P2

a) <u>Caso 2D</u>

$$\vec{r} = x\hat{x} + y\hat{y} \qquad \vec{a} = a_1\hat{x} + a_2\hat{y}$$

$$\vec{r}\cdot\vec{a} = \alpha$$

$$a_1 x + a_2 y = \alpha \longrightarrow \text{recta en } \mathbb{R}^2$$

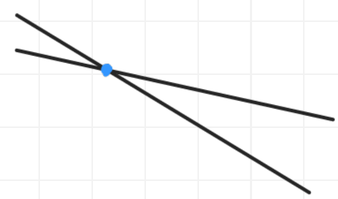
$a_1 \neq \lambda b_1$, ...

$$\vec{b} = b_1\hat{x} + b_2\hat{y}\ , \qquad \vec{a} \not\parallel \vec{b} \Rightarrow \vec{a} \neq \lambda\vec{b} \Rightarrow \frac{a_1}{a_2} \neq \frac{\lambda b_1}{\lambda b_2} = \frac{b_1}{b_2}$$

$$\vec{r}\cdot\vec{b} = \beta$$

$$b_1 x + b_2 y = \beta \longrightarrow \text{otra recta}$$

$$\begin{cases} y = (\alpha - a_1 x)/a_2 = \dfrac{\alpha}{a_2} - \dfrac{a_1}{a_2}x \\ y = (\beta - b_1 x)/b_2 = \dfrac{\beta}{b_2} - \dfrac{b_1}{b_2}x \end{cases}$$



→ se intersectan en un punto

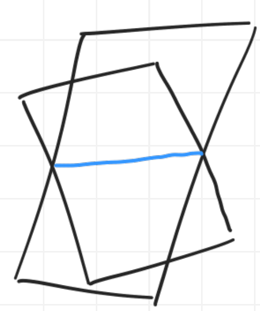
<u>Caso 3D</u>

$$\vec{r}\cdot\vec{a} = \alpha \longrightarrow a_1 x + a_2 y + a_3 z = \alpha \qquad (1)$$

$$\vec{r}\cdot\vec{b} = \beta \longrightarrow b_1 x + b_2 y + b_3 z = \beta \qquad (2)$$

$$(2)\cdot\frac{a_3}{b_3}$$

$$\frac{a_3 b_1}{b_3}x + \frac{a_3 b_2}{b_3}y + a_3 z = \frac{a_3\beta}{b_3}$$



$$(1) \longrightarrow a_3 z = \alpha - a_1 x - a_2 y$$

$$\frac{a_3 b_1}{b_3}x + \frac{a_3 b_2}{b_3}y + \alpha - a_1 x - a_2 y = \frac{a_3\beta}{b_3}$$

$$\left(\frac{a_3 b_1}{b_3} - a_1\right)x + \left(\frac{a_3 b_2}{b_3} - a_2\right)y = \frac{a_3\beta}{b_3} - \alpha$$

ec. de la recta

b) $\vec{a}\,,\ \hat{e}$ con $\|\hat{e}\| = 1 \longrightarrow \hat{e}\cdot\hat{e} = 1$

$$\vec{x}\times\vec{y} = -\vec{y}\times\vec{x} \qquad \text{(P1 b)}$$

$$\hat{e}(\vec{a}\cdot\hat{e}) + \hat{e}\times\underbrace{(\vec{a}\times\hat{e})}_{} = \hat{e}(\vec{a}\cdot\hat{e}) - (\overset{\vec{b}}{\vec{a}\times\hat{e}})\times\overset{\vec{c}}{\hat{e}}$$

$$= \cancel{\hat{e}(\underbrace{\vec{a}\cdot\hat{e}}_{\mathbb{R}})} - [\cancel{(\vec{a}\cdot\hat{e})\hat{e}} - (\hat{e}\cdot\hat{e})\vec{a}]$$

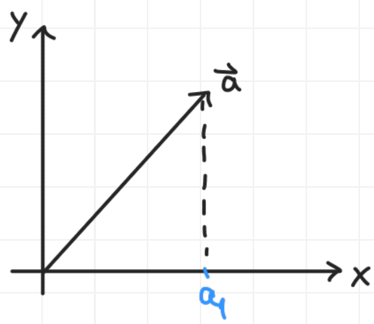
$$= (\cancel{\hat{e}\cdot\hat{e}}^{\to 1})\vec{a}$$

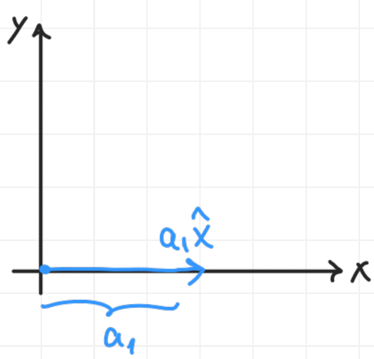
$$= \vec{a} \quad //$$

$$\vec{a} = \underbrace{\underbrace{\hat{e}(\vec{a}\cdot\hat{e})}_{\text{proyección de } \vec{a} \text{ sobre } \hat{e}}}_{\text{vector tangencial}} + \underbrace{\hat{e}\times(\vec{a}\times\hat{e})}_{\text{vector normal}}$$

ej: $\hat{e} = \hat{x} \qquad \vec{a} = a_1\hat{x} + a_2\hat{y}$

$\vec{a}\cdot\hat{x} = a_1 \qquad\qquad (\vec{a}\cdot\hat{x})\,\hat{x}$

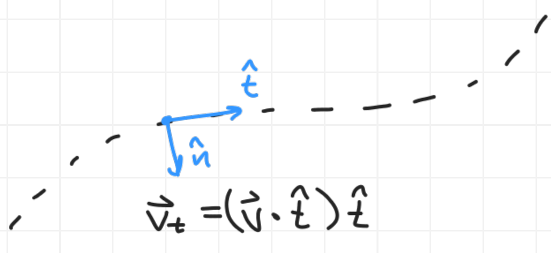




$$\vec{a}\times\hat{x} = (a_1\hat{x} + a_2\hat{y})\times\hat{x}$$
$$= a_2\,\hat{y}\times\hat{x}$$
$$= -a_2\,\hat{z}$$

$$\underbrace{\hat{b}}_{\text{binormal}} = \hat{t}\times\hat{n}$$

$$\hat{x}\times(\vec{a}\times\hat{x}) = \hat{x}\times(-a_2\hat{z})$$
$$= -a_2(\hat{x}\times\hat{z})$$
$$= a_2\hat{y}$$



$$\vec{v}_t = (\vec{v}\cdot\hat{t})\,\hat{t}$$

$$\vec{v}_n = \hat{n}\times(\vec{v}\times\hat{n})$$

$$\underbrace{\vec{a} = (\vec{a}\cdot\hat{t})\,\hat{t} + \overbrace{\hat{u}\times(\vec{a}\times\vec{u})}^{\text{ac. centrípeta}}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v\hat{v})$$

$$= \underbrace{\dot{v}\hat{v}}_{\vec{a}_t} + \underbrace{v\dot{\hat{v}}}_{\vec{a}_n}$$

$$\vec{a}_n = \vec{a} - \vec{a}_t$$

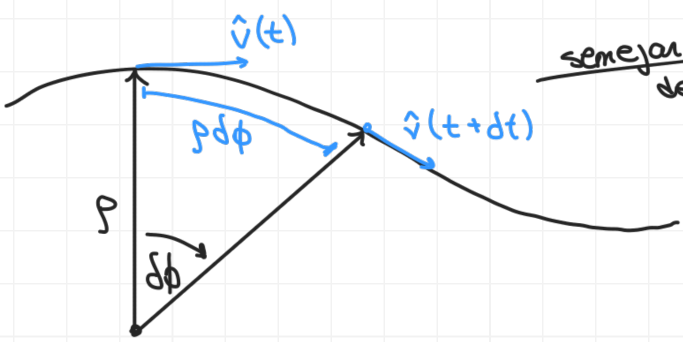
$$= \vec{a} - (\vec{a}\cdot\hat{v})\hat{v} \qquad (\hat{v} = \hat{t},\ \|\hat{v}\| = 1)$$

$$= (\hat{v}\cdot\hat{v})\vec{a} - (\vec{a}\cdot\hat{v})\hat{v}$$

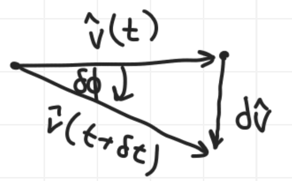
$$= \hat{v}\times(\vec{a}\times\hat{v})$$

$$= \hat{t}\times(\vec{a}\times\hat{t}) \quad //$$

$$\vec{a}_n = v\dot{\hat{v}} \longrightarrow \text{¿}\dot{\hat{v}}? \qquad (v \equiv \dot{s})$$



semejanza de Δ



$$\frac{ds}{\rho} = \frac{\|\delta\hat{v}\|}{\|\hat{v}\|} = \|\delta\hat{v}\|$$

$$\vec{a}_n = v\cdot\frac{d\hat{v}}{dt} = v\cdot\frac{d\hat{v}}{ds}\cdot\frac{ds}{dt}$$

$$= \underbrace{v^2\cdot\frac{1}{\rho}}_{a_c}\,\hat{n}$$

$$= a_c\hat{u}$$

$\hat{c} = \hat{c}(s)$, $\|\hat{c}\| = 1$

$\hat{c} = c_1 \hat{x} + c_2 \hat{y} + c_3 \hat{z}$

$c_1^2 + c_2^2 + c_3^2 = 1 \quad \Big/ \frac{d}{ds}$

$2c_1 \frac{dc_1}{ds} + 2c_2 \frac{dc_2}{ds} + 2c_3 \frac{dc_3}{ds} = 0$

$c_1 c_1' + c_2 c_2' + c_3 c_3' = 0$

$\hat{c} \cdot \frac{d\hat{c}}{ds} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \cdot \begin{pmatrix} c_1' \\ c_2' \\ c_3' \end{pmatrix} = c_1 c_1' + c_2 c_2' + c_3 c_3' \stackrel{!}{=} 0$ //