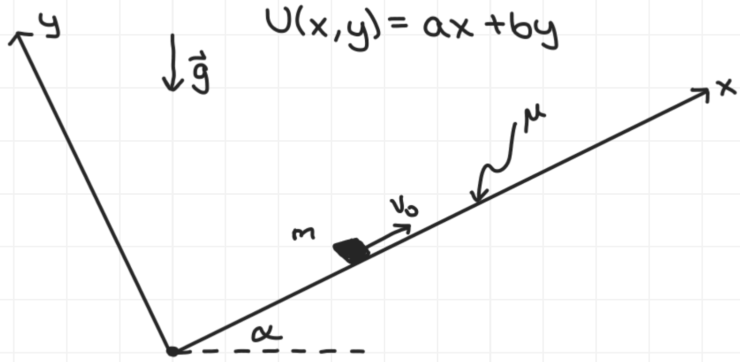
P1

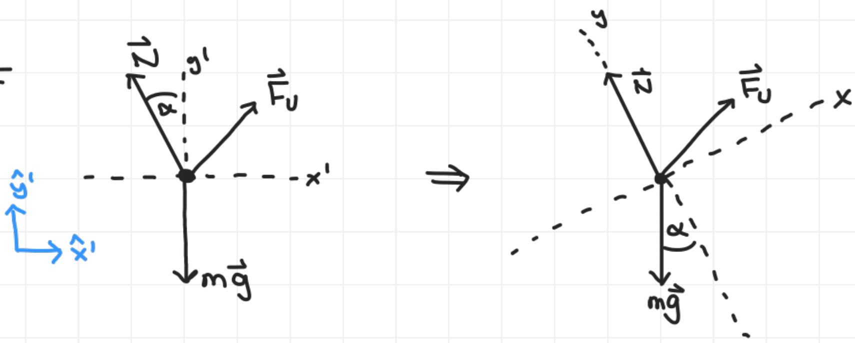
$U(x,y) = ax + by$

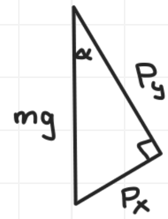


fuerzas sobre m
- peso
- contacto
- fuerza conservativa

a) determinar a y b tq. m en reposo.

<u>DCL</u>





$\sin\alpha = P_x / mg$

$\cos\alpha = P_y / mg$

<u>FUERZAS INVOLUCRADAS</u>

peso: $m\vec{g} = -mg(\sin\alpha\,\hat{x} + \cos\alpha\,\hat{y})$

contacto: $\vec{N} = N\hat{y}$ (no hay roce cinético por estar en reposo, y no hay roce dinámico por enunciado)

fuerza conservativa:

$$\vec{F}_U = -\nabla U = -\begin{pmatrix} \partial_x \\ \partial_y \end{pmatrix}(ax+by)$$

$$= -\begin{pmatrix} \partial_x(ax+by) \\ \partial_y(ax+by) \end{pmatrix} = -\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\vec{F}_U = -(a\hat{x} + b\hat{y})$$

<u>NEWTON</u>

reposo $\rightarrow \sum_i \vec{F}_i = \vec{F}_{neta} = \vec{0}$

- $\hat{x}\,\rfloor \quad -mg\sin\alpha - a = 0 \longrightarrow \boxed{a = -mg\sin\alpha}$

- $\hat{y}\,\rfloor \quad -mg\cos\alpha + N - b = 0 \longrightarrow N = b + mg\cos\alpha$

como se debe mantener el contacto $\rightarrow N > 0$

$$b + mg\cos\alpha > 0$$

$$\boxed{b > -mg\cos\alpha}$$

b) • rapidez v_0 inicial en $\hat{x}$, determinar distancia D recorrida hasta el reposo.
• calcular trabajo por cada fuerza

DIST. HASTA REPOSO

$\Delta E = W_{NC} \longrightarrow$ trabajo de fzas. no conservativas (roce)

$\Delta E = E_f - E_i$ (dif. de energía) $\qquad U_{tot} = U_g + U$

$$E_i = \underbrace{K_i + U_i}_{\text{total inicial}} \qquad K_i = \frac{1}{2}mv_0^2$$

$$U_i = mgx_i\sin\alpha + ax_i$$

$$E_f = \underbrace{K_f + U_f}_{\text{total final}} \qquad K_f = 0$$

$$U_f = mgx_f\sin\alpha + ax_f$$

$$D \equiv x_f - x_i$$

$$\Delta E = (0 + mgx_f\sin\alpha + ax_f) - \left(\frac{1}{2}mv_0^2 + mgx_i\sin\alpha + ax_i\right)$$

$$= mg\sin\alpha \cdot D + aD - \frac{1}{2}mv_0^2$$

$$\Delta E = D\cdot(a + mg\sin\alpha) - \frac{1}{2}mv_0^2$$

$$W_{NC} = W_{roce} \equiv \int_D \vec{F}_R \cdot d\vec{r} \qquad \vec{F}_R = -\mu N\hat{x}$$

$$d\vec{r} = dx\,\hat{x}$$

$$W_{roce} = \int_{x_i}^{x_f} -\mu N\hat{x}\cdot dx\,\hat{x} = -\mu N\int_{x_i}^{x_f} dx$$

$$W_{roce} = -\mu N D$$

$$\Delta E = W_{roce}$$

$$D \cdot (a + mg\sin\alpha) - \frac{1}{2} m v_0^2 = -\mu N D$$

$$D \cdot (a + mg\sin\alpha + \mu N) = \frac{1}{2} m v_0^2 \qquad a = -mg\sin\alpha$$

$$D \cdot \mu N = \frac{1}{2} m v_0^2 \qquad N = b + mg\cos\alpha$$

$$D = \frac{m v_0^2}{2\mu(b + mg\cos\alpha)}$$

<u>TRABAJOS</u>

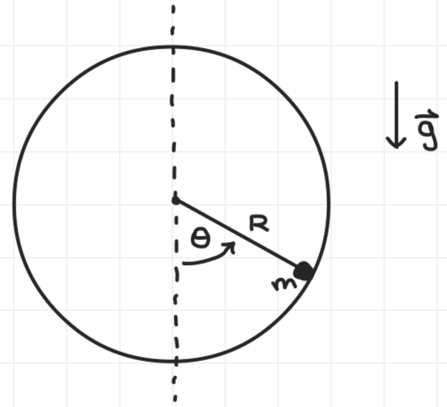
$\vec{N}$: $\quad W_{\vec{N}} = 0 \quad$ porque $\quad \vec{N} \perp d\vec{r} = dx\,\hat{x}$

$$W_{\vec{N}} = \int \vec{N} \cdot d\vec{r} = \int N\hat{y} \cdot dx\hat{x} = 0$$

$m\vec{g}$: $\quad W_{peso} = \int m\vec{g} \cdot d\vec{r} = \int -mg\sin\alpha \, dx = -mgD\sin\alpha$

$\vec{F}_U$: $\quad W_U = \int \nabla U \cdot d\vec{r} = \int -a \, dx = -aD = mgD\sin\alpha$

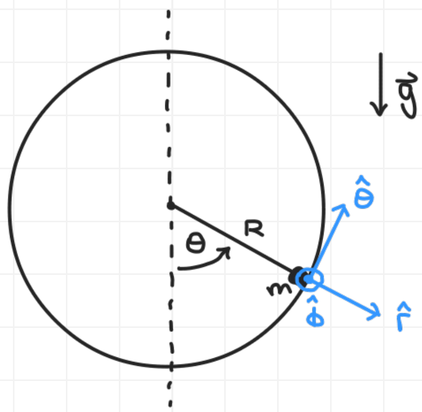
P2

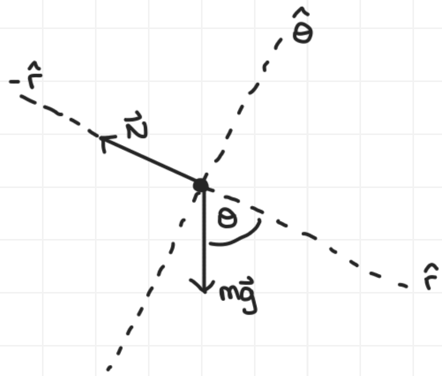


m se lanza por dentro del cascarón, desde θ_0 con velocidad inicial $\vec{v}_0 = v_0\hat{\phi}$

a) calcular $\dot{\phi}(\theta)$ sin despegue

COORDENADAS Y DCL





fuerzas involucradas

$$\vec{N} = -N\hat{r}$$

$$m\vec{g} = mg\left(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right)$$

aceleración en esféricas

$$\vec{a} = \left(\ddot{r} - r\dot{\phi}^2\sin^2\theta - r\dot{\theta}^2\right)\hat{r} + \left(2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta + r\ddot{\theta}\right)\hat{\theta} + \left(2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + r\ddot{\phi}\sin\theta\right)\hat{\phi}$$

como $r = R$, $\dot{r} = \ddot{r} = 0$

$$\vec{a} = \left(-R\dot{\phi}^2\sin^2\theta - R\dot{\theta}^2\right)\hat{r} + \left(-R\dot{\phi}^2\sin\theta\cos\theta + R\ddot{\theta}\right)\hat{\theta} + \underbrace{\left(2R\dot{\theta}\dot{\phi}\cos\theta + R\ddot{\phi}\sin\theta\right)}_{\frac{R}{\sin\theta}\frac{d}{dt}\left(\sin^2\theta\,\dot{\phi}\right)}\hat{\phi}$$

suma de fuerzas $\sum_i \vec{F}_i = m\vec{a}$

$$\hat{r}\rfloor \quad -N + mg\cos\theta = -mR\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) \qquad (1)$$

$$\hat{\theta}\rfloor \quad -mg\sin\theta = -mR\left(\dot{\phi}^2\sin\theta\cos\theta - \ddot{\theta}\right) \qquad (2)$$

$$\hat{\phi}\rfloor \quad 0 = \frac{R}{\sin\theta}\frac{d}{dt}\left(\sin^2\theta\,\dot{\phi}\right) \qquad (3)$$

de la ecuación en $\hat{\phi}$ (3)

$$\frac{d}{dt}\left(\sin^2\theta\,\dot{\phi}\right) = 0$$

$$(*) \qquad \sin^2\theta\,\dot{\phi} = cte = \sin^2\theta_0\,\dot{\phi}_0 \qquad \theta_0 \text{ conocido}$$

velocidad en esféricas

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\,\hat{\phi}$$

$$\vec{v}(t=0) = v_0\hat{\phi} = R\dot{\phi}_0\sin\theta_0\,\hat{\phi} \longrightarrow \dot{\phi}_0 = \frac{v_0}{R\sin\theta_0}$$

luego,

$$\dot{\phi}(\theta) = \frac{\sin^2\theta_0}{\sin^2\theta}\cdot\frac{V_0}{R\sin\theta_0}$$

$$\dot{\phi}(\theta) = \frac{V_0\sin\theta_0}{R\sin^2\theta}$$

b) calcular $\dot{\theta}(\theta)$

se puede reemplazar $\dot{\phi}(\theta)$ en la ec. de $\hat{\theta}$ (2):

$$\cancel{-m}g\sin\theta = \cancel{-m}R\left(\dot{\phi}^2\sin\theta\cos\theta - \ddot{\theta}\right)$$

$$g\sin\theta = R\left[\left(\frac{V_0\sin\theta_0}{R\sin^2\theta}\right)^2\sin\theta\cos\theta - \ddot{\theta}\right]$$

$$\frac{g}{R}\sin\theta = \frac{V_0^2\sin^2\theta_0}{R^2\sin^{\cancel{4}3}\theta}\cdot\cancel{\sin\theta}\cos\theta - \ddot{\theta}$$

$$\ddot{\theta} = \frac{V_0^2\sin^2\theta_0\cos\theta}{R^2\sin^3\theta} - \frac{g\sin\theta}{R} \qquad /\cdot\dot{\theta}$$

$$\dot\theta\,\frac{d\dot\theta}{dt}=\left(\frac{v_0^2\sin^2\theta_0\cos\theta}{R^2\sin^3\theta}-\frac{g\sin\theta}{R}\right)\frac{d\theta}{dt}\qquad /\int dt$$

$$\int\dot\theta\,d\dot\theta=\int\left(\frac{v_0^2\sin^2\theta_0\cos\theta}{R^2\sin^3\theta}-\frac{g\sin\theta}{R}\right)d\theta$$

$$\frac{1}{2}\left(\dot\theta^2-\cancelto{0}{\dot\theta_0^2}\right)=\frac{v_0^2\sin^2\theta_0}{R^2}\int\frac{\cos\theta}{\sin^3\theta}\,d\theta-\frac{g}{R}\int\sin\theta\,d\theta$$

$$\frac{1}{2}\dot\theta^2=\frac{v_0^2\sin^2\theta_0}{R^2}\int\frac{\cos\theta}{\sin^3\theta}\,d\theta-\frac{g}{R}\cdot-\cos\theta\Big|_{\theta_0}^{\theta}$$

$$\dot\theta^2=\frac{2v_0^2\sin^2\theta_0}{R^2}\underbrace{\int\frac{\cos\theta}{\sin^3\theta}\,d\theta}+\frac{2g}{R}(\cos\theta-\cos\theta_0)$$

$$u=\sin\theta\longrightarrow du=\cos\theta\,d\theta$$

$$\int\frac{\cos\theta}{\sin^3\theta}\,d\theta=\int\frac{du}{u^3}=-\frac{1}{2u^2}=-\frac{1}{2\sin^2\theta}\Big|_{\theta_0}^{\theta}=-\frac{1}{2}\left(\frac{1}{\sin^2\theta}-\frac{1}{\sin^2\theta_0}\right)$$

$$\dot\theta^2=\frac{2v_0^2\sin^2\theta_0}{R^2}\cdot\frac{1}{2}\left(\frac{1}{\sin^2\theta_0}-\frac{1}{\sin^2\theta}\right)+\frac{2g}{R}(\cos\theta-\cos\theta_0)$$

$$\dot\theta^2=\frac{v_0^2}{R^2}\left(1-\frac{\sin^2\theta_0}{\sin^2\theta}\right)+\frac{2g}{R}(\cos\theta-\cos\theta_0)$$

$$\dot\theta(\theta)=\sqrt{\frac{v_0^2}{R^2}\left(1-\frac{\sin^2\theta_0}{\sin^2\theta}\right)+\frac{2g}{R}(\cos\theta-\cos\theta_0)}$$

c) • si $\theta_0=\frac{\pi}{4}$ determinar v_0 para que la partícula alcance $\theta=2\pi/3$ y baje.
• mostrar que no se despega en el punto de altura máxima.

CÁLCULO DE v_0

en $\theta=2\pi/3$ alcanza el punto máximo, por lo que $\dot\theta(\theta=2\pi/3)=0$
de b) tenemos $\dot\theta(\theta)$, y $\theta_0=\pi/4$. Los valores de las fn. trigonométricas son

$$\sin\theta_0=\sin\frac{\pi}{4}=\frac{\sqrt2}{2}=\cos\theta_0$$

$$\sin\theta=\sin\frac{2\pi}{3}=\frac{\sqrt3}{2}\qquad\cos\theta=\cos\frac{2\pi}{3}=\frac{-1}{2}$$

reemplazando todo en $\dot{\theta}^2(\theta)$

$$0 = \frac{V_0^2}{R^2}\left(1 - \frac{1/2}{3/4}\right) + \frac{2g}{R}\left(-\frac{1}{2} - \frac{\sqrt{2}}{2}\right)$$

$$0 = \frac{V_0^2}{R}\left(1 - \frac{2}{3}\right) - g(1+\sqrt{2})$$

$$Rg(1+\sqrt{2}) = \frac{V_0^2}{3}$$

$$\boxed{V_0 = \sqrt{3Rg(1+\sqrt{2})}}$$

PRUEBA DE NO DESPEGUE

la ec. en $\hat{r}$ (1) es la única con info. sobre N
se puede evaluar en $\theta = 2\pi/3$

$$-N + mg\cos\theta = -mR\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right)$$

$$N - mg\cos\frac{2\pi}{3} = mR\left[\cancel{\dot{\theta}^2\left(\frac{2\pi}{3}\right)}^{\,0} + \left(\frac{V_0\sin\theta_0}{R\sin^2\theta}\right)^2_{\theta=2\pi/3}\sin^2\left(\frac{2\pi}{3}\right)\right]$$

$$N + \frac{1}{2}mg = mR\cdot\frac{V_0^2\sin^2(\pi/4)}{R^2\sin^2(2\pi/3)}$$

$$N = -\frac{1}{2}mg + \frac{mv_0^2}{R}\cdot\frac{1/2}{3/4}$$

$$N = m\left(\frac{2}{3R}\cdot 3Rg(1+\sqrt{2}) - \frac{1}{2}g\right)$$

$$N = mg\left(2 + 2\sqrt{2} - \frac{1}{2}\right)$$

$$N = mg\left(\frac{3}{2} + 2\sqrt{2}\right) > 0 \longrightarrow \text{la partícula no se despega.}$$