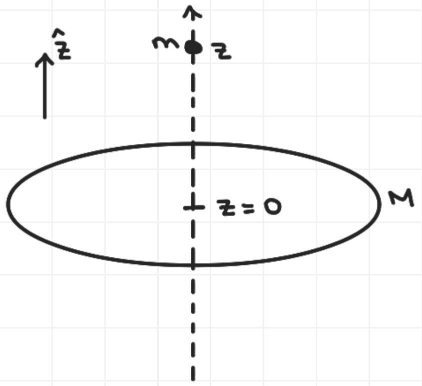
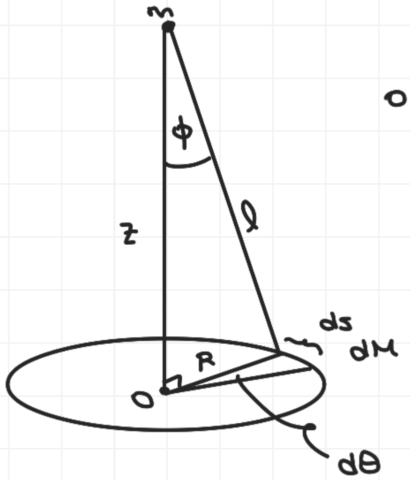
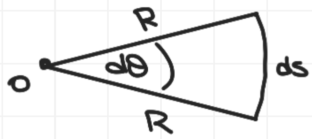
P1



situación.

a)





$$ds = R\,d\theta$$

M uniforme

↳ si ρ es densidad lineal de masa y es constante, entonces

$$\rho = \frac{\text{masa total}}{\text{largo}} = \frac{M}{2\pi R}$$

$$dM = \rho\, ds = \frac{M}{2\pi R} \cdot R\,d\theta = \frac{M\,d\theta}{2\pi}$$

fuerza gravitacional entre m y dM

$$dF = -\frac{GmdM}{\ell^2}$$

$$\ell^2 = R^2 + z^2$$

$$dF = \frac{-Gm\,dM}{z^2 + R^2}$$

proyección con respecto al centro

$$\cos\phi = \frac{z}{\ell} = \frac{z}{\sqrt{z^2+R^2}}$$

$$dF_z = \frac{-GmdM}{z^2+R^2} \cdot \cos\phi = -\frac{Gm\,dM}{z^2+R^2} \cdot \frac{z}{\sqrt{z^2+R^2}}$$

$$dF_z = \frac{-Gmz}{(z^2+R^2)^{3/2}} \cdot \frac{M\,d\theta}{2\pi}$$

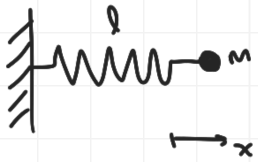
$$F_z = \int_0^{2\pi} \underbrace{\frac{dF_x}{d\theta}}_{dF_x} d\theta = \int_0^{2\pi} \frac{-Gmz}{(z^2+R^2)^{3/2}} \cdot \frac{M\,d\theta}{2\pi} = -\frac{GmMz}{(z^2+R^2)^{3/2}}$$

b) equilibrio en $z_{eq} = 0$ (si se deja inicialmente ahí la masa m estará en reposo)

perturbación pequeña $z(t) = z_{eq} + \eta(t) = \eta(t)$ con $\eta \ll 1$.

$$F_z = -\frac{GmM\eta}{(\underbrace{\eta^2}_{\to 0} + R^2)^{3/2}} = -\frac{GmM}{R^3} \cdot \eta$$

fuerza lineal



$F = -kx$

$$\cancel{m}\ddot{\eta} = -\frac{G\cancel{m}M}{R^3}\eta$$

$$\ddot{\eta} + \underbrace{\frac{GM}{R^3}}_{\omega^2}\eta = 0 \longrightarrow \ddot{\eta} + \omega^2\eta = 0$$

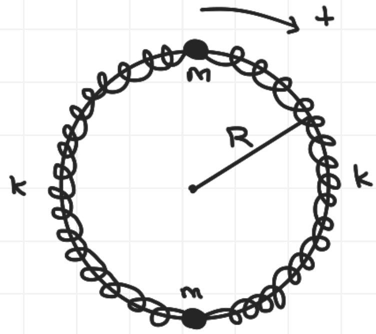
$\eta(t) = A\sin\omega t + B\cos\omega t$

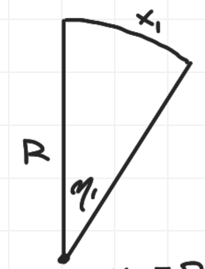
$\eta(0) = z_0 \ll 1 \longrightarrow B = z_0$

$\dot{\eta}(0) = 0 \longrightarrow A = 0$

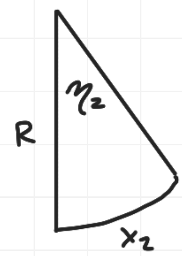
$$\eta(t) = z_0 \cos\left(\sqrt{\frac{GM}{R^3}}\, t\right)$$

P2





$x_1 = R\eta_1$



$x_2 = R\eta_2$

a) ec. de mov. según ángulos en equilibrio. $\Delta x = R(\eta_1 - \eta_2)$

$$\begin{cases} m\ddot{x}_1 = -k\cdot(x_1 - x_2) - k\cdot(x_1 - x_2) \\ m\ddot{x}_2 = -k\cdot(x_2 - x_1) - k(x_2 - x_1) \end{cases}$$

$$mR\ddot{\eta}_1 = -kR(\eta_1 - \eta_2) - kR(\eta_1 - \eta_2)$$

$$\hookrightarrow \ddot{\eta}_1 = -\frac{2k}{m}(\eta_1 - \eta_2)$$

$$\ddot{\eta}_2 = \frac{2k}{m}(\eta_1 - \eta_2)$$

$\omega \equiv \sqrt{\frac{k}{m}}$ frec. de oscilación

$$\begin{cases} \ddot{\eta}_1 + 2\omega^2(\eta_1 - \eta_2) = 0 & (1) \\ \ddot{\eta}_2 - 2\omega^2(\eta_1 - \eta_2) = 0 & (2) \end{cases}$$

sumando (1) y (2), y $\xi_1 \equiv \eta_1 + \eta_2$

$$\ddot{\eta}_1 + \ddot{\eta}_2 + 2\omega^2(\eta_1 - \eta_1 + \eta_2 - \eta_2) = 0$$

$$\ddot{\xi}_1 = 0$$

$$\boxed{\xi_1(t) = At + B}$$

restando (1) y (2), y $\xi_2 \equiv \eta_1 - \eta_2$

$$\ddot{\eta}_1 - \ddot{\eta}_2 + 2\omega^2(\eta_1 + \eta_1 - \eta_2 - \eta_2) = 0$$

$$\ddot{\xi}_2 + 4\omega^2\xi_2 = 0$$

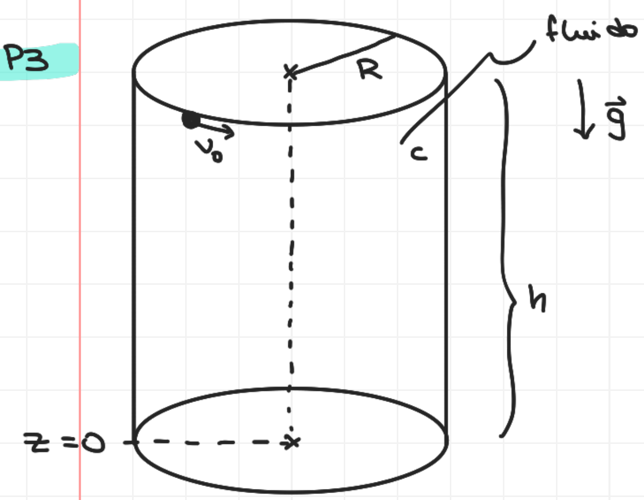
$$\xi_2(t) = C\sin(2\omega t) + D\cos(2\omega t)$$

$$\begin{aligned} \eta_1 + \eta_2 &= \xi_1 \\ \eta_1 - \eta_2 &= \xi_2 \end{aligned} \longrightarrow \eta_1 = \frac{\xi_1 + \xi_2}{2} \qquad \eta_2 = \frac{\xi_1 - \xi_2}{2}$$

$$\eta_1(t) = \frac{1}{2}\left[At + B + C\sin(2\omega t) + D\cos(2\omega t)\right]$$

$$\eta_2(t) = \frac{1}{2}\left[At + B - C\sin(2\omega t) - D\cos(2\omega t)\right]$$

P3



· con roce viscoso (fluido)

$$\vec{F}_v = -c\vec{v}$$

· sin roce de contacto

· $\vec{v}(t=0) = v_0\hat{\theta}$ (horizontal)

a) $\dot{z}(t)$ y $z(t)$

$$\left.\begin{aligned} \vec{r} &= \rho\hat{\rho} + z\hat{z} \\ \vec{v} &= \dot{\rho}\hat{\rho} + \rho\dot{\theta}\hat{\theta} + \dot{z}\hat{z} \\ \vec{a} &= (\ddot{\rho} - \rho\dot{\theta}^2)\hat{\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\hat{\theta} + \ddot{z}\hat{z} \end{aligned}\right\} \text{cilíndricas}$$

$$\vec{r}(t) = R\hat{\rho} + z\hat{z} \longrightarrow \vec{r}(t=0) = R\hat{\rho} + h\hat{z}$$

$$\vec{v}(t) = R\dot{\theta}\hat{\theta} + \dot{z}\hat{z} \longrightarrow \vec{v}(t=0) = R\dot{\theta}_0\hat{\theta} + \dot{z}_0\hat{z} = v_0\hat{\theta} \longrightarrow \begin{cases} \dot{\theta}_0 = v_0/R \\ \dot{z}_0 = 0 \end{cases}$$

$$\vec{a}(t) = -R\dot{\theta}^2\hat{\rho} + R\ddot{\theta}\hat{\theta} + \ddot{z}\hat{z}$$

fuerzas

$$\vec{F}_v = -c\vec{v} = -cR\dot{\theta}\hat{\theta} - c\dot{z}\hat{z}$$
$$\vec{N} = -N\hat{\rho}$$
$$m\vec{g} = -mg\hat{z}$$

2ª ley

$\hat{\rho})$ $\quad m(-R\dot{\theta}) = -N$

$\hat{\theta})$ $\quad mR\ddot{\theta} = -cR\dot{\theta}$

$\hat{z})$ $\quad m\ddot{z} = -c\dot{z} - mg$

de la ec. en $\hat{z}$:

$$\ddot{z} = \frac{d\dot{z}}{dt} = -\left(\frac{c}{m}\dot{z} + g\right)$$

$$\int \frac{d\dot{z}}{c\dot{z}/m + g} = \int - dt \qquad / \int_0^t$$

$$\int \frac{d\dot{z}}{c/m\,(\dot{z} + mg/c)} = -t$$

$$\int \frac{d\dot{z}}{\dot{z} + mg/c} = -\frac{ct}{m} \qquad \begin{array}{l} \dot{y} = \dot{z} + mg/c \\ d\dot{y} = d\dot{z} \end{array} \quad \begin{array}{l} \rightarrow \dot{z}_0 = 0 \\ \dot{y}_0 = mg/c \end{array}$$

$$\int \frac{d\dot{y}}{\dot{y}} = -\frac{ct}{m}$$

$$\ln(\dot{y}/\dot{y}_0) = -\frac{ct}{m}$$

$$\dot{y}(t) = \frac{mg}{c} e^{-\frac{ct}{m}}$$

$$\boxed{\dot{z}(t) = \frac{mg}{c}\left(e^{-ct/m} - 1\right)} \qquad / \int_0^t$$

$$\int \frac{dz}{dt} dt = \int \frac{mg}{c}\left(e^{-ct/m} - 1\right) dt$$

$$z - z_0 = \frac{mg}{c}\left(\int e^{-ct/m} dt - t\right)$$

$$z - h = \frac{mg}{c}\left[-\frac{m}{c}\left(e^{-ct/m} - 1\right) - t\right]$$

$$\boxed{z(t) = h - \frac{mg}{c}\left[\frac{m}{c}\left(e^{-ct/m} - 1\right) - t\right]}$$

b) $\dot{\theta}(t)$

de la ec. en $\hat{\theta}$ $\qquad m R \ddot{\theta} = -c R \dot{\theta}$

$$\ddot{\theta} = -\frac{c}{m}\dot{\theta} \qquad / \int$$

$$\int \frac{d\dot{\theta}}{\dot{\theta}} = \int -\frac{c}{m} dt$$

$$\ln(\dot{\theta}/\dot{\theta}_0) = -\frac{c}{m} t$$

$$\boxed{\dot{\theta}(t) = \frac{v_0}{R} e^{-ct/m}}$$

c) c para 1 vuelta si $h \to \infty$

$$\Delta\theta = \theta_f - \theta_0 = 2\pi \quad \leftarrow \text{condición.}$$

para caer una altura $h \to \infty$ se demora $t \to \infty$

$$\int_0^{\infty} \dot{\theta}\, dt = \int_0^{\infty} \frac{V_0}{R} e^{-ct/m}\, dt$$

$$\int_{\theta_0}^{\theta_f} d\theta = \frac{V_0}{R} \int_0^{\infty} e^{-ct/m}\, dt$$

$$2\pi = \frac{V_0}{R} \cdot -\frac{m}{c}(0 - 1)$$

$$\boxed{c = \frac{mV_0}{2\pi R}}$$