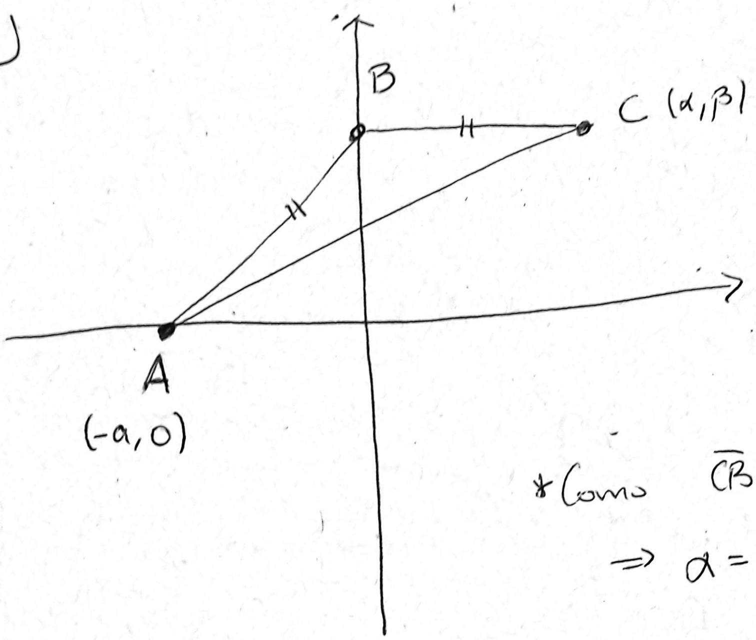
Aux 5 Calc.

P1)



$\Rightarrow \overline{CB} = \alpha$

$B = (0,\beta)$

* Como $A = (-a,0)$

$B = (0,\beta)$

$\Rightarrow \overline{AB} = \sqrt{a^2+\beta^2}$

* Como $\overline{CB} = \overline{AB}$

$\Rightarrow \alpha = \sqrt{a^2+\beta^2}$

$\Rightarrow \alpha^2 = a^2+\beta^2$

$\Rightarrow \alpha^2 - \beta^2 = a^2 \qquad \Rightarrow \dfrac{\alpha^2}{a^2} - \dfrac{\beta^2}{a^2} = 1$

Es una hipérbola //

- excentricidad: $e = \dfrac{\sqrt{a^2+a^2}}{a} = \sqrt{2}$
- directrices: $x = \pm \dfrac{a}{e} = \pm \dfrac{a}{\sqrt{2}}$
- focos: $F(\pm ae, 0) = (\pm a\sqrt{2}, 0)$ ▨

P2) $f(x) = \sqrt{1 - \frac{2}{1+x}} = \sqrt{\frac{x-1}{x+1}}$

Dominio: $(-\infty, -1) \cup [1, +\infty)$

Rec: $\mathbb{R}^{0}_{+} \setminus \{1\}$

Ceros: $\{1\}$

Paridad: $f(-x) = \sqrt{\frac{-x-1}{-x+1}} = \sqrt{\frac{-(x+1)}{-x+1}} = \sqrt{\frac{-(x+1)}{-(x-1)}} = \sqrt{\frac{x+1}{x-1}}$ $\therefore$ Ni par ni impar

Periodicidad: No es periódica

Sobreyect: No es sobreyectiva

Inyect: Sean $x_1, x_2 \in Dom(f)$, $x_1 \neq x_2$

$f(x_1) = f(x_2) \Rightarrow \sqrt{\frac{x_1-1}{x_1+1}} = \sqrt{\frac{x_2-1}{x_2+1}} \Rightarrow \frac{x_1-1}{x_1+1} = \frac{x_2-1}{x_2+1}$

$\Rightarrow (x_1-1)(x_2+1) = (x_1+1)(x_2-1) \Rightarrow x_1x_2 + x_1 - x_2 - 1 = x_1x_2 + x_2 - x_1 - 1$

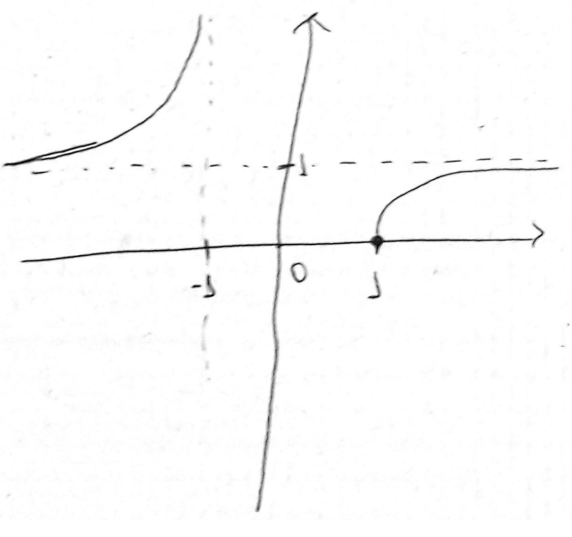
$\Rightarrow x_1 - x_2 = x_2 - x_1 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$ $\boxtimes$ $\therefore$ f inyect

Asíntotas: Verticales: $x = -1$

Horizontales: $y = 1$

Crec: • Creciente en $(-\infty, -1)$

• Creciente en $[1, +\infty)$



P3) a) Sabemos $\operatorname{sen}(\alpha-\beta)=\operatorname{sen}(\alpha)\cos(\beta)-\operatorname{sen}(\beta)\cos(\alpha)$

$$\Rightarrow \frac{\operatorname{sen}(\alpha-\beta)}{\cos(\alpha)\cos(\beta)} = \frac{\operatorname{sen}(\alpha)\cos(\beta)-\operatorname{sen}(\beta)\cos(\alpha)}{\cos(\alpha)\cos(\beta)}$$

$$= \frac{\operatorname{sen}(\alpha)\cancel{\cos(\beta)}}{\cos(\alpha)\cancel{\cos(\beta)}} - \frac{\operatorname{sen}(\beta)\cancel{\cos(\alpha)}}{\cancel{\cos(\alpha)}\cos(\beta)}$$

$$= \tan(\alpha)-\tan(\beta) \quad \boxtimes$$

b) Sean $x_1, x_2 \in (-\pi/2, \pi/2)$ tq $x_1 < x_2$

Luego $\left.\begin{array}{l} -\pi/2 < x_1 < \pi/2 \\ -\pi/2 < x_2 < \pi/2 \end{array}\right\}$ $\left.\begin{array}{l} -\pi/2 < x_1 < \pi/2 \\ -\pi/2 < -x_2 < \pi/2 \end{array}\right\}$ $-\pi < x_1 - x_2 < 0$

Además $\cos(x_1)>0$, $\cos(x_2)>0$, $\operatorname{sen}(x_1-x_2)<0$

$$\Rightarrow \tan(x_1)-\tan(x_2) = \frac{\operatorname{sen}(x_1-x_2)}{\cos(x_1)\cos(x_2)} < 0$$

$\Rightarrow \tan(x_1) < \tan(x_2)$ $\therefore$ $\tan(x)$ es creciente en $(-\pi/2, \pi/2)$ $\boxtimes$