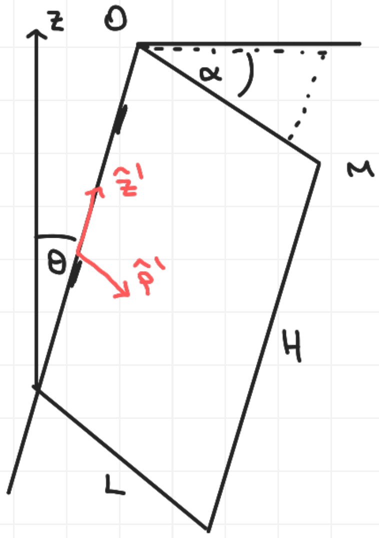
P1



$$E = \frac{1}{2} I \omega^2 + U$$

I: mom. de inercia de la puerta c/r al eje O

momento de inercia:

$$I = \int r^2 \, dm$$ r: distancia al eje ($r^2 = \rho^2$)

$$dm = \sigma \, dS \qquad \text{pero} \qquad \sigma = \frac{M}{LH}$$

$$dm = \frac{M}{LH} d\rho \, dz$$

$$I = \int_{-H/2}^{H/2} \int_{0}^{L} \rho^2 \cdot \frac{M}{LH} d\rho \, dz$$

$$= \frac{M}{LH} \int_{-H/2}^{H/2} dz \cdot \int_0^L \rho^2 \, d\rho$$

$$= \frac{M}{LH} \cdot H \cdot \frac{1}{3} L^3$$

$$I = \frac{1}{3} M L^2$$

centro de masas:

- puerta homogénea $\longrightarrow$ c.m. en el centro de la puerta

$$\vec{r}_{CM} = \frac{L}{2} \hat{\rho}'$$

energía potencial:

$$U = Mgz$$

$$z = \rho' \sin\theta \sin\psi + \phi' \sin\theta \cos\psi + z' \cos\theta$$

$$\rho' = \frac{L}{2} \qquad \phi' = 0 \qquad z' = 0$$

$$U_{cm} = Mg \cdot \frac{L}{2} \sin\theta \sin\psi$$

$$E_{cerrada} = E_{abierta}(\psi, \dot{\psi})$$

$$0 = \frac{1}{2} \cdot \frac{1}{3} ML^2 \cdot \dot{\psi}^2 + \frac{MgL}{2} \sin\theta \sin\psi$$

$$0 = \frac{1}{3} L \dot{\psi}^2 + g \sin\theta \sin\psi$$

$$\dot{\psi} = \sqrt{-\frac{3g}{L} \sin\theta \sin\psi}$$

resolución:

pos. inicial: $\psi = -\alpha = -\pi/2$
pos. final: $\psi = 0$

$$\frac{d\psi}{dt} = \sqrt{-\frac{3g}{L} \sin\theta \sin\psi}$$

$$\int_0^T dt = \int_{-\pi/2}^{0} \left(-\frac{3g}{L} \sin\theta \sin\psi\right)^{-1/2} d\psi$$

$$T = \left(\frac{3g}{L} \sin\theta\right)^{-1/2} \underbrace{\int_{-\pi/2}^{0} \frac{1}{\sqrt{-\sin\psi}} d\psi}_{J = 2,62}$$

$$T = \sqrt{\frac{L}{3g \sin\theta}} \cdot J$$

$$\sqrt{\sin\theta} = \frac{J}{T}\sqrt{\frac{L}{3g}}$$

$$\theta = \arcsin\left(\frac{J^2}{T^2}\cdot\frac{L}{3g}\right)$$

$$\theta = \arcsin\left(\frac{2{,}62^2}{2^2}\cdot\frac{1}{30}\right)$$

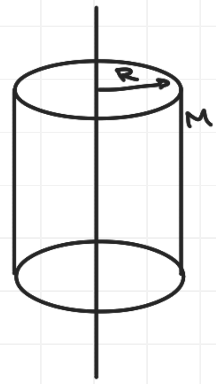
$$= \arcsin(0{,}057)$$

$$\theta = 3{,}279^\circ$$

P2

- cilindro hueco masa M_1 radio R_1 r.s.r. en la superficie interna de otro cilindro hueco de masa M_2 y radio R_2 $(R_1 \ll R_2)$
- cilindros con ejes paralelos; 2 es libre de rotar.
- calc. frec. peq. osc.

mom. inercia cascarón cilíndrico



$$I = \int r^2\, dm$$

$$= R^2 \int \sigma\, dS$$

$$= R^2 \cdot \frac{M}{2\pi R H} \cdot \int_0^{2\pi}\int_0^{H} R\, d\phi\, dz$$

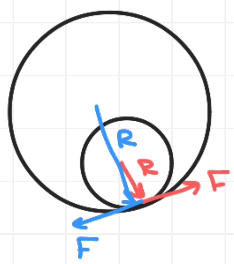
$$= \frac{MR}{2\pi H} \cdot R \int_0^{H} dz \cdot \int_0^{2\pi} d\phi$$

$$\boxed{I = MR^2}$$

$I_1 = M_1 R_1^2$ $\quad$ $I_2 = M_2 R_2^2$

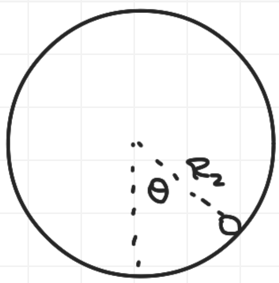
ecs. de fuerzas:

r.s.r $\longrightarrow$ F entre cilindros



$$\boxed{I_1 \alpha_1 = R_1 F} \qquad \boxed{I_2 \alpha_2 = -R_2 F}$$

$R_1 \ll R_2$ $\longrightarrow$



Luego, la condición de no deslizamiento entre cilindros:

$$R_2\theta_2 - R_1\theta_1 \approx R_2\theta \qquad \Big/ \frac{d^2}{dt^2}$$

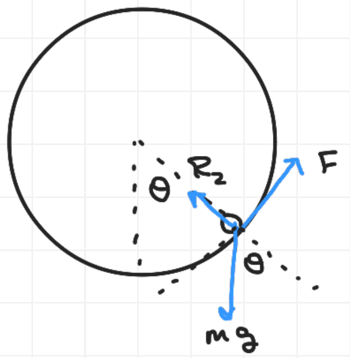
$$M_1R_1^2\ddot{\theta}_1 = R_1F \longrightarrow R_1\ddot{\theta}_1 = \frac{F}{M_1}$$

$$-M_2R_2^2\ddot{\theta}_2 = R_2F \longrightarrow -R_2\ddot{\theta}_2 = \frac{F}{M_2}$$

$$R_1\ddot{\theta}_1 - R_2\ddot{\theta}_2 = F\left(\frac{1}{M_1} + \frac{1}{M_2}\right)$$

$$-R_2\ddot{\theta} = F\left(\frac{1}{M_1} + \frac{1}{M_2}\right)$$

fuerzas sobre M_1



tangenciales: F y $-M_1 g\sin\theta$

$$\boxed{M_1(R_2\ddot{\theta}) = F - M_1 g\sin\theta}$$

$$-R_2\ddot{\theta} = \left(M_1R_2\ddot{\theta} + M_1 g\sin\theta\right)\left(\frac{1}{M_1} + \frac{1}{M_2}\right)$$

$$-R_2\ddot{\theta} = R_2\ddot{\theta} + g\sin\theta + \frac{M_1}{M_2}R_2\ddot{\theta} + \frac{M_1}{M_2}g\sin\theta$$

$$0 = \ddot{\theta}\left(2 + \frac{M_1}{M_2}\right)R_2 + \left(1 + \frac{M_1}{M_2}\right)g\sin\theta$$

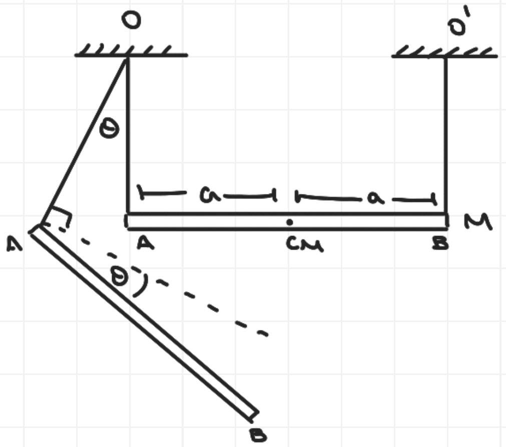
$$0 = \ddot{\theta}(2M_2 + M_1)R_2 + (M_1 + M_2)g\sin\theta$$

$$0 = \ddot{\theta} + \frac{M_1+M_2}{M_1+2M_2}\cdot\frac{g}{R_2}\sin\theta$$

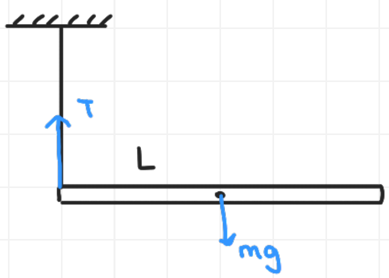
p.o. $\sin\theta \approx \theta$

$$\boxed{\omega = \sqrt{\frac{M_1+M_2}{M_1+2M_2}}\sqrt{\frac{g}{R_2}}}$$

P3



corte:



$$ma = T - mg$$

$$I\alpha = -Lmg \qquad L\alpha = a$$

$$mL\alpha = T - mg$$

$$mL \cdot -\frac{Lmg}{I} = T - mg$$

$$-\frac{m^2L^2g}{I} + mg = T$$

$$I = \int r^2\,dm = \lambda\int_0^{2L} r^2\,dr$$

$$= \frac{M}{2L}\cdot\frac{8L^3}{3} = \frac{4}{3}ML^2$$

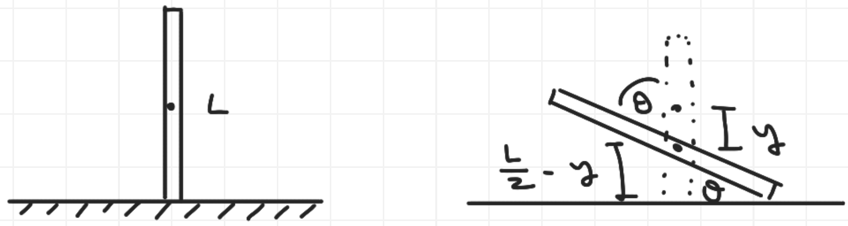
$$-\frac{M^{\cancel{2}}\cancel{L^2}g}{\frac{4}{3}\cancel{ML^2}} + Mg = T$$

$$-\frac{3}{4}Mg + Mg = T$$

$$\boxed{\frac{1}{4}Mg = T}$$

P4

- vara largo L masa M dist. uniforme
- inicialmente pos. vertical en mesa sin roce
- calcular rapidez del c.m. como fn. del ángulo de caída.



Como no hay fuerzas horizontales, el C.M. cae directamente hacia abajo.

Buscamos $v(\theta) \longrightarrow$ cons. de energía.

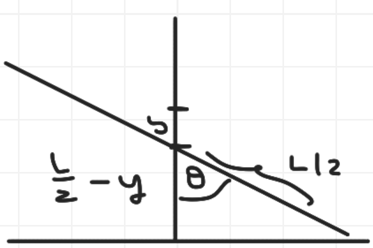
$$E_i = 0 + Mgz_{cm} = Mg\frac{L}{2}$$

$$K_f = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}M\dot{y}^2$$

$$U_f = Mg\left(\frac{L}{2} - y\right)$$

$$\frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}M\dot{y}^2 + \cancel{Mg\frac{L}{2}} - Mgy = \cancel{Mg\frac{L}{2}}$$

$$\boxed{I\dot{\theta}^2 + M\dot{y}^2 = 2Mgy}$$



$$\cos\theta = \frac{L/2 - y}{L/2}$$

$$\frac{L}{2}\cos\theta = \frac{L}{2} - y$$

$$\boxed{y = \frac{L}{2}(1 - \cos\theta)}$$

$$\dot{y} = \frac{L}{2}\dot{\theta}\sin\theta$$

$$\boxed{\frac{2\dot{y}}{L\sin\theta} = \dot{\theta}}$$

$$I \cdot \frac{4\dot{y}^2}{L^2 \sin^2\theta} + M\dot{y}^2 = 2Mg\frac{L}{2}(1-\cos\theta)$$

$$\left(\frac{4I}{L^2\sin^2\theta} + M\right)\dot{y}^2 = MgL(1-\cos\theta)$$

$$I = \int r^2\, dm$$

$$= \lambda \int_{-L/2}^{L/2} y^2\, dy \qquad \lambda = \frac{M}{L}$$

$$= \frac{M}{L} \cdot \frac{1}{3}\left[\frac{L^3}{8} + \frac{L^3}{8}\right]$$

$$= \frac{1}{12} M L^2$$

$$\left(\frac{4}{\cancel{L^2}\sin^2\theta} \cdot \frac{1}{12}\cancel{M}\cancel{L^2} + \cancel{M}\right)\dot{y}^2 = \cancel{M}gL(1-\cos\theta)$$

$$\boxed{\left(\frac{1}{3\sin^2\theta} + 1\right)\dot{y}^2 = gL(1-\cos\theta)}$$