname{arctg}(x)$

Def $x = tg(\theta)$

$\Rightarrow tg(\theta) \longrightarrow -\infty \Rightarrow \theta \longrightarrow -\pi/2$

$= \lim_{\theta \to -\pi/2} \theta$

$= -\pi/2$

$\therefore$ la recta $y = -\pi/2$ es asint. horizontal

2) $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{2+x+x^2}{1-x^2}\, e^{-1/x^2}$

$$= \lim_{x\to+\infty} \frac{2/x^2 + 1/x + 1}{1/x^2 - 1} \cdot \lim_{x\to+\infty} e^{-1/x^2}$$

$$= -1 \cdot 1 = -1$$

$\therefore$ la recta $y = -1$ es asint. horizontal

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∘ Veamos verticales

1) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{\sin(x)}{x(x-1)} = \lim_{x\to 1^-} \frac{1}{x-1} \cdot \frac{\sin(x)}{x} = -\infty$

$\therefore$ La recta $x=1$ es asint. vertical

2) ∘ $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \operatorname{arctg}(x) = \operatorname{arctg}(0) = 0$

∘ $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sin(x)}{x(x-1)} = \lim_{x\to 0^+} \frac{\sin(x)}{x} \cdot \lim_{x\to 0^+} \frac{1}{x-1} = 1 \cdot -1 = -1$

$\therefore \lim_{x\to 0} f(x)$ no existe $\nexists$

P2) • Dominio $f(x) = \ln\left(\sqrt{3e^{x/2}-2}\right) = \frac{1}{2}\ln\left(3e^{x/2}-2\right)$

1) El arg del log debe ser $>0 \implies \sqrt{3e^{x/2}-2} > 0$

2) la raiz necesita arg $>0 \implies 3e^{x/2}-2>0$

3) $3e^{x/2}-2>0 \iff e^{x/2} > 2/3$

$\iff x/2 > \ln(2/3)$

$\iff x > 2\ln(2/3)$

$\therefore \operatorname{Dom} f = \left(2\ln(2/3), +\infty\right)$

∘ Ceros

$$f(x) = 0 \iff \frac{1}{2}\ln\left(3e^{x/2} - 2\right) = 0$$

$$\iff \ln\left(3e^{x/2} - 2\right) = 0$$

$$\iff 3e^{x/2} - 2 = 1$$

$$\iff e^{x/2} = 1$$

$$\iff x = 0$$

∘ Ceros $f = \{0\}$

- Crecimiento: Sea $x, y \in \operatorname{Dom} f$ tq $x < y$ PDQ $f(x) < f(y)$

$\hookrightarrow x < y$

$\Leftrightarrow \frac{x}{2} < \frac{y}{2}$

$\Leftrightarrow e^{x/2} < e^{y/2}$

$\Leftrightarrow 3e^{x/2} - 2 < 3e^{y/2} - 2$

$\Leftrightarrow \ln(3e^{x/2} - 2) < \ln(3e^{y/2} - 2)$

$\Leftrightarrow \frac{1}{2}\ln(3e^{x/2} - 2) < \frac{1}{2}\ln(3e^{y/2} - 2)$

$\therefore x < y \Rightarrow f(x) < f(y)$

es decir

f es estrict. creciente //

• Recorrido Analizamos $y = f(x)$

$\hookrightarrow y = f(x)$

$\Leftrightarrow y = \frac{1}{2}\ln\left(3e^{x/2}-2\right)$

$\Leftrightarrow 2y = \ln\left(3e^{x/2}-2\right)$

$\Leftrightarrow e^{2y} = 3e^{x/2}-2$

$\Leftrightarrow \frac{e^{2y}+2}{3} = e^{x/2}$

$\Leftrightarrow \ln\left(\frac{e^{2y}+2}{3}\right) = \frac{x}{2}$

$\Leftrightarrow 2\ln\left(\frac{e^{2y}+2}{3}\right) = x$

$\times$ Se indefine si $\frac{e^{2y}+2}{3} < 0$, pero esto nunca ocurre

$\therefore \operatorname{Rec} f = \mathbb{R}$ //

- Asíntotas: Horizontal / Oblicua hacia $+\infty$

- $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\ln(3e^{x/2} - 2)}{2x}$

Notemos $\frac{\ln(3e^{x/2} - e^{x/2})}{2x} \leq \frac{\ln(3e^{x/2} - 2)}{2x} \leq \frac{\ln(3e^{x/2})}{2x} \quad \forall x > 2$

$\Leftrightarrow \frac{\ln(2e^{x/2})}{2x} \leq \frac{\ln(3e^{x/2} - 2)}{2x} \leq \frac{\ln(3e^{x/2})}{2x} \quad \forall x > 2$

$\Leftrightarrow \frac{\ln(2)}{2x} + \frac{1}{4} \leq \frac{\ln(3e^{x/2} - 2)}{2x} \leq \frac{\ln(3)}{2x} + \frac{1}{4} \quad \forall x > 2$

Por Sandwich, tomando $\lim_{x \to +\infty}$:

$$\frac{1}{4} \leq \lim_{x \to +\infty} \frac{\ln(3e^{x/2}-2)}{2x} \leq 1/4$$

$$\therefore \quad m := \lim_{x \to +\infty} \frac{f(x)}{x} = 1/4 \quad //$$

∘ Analizemos

↳ $\lim_{x \to +\infty} f(x) - mx = \lim_{x \to +\infty} \frac{1}{2} \ln(3e^{x/2} - 2) - x/4$

$= \frac{1}{2} \lim_{x \to +\infty} \ln(3e^{x/2} - 2) - x/2$

$= \frac{1}{2} \lim_{x \to +\infty} \ln(3e^{x/2} - 2) - \ln(e^{x/2})$

$= \frac{1}{2} \lim_{x \to +\infty} \ln\left(\frac{3e^{x/2} - 2}{e^{x/2}}\right)$

$= \frac{1}{2} \lim_{x \to +\infty} \ln\left(3 - \frac{2}{e^{x/2}}\right) = \frac{1}{2} \ln(3)$

$\therefore n = \lim_{x \to +\infty} f(x) - mx = \frac{\ln(3)}{2}$

$\therefore$ La recta $y = \frac{1}{4}x + \frac{\ln(3)}{2}$ es asint. oblicua

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• Ahora veamos las verticales

¿Cuándo $3e^{x/2} - 2$ se acerca al 0^+? Cuando $x \to 2\ln(2/3)^+$

$$\circ \lim_{x \to 2\ln(2/3)^+} f(x) = \lim_{x \to 2\ln(2/3)^+} \frac{1}{2}\ln\left(3e^{x/2} - 2\right)$$

Def $u = e^{x/2}$

$\Rightarrow u \longrightarrow 2/3^+$

$$= \lim_{u \to 2/3^+} \frac{1}{2}\ln(3u - 2)$$

Def $v = 3u - 2$

$\Rightarrow v \longrightarrow 0^+$

$$= \lim_{v \to 0^+} \frac{1}{2}\ln(v) = -\infty$$

$\therefore$ La recta $x = 2\ln(2/3)$ es asint. vertical //

• Bosquejo