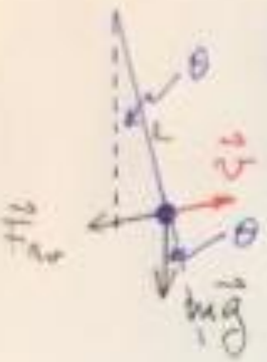
FI 2001 MECÁNICA SESIÓN 9 PALEITUNO 20/04/21 ①

PÉNDULO AMORTIGUADO POR ROCE VISCOSO



$\vec{v} = L\dot{\theta}\hat{\theta}$ ROCE VISCOSO: $-mc\vec{v}$

EC. MOVIMIENTO EN $\hat{\theta}$

$$mL\ddot{\theta} = -mg\,\mathrm{sen}\,\theta - mcL\ddot{\theta}$$

PARA OSCILACIONES CON θ PEQUEÑO $\mathrm{sen}\,\theta \sim \theta$

$$L\ddot{\theta} + cL\dot{\theta} + mg\theta = 0$$

si $\omega_0^2 = \frac{g}{L}$

$$\boxed{\ddot{\theta} + c\dot{\theta} + \omega_0^2\theta = 0}$$

C.I. $t=0$ $\underline{\theta = \theta_0}$ $L\underline{\dot{\theta} = v_0}$

SOLUCIÓN $\theta(t) = e^{st}$ $\dot{\theta} = s\theta$

$\ddot{\theta} = s^2\theta$

$$s^2 e^{st} + cse^{st} + \omega_0^2 e^{st} = 0$$

EC. CARACTERÍSTICA $\boxed{s^2 + cs + \omega_0^2 = 0}$

$$S_{1,2} = -\frac{c}{2} \pm \sqrt{\left(\frac{c}{2}\right)^2 - \omega_0^2}$$

a) $\left(\frac{c}{2}\right)^2 > \omega_0^2 \rightarrow c > 2\sqrt{\frac{g}{L}}$

EN ESTE CASO $\sqrt{\left(\frac{c}{2}\right)^2 - \omega_0^2} < \frac{c}{2}$

$\therefore$ S_1 y S_2 SON NEGATIVOS

SOLUCIÓN $\theta(t) = A e^{-|s_1|t} + B e^{-|s_2|t}$

A y B SE DETERMINAN CON LAS C.I.

LA OS

$t=0$ $\quad \theta_0 = A + B$

$$\frac{v_0}{L} = s_1 A + s_2 B$$

$s_1 \theta_0 - \frac{v_0}{L} = (s_1 - s_2) B \rightarrow \underline{\underline{B}} = \dfrac{s_1\theta_0 - \frac{v_0}{L}}{(s_1 - s_2)}$

$\underline{\underline{A}} = \theta_0 - B$

b) $\left(\frac{c}{2}\right)^2 = \frac{g}{L}$

SOLUCIÓN : $\theta(t) = (A+Bt)e^{-\frac{c}{2}t}$

$t=0$ $\quad \boxed{\theta_0 = A}$

$$\frac{v_0}{L} = Be^{-\frac{c}{2}t}\Big|_{t=0} - \frac{c}{2}(A+Bt)e^{-\frac{c}{2}t}\Big|_{t=0}$$

$$\frac{v_0}{L} = B - \frac{c}{2}A \rightarrow \boxed{B = \frac{v_0}{L} + \frac{c}{2}\theta_0}$$

c) $\left(\frac{c}{2}\right)^2 < \frac{g}{L}$

$$S_{1,2} = -\frac{c}{2} \pm \underset{\sqrt{-1}}{i}\sqrt{\underbrace{\frac{g}{L} - \left(\frac{c}{2}\right)^2}_{\omega_1^2}}$$

$$S_{1,2} = -\frac{c}{2} \pm i\omega_1$$

EN LA SOLUCIÓN APARECE TÉRMINOS DEL TIPO $e^{i\omega_1 t} = \cos\omega_1 t + i\,\mathrm{sen}\,\omega_1 t$

SOL. $\theta(t) = A\cos(\omega_1 t + \phi)e^{-\frac{c}{2}t}$

$$\theta_0 = A \cos\phi$$

$$\frac{v_0}{L} = -\omega_1 A \,\mathrm{sen}(\omega_1 t + \phi)\, e^{-\frac{c}{2}t}\Big|_{t=0} +$$

$$- \frac{c}{2} A \cos(\omega_1 t + \phi)\, e^{-\frac{c}{2}t}\Big|_{t=0}$$

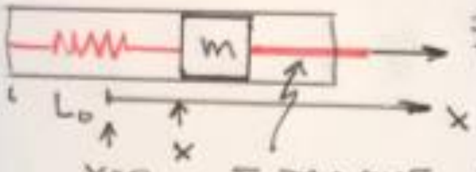
$$\frac{v_0}{L} = -\omega_1 A \,\mathrm{sen}\,\phi - \frac{c}{2} \underbrace{A \cos\phi}_{\theta_0}$$

$$\left(\frac{v_0}{L} + \frac{c}{2}\theta_0\right) = -\frac{\omega_1 \theta_0}{\cos\phi}$$

$$\underline{\underline{\cos\phi}} = \frac{-\omega_1 \theta_0}{\left(\frac{v_0}{L}\right) + \frac{c}{2}\theta_0}$$

$$\underline{\underline{A}} = \frac{\theta_0}{\cos\phi}$$

OSCILADOR ARMÓNICO AMORTIGUADO CON FORZAMIENTO SINUSOIDAL



$\vec{F}_0 = F_0 \cos \omega_2 t$

FUERZA DE ROCE VISCOSO $= -cm\dot{x}$

FUERZA ELÁSTICA DEL RESORTE $= -kx$

EC. MOVIMIENTO

$$m\ddot{x} = -kx - mc\dot{x} + F_0 \cos \omega_2 t$$

$$\ddot{x} + c\dot{x} + \underbrace{\frac{k}{m}}_{\omega_0^2} x = \frac{F_0}{m}\cos \omega_2 t$$

C.I $\quad t=0 \quad x = x_0$

$\qquad\qquad\quad \dot{x} = V_0$

SOLUCIÓN

$$x(t) = \underbrace{x_1(t)}_{\text{SOLUCIÓN DE EC. HOMOGÉNEA}} + \underbrace{x_2(t)}_{\text{SOLUCIÓN DE EC. PARTICULAR}}$$

EC. HOMOGÉNEA

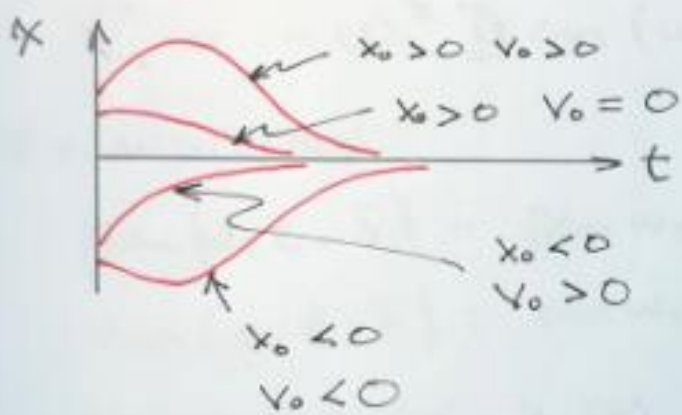
EC. CARACTERÍSTICA : $s^2 + cs + \omega_0^2 = 0$

$$s_{1,2} = -\frac{c}{2} \pm \sqrt{\left(\frac{c}{2}\right)^2 - \omega_0^2}$$

$$s_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2} \qquad \boxed{\gamma = \frac{c}{2}}$$

a) SOBRE-AMORTIGUAMIENTO $\boxed{\gamma^2 > \omega_0^2}$

$$\star \quad x_1(t) = A e^{s_1 t} + B e^{s_2 t} \qquad \begin{array}{l} s_1 < 0 \\ s_2 < 0 \end{array}$$



b) AMORTIGUACIÓN CRÍTICA $\boxed{\gamma^2 = \omega_0^2}$

$$\star \quad x_1(t) = (A + Bt)\, e^{-\frac{c}{2} t}$$

$\uparrow \gamma$

c) SUB-AMORTIGUACIÓN

$\gamma^2 < \omega_0^2 \qquad \omega_1^2 = \omega_0^2 - \left(\frac{c}{2}\right)^2$

$$s_{1,2} = -\frac{c}{2} \pm i\,\omega_1 \qquad i = \sqrt{-1}$$

$$\star \quad x(t) = A \cos(\omega_1 t + \phi)\, e^{-\frac{c}{2} t}$$

SOLUCIÓN PARTICULAR $x_2(t)$

$$x_2(t) = D\cos(\omega_2 t - \delta)$$

LA EC. QUE SE ESTÁ RESOLVIENDO

ES: $\boxed{\ddot{x} + c\dot{x} + kx = \frac{F_0}{m}\cos\omega_2 t}$ ★

$$\dot{x}_2 = -\omega_2 D \operatorname{sen}(\omega_2 t - \delta)$$

$$\ddot{x}_2 = -\omega_2^2 D\cos(\omega_2 t - \delta)$$

EXPANDIENDO

$$\operatorname{sen}(\omega_2 t - \delta) = \operatorname{sen}\omega_2 t\cos\delta - \cos\omega_2 t \operatorname{sen}\delta$$

$$\cos(\omega_2 t - \delta) = \cos\omega_2 t\cos\delta + \operatorname{sen}\omega_2 t \operatorname{sen}\delta$$

Y REEMPLAZANDO $x_2(t)$, $\dot{x}_2(t)$ Y $\ddot{x}_2(t)$ EN ECUACIÓN ★ SE OBTIENEN UNA SERIE DE TÉRMINOS CON $\operatorname{sen}\omega_2 t$ Y $\cos\omega_2 t$

$$[\ E\ ]\cos\omega_2 t + [\ F\ ]\operatorname{sen}\omega_2 t = 0$$

SI ESTO ES CIERTO $\forall t$

$$\therefore \quad E = 0 \quad y \quad F = 0$$

E: $\frac{F_0}{m} + (\omega_2^2 - \omega_0^2) D\cos\delta - c\omega_2 D \operatorname{sen}\delta = 0$

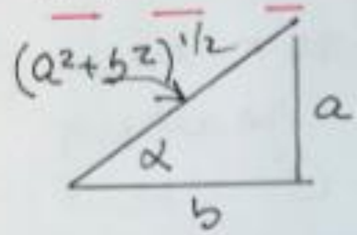
F: $(\omega_0^2 - \omega_2^2) D \operatorname{sen}\delta - c\omega_2 D\cos\delta = 0$

$$F = 0 \rightarrow \quad tg\,\delta = \frac{c\,\omega_z}{(\omega_0^2 - \omega_z^2)} \quad (**)$$

FALTA CONOCER D

De $E = 0$

$$*** \quad F_0 = m\left[(\omega_0^2 - \omega_z^2)\cos\delta + c\,\omega_z \operatorname{sen}\delta\right] D$$



$$tg\,\alpha = \frac{a}{b}$$

$$\cos\alpha = \frac{b}{(a^2+b^2)^{1/2}} \qquad \operatorname{sen}\alpha = \frac{a}{(a^2+b^2)^{1/2}}$$

De $(**)$

$$\cos\delta = \frac{\omega_0^2 - \omega_z^2}{\left[(\omega_0^2-\omega_z^2)^2 + c^2\omega_z^2\right]^{1/2}}$$

$$\operatorname{sen}\delta = \frac{c\,\omega_z}{\left[(\omega_0^2-\omega_z^2)^2 + c^2\omega_z^2\right]^{1/2}}$$

REEMPLAZANDO $\cos\delta$ y $\operatorname{sen}\delta$ EN $***$

$$F_0 = m\,\frac{(\omega_0^2-\omega_z^2)(\omega_0^2-\omega_z^2) + c\,\omega_z\,c\,\omega_z}{\left[(\omega_0^2-\omega_z^2)^2 + c^2\omega_z^2\right]^{1/2}}\,D$$

$$D = \frac{F_0}{m}\left[\frac{1}{\underbrace{\left[(\omega_0^2-\omega_2^2)^2 + c^2\omega_2^2\right]^{1/2}}_{G}}\right]$$

SI $c = 0$ PARA $\omega_2^2 = \omega_0^2 = \frac{K}{m}$

$D \to \infty$ $\omega_2 = \sqrt{\frac{K}{m}}$

SI $c > 0$ $\exists$ UNA FRECUENCIA ANGULAR PARA LA CUAL D ES MAX

(FRECUENCIA DE RESONANCIA)

D ES MÁXIMO CUANDO EL DENOMINADOR G ES MÍNIMO

$$G = \left[(\omega_0^2 - \omega_2^2)^2 + c^2\omega_2^2\right]^{1/2}$$

MIN DE G $\Rightarrow$ $\frac{dG}{d\omega_2} = 0$

$$\frac{dG}{d\omega_2} = \frac{1}{2}\left[\ \right]^{-1/2}\left[\cancel{2}(\omega_0^2-\omega_2^2)(-\cancel{2}\cancel{\omega_2}) + 2c^2\cancel{\omega_2}\right] = 0$$

$$\omega_0^2 - \omega_2^2 = 2c^2$$

RESONANCIA $\to$ $\omega_2 = \left[\omega_0^2 - 2c^2\right]^{1/2}$ $\omega_0^2 = \frac{K}{m}$

SI $\frac{K}{m} \leq 2c^2$ NO HAY RESONANCIA